Distributed Dynamic Spectrum Leasing (D-DSL) for Spectrum Sharing over Multiple Primary Channels

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***Abstract*—We propose a new architecture for dynamic spectrum sharing called the distributed dynamic spectrum leasing (D-DSL) and a game-theoretic framework for its implementation on a cognitive radio network. In D-DSL, each available frequency channel is assigned to a primary user who may independently lease the channel to secondary users. Secondary users are allowed to transmit in multiple channels simultaneously. We establish conditions for the system to reach an equilibrium and analyze the robustness of the proposed game theoretic D-DSL in time-varying channels. For the same amount of available primary spectrum, D-DSL leads to better overall performance compared to previously proposed Centralized-DSL.**

***Index Terms*—Cognitive radios, D-DSL, distributed dynamic spectrum leasing, DSL, dynamic spectrum leasing, game theory, time-varying channels, Rayleigh fading.**

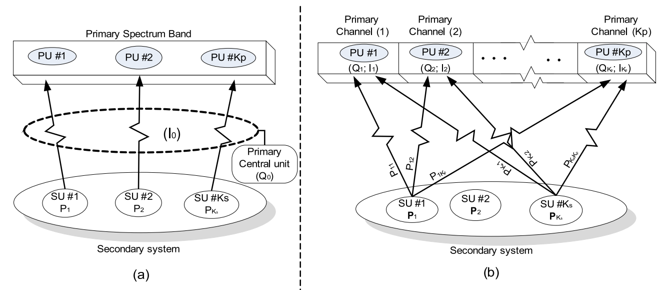

Fig. 1. a) The centralized dynamic spectrum leasing (C-DSL); b) the distributed dynamic spectrum leasing (D-DSL).

I. Introduction

THE rapid growth of wireless communication has resulted in an increasing demand for the wireless bandwidth. On the other hand some allocated spectrum bands, such as those allocated for television broadcasting or paging systems have found to be underutilized as noted in several recent studies by the Federal Communications Commission (FCC) [1]. This has led the FCC to allow unlicensed wireless users to access the licensed spectrum bands under the concept of spectrum sharing. The considerable progress made in transition of cognitive radios [2], [3] from a technical concept to reality over last few years has positioned dynamic spectrum sharing (DSS) as a viable technology for better spectrum utilization.

Under the proposed Dynamic Spectrum Sharing, the spectrum owner (called the primary user) allows the unlicensed users (called the secondary users) to dynamically access the so-called white spaces in its spectrum. In almost all existing proposals, the secondary users are solely responsible for the interference management and coexistence in the primary spectrum band. These proposals are termed as Dynamic Spectrum Access (DSA). Recently in [4]–[7], the authors introduced the concept of Dynamic Spectrum Leasing (DSL) in which the primary users, as well as the secondary users, are involved in managing the interference. In [4], the authors introduced the Centralized Dynamic Spectrum Leasing (C-DSL), as shown in Fig. 1(a), in which a central unit measure the total interference I_0 from all secondary users and sets a *common* interference cap (IC), denoted by Q_0, that is valid for all primary users. The interference cap Q_0 is the maximum interference that the primary system is willing to tolerate from all secondary transmissions. This framework is termed as Centralized-DSL in [8] due to the assumed central coordination among primary users.

In this paper we introduce a new framework called the Distributed Dynamic Spectrum Leasing (D-DSL). In contrast to the C-DSL, each primary user in a D-DSL system sets its own interference cap depending on the interference level from the secondary users. Note that, it is assumed that each primary user is assigned its own frequency channel in which it transmits. On the other hand, the secondary users are autonomous agents that are allowed to transmit simultaneously in more than one spectrum band in order to capitalize on and fully utilize the available spectrum opportunities. We model this scenario as a spectrum sharing game. The non-cooperative game in this new framework will not only be between the primary and the secondary users as in C-DSL, but also be a non-cooperative game among primary users themselves.

This paper develops a new game-theoretic framework for such a D-DSL system by identifying suitable payoff functions for both primary and secondary users, and establishes conditions for existence of an equilibrium point that can be reached via adaptive best-response of users. As in previous work on DSL, the proposed D-DSL can be implemented with the same inter-system control information exchanges assumed in [5]–[7]. We study the system performance as a function of the secondary system size and number of primary channels. We also investigate the robustness of the D-DSL system to time-varying channel fading. Using computer simulations we evaluate the performance of the proposed D-DSL and we show the improvement that the secondary system rate may achieve by increasing the number of degrees of freedom.

The remainder of this paper is organized as follows: Section II describes the D-DSL framework and the system model. Section III presents the game theoretic formulation. Section IV

Manuscript received April 18, 2010; revised July 16, 2010 and September 28, 2010; accepted October 8, 2010. The associate editor coordinating the review of this paper and approving it for publication was R. K. Mallik.

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Digital Object Identifier 10.1109/TWC.2010.110310.100638

discusses the existence and uniqueness of a Nash equilibrium in the proposed game. In section V, we evaluate the performance of a spectrum sharing network based on the proposed D-DSL and we compare it to the C-DSL performance. Finally Section VI concludes the paper by summarizing the results.

II. DSL - Based Cognitive Radio System Model

We assume that there are K_p primary communication channels, where a channel here specifically means a distinct frequency band. Without loss of generality, we assume that there are exactly K_p primary transmitter receiver pairs (links) and each of them is allocated to a unique primary channel. Depending on primary traffic patterns these channels may not be fully utilized all the time by the primary users and the primary users may be able to tolerate some interference without affecting their QoS. Under this scenario, it would make sense for the primary users to lease their spectrum to others, called the secondary users, where they may transmit on the leased channels without violating the primary QoS. We assume that there are K_s secondary transmitter-receiver pairs (links) that are active. In this paper we introduce the concept of D-DSL in which the j-th primary user, for $j \in \mathcal{K}_p$, measures the total secondary interference I_j on its channel and dynamically adjust its own interference cap, denoted by Q_j, that is applicable for only the j-th channel. The motivation for the primary users could be the monetary reward obtained by allowing the secondary users access its channel.

As shown in Fig. 1(b), each secondary user now has the opportunity to communicate over multiple primary channels by allocating its transmit power appropriately for different primary channels. We denote the k-th secondary user's power vector $\mathbf{p}_k = \left(p_{k,1}, p_{k,2}, \cdots, p_{k,K_p}\right)^T$, for $k \in \mathcal{K}_s$, where $p_{k,j}$ is the power it allocates to communicate over the j-th primary channel. As each primary user dynamically changes its interference cap, Q_j, the secondary users try to fully utilize each channel by adjusting their transmit powers $p_{k,j}$'s without violating the QoS requirement of the channel owner. When a primary user raises its interference cap, it encourages the secondary users to prefer that particular channel over the other primary channels. This leads to a non-cooperative game among the primary users in which the reward of a primary user should be an increasing function of the interference cap.

For each $j \in \mathcal{K}_p$, the primary user in the j-th channel will be labeled by 0 and the secondary links (transmitter-receivers pairs) are labeled from 1 through K_s. The set of user indices (primary and secondary) in every channel is denoted by $\mathcal{K}_c$, i.e. $\mathcal{K}_c = \{0\} \cup \mathcal{K}_s$. The time varying channel gain between the k-th receiver in the j-th channel and k'-th transmitter is denoted by $h^{(j)}_{k,k'}(i)$ for $k, k' \in \mathcal{K}_c$ and $j \in \mathcal{K}_p$ where i represents the symbol index. Note that the channel gains are considered independent between the users with a constant gain during one symbol transmission, i.e. $h^{(j)}_{k,k'}(t) \approx h^{(j)}_{k,k'}(i)$ when $t \in [(i-1)T_s, iT_s)$ where T_s represents the symbol period. Let $A^{(j)}_{k,k'}(i) = h^{(j)}_{k,k'}(i)\sqrt{p_{k',j}}$ be the received signal amplitude for $k, k' \in \mathcal{K}_c$.

The discrete-time representation of the received signal at the primary receiver in the j-th channel during the i-th symbol transmission is

$$\mathbf{r}_{0,j} = A^{(j)}_{0,0}(i) b_{0,j}(i) \mathbf{s}^{(p)}_0 + \sum_{k \in \mathcal{K}_s} A^{(j)}_{0,k}(i) b_{k,j}(i) \mathbf{s}^{(p)}_k + \sigma_{0,j}\mathbf{n}_{0,j}, \tag{1}$$

where $\mathbf{r}_{0,j} = \left(r^{(j)}_{0,1}, \cdots, r^{(j)}_{0,M}\right)^T$ and $\mathbf{s}^{(p)}_k = \left(s^{(p)}_{k,1}, \cdots, s^{(p)}_{k,M}\right)$ are the vector representation of the primary received signal $r_{0,j}(t)$ and the deterministic signaling waveform $s_k(t)$ with respect to the M-dimensional primary basis, $\sigma^2_{0,j}$ is the variance of primary receiver noise in the j-th channel and $\mathbf{n}_{0,j} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I_M})$. Similarly, a discrete-time representation of the received signal at the k-th secondary receiver in the j-th channel during the i-th symbol interval can be written as

$$\mathbf{r}_{k,j} = \sum_{k' \in \mathcal{K}_s} A^{(j)}_{k,k'}(i) b_{k',j}(i) \mathbf{s}^{(s)}_{k'} + A^{(j)}_{k,0}(i) b_{0,j}(i) \mathbf{s}^{(s)}_0 + \sigma_{k,j}\mathbf{n}_{k,j}, \tag{2}$$

where $\mathbf{r}_{k,j} = \left(r^{(j)}_{k,1}, \cdots, r^{(j)}_{k,N}\right)^T$ and $\mathbf{s}^{(s)}_k = \left(s^{(s)}_{k,1}, \cdots, s^{(s)}_{k,N}\right)^T$ are the vector representation of the secondary received signal $r_{k,j}(t)$ and $s_k(t)$ with respect to the N secondary basis.

In the following we assume that the primary transmissions are modulated as binary phase shift keying (BPSK), and the detectors are based on the matched filter (MF) outputs. Therefore the primary decision in the j-th channel is given by $\hat{b}_{0,j} = sgn\left(y_{0,j}\right)$ where $y_{0,j} = \left(\mathbf{s}^{(p)}_0\right)^T \mathbf{r}_{0,j} = A^{(j)}_{0,0}(i) b_{0,j}(i) + \sum_{k \in \mathcal{K}_s} \rho^{(p)}_{0,k} A^{(j)}_{0,k}(i) b_{k,j}(i) + \sigma_{0,j}\eta_{0,j}$, with $\rho^{(p)}_{0,k} = \left(\mathbf{s}^{(p)}_0\right)^T \mathbf{s}^{(p)}_k$ and $\eta_{0,j} \sim \mathcal{CN}(0,1)$. We can observe that the total secondary interference I_j from all secondary links to the primary signal for $j \in \mathcal{K}_p$ is $I_j = \sum_{k \in \mathcal{K}_s} \left(\rho^{(p)}_{0,k}\right)^2 \left|h^{(j)}_{0,k}(i)\right|^2 p_{k,j} = \sum_{k \in \mathcal{K}_s} \left|\tilde{A}_{k,j}\right|^2 p_{k,j}$ where $\tilde{A}_{k,j} = \rho^{(p)}_{0,k} h^{(j)}_{0,k}(i)$ is the k-th secondary transmitter's effective channel coefficient on the j-th primary channel. Similarly the k-th secondary link estimates its symbols on the j-th channel as $\hat{b}_{k,j} = sgn\left(y_{k,j}\right)$, where $y_{k,j} = \left(\mathbf{s}^{(s)}_k\right)^T \mathbf{r}_{k,j} = A^{(j)}_{k,k}(i) b_{k,j}(i) + \sum_{k' \in \mathcal{K}_c \setminus k} \rho^{(s)}_{k,k'} A^{(j)}_{k,k'}(i) b_{k',j}(i) + \sigma_{k,j}\eta_{k,j}$ with $\rho^{(s)}_{k,k'} = \left(\mathbf{s}^{(s)}_k\right)^T \mathbf{s}^{(s)}_{k'}$ for $k' \in \mathcal{K}_c$, and $\eta_{k,j} \sim \mathcal{CN}(0,1)$

III. A Game Model for D-DSL

In the proposed D-DSL game model, the users interact with each other by adjusting the tolerable interference caps of the primary users and the transmit powers of the secondary users in each channel. Each user (primary or secondary) chooses its action in order to maximize its own utility. We model the D-DSL system as the following non-cooperative game $G = \left(\mathcal{K}, \mathcal{A}, u_{(.,.)}\right)$. The player set is $\mathcal{K} = \mathcal{K}_s \cup \mathcal{K}_p$ where we represent the secondary players by the index k for $k \in \mathcal{K}_s$ and the primary players by j for $j \in \mathcal{K}_p$. Let $\mathcal{A} = \mathcal{A}_{s,1} \times \mathcal{A}_{s,2} \times \cdots \times \mathcal{A}_{s,K_s} \times \mathcal{A}_{p,1} \cdots \times \mathcal{A}_{p,K_p}$ be the action space of the non-cooperative game G, where $\mathcal{A}_{s,k} = \mathcal{P}_{k,1} \times \cdots \times \mathcal{P}_{k,K_p}$ for $k \in \mathcal{K}_s$ represents the action space of the k-th secondary user with $\mathcal{P}_{k,j} = [0, \overline{P}_k]$ and $\mathcal{A}_{p,j} = [0, \overline{Q}_j]$ for $j \in \mathcal{K}_p$ represent the action set of the

j-th primary user. The upper limits of the action sets $\overline{P}_k$ and $\overline{Q}_j$ represent, respectively, the maximum transmission power of the k-th secondary user and the maximum tolerable interference cap of the j-th primary user. We denote the action vector of all users by $\mathbf{a} = [\mathbf{p}_1, \cdots, \mathbf{p}_{K_s}, Q_1, \cdots, Q_{K_p}]^T$ where $Q_j \in \mathcal{A}_{p,j}$ for $j \in \mathcal{K}_p$ and $\mathbf{p}_k = [p_{k,1}, \cdots, p_{k,K_p}]^T$ is the action set of the k-th secondary user for $k \in \mathcal{K}_s$ on K_p primary channels with $p_{k,j} \in \mathcal{P}_{k,j}$ and $\|\mathbf{p}_k\|_1 \leq \overline{P}_k$ where $\|\mathbf{x}\|_1$ represents the L^1 norm of vector $\mathbf{x}$. For notational convenience, we refer to the action vector excluding that of the k-th secondary player by $\mathbf{a}_{-s,k}$ for $k \in \mathcal{K}_s$ and the action vector excluding the j-th primary player by $\mathbf{a}_{-p,j}$ for $j \in \mathcal{K}_p$. We denote by $u_{s,k}(\mathbf{p}_k, \mathbf{a}_{-s,k})$ for $k \in \mathcal{K}_s$, the k-th secondary user's utility function, and by $u_{p,j}(Q_j, \mathbf{a}_{-p,j})$ for $j \in \mathcal{K}_p$, the j-th primary user's utility function.

At any given time t, the *assumed* worst-case target SINR of the j-th primary user is defined as follows:

$$\overline{\gamma}_{p,j} = \frac{\left|h_{0,0}^{(j)}(i)\right|^2 p_{0,j}}{Q_j + \sigma_{0,j}^2}. \quad (3)$$

Note that the worst case SINR is a function of the maximum tolerable interference by the primary user.

In dynamic spectrum leasing, the primary and secondary users interact with each other through the utility functions which are functions of mutual interference. We choose the utility function of the j-th primary user for $j \in \mathcal{K}_p$ to be [4]:

$$u_{p,j}\left(Q_j, \mathbf{a}_{-p,j}\right) = \left(\overline{Q}_j - \left(Q_j - I_j\left(\mathbf{a}_{-p,j}\right)\right)\right) Q_j. \quad (4)$$

The SINR of the k-th secondary user in the j-th channel can be found as:

$$\gamma_{k,j} = \frac{\left|h_{k,k}^{(j)}(i)\right|^2 p_{k,j}}{\sum_{k' \in \mathcal{K}_c \backslash k} \left(\rho_{k,k'}^{(s)}\right)^2 \left|h_{k,k'}^{(j)}(i)\right|^2 p_{k',j} + \sigma_{k,j}^2} = \frac{p_{k,j}}{N_{k,j}}, \quad (5)$$

where $N_{k,j} = \frac{\sum_{k' \in \mathcal{K}_c \backslash k} \left(\rho_{k,k'}^{(s)}\right)^2 \left|h_{k,k'}^{(j)}(i)\right|^2 p_{k',j} + \sigma_{k,j}^2}{\left|h_{k,k}^{(j)}(i)\right|^2}$. A natural objective of each secondary user would be to maximize its SINR on each channel without violating the interference cap Q_j set by the j-th primary user. A reasonable utility function for the secondary user should be a monotonically increasing function of the SINR $\gamma_{k,j}$. At the same time it should be a decaying function of $I_j - Q_j$ for every $j \in \mathcal{K}_p$. To that end, we propose the following secondary user utility function, for $k \in \mathcal{K}s$:

$$\begin{aligned} u_{s,k}\left(\mathbf{p}_k, \mathbf{a}_{-s,k}\right) &= \sum_{j \in \mathcal{K}_p} u_{k,j}\left(p_{k,j}, \mathbf{a}_{-s,k}\right) \\ &= \sum_{j \in \mathcal{K}_p} \left(Q_j - \lambda_j I_j\right) f_{k,j}\left(p_{k,j}\right), \end{aligned} \quad (6)$$

where $u_{k,j} = \left(Q_j - \lambda_j I_j\right) f_{k,j}\left(p_{k,j}\right) = \left(Q_j - \lambda_j I_{j,-k} - \lambda_j \left|\tilde{A}_{k,j}\right|^2 p_{k,j}\right) f_{k,j}\left(p_{k,j}\right)$ is the partial utility that the k-th user obtains by transmitting on the j-th channel and λ_j is a positive coefficient which controls how strictly the secondary users need to obey the j-th primary interference cap Q_j. The reward function $f_{k,j}(.)$ of the k-th secondary user on the j-th channel should be an increasing function of $p_{k,j}$ (i.e. $f'_{k,j}\left(p_{k,j}\right) \geq 0$). Without loss of generality $f_{k,j}(0) = 0$ since when there is no transmission, there is no rewards for the secondary users. Note that $I_{j,-k} = I_j - \left|\tilde{A}_{k,j}\right|^2 p_{k,j}$ is the interference from the secondary users on the j-th primary channel excluding the interference caused by the k-th secondary link.

IV. The Existence and Uniqueness of a Nash Equilibrium in the Proposed D-DSL Game

In this section we investigate the existence and uniqueness of an equilibrium in the above D-DSL game $G = \left(\mathcal{K}, \mathcal{A}, u_{(.,.)}\right)$. Where each user tries to maximize its own utility function defined earlier in (4) and (6). The Nash equilibrium is a predictable and stable outcome for the non-cooperative game D-DSL [9].

Definition 1: A strategy profile vector $\mathbf{a} = [a_1, a_2, \cdots, a_K]^T$ is a Nash equilibrium for the game $G = (\mathcal{K}, \mathcal{A}_k, u_k)$ if, for all players $k \in \mathcal{K}$, $u_k(a_k, \mathbf{a}_{-k}) \geq u_k(a'_k, \mathbf{a}_{-k})$ for all $a'_k \in \mathcal{A}_k$.
However such a point may not necessarily exist in a game.

Proposition 1: There exists a Nash equilibrium for the D-DSL game $G = \left(\mathcal{K}, \mathcal{A}, u_{(.,.)}\right)$.

Proof: To prove the existence of the Nash equilibrium it is sufficient to show that the action space of each player is a nonempty compact convex subsets of an Euclidian space $\mathbb{R}^n$ and the primary-secondary utility functions are continuous in $\mathbf{a}$ and quasi-concave in a_k [9]. Clearly the action spaces of primary and secondary users are compact convex nonempty sets (they are closed and bounded finite sets). The primary utility function $u_{p,j}\left(Q_j, \mathbf{a}_{-p,j}\right)$ is continuous in $\mathbf{a}$ and concave in Q_j. The secondary utility function $u_{s,k}\left(\mathbf{p}_k, \mathbf{a}_{-s,k}\right)$ is continuous in the action vector $\mathbf{a}$ and it is concave if the reward functions $f_{k,j}(p_{k,j})$ are concave in $p_{k,j}$. Thus with concave reward functions, all the necessary conditions for the existence of a Nash equilibrium are satisfied. ■

In dynamic spectrum leasing the goal of each secondary user is to maximize the rate it can achieve. To that end, we will set the reward function in (6) to be $f_{k,j}(p_{k,j}) = W_{k,j} \log\left(1 + \gamma_{k,j}\right)$ where $\gamma_{k,j}$ is the k-th secondary user's received SINR on the j-th channel as defined in (5) and $W_{k,j}$ is a positive weighting coefficient which can be taken to be proportional to the bandwidth of channel j.

The best response of a particular player is the reaction that maximizes its own utility function for a fixed action vector of the other players:

Definition 2: The best response of the k-th player $r_k : \mathcal{A}_{-k} \rightarrow \mathcal{A}_k$ is the set:

$$\begin{aligned} r_k(\mathbf{a}_{-k}) = \{a_k \in \mathcal{A}_k : & u_k(a_k, \mathbf{a}_{-k}) \geq u_k(a'_k, \mathbf{a}_{-k}) \\ & \text{for all } a'_k \in \mathcal{A}_k\}. \end{aligned} \quad (7)$$

The best response of the primary users in a D-DSL game is the *unique* Q_j^* that maximizes the primary utility function $u_{p,j}(Q_j)$. The uniqueness in the best response is due to the fact that the primary utility function is concave. It can be shown that $Q_j^*\left(I_j\right) = \frac{\overline{Q}_j + I_j}{2}$ for all $j \in \mathcal{K}_p$. Since the primary utility function is an increasing function when $Q_j \leq Q_j^*$, if Q_j^*

exceeds the maximum interference cap $\overline{Q}_j$, the j-th primary users will set their interference cap to be $Q_j = \overline{Q}_j$. Therefore the best response of the j-th primary user is $r_{p,j}(\mathbf{a}_{-p,j}) = \min\{Q_j^*(I_j), \overline{Q}_j\}$.

In order to determine the best response for a primary user, the only quantity needed is the total interference level I_j which can be easily estimated at the primary receiver.

On the other hand, the best response of the k-th secondary user is $\mathbf{r}_{s,k}(\mathbf{a}_{-s,k}) = \mathbf{p}_k^* = \left(p_{k,1}^*, p_{k,2}^*, \cdots, p_{k,K_p}^*\right)^T$ where $\mathbf{p}_k^*$ is the *unique* transmitted power vector that maximizes the *concave* secondary utility function. It can be shown that $\mathbf{p}_k^*$ should satisfies the following Karush-Kuhn-Tucker (KKT) conditions [10]: 1) $\mu_k \leq 0$, 2) $u'_{k,j}(p_{k,j}) + \mu_k = 0$ for all $j \in \mathcal{K}_p$, 3) $\mu_k \left(\sum_{j\in\mathcal{K}_p} p_{k,j} - \overline{P}_k\right) = 0$ and 4) $\sum_{j\in\mathcal{K}_p} p_{k,j} \leq \overline{P}_k$.

Note that the KKT conditions are only necessary conditions to maximize the secondary utility. Let $\mathbf{L}(\mathbf{p}_k, \mu_k)$ be the Hessian matrix of the Lagrangian function $l(\mathbf{p}_k, \mu_k) = u_{s,k}(\mathbf{p}_k) + \mu_k\left(\|\mathbf{p}_k\| - \overline{P}_k\right)$ with respect to $\mathbf{p}_k$. In this special case, $\mathbf{L}(\mathbf{p}_k^*, \mu_k^*)$ is negative definite on $\mathbb{R}^{K_p}$. Thus the extremum point $\mathbf{p}_k^*$ is indeed a global maximum [10]. We observe that the best response of the k-th secondary user is a function of the primary interference caps Q_j and the residual interferences $I_{j,-k}$ from all other secondary users on the j-th channel, for all $j \in \mathcal{K}_p$. To obtain these two quantities, we assume that each primary user periodically broadcasts Q_j and I_j. Knowing the total interference level I_j, the secondary user can compute the residual interference $I_{j,-k} = I_j - \left|\tilde{A}_{k,j}\right|^2 p_{k,j}$ since it knows its own transmit power $p_{k,j}$ and it may estimate the channel state information $\tilde{A}_{k,j}$ if the reverse link signals are available to the secondary system.

To sum up, each primary user measures the total secondary interference I_j and sets its interference cap Q_j^* according to the primary user's best response $r_{p,j}(\mathbf{a}_{-p,j})$. The secondary users receive Q_j^* and I_j from all the primary users and adjust their new transmit power vectors $\mathbf{p}_k^*$ that satisfy the KKT conditions defined earlier. The convergence of the algorithm is guaranteed due to the existence and uniqueness of the Nash equilibrium.

V. Performance of a D-DSL based Spectrum Sharing System

In the following we investigate the performance at Nash equilibrium of the proposed D-DSL based DSS system. Unless stated otherwise, the system parameters are set as follows: the receiver noise variance is set to be $\sigma_{k,j}^2 = 1$ for $j \in \mathcal{K}_p$ and $k \in \mathcal{K}_c$, the primary user target SINR is $\overline{\gamma}_{p,j} = 1$, the weighting coefficient is $\lambda_j = 1$, the cross correlation coefficients are assumed to be the same $\rho_{0,k}^{(p)} = \rho_{k,0}^{(s)} = \rho_{k,k'}^{(s)} = 1$ for all $k, k' \in \mathcal{K}_s$ and $W_{k,j} = W = 1$.

A. D-DSL Performance in Quasi-static and Time-varying Channels

We will assume Rayleigh distributed channel fading coefficients with normalized channel coefficients, i.e. $\mathbb{E}\left[\left|h_{k,k'}^{(j)}(i)\right|^2\right] = 1$. The maximum possible primary interference caps are $\overline{Q}_j = 20$ and the maximum secondary transmit powers are $\overline{P}_k = 10$.

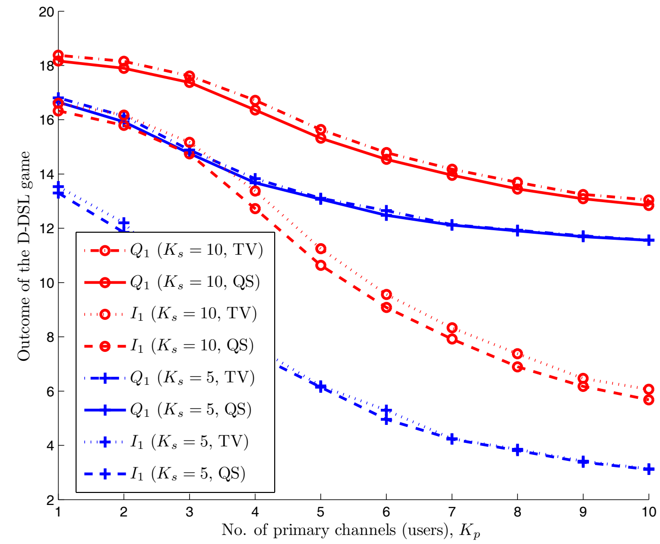

Fig. 2. The Game outcome as a function of the secondary system size K_s in a Rayleigh distributed channel fading, where $\overline{Q}_j = 20$ and $\overline{P}_k = 10$.

We investigate the performance with both Quasi-static (QS) fading where the channel state information is constant for the duration of a block and time-varying (TV) fading where the Rayleigh fading channel coefficients are correlated through a first order Guass-Markov process [11], described via $h_{k,k'}^{(j)}(i) = \sqrt{1-\epsilon}\, h_{k,k'}^{(j)}(i-1) + \epsilon w_{k,k'}^{(j)}(i)$, $w_{k,k'}^{(j)}(i)$ are iid $\mathcal{CN}(0, \sigma_h^2)$ and ϵ is the channel variation rate. We assume that each receiver updates the channel state information (CSI) periodically every L samples. The receiver decisions will thus be based on the estimated CSI defined as: $\hat{h}_{k,k'}^{(j)}(i) = h_{k,k'}^{(j)}(\lfloor i/L \rfloor L)$. In all time-varying simulation results we set the channel estimate period to $L = 10$ and the channel variation rate to $\epsilon = 0.1$. In all simulations the results are averaged over 2000 channel coefficients using Monte Carlo methods.

Figure 2 shows how the interference cap Q_j^* and the total secondary interference I_j^* decrease with increasing number of primary channels K_p in the presence of both time-varying and quasi-static channels. In time-varying case, values of Q_j and I_j are slightly higher than those in the quasi-static case. This is due to the incomplete information that causes a deviation from the actual Nash equilibrium. As seen in Fig. 3, the secondary system has the incentive to keep K_s small enough to maintain a minimum QoS guarantee for all its users. Note that, due to the higher degrees of freedom available in a D-DSL system, the secondary system is able to achieve a better sum and per-user rates compared to those achieved in a single channel scenario.

Depending on the application, a secondary user may require a minimum rate to achieve a least acceptable QoS requirement. We denote this minimum rate for the secondary users by R_{min}. Because of the dependency on the random fading coefficients, at any given time a particular user may

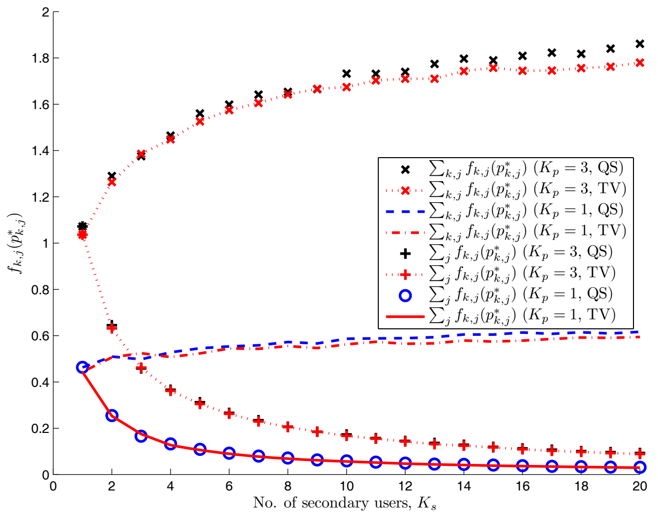

Fig. 3. The sum-rate and per-user rate of the secondary system at Nash equilibrium as a function of secondary system size K_s, in a Rayleigh distributed channel fading, where $\overline{Q}_j = 20$ and $\overline{P}_k = 10$.

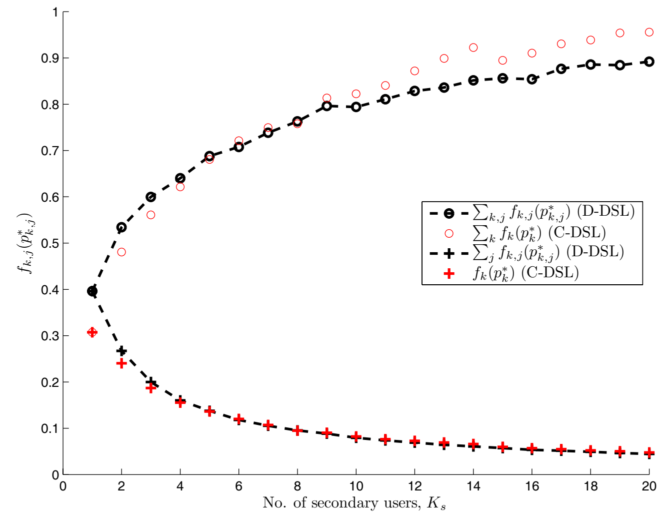

Fig. 5. The sum-rate and per-user rate of the secondary system at Nash equilibrium of D-DSL system compared to the C-DSL system, where $K_p = 3$, $\overline{Q}_0^{(\text{C-DSL})} = 18$, $\overline{Q}_j^{(\text{D-DSL})} = 2$, $\overline{P}_k = 10$, $\overline{\gamma}_{p,j}^{(\text{C-DSL})} = 1$ and $\overline{\gamma}_{p,j}^{(\text{D-DSL})} = 7$.

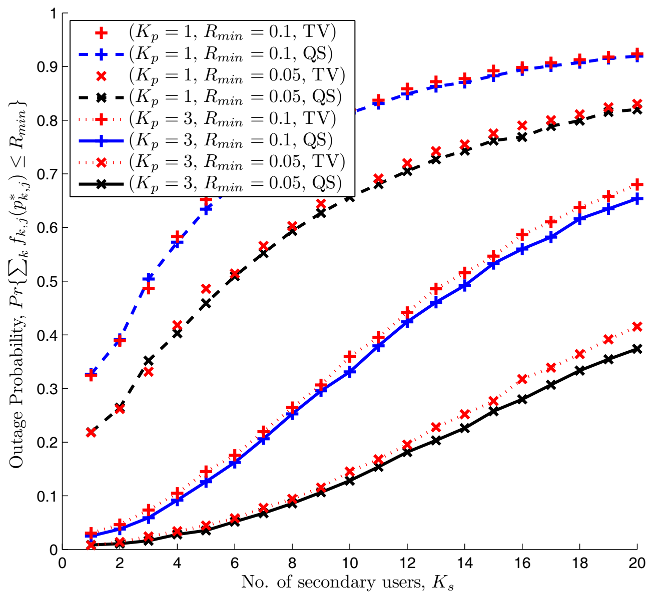

Fig. 4. The outage probability at Nash equilibrium as a function of secondary system size K_s, in a Rayleigh distributed channel fading, where $\overline{Q}_j = 20$ and $\overline{P}_k = 10$.

or may not achieve the rate found in Fig. 3. We define the outage probability as $Pr\{\sum_j f_{k,j}(p^*_{k,j}) \leq R_{min}\}$, which is the probability that a particular secondary user does not achieve the minimum required QoS. Figure 4 shows the outage probability of the secondary system. As one would expect the outage probability of the secondary users increases as the minimum QoS requirement R_{min} increases or as the number of primary channels decreases. Thus the average rate achieved by the secondary users shown in Fig. 3 should be interpreted in conjunction with the outage probability shown in Fig. 4. For instance for quasi-static channel, according to Fig. 3 and Fig. 4, with an average minimum rate of $R_{min} = 0.05$ the single channel D-DSL system can support up to 3 secondary users with an outage probability of 0.35 compared to the 3-channel D-DSL system which can handle up to 9 secondary users with a lower outage probability ($P_{out} = 0.1$).

B. A Fair Comparison Between C-DSL and D-DSL

In C-DSL, all K_p primary users transmit on the same channel. Each primary link measures the total secondary interference I_j at its receiver and reports to a central unit. The central unit determines the worst secondary interference $I_0 = \max_{j \in \mathcal{K}_p} I_j$, chooses the corresponding best interference cap Q_0^* and broadcasts both values to the secondary system.

To compare the performance of a D-DSL system against that of C-DSL, we allocate the same bandwidth for both systems. We assume that the bandwidth is small enough so that the channel coefficients are frequency non-selective, but only vary due to spatial distribution of the users. In Fig. 6 and Fig. 5, the 3-user ($K_p = 3$) C-DSL based primary system allocates a bandwidth of $W_k = 3W = 3$ units for all users. The noise variance at the receivers is $\sigma^2_{k,j} = 3$, the maximum interference cap in C-DSL is set to $\overline{Q}_0 = 18$ and the primary target SINR's are $\overline{\gamma}_{p,j}^{(\text{C-DSL})} = 1$. On the other hand, in the D-DSL system, the j-th primary user solely occupies its own channel with a bandwidth $W_{k,j} = W = 1$ unit for $j \in \{1, 2, 3\}$. To maintain the same data rate for the primary users in both systems with the same primary transmit powers, in D-DSL, the corresponding maximum interference caps are $\overline{Q}_j = 2$, the primary target SINR's are $\overline{\gamma}_{p,j}^{(\text{D-DSL})} = 7$ and the noise variance at the receivers is $\sigma^2_{k,j} = 1$. The maximum secondary transmit powers are assumed to be $\overline{P}_k = 10$.

Figure 5 shows the sum-rate and the per-user rate of the secondary users in the two systems. As can be seen, the secondary system sum-rate performance in C-DSL is slightly better compared to the D-DSL especially for large K_s, although the difference in per-user rates is almost negligible. However, the D-DSL outperforms the C-DSL in terms of outage probability as seen in Fig. 6 especially for relatively small K_s values. This we believe is due to the spatial/frequency diversity achieved in D-DSL. Note that, for large K_s the outage probability of the

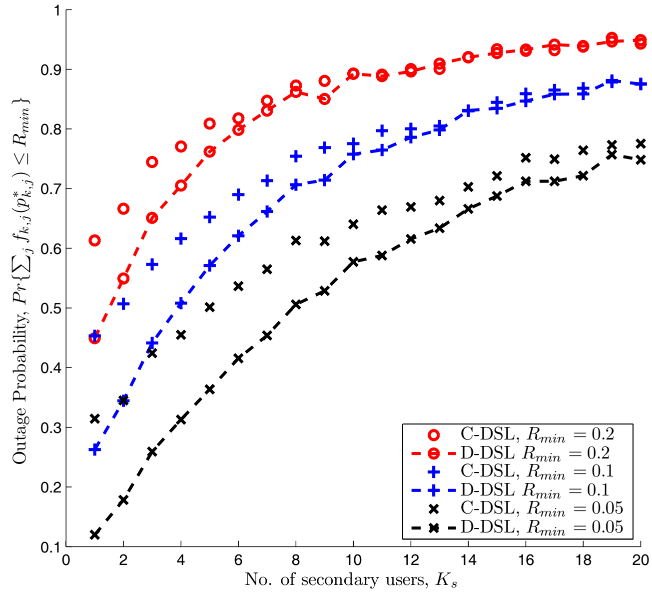

Fig. 6. The outage probability of the secondary system at Nash equilibrium of D-DSL system compared to the C-DSL system, where $K_p = 3$, $\overline{Q}_0^{(\text{C-DSL})} = 18$, $\overline{Q}_j^{(\text{D-DSL})} = 2$, $\overline{P}_k = 10$, $\overline{\gamma}_{p,j}^{(\text{C-DSL})} = 1$ and $\overline{\gamma}_{p,j}^{(\text{D-DSL})} = 7$.

D-DSL matches that of the C-DSL system. However, as seen by Fig. 5 and Fig. 6, in this region the outage probabilities are too high for a realistic system to be operated.

VI. Conclusion

In this paper, we proposed a new concept of distributed dynamic spectrum leasing for DSS in a primary spectrum divided into multiple frequency channels and developed a game-theoretic framework to model dynamic spectrum sharing based on such a D-DSL architecture. In the proposed D-DSL networks, the secondary users prefer a primary system with a large number of distinct frequency channels. We showed that due to the multiple degrees of freedom available in a D-DSL system, the secondary system can achieve a better outage probability compared to those in a C-DSL network, possibly at the expense of additional implementation complexity. We also investigated, through simulations, the robustness of the proposed D-DSL game to time-varying fading, and showed that this causes a slight deviation from the actual Nash equilibrium due to delayed CSI.

Acknowledgment

This research was supported in part by the Space Vehicles Directorate of the Air Force Research Laboratory (AFRL), Kirtland AFB, Albuquerque, NM and the National Science foundation (NSF) under the grant CCF-0830545.

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