Iterative Low-Complexity Multiuser Detection and Decoding for Coded UWB Systems

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***Abstract*— In general, ultra wideband (UWB) signals are transmitted using very short pulses in time domain, thus promising very high data rates. In this paper, a receiver structure is proposed for decoding multiuser information data in a convolutionally-coded UWB system. The proposed iterative receiver has three stages: a pulse detector, a symbol detector, and a channel decoder. Each of these stages output soft values, which are used as *a priori* information in the next iteration. Simulation results show that the proposed system can provide performance very close to a single-user system.**

I. Introduction

The impulse-radio ultra wideband (UWB) systems transmit very short pulses with a duration of a few hundred picoseconds. Hence, these systems have a very large bandwidth. The United States Federal Communications Commission (FCC) defines a UWB system as any radio system with a fractional bandwidth greater than 20 percent of the center frequency or a -10dB bandwidth greater than 500MHz. Time-hopping impulse radios (TH-IRs) have drawn considerable attention among both researchers and practitioners over the past few years. An additional time shift at the start of each pulse, which is known to the receiver, helps to avoid the catastrophic collisions between two users on the same channel.

Most of the previous work on multiuser detection focused on uncoded UWB systems. In this paper, we study an iterative receiver structure for decoding multiuser information data in convolutionally-coded systems. This work is based on the iterative technique proposed in [1] for code division multiple access (CDMA) systems. It is already noted in [2] that almost all multiuser detectors designed for direct-sequence CDMA (DS-CDMA) systems can be used in a TH-IR system with or without slight modifications. In this paper, the soft-input soft-output (SISO) multiuser detector used in [1] is replaced by the low-complexity, turbo detector discussed in [3].

The remainder of the paper is organized as follows: In Section II, we describe both continuous-time and discrete-time models of the transmitted and received signals in TH-IR systems. In Section III, the iterative multiuser receiver structure is discussed. In Section IV, we provide simulation results of the proposed receiver structure. Section V presents conclusions.

II. Signal Model and the System Description

The transmitted signal of the k^{th} user in a TH-IR system is described by the following general model [3]:

$$s_{tr}^{k}(t) = \sum_{j=-\infty}^{\infty} b_k^{\lfloor j \backslash N_f \rfloor} w_{tr}(t - jT_f - c_j^k T_c), \tag{1}$$

where $w_{tr}(t)$ is the transmitted UWB pulse; $\{b_k^j\}$ is the binary sequence of information symbols transmitted by the k^{th} user, where $b_k^j \, \epsilon \, \{+1, -1\}$; $\{c_j^k\}$ is a pseudorandom time-hopping sequence of the k^{th} user taking values in $\{0, 1, \ldots, Nc-1\}$; and N_c is the number of chips in which a pulse can take its position. These $\{c_j^k\}$ provide an additional shift of $c_j^k T_c$ seconds to the j^{th} pulse of the k^{th} user, T_f is the nominal pulse repetition time, N_f is number of pulses used to transmit one information symbol, and $\lfloor x \rfloor$ denotes the closest integer less than or equal to x. The received signal at the antenna output is

$$r(t) = \sum_{k=1}^{K} A_k \sum_{j=-\infty}^{\infty} b_k^{\lfloor j \backslash N_f \rfloor} w_{rx}(t - jT_f - c_j^k T_c) + n(t), \tag{2}$$

where $w_{rx}(t)$ is the received pulse, $n(t)$ is the additive noise, and A_k is the received amplitude of the signal of the k^{th} user. The received signal (2) is passed through a linear filter matched to $w_{rx}(t)$, and the output of this filter is sampled every T_c seconds [3]. A sufficient statistic for detecting the i^{th} information symbols of all users is given by $\mathbf{r}[i]$, which is obtained by stacking up all the samples corresponding to the i^{th} frame:

$$\mathbf{r}[i] = \mathbf{S}[i]\mathbf{A}\mathbf{b}[i] + \mathbf{n}[i] \tag{3}$$

where $\mathbf{S}$ is a $N_c N_f \times K$ matrix whose non-zero elements in the k^{th} column are placed at indices representing the time instances where pulses from the k^{th} user are received [3], $\mathbf{A}$ = diag$(A_1, ..., A_K)$ is a $K \times K$ diagonal matrix with the gains

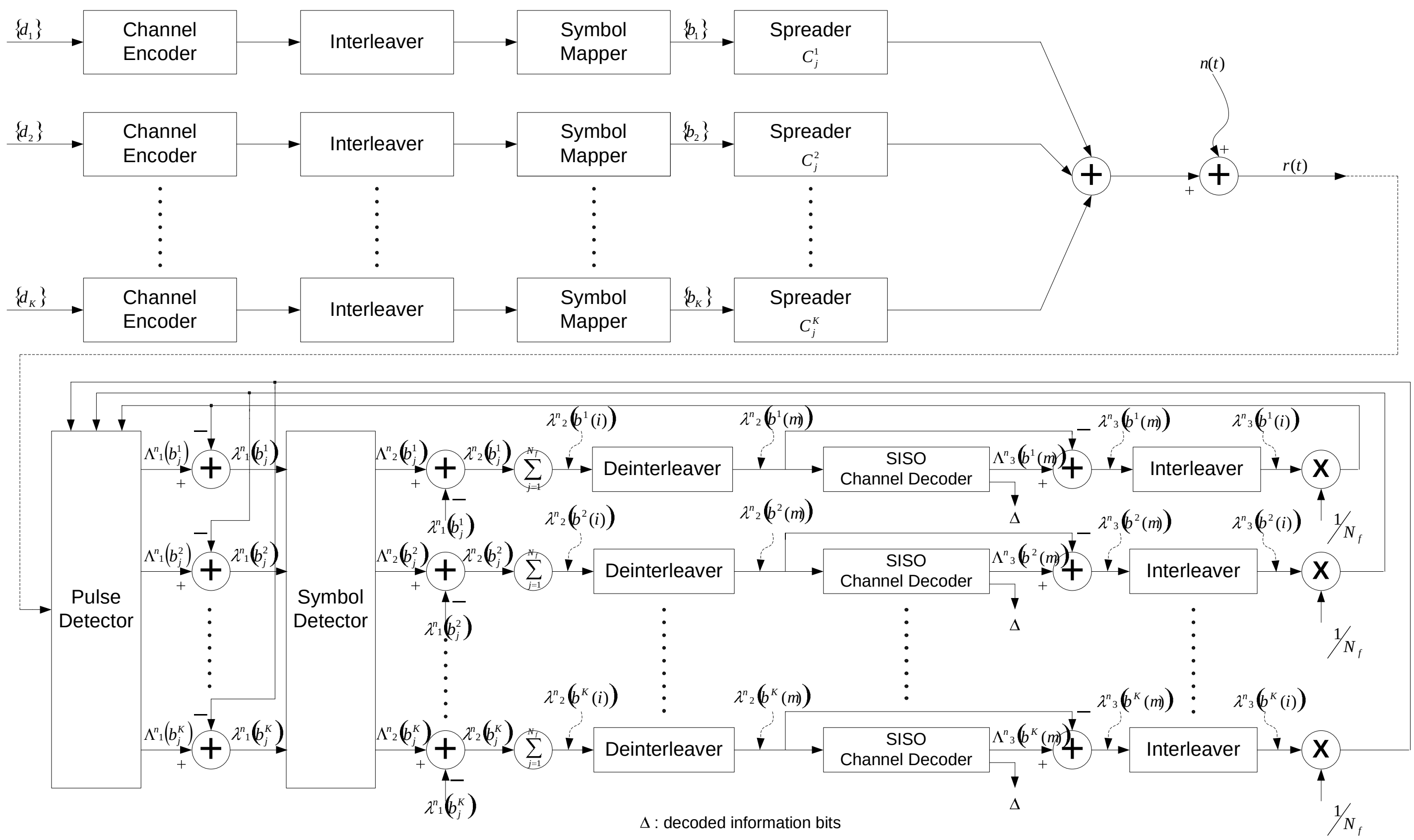

Fig. 1. A coded UWB system with iterative multiuser receiver

between the transmitter and the receiver on its diagonal, and $\mathbf{b}[i]$ is a K-vector whose k^{th} element is the i^{th} information symbol transmitted by the k^{th} user. The receiver noise $\mathbf{n}[i]$ is a zero mean Gaussian random vector with correlation matrix $\sigma_n^2\mathbf{I}$, where $\sigma_n^2 = N_o$. If $l(j,k)$ is the time index at which the j^{th} pulse from the k^{th} user was transmitted, and then from (3), $r_{l(j,k)}$ obeys the following model [3]:

$$r_{l(j,k)} \quad = \quad \mathbf{1}_{K_k^j+1}\mathbf{A}_k^j\mathbf{b}_j^k + n_{l(j,k)} \tag{4}$$

where the information symbols colliding with the j^{th} pulse of the k^{th} user is denoted by the vector $\mathbf{b}_j^k = \left[b_j^k, b_j^{f_j^k(1)}, ..., b_j^{f_j^k(K_k^j)}\right]$, the f_j^k's are the indices of the users colliding with the j^{th} pulse of the k^{th} user, K_k^j is the number of users colliding with the j^{th} pulse of the k^{th} user, $\mathbf{1}_{K_k^j+1}$ is the $1 \times (K_k^j+1)$ vector of all ones, the diagonal matrix $\mathbf{A}_k^j$ = diag $\left[A_k, A_{f_j^k(1)}, ..., A_{f_j^k(K_k^j)}\right]$ contains the amplitudes of the pulses received from the k^{th} user and the users colliding with the j^{th} pulse of the k^{th} user, and $n_{l(j,k)}$ is the additive white Gaussian noise with zero mean and $\sigma_n^2 = N_0$.

III. Iterative Multiuser Receiver

The proposed iterative multiuser receiver has three stages: a pulse detector, a symbol detector, and a channel decoder. This receiver structure works on the same principle as that of other turbo algorithms [4], [5], except that it has three stages that exchange the soft values compared to others that usually have only two stages. The first two stages cancel the interference among the interfering users, and the third stage helps in error correction of received bits resulting from transmission noise. The soft input soft output (SISO) channel decoder at the third stage has a code bit interleaver, and a deinterleaver to reduce the influence of error busts. In Fig. (1), interleaved code bits and a deinterleaved code bits are denoted by $b^k(i)$ and $b^k(m)$, respectively. The pulse detector at the first stage takes the output of the linear-matched filter front-end as its input. It will be shown that at the n^{th} iteration, the pulse detector computes the *a posteriori* log-likelihood ratio (LLR) of the pulse b_j^k, denoted as $\Lambda_1^n(b_j^k)$, given the received signal, the other users information about the transmitted bits, and the *a priori* LLR b_j^k obtained from the channel decoder:

$$\Lambda_1^n(b_j^k) \quad = \quad \lambda_1^n(b_j^k) + \lambda_3^{n-1}(b_j^k), \tag{5}$$

where $\lambda_1^n(b_j^k)$ is the extrinsic information, and $\lambda_3^{n-1}(b_j^k)$ represents the *a priori* LLR of b_j^k, which is computed by the channel detector at the $(n-1)^{th}$ iteration. For the first iteration, we set $\lambda_3^0(b_j^k) = 0$ for $k = 1, 2, ..., K$ and $j = 1, 2, ..., N_f$.

The symbol detector computes the *a posteriori* LLR of b_j^k, denoted by $\Lambda_2^n(b_j^k)$, given the soft information $\lambda_1^n(b_j^k)$ provided from the pulse detector:

$$\Lambda_2^n(b_j^k) \quad = \quad \lambda_2^n(b_j^k) + \lambda_1^n(b_j^k), \tag{6}$$

where $\lambda_2^n(b_j^k)$ is the extrinsic information that is fed to the channel decoder and $\lambda_1^n(b_j^k)$ is the intrinsic information received from the pulse detector at the first stage.

Similarly, it will be shown that at the n^{th} iteration, the channel decoder computes the *a posteriori* LLR of b_j^k, denoted by $\Lambda_3^n(b_j^k)$, given the soft information $\lambda_2^n(b_j^k)$ from the symbol detector.

$$\Lambda_3^n(b_j^k) \quad = \quad \lambda_3^n(b_j^k) + \lambda_2^n(b_j^k) \tag{7}$$

where $\lambda_3^n(b_j^k)$ is the extrinsic information and $\lambda_2^n(b_j^k)$ is the intrinsic information delivered from the symbol detector.

A. Pulse Detector

The first stage of the proposed iterative turbo multiuser receiver is a pulse detector similar to that in [3]. The information symbols corresponding to each pulse interval of the k^{th} user b_j^k's are assumed to be independent. Hence, *a priori* information $b_1^k = ... = b_{N_f}^k$ is ignored in the pulse detector. At the n^{th} iteration, the pulse detector computes the *a posteriori* LLR of b_j^k, given the received signal, the information about the transmitted bits from other users, and the *a priori* information about b_j^k obtained from the channel decoder as

$$\begin{aligned}
\Lambda_1^n(b_j^k) &= \log \frac{P(b_j^k = 1|\mathbf{r})}{P(b_j^k = -1|\mathbf{r})} \\
&= \log \frac{P(\mathbf{r}|b_j^k = 1)}{P(\mathbf{r}|b_j^k = -1)} + \log \frac{P(b_j^k = 1)}{P(b_j^k = -1)} \\
&= \underbrace{\log \frac{p(r_{l(j,k)}|b_j^k = 1)}{p(r_{l(j,k)}|b_j^k = -1)}}_{\lambda_1^n(b_j^k)} + \log \frac{P(b_j^k = 1)}{P(b_j^k = -1)} \qquad (8) \\
&= \lambda_1^n(b_j^k) + \lambda_3^{n-1}(b_j^k),
\end{aligned}$$

where $r_{l(j,k)}$ is defined in (4), and it is assumed that each pulse is modulated with an independent symbol; the *a priori* LLR, $\log \frac{p(r_{l(j,k)}|b_j^k=1)}{p(r_{l(j,k)}|b_j^k=-1)}$, is defined in (9) at the bottom of this page and can be derived from (4). In (9), $[\mathbf{b}]_g$ denotes the g^{th} element of $\mathbf{b}$.

B. Symbol Detector

The symbol detector is essentially the same as that discussed in [3]. The symbol detector of [3] exploits the fact that $b_1^k = ... = b_{N_f}^k$. It computes the *a posteriori* LLR of b_j^k, given the information from the pulse detector as

$$\begin{aligned}
\Lambda_2^n(b_j^k) &= \log \frac{P(b_j^k = 1|\lambda_1^n(b_j^k), j = 1, ..., N_f)}{P(b_j^k = 1|\lambda_1^n(b_j^k), j = 1, ..., N_f)} \qquad (10) \\
&= \log \frac{P(b_1^k = ...b_{N_f}^k = 1|\lambda_1^n(b_j^k), j = 1, ..., N_f)}{P(b_1^k = ...b_{N_f}^k = -1|\lambda_1^n(b_j^k), j = 1, ..., N_f)} \\
&= \sum_{l=1}^{N_f} \log \frac{P(b_l^k = 1|\lambda_1^n(b_l^k))}{P(b_l^k = -1|\lambda_1^n(b_l^k))} \\
&= \underbrace{\sum_{l=1, l\neq j}^{N_f} \log \frac{P(b_l^k = 1|\lambda_1^n(b_l^k))}{Pr(b_l^k = -1|\lambda_1^n(b_l^k))}}_{\lambda_2^n(b_j^k)} + \lambda_1^n(b_j^k) \\
&= \underbrace{\sum_{l=1, l\neq j}^{N_f} \lambda_1^n(b_l^k)}_{\lambda_2^n(b_j^k)} + \lambda_1^n(b_j^k). \qquad (11)
\end{aligned}$$

The output of the symbol detector $\Lambda_2^n(b_j^k)$ contains the extrinsic information $\lambda_2^n(b_j^k)$ used by the channel decoder as the *a priori* information (after deinterleaving) and the intrinsic information $\lambda_1^n(b_j^k)$ from all pulses from the same user k bearing the same information symbol.

C. SISO Channel Decoder

The SISO channel decoder stage has a bank of K single-user decoders, as shown in Fig. (1). A code bit interleaver and a deinterleaver are used to reduce the influence of error busts at the output and input of each channel decoder. The input to each of these decoders is the deinterleaved *a priori* output from the symbol detector. The channel decoder computes the LLR of the coded bits and the information bits. The convolutional encoder with a binary rate of $\frac{k_o}{n_o}$ at the transmitter end encodes a block of k_o information bits and outputs n_o coded bits. At time t, if the input to the encoder is $\mathbf{d}_t = [d_t^1, ..., d_t^{k_o}]$, then the output is $\mathbf{b}_t = [b_t^1, ..., b_t^{n_o}]$. Denote $\mathbf{d}(s', s)$ as the input

$$\log \frac{p(r_{l(j,k)}|b_j^k = 1)}{p(r_{l(j,k)}|b_j^k = -1)} = \log \frac{\sum_{\mathbf{b}\epsilon\{\pm1\}^{K_k^j}} e^{\frac{(r_{l(j,k)} - \mathbf{1}A_k^j[1 \ \ \mathbf{b}]^T)^2}{2\sigma_n^2}} \prod_{g=1}^{K_k^j} 1 + [\mathbf{b}]_g \tanh\left(\frac{1}{2}\lambda_3^{n-1}\left(b_j^{f_j^k(g)}\right)\right)}{\sum_{\mathbf{b}\epsilon\{\pm1\}^{K_k^j}} e^{\frac{(r_{l(j,k)} - \mathbf{1}A_k^j[-1 \ \ \mathbf{b}]^T)^2}{2\sigma_n^2}} \prod_{g=1}^{K_k^j} 1 + [\mathbf{b}]_g \tanh\left(\frac{1}{2}\lambda_3^{n-1}\left(b_j^{f_j^k(g)}\right)\right)} \tag{9}$$

bits which results in the transition of the encoder trellis state $S_{t-1} = s'$ to $S_t = s$ and outputs the n_o coded bits $\mathbf{b}(s', s)$. Let τ be the code block length (after padding zeros). The output of the channel encoder at time t is denoted by $\mathbf{b}_t$. We use the notation [1]

$$P[\mathbf{b}_t(s', s)] \triangleq P[\mathbf{b}_t = \mathbf{b}(s', s)] \quad (12)$$

to define the forward and backward recursions as in [1], [6]

$$\alpha_t(s) = \sum_{s'} \alpha_{t-1} P[\mathbf{b}_t(s', s)], \quad t = 1, 2, ..., \tau \quad (13)$$

$$\beta_t(s) = \sum_{s'} \beta_{t+1} P[\mathbf{b}_{t+1}(s', s)], \quad t = \tau - 1, \tau - 2, ..., 0 \quad (14)$$

with boundary conditions $\alpha_o(0) = 1, \ \alpha_o(s \neq 0) = 0$, and $\beta_\tau(0) = 1, \ \beta_\tau(s \neq 0) = 0$. To obtain a numerically stable algorithm, the parameters $\alpha_t(s)$ and $\beta_t(s)$ can be scaled as the computation proceeds as that explained in [1]. i.e. we compute $\tilde{\alpha}_t(s)$, which is the scaled version of $\alpha_t(s)$ as follows: we first set $\hat{\alpha}_1(s) = \alpha_1(s)$ and $\tilde{\alpha}_1(s) = c_t \hat{\alpha}_1(s)$ with $c_1 \triangleq \frac{1}{\sum_s \hat{\alpha}_1(s)}$. Then, $\tilde{\alpha}_t(s)$ is computed using

$$\hat{\alpha}_t(s) = \sum_{s'} \tilde{\alpha}_{t-1}(s') P[\mathbf{b}_t(s, s')]. \quad (15)$$

Similarly,

$$\hat{\beta}_t(s) = \sum_{s'} \tilde{\beta}_{t+1}(s') P[\mathbf{b}_{t+1}(s, s')], \quad (16)$$

$$\tilde{\beta}_t(s) = c_t \hat{\beta}_t(s). \quad (17)$$

The SISO channel decoder of the k^{th} user outputs the *a posteriori* LLR of the code bit b_t^q, for $q = 1, 2, ..., n_o$ computed as (note that we have suppressed the user index k)

$$\begin{aligned}\Lambda_3[b_t^q] &\triangleq \log \frac{\sum_{S_q^+} \alpha_{t-1}(s')\beta_t(s) \prod_{i=1}^{n_o} P[b_t^i(s', s)]}{\sum_{S_q^-} \alpha_{t-1}(s')\beta_t(s) \prod_{i=1}^{n_o} P[b_t^i(s', s)]} \\ &= \log \frac{\sum_{S_q^+} \alpha_{t-1}(s')\beta_t(s) \prod_{i \neq q} P[b_t^i(s', s)]}{\sum_{S_q^-} \alpha_{t-1}(s')\beta_t(s) \prod_{i \neq q} P[b_t^i(s', s)]} \\ &+ \log \frac{P[b_t^q = +1]}{P[b_t^q = -1]}, \quad (18)\end{aligned}$$

where S_q^+ is defined as the set of state pairs (s', s) such that the q^{th} bit of the code symbol $\mathbf{b}(s', s)$ is $+1$ and similarly for S_q^-. Hence, we can write $\Lambda_3[b_t^q]$ as

$$\Lambda_3[b_t^q] = \lambda_3[b_t^q] + \lambda_2^n[b_t^q]$$

where $\lambda_3[b_t^q]$ is the extrinsic information of the SISO channel decoder, and $\lambda_2^n[b_t^q]$ is the *a priori* information provided by the symbol detector.

We need the LLR of information bits as the final output of the channel decoder in the last iteration. This can be obtained by modifying (18): instead of summing over S_q^+ and S_q^-, the summation is performed over U_q^+, the set of state pairs (s', s), such that the q^{th} bit of the information symbol $\mathbf{d}(s', s)$ is $+1$ and similarly for U_q^-. Then, we have

$$\begin{aligned}\Lambda_3[d_t^q] &\triangleq \log \frac{\sum_{U_q^+} \alpha_{t-1}(s')\beta_t(s) \prod_{i=1}^{n_o} P[b_t^i(s', s)]}{\sum_{U_q^-} \alpha_{t-1}(s')\beta_t(s) \prod_{i=1}^{n_o} P[b_t^i(s', s)]} \\ &= \log \frac{\sum_{U_q^+} \alpha_{t-1}(s')\beta_t(s) \prod_{i \neq q} P[b_t^i(s', s)]}{\sum_{U_q^-} \alpha_{t-1}(s')\beta_t(s) \prod_{i \neq q} P[b_t^i(s', s)]} \\ &+ \log \frac{P[b_t^q = +1]}{P[b_t^q = -1]}. \quad (19)\end{aligned}$$

Note that the input to the pulse detector is the soft values of the pulses not the symbols. However, the channel decoder outputs the LLR of coded bits $\lambda_3[b_t^q]$. Since it is assumed that the pulses are independent in the pulse detector, we divide the channel decoder output by N_f and feed them back to the pulse detector as the *a priori* information of the N_f pulses $\lambda_3[b_j^k]$ (after interleaving). The iteration then continues through these three stages.

IV. Simulations

In this section we present the simulation results of the proposed iterative multiuser detector. We evaluate the average bit error rate of the proposed iterative receiver as a function of signal-to-noise ratio (SNR) per pulse, and the number of users in the system. The UWB system considered has $N_f = 10$, $N_c = \{10, 20\}$ and the UWB pulses are generated as discussed in [7]. All user information bits are encoded with a rate - $\frac{1}{2}$ convolutional encoder having constraint length of $\nu = 5$ and the generator matrix [23, 35] in octal notation. The information block size is 128 bits. We use the same random interleaver for each user. We also assume that all users have equal received power.

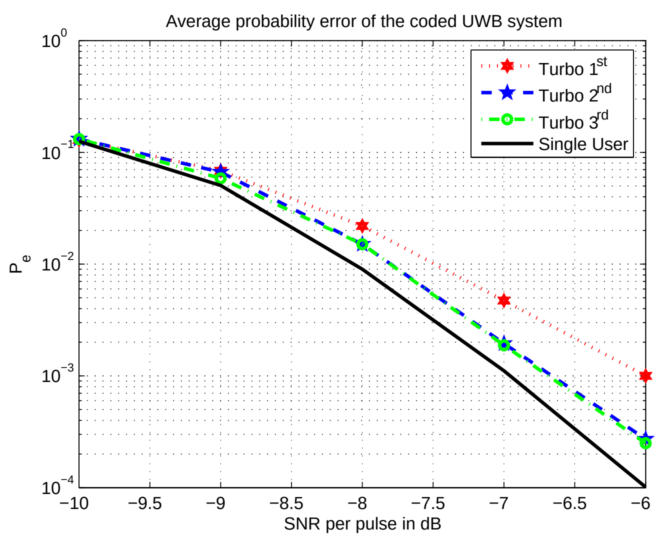

Fig. 2. The average probability of the turbo UWB receiver with $N_c = 20, N_f = 10$, and $K = 20$.

Figures (2) and (3) show the average bit error rates of the proposed iterative multiuser receiver as a function of SNR per pulse with $K = 20$ and $K = 10$ users, respectively. In Fig. (2), the number of users in the system is $K = 20$ with $N_c = 20$ and $N_f = 10$. Performance has increased greatly after only two iterations, but there is still a performance degradation compared to a single-user system due to residual multiple-access interference.

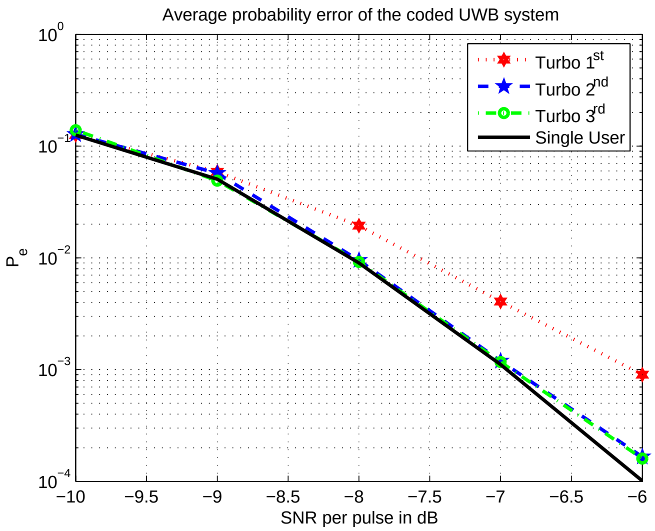

Fig. 3. The average probability of the turbo UWB receiver with $N_c = 20, N_f = 10$, and $K = 10$.

In Fig. (3), the number of users in the system is $K = 10$ with $N_c = 20$ and $N_f = 10$. It can be seen that the proposed iterative receiver system performs close to the single-user system after only two iterations. The performance in Fig. (3) is comparatively better than the performance in Fig. (2) as the number of users considered in Fig. (3) is less than that in Fig. (2).

In Fig. (4), we demonstrate the average bit error rates of the iterative multiuser receiver as a function of SNR per pulse with $N_f = 10$, $N_c = 10$, for $K = 10$ users. Since N_c, the number of chips per pulse, is less than those considered in Fig. (3), there is more interference in this system. As a result, in the first iteration, the performance of this system has considerably degraded compared to the system considered in Fig. (3). However, the performance of the proposed receiver quickly improves with more iterations, although still the final performance is not as good as that seen in Fig. (3).

V. Conclusions

In this paper, we have proposed an iterative receiver structure for decoding multiuser information data in a convolutionally-coded UWB system. At each iteration, extrinsic information is extracted from a pulse detector, symbol

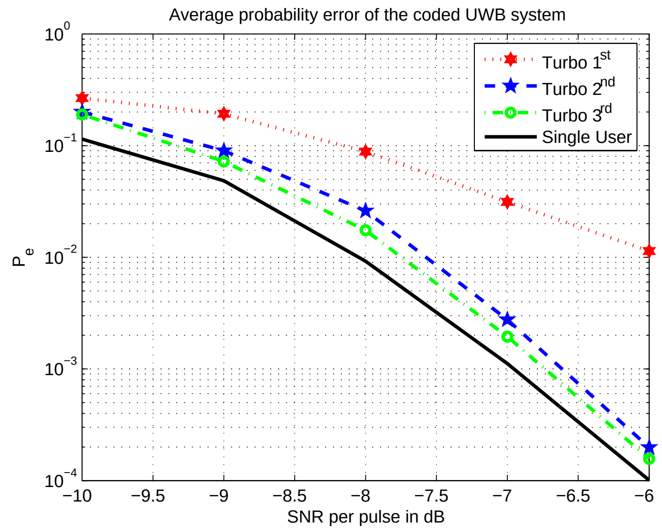

Fig. 4. The average probability of the turbo UWB receiver with $N_c = 10, N_f = 10$, and $K = 10$.

detector, and a bank of SISO channel decoders and then is passed as *a priori* information to the next stage. The low-complexity multiuser detector comprised of the pulse and symbol detector cancels interference among different users. The performance of the proposed receiver is demonstrated via simulation results. It is seen that the proposed iterative receiver offers a performance that is close to that of the single-user bound at high SNR values.

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