Dynamic Spectrum Leasing: A New Paradigm for Spectrum Sharing in Cognitive Radio Networks

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***Abstract*—We recently proposed the dynamic spectrum leasing (DSL) paradigm for dynamic spectrum access in cognitive radio networks. In this paper, we formalize this concept by developing a general game-theoretic framework for the DSL and by carefully identifying requirements for the coexistence of primary and secondary systems via spectrum leasing. In contrast to hierarchical spectrum access, spectrum owners in proposed DSL networks, which are denoted as primary users, dynamically adjust the amount of secondary interference that they are willing to tolerate in response to the demand from secondary transmitters. The secondary transmitters, in turn, opportunistically attempt to achieve maximum possible throughput, or another suitably defined reward, while not violating the interference limit that is set by the primary users. The new game-theoretic model, however, allows the secondary users to encourage the spectrum owners to push the interference cap upward based on demand. We have proposed a general structure for the utility functions of primary users and secondary users that allows the primary users to control the price and the demand for spectrum access based on their required quality of service (QoS). We establish that, with these utility functions, the DSL game has a unique Nash equilibrium to which the best response adaptation finally converges. Moreover, it is shown that the proposed coexistence and best response adaptations can be achieved with no significant interaction between the two systems. In fact, it is shown that the only requirement is that the primary system periodically broadcasts two parameter values. We use several examples to illustrate the system behavior at the equilibrium and use the performance at the equilibrium to identify suitable system design parameters.**

***Index Terms*—Cognitive radios, dynamic spectrum access (DSA), dynamic spectrum leasing (DSL), dynamic spectrum sharing, game theory, power control.**

I. INTRODUCTION

IN RECENT YEARS, it has been observed that the scarcity of the radio spectrum is mainly due to the inefficiency of traditional static spectrum-allocation policies [2], [3]. This has prompted proposals for various dynamic spectrum-access (DSA) approaches that can be primarily grouped into three main classes: 1) open sharing, 2) hierarchical access, and 3) dynamic exclusive use [2], [4]. The open-sharing approach advocates a model that is similar to the highly successful industrial, science, and medicine bands. The hierarchical spectrum access, on the other hand, attempts to improve the spectrum utilization in current allocations. The hierarchical access in which secondary users are allowed to opportunistically access the spectrum on the basis of no interference to the primary (licensed) users is arguably the method that has received the most attention in the recent literature. Various spectrum-underlay and overlay schemes have been proposed and investigated in recent years to achieve such hierarchical DSA in cognitive radio networks (see [5]–[10] and references therein). Cognitive radios have been chosen as an enabling platform to realize such dynamic spectrum sharing due to their built-in cognition that can be used to observe, learn from, and adjust to the radio-frequency interference environment [11]–[13].

In DSA, it is assumed that there is a primary system that owns the spectrum rights. The existing literature in underlay- and overlay-based secondary networks, however, mainly imposes the burden of interference management on the secondary system. In particular, it is assumed that there is a maximum interference level that the primary system is willing to tolerate, and the secondary power/activity is to be adjusted within this constraint. In [1], on the other hand, we proposed a new concept of *dynamic spectrum leasing* (DSL) as an approach for better spectrum utilization. Spectrum leasing is one of the solutions that has been suggested under the third option of the dynamic exclusive-use model in which the spectrum licensees are granted the rights to sell or trade their spectrum to third parties [2], [4]. As opposed to passive spectrum sharing by the primary users as in hierarchical DSA, leasing means that the primary users have an incentive (e.g., monetary rewards as leasing payments) to allow secondary users to access their licensed spectrum. However, until [1], spectrum leasing has only been identified as a static, or offline, sharing technique, with the possible exception of [14]. On the other hand, in [1], we proposed to achieve DSL by allowing the primary users to dynamically adjust the extent to which they are willing to lease their spectrum. Thus, the proposed DSL approach is well suited for spectrum underlay systems in which both primary and secondary systems are expected to coexist simultaneously. However, unlike in hierarchical-access systems considered in the existing literature, the primary users in a

Manuscript received October 2, 2009; revised December 18, 2009; accepted January 29, 2010. Date of publication February 17, 2010; date of current version June 16, 2010. This work was supported in part by the National Science Foundation under Grant CCF-0830545 and in part by the Air Force Research Laboratory Space Vehicles Directorate. The contribution of the University of Vigo was supported by the Spanish Government under Project SPROACTIVE (Ref. TEC2007-68094-C02-01/TCM) and Project COMONSENS (Consolider-Ingenio 2010 CSD2008-00010). The review of this paper was coordinated by Dr. Y.-C. Liang.

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Digital Object Identifier 10.1109/TVT.2010.2042741

DSL network actively adapt the maximum secondary interference that they are willing to tolerate, which is known as the interference cap (IC), according to the observed RF environment and their required QoS. At this point, it is also worth pointing out that the spectrum leasing considered in [14] differs from our DSL approach in several ways. Most importantly, it relies on cooperative communication involving primary and secondary systems, whereas the proposed DSL scheme does not.

In this paper, we formalize our proposed DSL framework for cognitive radios [1]. Specifically, we first present a signal and system model for coexistence of primary and secondary systems under the DSL. Next, we develop a more-general game-theoretic formulation to model the interactions among primary and secondary systems that better capture the realities of such a DSL network. We propose a general structure for a suitable class of utility functions for both primary and secondary systems that reflect the demand for spectrum access by the secondary users, with their payoffs in terms of a suitable performance measure and the primary-user QoS requirements. We establish the conditions under which the proposed game-theoretic formulation has a unique Nash equilibrium (NE) to which both primary and secondary best-response adaptations would converge.

Naturally, any DSL system requires each system to know a certain amount of information about the other system. While in hierarchical-access systems it is usually assumed that only the secondary system needs to be aware of the primary operation, in a DSL network, both systems will be aware of each other. However, it may arguably be desirable to minimize the awareness that the primary system needs to have on the secondary operation. In this paper, we show that, indeed, successful DSL can be achieved while still relegating most of the interference management burden to the secondary and primary systems having to periodically broadcast only two parameter values: its tolerable IC and the total interference it is currently experiencing from the secondary transmissions. These are quantities that are readily available at the primary users (or can easily be estimated). Thus, we believe that the proposed DSL framework is indeed a viable solution for active spectrum sharing in cognitive radio networks.

The rest of this paper is organized as follows: In Section II, we introduce a signal and system model for the proposed DSL network. Next, in Section III, we develop a noncooperative game for DSL. In this section, we propose a general class of utility functions that are suitable for DSL and establish conditions under which the spectrum leasing game will converge to an NE. In Section IV, we use several example DSL systems to illustrate the performance characteristics of the proposed game-theoretic DSL scheme. Specifically, we investigate the primary and secondary system coexistence within each other's required performance QoS constraints and, based on that, provide design guidelines for a DSL network. We also investigate the robustness of the best-response adaptations to time-varying channel fading conditions and the effect of this on the system equilibrium. Finally, Section V concludes this paper by summarizing our results and discussing possible future work.

II. System Model for Dynamic Spectrum Leasing

We assume that there is one primary wireless communication system that owns the license rights to the spectrum band of interest. The users in this primary system, however, may not be using its spectrum completely all the time or may be able to tolerate a certain amount of additional cochannel interference without compromising required QoS constraints, leading to inefficient utilization of the radio spectrum. For simplicity of exposition, we focus on a particular channel in the primary system that is allocated to a single primary user (for example, as in frequency-division multiple access). Thus, there is only one primary transmitter of interest, and there are K secondary transmitters who are interested in accessing this spectrum band of interest to the maximum possible extent. The primary user is denoted as user 0, and the secondary users are labeled as users 1 through K. There are one primary receiver and one secondary receiver of interest.[1] The channel gain between the kth transmitter (either primary or secondary) and the common secondary receiver is denoted by h_{sk}, and that between the kth transmitter and the primary receiver is denoted by h_{pk}, for $k = 0, 1, \ldots, K$. Throughout the analysis in this paper, we assume fading to be quasi-static so that the coefficients stay fixed for certain duration of time, after which, they change to a new set of values. It should be mentioned that the quasi-static fading model is frequently used in modeling many wireless communication environments [16]. Our model can also be complemented with a channel estimation and tracking algorithm to cope with slowly time-varying situations, and as we will show later, the performance of the proposed DSL scheme is fairly robust against such time-varying fading.

The primary user is assumed to adapt its IC, which is denoted by Q_0, which is the maximum total interference that the primary user is willing to tolerate from secondary transmissions at any given time. By adjusting this IC Q_0, the primary user can control the total transmit power that the secondary users impose on its licensed channel. The motivation for the primary user can be, for instance, the monetary reward that is obtained by allowing secondary users to access its licensed spectrum. In essence, then, the IC determines how much secondary-user activity that the primary user is ready to allow, and thus, its reward should be an increasing function of the IC. However, we impose the realistic constraint that the primary user should always maintain a target signal-to-interference-plus-noise ratio (SINR) to ensure its required transmission QoS. Moreover, an unnecessarily large IC by the primary user could hinder both the secondary system's and other primary transmitters' (although, for simplicity, not included in the current model) performance due to the resulting high interference.

The goal of secondary transmitters is to capitalize on the allowed spectrum activity by the primary system by fully utilizing the interference margin. Each secondary user may be assumed to act in its own interest to maximize its own utility. However, their transmission power must be carefully regulated to ensure low interference to the primary user (within the IC),

[1]Generalization to more than one secondary receiver is straightforward and is reported in [15].

as well as to other secondary users. We use p_k to represent the transmission power of the kth user for $k = 0, 1, \ldots, K$.

Throughout this paper, we will assume that both primary and secondary receivers are equipped with conventional matched-filter (MF) receivers.[2] The signal received at the primary and secondary receivers can be, respectively, written as

$$r_p(t) = A_0 b_0 s_0(t) + \sum_{k=1}^{K} \Theta_k A_k b_k s_k(t) + \sigma_p n_p(t) \quad (1)$$

$$r_s(t) = \sum_{k=1}^{K} B_k b_k s_k(t) + B_0 b_0 s_0(t) + \sigma_s n_s(t) \quad (2)$$

where $n_.(t)$ is the additive white Gaussian noise (AWGN) with unit spectral height, $\sigma_.^2$ is the variance of the receiver noise, and A_k and B_k, for $k = 0, 1, \ldots, K$, represent the received signal amplitude at the primary and secondary receivers, respectively, and are defined as $A_k \doteq h_{pk}\sqrt{p_k}$ and $B_k \doteq h_{sk}\sqrt{p_k}$, respectively. Θ_k is a Bernoulli random variable such that $Prob(\Theta_k = 1) = q_k$ and $Prob(\Theta_k = 0) = 1 - q_k$, representing the randomness in secondary-user collisions with the primary transmission. In an overlay DSA system, the secondary users are prohibited from transmitting whenever primary users are using the spectrum. Thus, in an overlay system, the secondary interference will be present at the primary receiver only when a secondary transmitter makes a mistake in its white-space detection procedure. Hence, q_k can be interpreted as the false-alarm probability of the white space detector at the kth secondary transmitter in an overlay system. On the other hand, in an underlay dynamic spectrum-sharing system, the secondary users are allowed to transmit always without regard to the timings of the primary transmissions albeit at a low power level. In this case, the secondary interference is always present at the primary receiver. We can use the above model to capture this situation simply by assuming that $q_k = 1$ (or $\Theta_k = 1$ with probability 1). Hence, models (1) and (2) are general enough to be applicable for both underlay and overlay cognitive operations, although we envision for DSL to be more meaningful in spectrum underlay systems.

Assuming M discrete-time projections $r_m^{(p)} = \langle r_p(t), \psi_m^{(p)}(t)\rangle$, for $m = 1, 2, \ldots, M$ of the continuous-time received signal $r_p(t)$ onto a set of M orthonormal directions specified by $\psi_1^{(p)}(t), \ldots, \psi_M^{(p)}(t)$, and letting $\mathbf{r}^{(p)} = (r_1^{(p)}, \ldots, r_M^{(p)})^T$, we obtain the following discrete-time representation of the received signal at the primary receiver:

$$\mathbf{r}^{(p)} = A_0 b_0 \mathbf{s}_0^{(p)} + \sum_{k=1}^{K} \Theta_k A_k b_k \mathbf{s}_k^{(p)} + \sigma_p \mathbf{n}^{(p)}$$

where $\mathbf{s}_k^{(p)} = (s_{k1}^{(p)}, \ldots, s_{kM}^{(p)})$, for $k = 0, 1, \ldots, K$, is the M-vector representation of $s_k(t)$ in the M-dimensional basis employed by the primary system, where $s_{km}^{(p)} = \langle s_k(t), \psi_m^{(p)}(t)\rangle$, and $\mathbf{n}^{(p)} \sim \mathcal{N}(\mathbf{0}, \mathbf{I}_M)$. Analogously, a discrete-time representation of $r_s(t)$ with respect to an N-dimensional orthonormal basis $\psi_1^{(s)}(t), \ldots, \psi_N^{(s)}(t)$ used by the secondary system can be written as

$$\mathbf{r}^{(s)} = \sum_{k=1}^{K} B_k b_k \mathbf{s}_k^{(s)} + B_0 b_0 \mathbf{s}_0^{(s)} + \sigma_s \mathbf{n}^{(s)}$$

where $\mathbf{r}^{(s)} = (r_1^{(s)}, \ldots, r_N^{(s)})^T$, $r_n^{(s)} = \langle r_s(t), \psi_n^{(s)}(t)\rangle$, for $n = 1, 2, \ldots, N$ is the projection of the received signal at the secondary receiver onto the nth orthonormal basis function $\psi_n^{(s)}(t)$, $\mathbf{s}_k^{(s)} = (s_{k1}^{(s)}, \ldots, s_{kN}^{(s)})$, for $k = 0, 1, \ldots, K$ is the N-vector representation of $s_k(t)$ with respect to the N-dimensional basis employed by the secondary system with $s_{kn}^{(s)} = \langle s_k(t), \psi_n^{(s)}(t)\rangle$, and $\mathbf{n}^{(s)} \sim \mathcal{N}(\mathbf{0}, \mathbf{I}_N)$.

With the conventional MF detector at the primary and secondary receivers, decisions are taken based on the matched-filtered signals $y_0^{(p)} = (\mathbf{s}_0^{(p)})^T \mathbf{r}^{(p)}$ and $y_k^{(s)} = (\mathbf{s}_k^{(s)})^T \mathbf{r}^{(s)}$, respectively. Note that

$$y_0^{(p)} = A_0 b_0 + \sum_{k=1}^{K} \Theta_k \rho_{0k}^{(p)} A_k b_k + \sigma_p \eta^{(p)}$$

$$y_k^{(s)} = B_k b_k + \sum_{j=1, j\neq k}^{K} \rho_{kj}^{(s)} B_j b_j + \rho_{k0}^{(s)} B_0 b_0 + \sigma_s \eta_k^{(s)}$$

with $\rho_{0k}^{(p)} = (\mathbf{s}_0^{(p)})^T \mathbf{s}_k^{(p)}$, $\rho_{kj}^{(s)} = (\mathbf{s}_k^{(s)})^T \mathbf{s}_j^{(s)}$, for $j = 0, 1, \ldots, K$. Note that the noise terms $\eta^{(p)}$ and $\eta_k^{(s)}$ follow a $\mathcal{N}(0, 1)$.

It is straightforward to observe that the total secondary interference I_0 from all secondary transmissions to the primary user is given by

$$I_0 = \sum_{k=1}^{K} \tilde{A}_k^2 p_k \quad (3)$$

where $\tilde{A}_k = \sqrt{q_k}\rho_{0k}^{(p)} h_{p_k}$ is the effective channel coefficient of the kth secondary user as seen by the primary receiver. Similarly, the total interference from all secondary users to the kth user signal, excluding the primary user, will be denoted by

$$i_k = \sum_{j=1, j\neq k}^{K} \left(\rho_{kj}^{(s)}\right)^2 h_{sj}^2 p_j.$$

III. Game Model for Dynamic Spectrum Leasing

A. Game Model

In the proposed DSL-based cognitive radio network, the primary and secondary users interact with each other by adjusting their IC and transmit power levels, respectively, to maximize each other's own utility. Hence, game theory provides a natural framework to model and analyze this system. In fact, we may formulate the above system as in the following noncooperative game $(\mathcal{K}, \mathcal{A}_k, u_k(\cdot))$.

1) Players: $\mathcal{K} = \{0, 1, 2, \ldots, K\}$, where we assume that the 0th user is the primary user, and $k = 1, 2, \ldots, K$ represents the kth secondary user.

[2]The signal model below is general enough to allow for the extensions to more sophisticated multiuser receivers and will be considered in a follow-up paper.

2) Action space: $\mathcal{P} = \mathcal{A}_0 \times \mathcal{A}_1 \times \cdots \times \mathcal{A}_K$, where $\mathcal{A}_0 = \mathcal{Q} = [0, \bar{Q}_0]$ represents the primary user's action set, and $\mathcal{A}_k = \mathcal{P}_k = [0, \bar{P}_k]$, for $k = 1, 2, \ldots, K$ represents the kth secondary user's action set. Note that $\bar{Q}_0$ and $\bar{P}_k$ respectively represent the maximum possible IC of the primary user and the maximum transmission power of the kth secondary user (as determined by the system and regulatory considerations). The lower limit of these action sets being zero indicates that, at times, a secondary user may turn off its transmission, or the primary user may not be willing to tolerate any interference from the secondary system at all. We denote the action vector of all users by $\mathbf{a} = (Q_0, p_1, \ldots, p_K)^T$, where $Q_0 \in \mathcal{Q}$, and $p_k \in \mathcal{P}_k$. It is customary to denote the action vector excluding the kth user, for $k = 0, 1, 2, \ldots, K$, by $\mathbf{a}_{-k}$.
3) Utility function: We denote by $u_0(Q_0, \mathbf{a}_{-0})$ the primary user's utility function and by $u_k(p_k, \mathbf{a}_{-k})$, for $k = 1, 2, \ldots, K$, the kth secondary user's utility function.

At any given time, the primary user's target SINR is defined in terms of its assumed worst-case secondary interference, i.e.,

$$\bar{\gamma}_0 = \frac{h_{p0}^2 p_0}{Q_0 + \sigma_p^2}, \tag{4}$$

Note that, since Q_0 is the maximum possible interference from secondary users that the primary user is willing to tolerate, $\bar{\gamma}_0$ represents the least-acceptable transmission quality of the primary user. On the other hand, the primary user's actual instantaneous SINR is given by

$$\begin{aligned} \gamma_0 &= \frac{h_{p0}^2 p_0}{\sum_{k=1}^{K} \tilde{A}_k^2 p_k + \sigma_p^2} \\ &= \bar{\gamma}_0 \left(1 + \frac{Q_0 - I_0}{I_0 + \sigma_p^2}\right). \end{aligned} \tag{5}$$

One of the main features of the DSL approach is to take into account the coupling of the primary system with the secondary-user system in terms of mutual interference. However, the awareness of the primary system to the secondary network must be kept low enough to avoid an excessive overhead and complexity of the network.

The primary user is expected to obtain a reward from the secondary network, thus motivating the leasing of the owned spectrum. Moreover, the reward function for the primary system will be, in general, increasing with the demand that is seen from the secondary network as it occurs in the trade market. On the other hand, the reward is expected to grow with the allowed interference since the secondary system has more resources to exploit in this case. With these points in mind, we introduce the following utility function for the primary user:

$$\begin{aligned} u_0(Q_0, \mathbf{a}_{-0}) &= \left(\bar{Q}_0 - (Q_0 - I_0(\mathbf{a}_{-0}))\right) Q_0 \\ &= u_0(Q_0, I_0). \end{aligned} \tag{6}$$

Note that (6) essentially assumes that the utility of the primary user is proportional to both demand and its IC Q_0. The demand is taken to be increasing when the extra interference margin $Q_0 - I_0$ decreases. This discourages the primary user to swamp all other transmissions (both primary and secondary) by setting an IC too large, which will lead to higher transmission power according to (4). Additionally, the described reward function depends on just the parameter I_0 of the secondary system, which can be easily estimated, as we will see later in this section, avoiding the need for detailed channel-state information (CSI) from the secondary network. We believe that this model for primary utility is more sensible in a DSL cognitive radio network compared with, for example, what was used in [1]. It is also worth noting that this u_0 is continuous in $\mathbf{a}$ and concave in Q_0.

At the secondary receiver, the received SINR of the kth secondary user, for $k = 1, 2, \ldots, K$, is given by

$$\begin{aligned} \gamma_k &= \frac{|h_{sk}|^2 p_k}{i_k + \left(\rho_{k0}^{(s)}\right)^2 |h_{s0}|^2 p_0 + \sigma_s^2} \\ &= \frac{|h_{sk}|^2 p_k}{i_k + \tilde{\sigma}_s^2} = \frac{p_k}{N_k} \end{aligned}$$

where, as defined earlier, i_k is the total secondary interference, $\tilde{\sigma}_s^2 = (\rho_{k0}^{(s)})^2 |h_{s0}|^2 p_0 + \sigma_s^2$ is the effective noise that is seen by the kth user, and $N_k = (i_k + \tilde{\sigma}_s^2)/|h_{sk}|^2$.

The (selfish) objective of each secondary user is to maximize a given utility function (for example, throughput) that depends on its own SINR without violating the primary-user IC. Observe from (5) that as long as the secondary-user interference I_0 is below the IC Q_0 that is set by the primary user, the required QoS of the primary user will be guaranteed. Therefore, any utility function in a reasonable communication system will be a monotonically increasing function of the received SINR γ_k, and it should be a fast-decaying function of $I_0 - Q_0$ when this difference is positive. To ensure this, the secondary utility function will be formed by two terms: 1) a selfish reward function depending on the received SINR and 2) a penalization term depending on $I_0 - Q_0$. Motivated by these arguments, we propose the following form for the secondary-user utility function:

$$\begin{aligned} u_k(p_k, \mathbf{a}_{-k}) &= (Q_0 - \lambda_s I_0) f(p_k) \\ &= \left(Q_0 - \lambda_s I_{0,-k} - \lambda_s \tilde{A}_k^2 p_k\right) f(p_k) \end{aligned} \tag{7}$$

where $f(\cdot)$ is a suitable nonnegative reward function, λ_s is a suitably chosen positive (pricing) coefficient, $I_{0,-k} = \sum_{j=1, j\neq k}^{K} \tilde{A}_j^2 p_j$ is the total secondary interference to the primary user excluding that from the kth secondary user, and $\tilde{A}_k$ is the effective channel coefficient of the kth secondary user at the primary receiver [see (3)]. Note that the penalization term has been chosen linear on I_0 to allow simpler analytical derivations. However, the global system behavior is similar for other steps like penalization functions. In (7), the coefficient λ_s essentially controls how strictly secondary users need to adhere to the primary user's IC and allows the system designer to dimensionate the network for the maximum expected number of secondary users, as we will see in the simulation section.

The proposed utility function (7) leaves the performance metrics of the secondary system to be arbitrary by allowing for any reasonable reward function $f(\cdot)$ that will satisfy the conditions to be set forth in the next section. Without loss of generality, we may assume that the reward function $f(p_k)$ satisfies $f(0) = 0$ and $f'(0) > 0$ since when the received SINR of a user vanishes, no useful communication is possible for that user.

B. Existence of an NE in the DSL Game

In the following, we investigate equilibrium strategies on the proposed DSL game $G = (\mathcal{K}, \mathcal{A}_k, u_k)$, where users are interested in maximizing the following utility functions:

$$\text{primary-user utility:} \quad u_0(Q_0, \mathbf{a}_{-0})$$

$$\text{secondary-user utility:} \quad u_k(p_k, \mathbf{a}_{-k}) \quad \text{for } k = 1, 2, \ldots, K.$$

The most commonly used equilibrium concept in noncooperative game theory is the NE.

Definition 1: A strategy vector $\mathbf{a} = (a_0, a_1, \ldots, a_k)$ is an NE of the primary–secondary-user DSL game $G = (\mathcal{K}, \mathcal{A}_k, u_k)$ if, for every $k \in \mathcal{K}$, $u_k(a_k, \mathbf{a}_{-k}) \geq u_k(a'_k, \mathbf{a}_{-k})$ for all $a'_k \in \mathcal{A}_k$.

In essence, at an NE, no user has an incentive to unilaterally change its own strategy when all other users keep their strategies fixed. Hence, the NE can be viewed as a stable outcome where a game might end up when noncooperative users adjust their strategies according to their self-interests. In fact, the best response correspondence of a user gives the best reaction strategy that a rational user would choose to maximize its own utility, in response to the actions that are chosen by other users.

Definition 2: The user k's best response $r_k : \mathcal{A}_{-k} \longrightarrow \mathcal{A}_k$ is the set

$$r_k(\mathbf{a}_{-k}) = \{a_k \in \mathcal{A}_k : u_k(a_k, \mathbf{a}_{-k}) \geq u_k(a'_k, \mathbf{a}_{-k}) \quad \text{for all } a'_k \in \mathcal{A}_k\}.$$

Note that the primary-user action set is of the form $\mathcal{A}_0 = \mathcal{Q} = [0, \bar{Q}_0]$, where $\bar{Q}_0$ is the maximum IC that is determined by the required minimum QoS and the maximum possible transmit power of the primary user. Clearly, $\mathcal{A}_0$ is both compact and convex. Similarly, for all $k = 1, \ldots, K$, the secondary-user strategy sets are of the form $\mathcal{A}_k = \mathcal{P}_k = [0, \bar{P}_k]$. Again, it is easy to observe that all secondary-user action sets are convex and compact (being closed and bounded real intervals). Furthermore, both $u_0(\mathbf{a})$ and $u_k(\mathbf{a})$ are continuous in the action vector $\mathbf{a}$, and u_0 is concave in Q_0. For the existence of an NE, the only other condition that we need to ensure is the quasi-concavity of u_k's in p_k for $p_k \geq 0$, for $k = 1, 2, \ldots, K$.

Let us define

$$\phi_k(\gamma_k) = \frac{I_{0,-k}}{Q_0} + \frac{\tilde{A}_k^2 N_k}{Q_0}\left(\gamma_k + \frac{g(\gamma_k)}{g'(\gamma_k)}\right)$$

where $g(\gamma_k) = f(N_k \gamma_k)$ is the reward function with respect to the SINR. Then, it can be seen that u_k has a local maximum that is, indeed, a global maximum if $\phi_k(\gamma_k) = 1/\lambda_s$ has only one solution for $p_k \in \mathcal{P}_k$. Clearly, $\phi_k(\gamma_k) = 1/\lambda_s$ has a solution if $\phi_k(0) \leq (1/\lambda_s) < \lim_{\gamma_k \to \infty} \phi_k(\gamma_k)$, and moreover, this solution is, indeed, a global maximum if, in addition, $\phi'_k(\gamma_k) > 0$ for $\gamma_k > 0$. It can be easily verified that $\phi'_k(\gamma_k) > 0$ will be true if the reward function is such that $(g(\gamma_k)g''(\gamma_k)/(g'(\gamma_k)^2)) < 2$ for all $\gamma_k > 0$. Note that this is trivially true for any reward function that is concave in γ_k since, in that case, $g'' \leq 0$. Note also that $\phi_k(0) = (I_{0,-k}/Q_0)$ and $\lim_{\gamma_k \to \infty} \phi_k(\gamma_k) = \infty$ if $\lim_{\gamma_k \to \infty}(g(\gamma_k)/g'(\gamma_k)) > -\infty$. Hence, if reward function f (or, equivalently, g) and the coefficient λ_s satisfy the following conditions, u_k, indeed, has a local maximum that is a global maximum.

1) $g(0) = 0$, $g'(0) > 0$, and $\lim_{\gamma_k \to \infty} \frac{g(\gamma_k)}{g'(\gamma_k)} > -\infty$.
2) $\frac{g(\gamma_k)g''(\gamma_k)}{(g'(\gamma_k))^2} < 2$ for all $\gamma_k > 0$.
3) $0 < \lambda_s \leq \frac{Q_0}{I_{0,-k}}$.

Theorem 1: With $\mathcal{A}_k$'s and u_k's as defined above, the DSL game has an NE if conditions 1–3 are satisfied.

Proof: From the well-known result due to Debreu *et al.* [17], an NE exists in game $G = (\mathcal{K}, \mathcal{A}_k, u_k)$ if, for all $k = 0, 1, \ldots, K$, $\mathcal{A}_k$ is a nonempty, convex, and compact subset of some Euclidean space $\mathbb{R}^N$ and $u_k(\mathbf{p})$ is continuous in $\mathbf{p}$ and quasi-concave in p_k. Thus, from the above discussion, it follows that the above primary–secondary-user DSL game G will have at least one NE. ∎

Clearly, the above DSL game model is general enough to allow for various secondary reward functions g that may satisfy the above conditions. In general, choosing the most suitable secondary-user performance metric and the associated reward function in a cognitive radio network can itself be a nontrivial task [18]. Although we do not delve into this issue here, for illustrative purposes, in the remainder of this paper, we consider the following two specific reward functions:

$$g_k^{(1)}(\gamma_k) = W_k \log(1 + \gamma_k)$$

$$g_k^{(2)}(\gamma_k) = R_k \frac{C_{\text{BSC}}(P_e(\gamma_k))}{p_k}$$

where W_k and R_k are the bandwidth and the data rate of user k, respectively, $P_e(\gamma_k)$ is the probability of bit error with a received SINR of γ_k, and $C_{\text{BSC}}(P_e)$ is the capacity of a binary symmetric channel with crossover probability P_e, which can be written in terms of the binary entropy function $H(P_e) = -P_e \log_2 P_e - (1 - P_e) \log_2(1 - P_e)$ as $C_{\text{BSC}}(P_e) = 1 - H(P_e)$. Both these reward functions can be justified in a wide variety of contexts. For example, $g^{(1)}$ is a measure of user k's capacity in the presence of all other users, and $g^{(2)}$ is a measure of its throughput per unit power. The reward function $g^{(1)}$ can be justified in a DSL application in which the secondary users are mainly concerned with getting access to the spectrum, and their power consumption is not a major concern. On the other hand, $g^{(2)}$ is suitable when secondary users are interested in achieving the best throughput per unit energy spent. Note that the function $g^{(2)}$ proposed here is arguably better than a similar utility function proposed in [19] and often used by many thereafter. For example, the utility function defined in [19] is based on an efficiency function that was defined in an *ad hoc* way to avoid a degenerate behavior as the user transmit power vanishes. However, the proposed reward function $g^{(2)}$ avoids this degeneracy and has the natural meaning of throughput per unit energy, as was intended in [19]. Indeed, it can be shown that

$$g_k^{(2)}(0) = \lim_{\gamma_k \to 0} g_k^{(2)}(\gamma_k)$$
$$= -\lim_{\gamma_k \to 0} \frac{P_e'(\gamma_k) H'(P_e)}{N_k} = 0$$

since $H'(P_e) = \log_2((1-P_e)/P_e)$, $P_e(0) = 1/2$, and $P_e'(0) < \infty$ for any practical communication receiver. For concreteness, throughout the remainder of this paper, we will assume that $P_e(\gamma_k) = (1/2)\exp(-\gamma_k)$ (i.e., binary phase-shift keying modulation with an MF receiver).

C. Best-Response Adaptations and Implementation Issues

Primary User: Since the best response by a player in a game is a strategy that maximizes its own utility given all other players' actions, the best response of the primary user in the above DSL game is obtained by setting $u_0'(Q_0) = 0$. The unique interior solution is given by

$$Q_0^*(I_0) = \frac{\bar{Q}_0 + I_0}{2}.$$

Note that, since $u_0(Q_0)$ is monotonically increasing for $Q_0 < Q_0^*$, if the maximum IC is such that $\bar{Q}_0 < Q_0^*$, the best response of the primary user would be to set the IC to $Q_0 = \bar{Q}_0$. Hence, the primary user's best response is given by

$$r_0(\mathbf{a}_{-0}) = r_0(I_0) = \min\left\{\bar{Q}_0, Q_0^*(I_0)\right\}.$$

We observe that, to determine its best response for a chosen power vector $\mathbf{a}_{-0}$ by the secondary users, the only quantity that the primary user needs to know is the total secondary interference at the primary receiver denoted by I_0 given in (3). This parameter can, indeed, be estimated at the primary receiver by using any standard SNR estimation algorithm, either data-aided if the primary is able to decode its own signal or nondata-aided in the other case.

Secondary Users: On the other hand, the best response of the kth secondary user to the transmit power of other secondary users as well as the IC set by the primary user is given by the (unique) solution $p_k = p_k^*(Q_0, I_{0,-k}, i_k)$ to the following:

$$\phi_k(\gamma_k) - \frac{1}{\lambda_s} = 0.$$

Since u_k is quasi-concave in p_k, if $p_k^*(Q_0, I_{p,-k}) > \bar{P}_k$, where $\bar{P}_k$ is the kth user's maximum possible transmit power, its best response is to set its transmit power to $p_k = \bar{P}_k$. Hence, we have the best response of the kth secondary user for $k = 1, 2, \ldots, K$, i.e.,

$$r_k(\mathbf{a}_{-k}) = \min\left\{\bar{P}_k, p_k^*(Q_0, I_{0,-k}, i_k)\right\}.$$

Observe that, in general, the best response of the kth secondary user is a function of the primary IC Q_0, the residual interference $I_{0,-k}$ from all other secondary users to the primary user, and the total interference from all secondary and primary users to the kth user's received signal at the secondary receiver i_k. Like in the primary case, the secondary system can estimate i_k without much difficulty using standard SNR estimation algorithms. To obtain the knowledge of Q_0 and $I_{0,-k}$, we assume that the primary system periodically broadcasts Q_0 and I_0. Note that this is the only interaction that the primary system will need to have with the secondary system. Since these two quantities are readily available to the primary system, we believe that the periodic broadcast of these quantities, informing the secondary system what it needs to know to avoid severe conflicts with primary transmissions, is a reasonable expectation for a future cognitive radio system that expects to harvest spectrum leasing gains. Observe that knowing I_0, each secondary user can compute the residual interference $I_{0,-k} = I_0 - \tilde{A}_k^2 p_k$ if it can estimate the CSI $\tilde{A}_k$. This quantity may be estimated if the reverse link signals are available in the same band. Otherwise, the secondary receiver does not necessarily need the CSI of its link with the primary receiver, as we will demonstrate in our simulation results, since the approximation $I_{0,-k} \approx I_0$ performs well in practice, particularly when the number of secondary users K is sufficiently large.

In the above discussion, we have assumed the quasi-static fading in which fading realizations stay fixed for a period of duration and then change to new values. This facilitated the NE analysis without having to deal with time-varying channel coefficients. While quasi-static assumption may be justified in certain channel environments, sometimes, it is likely that the channel coefficients may slowly vary in time. It is easy to see that for the best-response adaptations to converge to an NE, the rate of adaptations needs to be faster than the time variations of the channel. One may expect that, in the presence of channel variations, the convergence may be slowed or may even not occur. However, as we will demonstrate in Section IV, the proposed DSL game has the desired property of being tolerant toward slow time variations of the channel state. Moreover, the NE of the proposed DSL game is robust against small channel-estimation errors. This is also a desired property since, in practice, the channel coefficients need to be estimated, and these estimations are almost always not perfect.

IV. Performance Analysis of a Dynamic Spectrum Leasing System

In the following, we consider a DSL cognitive radio system that employs the proposed game-theoretic framework for their interactions. Our goal is to investigate the behavior of the primary and secondary systems at the equilibrium. It is to be noted that the NE can reasonably be expected to be the natural outcome of the system when it reaches the steady state. Thus, the performance of the system is to be considered as its performance at the NE.

To illustrate the characteristics of the NE in this primary–secondary-user DSL game, we first consider a simplified scenario with identical secondary users. This scenario allows analytical determination of the NE state and its general behavior. We analyze next a more-general scenario with nonidentical secondary users and fading channels by means of simulations.

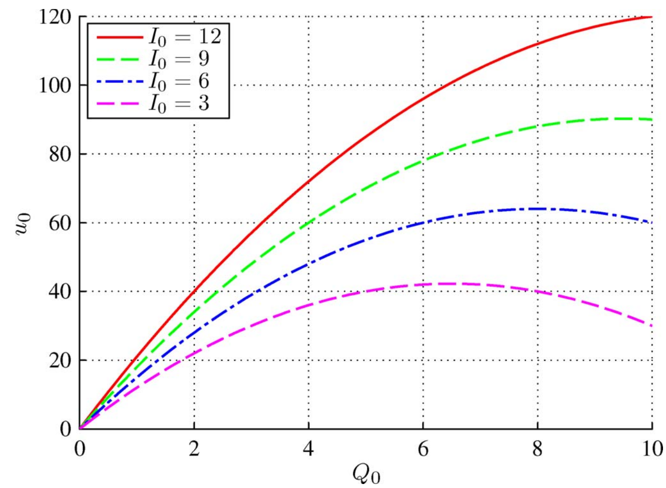

Fig. 1. Primary-user utility u_0 for a fixed secondary interference I_0 in a single-user secondary system.

A. Stationary System With Identical Secondary Users

When all secondary users have the same cross-correlation coefficients, it is possible to characterize the best response correspondences of primary and secondary users to graphically visualize the NE. If $\rho_{0k}^{(p)} = \rho_0^{(p)}$, $\rho_{k0}^{(s)} = \rho_0^{(s)}$, and $\rho_{kj}^{(s)} = \rho^{(s)}$, for all $k, j = 1, 2, \ldots, K$, the same collision probabilities $q_k = q$ for all k and all channels are AWGN: $h_{sk} = h_{pk} = 1$ for all $k = 0, 1, \ldots, K$. Then, $\tilde{A}_k = \tilde{A}$ for all k. By symmetry, in this case, all secondary users must have the same power $p_k = p^*$ at the NE (equivalently, the same SINR $\gamma_k = \gamma^*$). Thus, the NE is characterized by the intersection (Q_0^*, p^*) of the following two curves:

$$Q_0 = r_0(p) = \frac{\bar{Q}_0 + K\tilde{A}^2 p}{2} \tag{8}$$

$$\begin{aligned} p &= r_s(Q_0) \\ &= (\text{solution to equation } \psi_{Q_0}(p) = 0) \end{aligned} \tag{9}$$

where

$$\psi_{Q_0}(p) = Kp + \frac{f(p)}{f'(p)} - \frac{Q_0}{\lambda_s \tilde{A}^2}. \tag{10}$$

Combining (8) and (9), the Nash power p^* of the secondary users is given by the solution to the following:

$$K\left(1 - \frac{1}{2\lambda_s}\right)p + \frac{f(p)}{f'(p)} - \frac{\bar{Q}_0}{2\tilde{A}^2\lambda_s} = 0. \tag{11}$$

Fig. 1 shows the primary utility function for fixed secondary network actions in a single secondary-user system, that is, $K = 1$, assuming that $\bar{Q} = Q_{\max} = 10$, $\bar{P}_1 = 12$, $W_1 = 1$, $q_1 = 1$, $\rho_{01}^{(p)} = \rho_{10}^{(s)} = 1$, $\lambda_s = 1$, $\bar{\gamma}_0 = 1$, $q_1 = 1$, $h_{p1} = 1 = h_{p0} = h_{s0} = h_{s1} = 1$, and $\sigma_s^2 = \sigma_p^2 = 1$.

On the other hand, for the setup described, secondary utility and best response depends on the considered reward function $g(\gamma)$. First, Fig. 2(a) and (b) assumes the secondary reward function $g(\gamma) = g^{(1)}(\gamma) = \log(1+\gamma)$. In Fig. 2(a), we can see the concavity of the secondary utility function for a fixed primary response and, thus, the existence of a best response. The

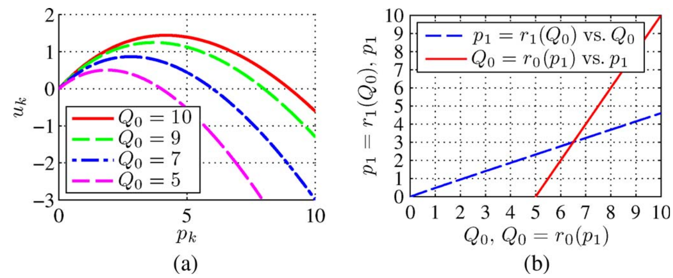

Fig. 2. Secondary utility and the best-response functions in a single secondary user dynamic spectrum leasing network with $f(\gamma) = f^{(1)}(\gamma) = \log(1+\gamma)$. (a) Secondary user utility u_k for a fixed primary IC Q_0. (b) Primary- and secondary-user best-response functions in a single secondary-user DSL system when $f(\gamma) = f^{(1)}(\gamma) = \log(1+\gamma)$ and $\lambda_s = 1$.

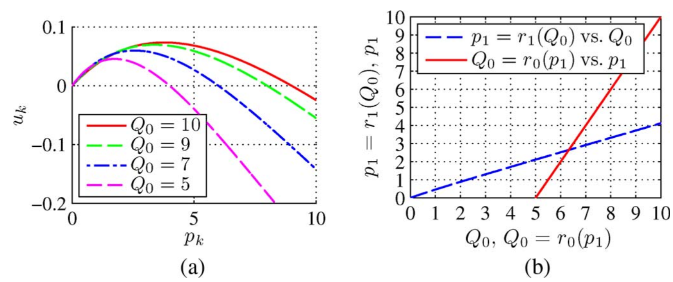

Fig. 3. Secondary utility and the best response functions in a single secondary-user DSL network when $f(p_1) = f^{(2)}(p_1) = (R_1/p_!)$ $C_{\mathrm{BSC}}(P_e(\gamma_1))$ and $\lambda_s = 1$. (a) Secondary-user utility for a fixed primary IC. (b) Corresponding best-response functions.

primary and secondary best-response curves $Q_0 = r_0(p_1)$ and $p_1 = r_1(Q_0)$ for the setup described are presented in Fig. 2(b). Of course, the intersection of these two best-response curves specifies the NE for this system: $(Q^*, p_1^*) = (6.505, 3.010)$.

Similarly, Fig. 3(a) and (b) shows the secondary-user utility for a fixed primary IC and the best-response functions, respectively, when the secondary utility function is chosen to be $g(\gamma) = g^{(2)}(\gamma) = R(C_{\mathrm{BSC}}(P_e(\gamma))/p)$ with $R_1 = 1$ and all other parameters being the same as in the previous figures. From Fig. 3(a), we observe that the secondary utility function is still concave in secondary power. The best-response curves in Fig. 3(b) are characterized by (8) and (9), where now, $g(\gamma) = g^{(2)}(\gamma)$. Fig. 3(b) shows that the NE in this system is $(Q^*, p_1^*) = (6.325, 2.650)$. Note that this NE shows that due to the penalty for increasing transmit power in the secondary system, the secondary user now settles for a slightly lower transmit power level compared with the earlier situation in which it was not concerned with power expenditure. As a result, the primary user is also better off by slightly lowering its IC so that it keeps the demand high.

It is of interest to investigate the equilibrium behavior of this DSL system as a function of the secondary system size K. In Fig. 4(a), we show the allowed IC Q_0 and the actual secondary interference I_0 at the system equilibrium for a system such that $f(p) = f^{(1)}(p) = \log(1 + (p/N(p)))$, where $N(p) = N_k = (K-1)p + \rho_{sp}(Q_0 + \sigma_p^2) + \sigma_s^2$, $\bar{Q} = 10$, $\bar{P}_k = 10$, $W_k = 1$, $R_k = 1$, $\bar{\gamma}_0 = 1$, $q_k = 1$, $\rho_{0k}^{(p)} = \rho_{kj}^{(s)} = 1$, $h_{pk} = h_{sk} = 1$ for all k, and $\sigma_s^2 = \sigma_p^2 = 1$. From Fig. 4(a), we can observe how the total interference I_0 increases with increasing K and how, in turn, the primary user also increases its IC to maximize

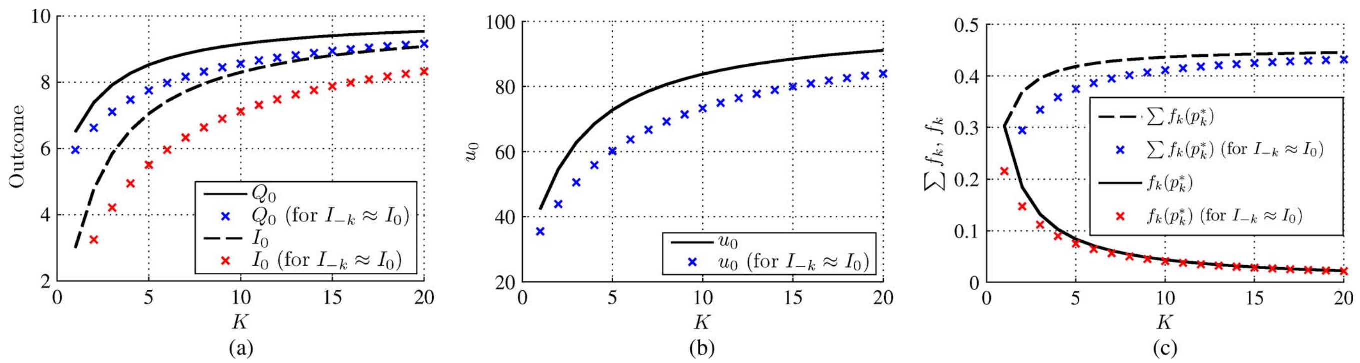

Fig. 4. System performance of the DSL game at the NE, with both exact CSI and using the approximation $I_{0,-k} \approx I_0$, as a function of secondary system size K, assuming identical secondary users, when $f(p) = f^{(1)}(p)$. (a) Game outcome. (b) Primary-user utility. (c) Sum rate and per-user rate achieved by the secondary system at the NE.

its utility. It is also of interest to note that the safety margin $Q_0 - I_0$ is large for a smaller number of users and seems to monotonically decrease with increasing K. This, we believe, is essentially due to the fact that the number of degrees of freedom in a multiuser system is being proportional to the number of users. When the number of secondary users K is large, the interference that is generated by the secondary system I_0 is close to the IC Q_0 and yet, as desired, is always below it. Fig. 4(a) shows the game outcomes when exact CSI for the primary system is available at each secondary user (via estimation) so that the exact $I_{0,-k}$ is used in its best-response adaptation, as well as when this CSI to the primary is not available, so that the secondary user employs the approximation $I_{0,-k} \approx I_0$. As we may observe from Fig. 4(a), the system that does not rely on the knowledge of CSI demonstrates the same performance trends at the equilibrium. In particular, still, the DSL game converges to an NE that does not violate the primary IC. It seems that the only effect of not having the exact $I_{0,-k}$ is that the safety margin $Q_0 - I_0$ at the equilibrium is slightly larger. This is essentially due to the fact that each secondary user believes an exaggerated residual interference $I_{0,-k}$, making it decrease its power.

Fig. 4(b) and (c) shows the primary and secondary utilities at the NE of the system considered in Fig. 4(a) as a function of the secondary system size. Again, we have shown the utilities that are achieved when exact CSI for the primary system is available at each secondary user (via estimation) so that the exact $I_{0,-k}$ is used in its best response adaptation, as well as when this CSI to the primary is not available, so that the secondary user employs the approximation $I_{0,-k} \approx I_0$. In particular, as seen in Fig. 4(b), the primary utility u_0^* at the NE typically increases with the number of secondary users K. However, the rate of increase decreases with increasing K. Thus, from a design point of view, we may argue that the primary user might prefer the system to operate at a point where its rate of utility increase is above a certain threshold value. However, the primary system cannot explicitly impose this on the secondary system, and indeed, it is not a requirement. The only requirement is that $I_0 \leq Q_0$. However, as we see next in Fig. 4(c), the secondary system has the incentive to ensure that K is not too high. It is also observed from Fig. 4(b) that the equilibrium utility of the primary user is decreased when exact CSI is not available at the secondary users.

Fig. 4(c) shows both the sum rate $\sum_{k=1}^{K} f_k(p_k^*)$ and the per-user rate $(1/K)\sum_{k=1}^{K} f_k(p_k^*)$ achieved by the secondary system with and without exact CSI. As was the case with primary utility, the secondary utilities are also slightly reduced in the absence of CSI. However, as we observe from Fig. 4(c), this performance degradation seems to be small when the secondary system size is sufficiently large. Note that, from a system point of view, the secondary system would prefer to maximize the sum rate. As we see from Fig. 4(c), the sum rate monotonically increases with K with and without CSI. Thus, at a first glance, allowing more secondary users to simultaneously operate seems to be the preferred solution. However, Fig. 4(c) also shows that the per-user rate is monotonically decreasing in K, leading to decreasing incremental gains in the sum rate as additional secondary users are added to the system. Depending on the application and the QoS requirement of the secondary system, each secondary user will have a minimum required rate (in bits per transmission), below which, the transmissions would be useless. Thus, we note that this QoS requirement will determine the maximum number of secondary users K that the secondary system would want to support at any given time. For example, if the minimum per-user rate required is 0.1 b/s, the optimal K would be $K^* = 4$, assuming exact CSI. If, on the other hand, the rate threshold was reduced to 0.025 b/s, the secondary system may allow up to $K = 18$ secondary users to simultaneously operate.

B. DSL Network Under Quasi-Static Fading Channels

In the presence of wireless channel fading, the NE power profile of the DSL system will depend on the observed channel state realization. In particular, it is expected that, in this case, the NE transmit power of individual secondary users will be different from each user. In Fig. 5(a), we have shown the game outcome at the NE in the presence of channel fading as a function of the number of secondary users K, both with and without CSI (when there is no CSI, again, we use the approximation $I_{0,-k} \approx I_0$). Note that Fig. 5(a) assumes $f(p_k) = f^{(1)}(p_k)$ with $\bar{Q} = 10$, $\bar{P}_k = 10$, $W_k = 1$, $R_k = 1$, $\bar{\gamma}_0 = 1$, $q_k = 1$, $\rho_{0k}^{(p)} = \rho_{kj}^{(s)} = 1$, and $\sigma_s^2 = \sigma_p^2 = 1$, as before. Fig. 5(b) and (c) shows the corresponding primary- and secondary-user utilities achieved at the NE in the presence of channel fading. In obtaining Fig. 5, we have assumed all channel gains in the

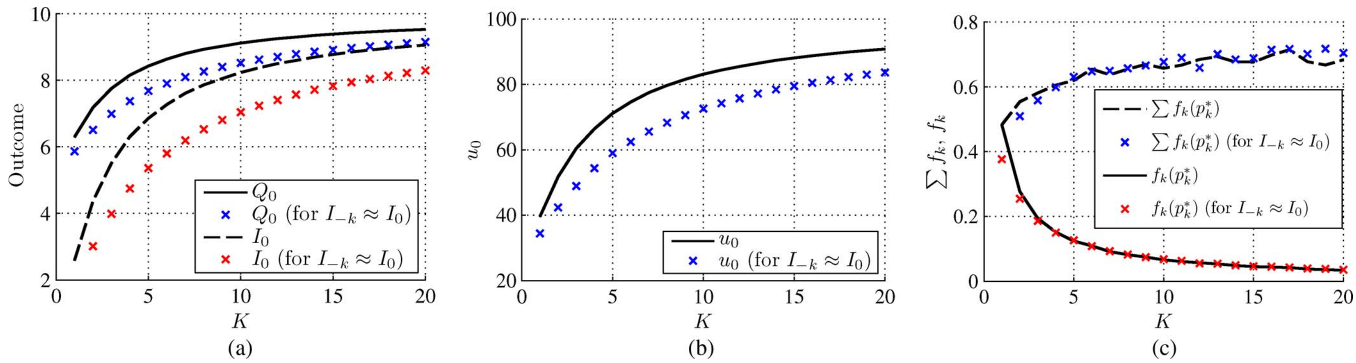

Fig. 5. System performance of the DSL game at the NE, with both exact CSI and using the approximation $I_{0,-k} \approx I_0$, as a function of secondary system size K in the presence of Rayleigh-distributed quasi-static channel fading when $f(p) = f^{(1)}(p)$ and $\lambda_s = 1$. (a) Game outcome. (b) Primary-user utility. (c) Sum rate and per-user rate achieved by the secondary system at the NE.

system to be Rayleigh-distributed with all channel coefficients normalized so that $\mathbb{E}\{h^2\} = 1$. This essentially allows us to consider, with no loss in generality, the transmit power p_k to be equal to the average received power (averaged over fading). Note that, due to interference averaging in the presence of fading, in this case, the secondary system is able to achieve better sum- and per-user rates compared with those with nonfading channels.

Note that when the reward function $f = f^{(1)}$, the reward for a secondary user is the capacity (in bits per second) that it can achieve, assuming that all other transmissions (both primary and secondary) are purely noise. In the presence of channel fading, this capacity is a random quantity that is determined by the fading coefficients of all users. As we saw earlier with identical users, the per-user reward is typically a decreasing function of the increasing secondary system size. The interpretation is simple: Essentially, all secondary users in the system must share the allowed interference level that is set by the primary system. As we mentioned earlier, a secondary user may require a minimum capacity to ensure at least an acceptable QoS for its applications. In Fig. 6, we show the maximum secondary system size (i.e., K) in the presence of fading for different QoS requirements in the secondary system as a function of the (weighting) coefficient λ_s. Note that in Fig. 6, we have set $W_k = W = 1$ so that the secondary reward with $f = f^{(1)}$ has the meaning of spectral efficiency in bits per second per Hertz (or the normalized capacity). All other parameter values are the same as those assumed in Fig. 5. The minimum-transmission quality for the secondary system is defined as the average (over fading) minimum reward that is achieved by a user at the equilibrium. We denote this minimum required QoS for user k as $f_{\min,k}$ and, in all simulation results below, assume that $f_{\min,k} = f_{\min}$ for all secondary users.

As one would expect, as the minimum QoS requirement $f_{\min}$ increases, the number of secondary users who can simultaneously transmit decreases. In addition, the maximum secondary system size also decreases, albeit slowly, as the pricing coefficient λ_s increases. As we may observe from Fig. 6, the greatest impact of the coefficient λ_s is on the primary system. We have included in Fig. 6 the maximum tolerable secondary system size by the primary system before the IC is exceeded at the equilibrium. Fig. 6 shows that when $\lambda_s < 1$, there is a high likelihood that the IC might be exceeded by even a relatively

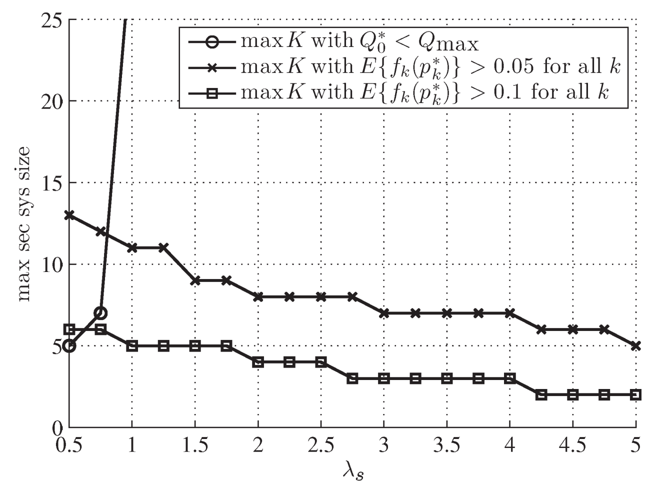

Fig. 6. Maximum secondary system size for a required QoS requirement $f_{\min}$ for all secondary users and the maximum secondary system size supportable without violating the primary IC as a function of the weighting coefficient λ_s, assuming channel fading.

smaller size secondary system. While smaller λ_s would result in higher utilities for the primary system (we have not shown these plots to save space), this comes at the price of violating the interference condition. Thus, the risk with smaller λ_s values is that, depending on the secondary QoS requirement $f_{\min}$, the secondary system may opt to operate at a number of simultaneous users K that could easily violate the interference condition. However, as we observe from Fig. 6 when $\lambda_s \geq 1$, the number of secondary users who simultaneously transmit without violating the primary interference condition dramatically increases, leaving the primary system with enough of a safety margin in case the secondary system opts for a large number of simultaneous users. Thus, we believe that in a proposed DSL network, the primary system must set the pricing coefficient λ_s based on how strictly it wants the secondary users to adhere to the maximum IC condition. If the primary system is also based on a certain amount of cognition, it is reasonable to expect that it may adjust its (pricing) coefficient λ_s to maximize its profits by dynamically adapting optimal λ_s based on its estimation of how many secondary users are in the secondary system.

On the other hand, Fig. 6 shows the maximum number of secondary users who can, on average, coexist while achieving the minimum required transmission quality. However, at times,

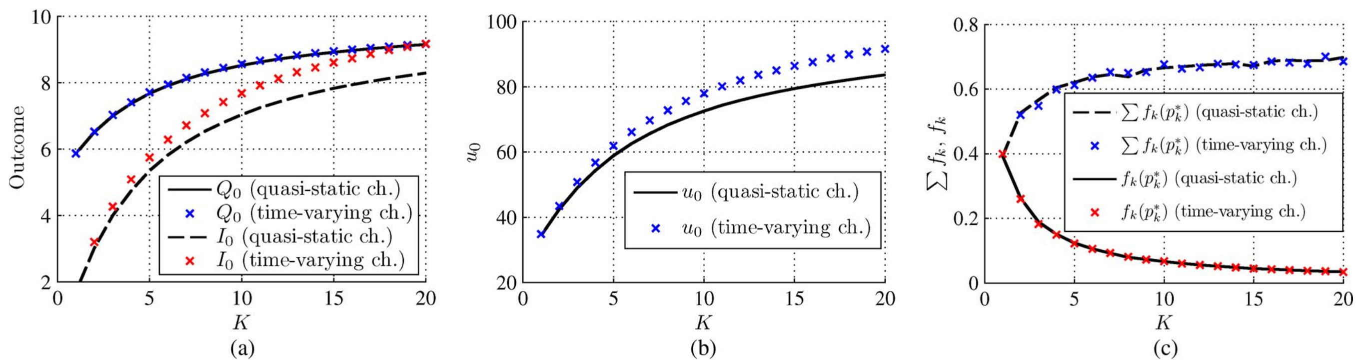

Fig. 7. System performance of the DSL game at the NE in the presence of slow time-varying channel fading when $f(p) = f^{(1)}(p)$ using the approximation $I_{0,-k} \approx I_0$. (a) Game outcome. (b) Primary-user utility. (c) Sum rate and per-user rate achieved by the secondary system at the NE.

depending on the fading statistics, a particular user may or may not meet the minimum transmission quality at the system equilibrium. When this occurs, we say that the user is in outage, and thus, the probability of outage for user k is defined as $\Pr(f_k(p_k^*) < f_{\min,k})$.

C. DSL Network Under Time-Varying Fading Channels

In Section IV-B, we have assumed that the fading coefficients are essentially quasi-static so that they remain constant during the best-response adaptations. However, in practice, these fading coefficients may slowly change during the best-response iterations. In these circumstances, transceivers may need to employ a channel-tracking algorithm to update the estimated fading coefficients. In Fig. 7, we investigate the effect of slowly varying channel coefficients on the DSL game. We model the variations of the channel coefficients with a first-order Gauss–Markov process [20] so that the fading coefficients of the $(n+1)$th best-response adaptation are related to those of the nth iteration as follows:

$$h_{\cdot k}^{(n+1)} = \sqrt{1-\epsilon^2} h_{\cdot k}^{(n)} + \epsilon w_{\cdot k}^{(n)} \tag{12}$$

where $w_{\cdot k}^{(n)}$ is a complex white Gaussian random process of variance $\sigma_{h_{\cdot k}}^2$, independent among channel coefficients $h_{\cdot k}$, ϵ is a parameter indicating the temporal variation rate of the channel, and initial $h_{\cdot k}^{(0)}$ is chosen to be complex Gaussian. It is easy to verify that $\sqrt{1-\epsilon^2}$ represents the temporal correlation of the channel coefficients between two best-response iterations. Fig. 7(a) shows the DSL game outcome when the coefficients are time-varying with an $\epsilon = 0.1$, as compared with a quasi-static system in which fading is constant throughout (i.e., $\epsilon = 0$). It is assumed that the slowly time-varying system only updates the fading coefficients once in every $L = 10$ iterations (of course, the quasi-static system always has exact coefficients since they stay fixed throughout the iterations and, thus, correspond to $L = 1$). Fig. 7(a) shows that the primary IC is basically insensitive against assumed slow channel variations. However, the corresponding secondary interference I_0 at the NE is usually larger in the presence of channel variations, particularly for a large number of secondary users. The reason for this is that the game response falls somewhat behind compared with the channel variations. Of course, this effect could

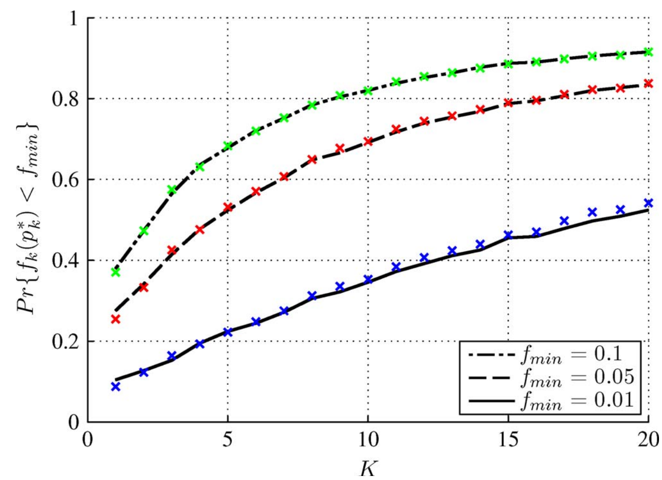

Fig. 8. Outage probability $\Pr(f_k(p_k^*) < f_{\min})$ of a typical secondary user at the NE of the DSL game in both (lines) quasi-static and (marks) slow time-varying fading channels, as a function of secondary system size K for a required QoS requirement $f_{\min}$.

be reduced by allowing more frequent channel adaptations (i.e., small L). The effect of this increased I_0 is to reduce the safety margin $(Q_0 - I_0)$ that the primary receiver has in terms of its tolerable interference level. However, as Fig. 7(a) shows, unless the number of secondary users is relatively large, still, the IC Q_0 is not violated by the increased interference I_0. Thus, we conclude that as long as the channel variations are sufficiently slow and/or coefficient adaptations are fast enough, the DSL game can still reach an acceptable equilibrium state.

Fig. 7(b) and (c) shows the corresponding primary and secondary utilities in the presence of slowly time-varying channel fading at the NE outcome shown in Fig. 7(a). Fig. 7(b) shows that the primary utility at the NE is slightly increased when fading is slow time-varying, compared with that with static channels. This is a direct consequence of the increased interference level I_0 seen in Fig. 7(a) that reduced the safety margin. From primary utility function (6), we can see that the reduced safety margin may lead to increased primary utility. However, it is to be noted that this itself may not be a good outcome if the number of secondary users is too large since the secondary interference may violate the IC. As can be seen from Fig. 7(c), both the sum and per-user rates that are achieved by the secondary system are reasonably robust against slow time variations in fading.

Fig. 8 shows the outage probability of a typical secondary user as the system size increases with both quasi-static, as well as slow time-varying [according to (12)] channel fading. It is seen from Fig. 8 that the outage probability increases with K as well as with the minimum QoS requirement. However, as one may have predicted from the insensitivity of secondary-user utilities to slow channel variations observed in Fig. 7(c), the outage probabilities are robust against the channel time variations. The maximum secondary system size that can be supported according to Fig. 6 thus needs to be interpreted in conjunction with the outage probabilities shown in Fig. 8. For example, although, as shown in Fig. 6, about five secondary users can, on average, meet the $f_{\min=} = 0.1$ QoS requirement, according to Fig. 8, each of these users may be in outage about 70% of the time. This, of course, is the price of operating as a secondary system.

V. Conclusion

In this paper, we have proposed the concept of DSL as a new paradigm for DSA in cognitive radio networks. As opposed to the hierarchical DSA networks, the proposed DSL networks provide an incentive for the primary users who own the spectrum to actively allow secondary spectrum access whenever it is feasible. In our proposed framework, this is achieved by defining a utility function for the primary system that is proportional to both the demand (for interference) as well as the amount of total interference that it is willing to tolerate. The rationale behind the proposed utility is that the more secondary interference that the primary user is willing to tolerate, the higher its reward must be. On the other hand, if the IC set by the primary user is higher than the actual secondary interference that exists in the system, then the demand for interference by the secondary system must decrease, and the primary utility must be proportional to this demand. For the secondary users, their utility must be proportional to a suitably chosen reward function f, as well as the achieved interference margin with respect to the primary system. The higher the interference margin, the safer the secondary operation without violating the primary QoS, and, hence, the rationale for its utility to be proportional to the interference margin. We have formulated the DSL cognitive system as a noncooperative DSL game between the primary and secondary users and established a basic result on the existence of a unique NE. Specifically, we have established the general condition on the reward function f to ensure the existence of an equilibrium.

Next, we have considered several example cognitive radio DSL networks in detail to investigate the behavior of the proposed system. In particular, we have shown that, in the case of identical users, the proposed DSL game can be solved to obtain the NE action profile as the solution to a single equation. In such a system, we have observed that the proposed DSL naturally leads to a design that will determine the maximum number of secondary users based on required minimum QoS criteria. In the presence of fading, we have observed that the achieved secondary sum rate could be considerably higher than that without fading. This was due to interference averaging effect due to fading that de-emphasized the interference among users leading to a better SINR.

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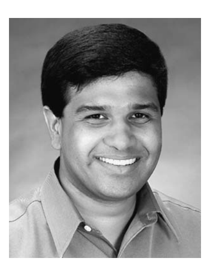

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