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An Asymptotically Full Rate Cooperative Communication Scheme for DS-CDMA Systems with Non-orthogonal Codes

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Abstract

It is observed that previously proposed cooperative communication schemes do not increase throughput in the high Signal-to-Noise Ratio (SNR) region although they improve the outage probability. In this paper we introduce a new cooperative communications protocol for direct-sequence code-division-multiple-access (DS-CDMA) systems that increases the SNR range over which there is no significant throughput loss compared to direct transmission. This eliminates the necessity for switching between more than one scheme which leads to higher system complexity. The proposed new scheme has a high spectral efficiency (reaching one asymptotically with the SNR and block size) at the cost of extra spreading sequences. However the channels are allowed to be non-orthogonal, and we analyze the effect of that on cooperative diversity under the LMMSE receiver. We present both achievable and outage throughput analysis under different cross correlations between user codes. From our results we show that the proposed scheme improves the throughput and the outage throughput performance in high SNR at the expense of only a negligible loss in the low SNR region compared to the previously proposed reference scheme.

1. Introduction

Demand for better wireless quality of service (QoS) and higher efficiencies are further fueled due to proliferation of new systems such as Bluetooth, WiFi and wireless sensor networks. One of the main issues in wireless communications is fading, and this can be overcome by extracting diversity gain in time, frequency and space. Cooperative communication has emerged as a promising technique in extracting the spatial diversity. The enormous potential of space diversity through multiple antennas were pointed out in [1] and [2]. The concept of cooperative diversity was introduced in [3], [4] and [5] as an approach to achieve virtual MIMO communication [6, 7] via relay channels. It has been shown that due to the loss in multiplexing gain cooperative diversity is not effective in achieving high throughput in the high SNR regime [3], [4], [5]. The fundamental tradeoff between diversity and multiplexing for MIMO systems and for cooperative channels was discussed in [8] and [9], respectively. Cooperative diversity under decode-and-forward (DF) relaying in the low SNR regime was analyzed in [3] and [4]. In [5] the cooperative diversity was analyzed in both high and low SNR regimes. In both cases, however, frequency flat slow fading channels were considered. The schemes discussed in [5] exploit the full diversity available. But, they are not optimal in achieving the diversity-multiplexing tradeoff (DMT). In [10] efficient protocols which achieves the optimal DMT, especially for a smaller number of relays, for low multiplexing gains were proposed. In [11] a protocol called bursty-amplify-and-forward (BAF) that achieves the outage capacity under low SNR and low probability of outage was presented.

In this paper we consider non-orthogonal communication systems with binary modulation and propose a new cooperative communication scheme that achieves the single-antenna multiplexing gain asymptotically with block size L. We consider two performance criteria, namely the achievable and outage throughputs, and compare them with that of no cooperation (denoted by NC), and the scheme proposed in [4] with full cooperation (denoted by CS1). We analyze and compare the performance of these schemes with the linear minimum-mean-squared-error (LMMSE) receivers. Results show that our proposed scheme improves the performance for high SNR values at the expense of only a negligible loss in the low SNR region compared to that of CS1.

The rest of this paper is organized as follows: Section 2 presents the transmission model and derives the throughput of all three schemes under the linear minimum-mean-squared-error (LMMSE) receiver. In section 3 achievable and outage throughput results are presented. Section 4 concludes the discussion by summarizing our results.

2. System model

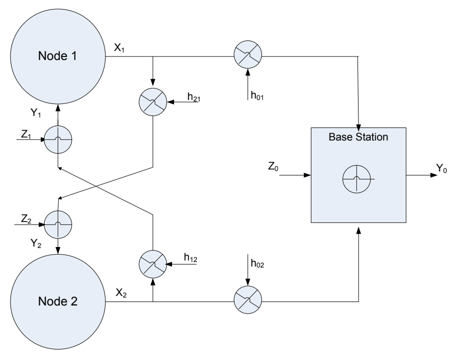

Figure 1: System model.

We consider a DS-CDMA wireless communication system with a single base station (BS) and two full-duplex cooperative sources. The signalling structure is as shown in Fig.1. We assume BPSK transmission under a slow Rayleigh fading flat channel and analyze the system under the symmetric situation in which both users need the same throughput. The received baseband signals at the BS, and node j, for $j = 1, 2$, (two users) can be written as, $Y_0(t) = h_{0j}(t)X_j(t) + h_{0j'}(t)X_{j'}(t) + Z_0(t)$, $Y_j(t) = h_{jj'}(t)X_{j'}(t) + Z_j(t)$, where $Y_0(t)$, $Y_1(t)$ and $Y_2(t)$ are the received signals at the BS, node 1 and node 2, respectively, $X_j(t)$ for $j = 1, 2$ is the transmitted signal of user j that is subjected to the same average transmit power constraint P, and $Z_0(t)$ and $Z_j(t)$ are the additive white Gaussian channel noise at the BS and node j, respectively. It is assumed that $Z_j(t)$, for $j = 0, 1, 2$, is zero mean complex white Gaussian with variance σ_j^2. The fading coefficient $h_{jj'}$ from node j' to node j is zero-mean complex Gaussian with variance $\sigma_{jj'}^2$ and is assumed to remain constant over a block of L symbol periods. We assume that the fading coefficients are known only at the respective receivers. The nominal SNR is defined as $SNR = \frac{P}{\sigma_0^2}$.

2.1. Direct transmission scheme (No cooperation)

Under no cooperation, in discrete time, the sequence of symbols transmitted by node j, for $j = 1, 2$, is simply given by $X_j = \{a_{j,1}b_j(1)c_{jj}, \quad a_{j,2}b_j(2)c_{jj}, \ldots\}$ where $b_j(i)$ is the i-th bit of user j, $c_{jj'}$ is the spreading code of user j for transmitting user j''s data, and $a_{j,i}$ is the amplitude of the i-th symbol of user j with $a_{j,i}^2 \leq P$ for $\forall i$. The cross-correlation between any two signalling codes is denoted by ρ and the spreading sequences are normalized to have unit energy. For the symbol period i, the discrete time received signals at the BS after matched filtering and despreading can be written in matrix form as (assuming synchronous transmissions)

$$\underline{y}_0^{(nc)}(i) = \mathbf{RA}\underline{b}(i) + \sigma_0 \underline{n}(i), \tag{1}$$

where

$$\underline{y}_0^{(nc)}(i) = \begin{pmatrix} y_{01}^{(nc)}(i) \\ y_{02}^{(nc)}(i) \end{pmatrix}, \quad \mathbf{R} = \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}, \quad \underline{b}(i) = \begin{pmatrix} b_1(i) \\ b_2(i) \end{pmatrix},$$
$$\mathbf{A} = \begin{pmatrix} h_{01}a_{1,i} & 0 \\ 0 & h_{02}a_{2,i} \end{pmatrix}, \quad \underline{n}(i) = \begin{pmatrix} n_1(i) \\ n_2(i) \end{pmatrix}, \tag{2}$$

with $Cov(\underline{n}(i)) = \mathbf{R}$ and $y_{0j}^{nc}(i)$ is the output at symbol time i of the matched-filter matched to c_{jj} at the base station, for $j = 1, 2$. We assume that the BS is equipped with a linear MMSE detector, so that the symbol decisions are given by $\underline{\hat{b}}_0^{(nc)}(i) = sgn(\mathbf{M}\underline{y}_0^{(nc)}(i))$ where [12],

$$\mathbf{M} = (\mathbf{R} + \sigma_0^2 \mathbf{A}^{-2})^{-1}. \tag{3}$$

The resultant probability of error at the BS for user j is

$$Pe_{0j}^{(nc)} = \frac{1}{2}Q\left(\frac{B_j + B_{j'}}{\nu_j}\right) + \frac{1}{2}Q\left(\frac{B_j - B_{j'}}{\nu_j}\right), \tag{4}$$

where

$$B_k = \mathbf{A}_{k,k}[\mathbf{MR}]_{jk}, \quad k\epsilon\{1, 2\}, \tag{5}$$
$$\text{and} \quad \nu_j^2 = \sigma_0^2[\mathbf{MRM}]_{jj}. \tag{6}$$

2.2. Cooperative transmission scheme 1 (CS1)

This refers to the cooperative communication scheme proposed in [4] with full cooperation. For completeness, here we briefly describe this scheme. Each node receives an attenuated and noisy version of its partner's transmitted signal and retransmits a combination of the estimated symbols of the other node and its own data. We denote by L_c the number of periods over which the combined bits are sent in one block of length L. Thus full cooperation implies $L_c = \frac{L}{2}$. For $L = 2$ and $L_c = 1$ the sequence of symbols transmitted by node j is $X_j = \left\{a_{j,i}b_j(i)c_{jj}, \quad a_{j,i+1}b_j(i)c_{jj} + \hat{a}_{j,i+1}\hat{b}_{j'}(i)c_{jj'}\right\}$ where $\hat{a}_{j,i}$ is the amplitude of the estimated bit transmissions and $\hat{b}_j(i)$ is the estimated bits by the partner j'. Note that, in this scheme $c_{jj'} = c_{j'j'}$ for $j, j'\epsilon\{1, 2\}$. The power constraint for node j is $[L_c(a_{j,i}^2 + a_{j,i+1}^2 + \hat{a}_{j,i+1}^2)] \leq LP$. Both the destination and the partner receive the symbols during the odd periods, $i = 2m - 1$, and only the destination receives the cooperatively sent symbols during the even periods, $i = 2m$, for $m = 1, 2, 3, ..$ where m is the symbol index. Thus, discrete time transmitted and received signals during odd periods are given by $X_j(2m-1) = a_{j,2m-1} b_j(m) c_{jj}$, $Y_{j'}^{(cs1)}(2m-1) = h_{j'j} X_j(2m-1) + Z_{j'}(2m-1)$, and $Y_0^{(cs1)}(2m-1) = h_{0j} X_j(2m-1) + h_{0j'} X_{j'}(2m-1) + Z_0(2m-1)$. After chip matched filtering and despreading, the received signal $Y_0^{(cs1)}(2m-1)$ can be written in matrix form as $\underline{y}_0^{(cs1)}(2m-1) = \underline{y}_0^{nc}(2m-1)$ where $\underline{y}_0^{nc}(i)$ is given in (1). During even periods, the transmitted signal of user j is $X_j(2m) = a_{j,2m} b_j(m) c_{jj} + \hat{a}_{j,2m} \hat{b}_{j'}(m) c_{jj'}$, and the received signal at the BS is $Y_0^{(cs1)}(2m) = h_{0j} X_j(2m) + h_{0j'} X_{j'}(2m) + Z_0(2m)$. After chip matched filtering the received signal $Y_0^{(cs1)}(2m)$ can be written in matrix form as $\underline{y}_0^{(cs1)}(2m) = [y_{01}^{(cs1)}(2m) \quad y_{02}^{(cs1)}(2m)]^T = \mathbf{R}\mathbf{A}^{(cs1)}(\theta_1, \theta_2, m)\underline{b}(2m) + \sigma \underline{n}(2m)$ where

$$\mathbf{A}^{(cs1)}(\theta_1, \theta_2, m) = \begin{pmatrix} h_{01} a_{1,2m} + \theta_1 h_{02} \hat{a}_{2,2m} & 0 \\ 0 & \theta_2 h_{01} \hat{a}_{1,2m} + h_{02} a_{2,2m} \end{pmatrix},$$

and $\theta_1, \theta_2 \epsilon \{+1, -1\}$ are Bernoulli with parameters determined by the error probability of the detector used at each cooperating node to detect partner's symbols. Suppose that we are interested in decoding of user j's data, for $j \epsilon 1, 2$. During odd periods the partner j' makes a hard estimate of the received symbol, $\hat{b}_j^{(cs1)}(2m-1) = sgn(y_{j'}^{(cs1)}(2m-1))$ with an error probability of $Pe_{j'j}^{(cs1)} = Q\left(\frac{h_{j'j} a_{j,2m-1}}{\sigma_{j'}}\right)$ where $y_{j'}^{(cs1)}(2m-1)$ is the signal after matched filtering of $Y_{j'}^{(cs1)}(2m-1)$. During the odd periods the received signal at the base station after the matched filtering is $\underline{y}_0^{(cs1)}(2m-1)$. The output of the LMMSE filter is

$$\begin{aligned} z_j^{(cs1)}(2m-1) &= [\mathbf{M}\underline{y}_0^{(cs1)}(2m-1)]_j \\ &= B_j b_j(m) + B_{j'} b_{j'}(m) + \tilde{n}_j(2m-1), \end{aligned}$$

where $\tilde{n}_j(2m-1) \sim \mathcal{N}(0, \sigma_0^2[\mathbf{MRM}]_{jj})$ and $\mathbf{M}$ and B_j are given in (3) and (5)) respectively. During even periods the output of the matched filter bank is $\underline{y}_0^{(cs1)}(2m)$. The output of the LMMSE filter is then given by

$$z_j^{(cs1)}(2m) = [\mathbf{M}\underline{y}_0^{(cs1)}(2m)]_j = B_j^{'} b_j(m) + B_{j'}^{'} b_{j'}(m) + \tilde{n}_j(2m), \tag{7}$$

where $\mathbf{M}^{(cs1)} = ((\mathbf{R} + \sigma_0^2 \mathbf{A}^{(cs1)}(\theta, \theta', m))^{-2})^{-1}$, $\tilde{n}_j(2m) \sim \mathcal{N}(0, \sigma_0^2[\mathbf{M}^{(cs1)}\mathbf{R}\mathbf{M}^{(cs1)}]_{jj}$ and $B_k^{'}(\theta_1, \theta_2) = \mathbf{A}^{(cs1)}(\theta_1, \theta_2, m)_{k,k}[\mathbf{M}^{(cs1)}\mathbf{R}]_{jk}$, for $k \epsilon \{1, 2\}$. Next, $z_j^{(cs1)}(2m-1)$ and $z_j^{(cs1)}(2m)$ are linearly combined to obtain

$$\xi_j(m) = [B_j \quad \lambda B_j^{'}(\theta_1, \theta_2)][z_j^{(cs1)}(2m-1) \quad z_j^{(cs1)}(2m)]^T, \tag{8}$$

where $0 \leq \lambda \leq 1$ is a parameter. The final decisions at the BS are then given by, for $j = 1, 2$,

$$\begin{aligned} \hat{\hat{b}}_j(m) &= sgn(\xi_j(m)) \\ &= sgn(\hat{B}_j(\theta_1, \theta_2) b_j(m) + \tilde{B}_j(\theta_1, \theta_2) b_{j'}(m) + \tilde{n}_T) \end{aligned}$$

where $\hat{B}_j(\theta_1, \theta_2) = (B_j^2 + \lambda B_j^{'2}(\theta_1, \theta_2))$, $\tilde{B}_j(\theta_1, \theta_2) = (B_j B_{j'} + \lambda B_j^{'}(\theta_1, \theta_2) B_{j'}^{'}(\theta_1, \theta_2))$ and $\tilde{n}_T(\theta_1, \theta_2) = B_j \tilde{n}_1 + B_j^{'}(\theta_1, \theta_2)\tilde{n}_j^{'} \sim \mathcal{N}(0, \sigma_0^2(B_j^2[\mathbf{MRM}]_{11} + B_j^{'2}[\mathbf{M}^{(cs1)}\mathbf{R}\mathbf{M}^{(cs1)}]_{11})$. Let $\sigma_T^2(\theta_1, \theta_2) = \sigma_0^2(B_j^2[\mathbf{MRM}]_{11} + B_j^{'2}[\mathbf{M}^{(cs1)}\mathbf{R}\mathbf{M}^{(cs1)}]_{11}$. The probability of error for user j averaged over θ_1 and θ_2 becomes,

$$Pe_{0j}^{(cs1)} = (1 - Pe_{j'j}^{(cs1)})(1 - Pe_{jj'}^{(cs1)})Pe_{++,j} + (1 - Pe_{j'j}^{(cs1)})Pe_{jj'}^{(cs1)}Pe_{+-,j} + Pe_{j'j}^{(cs1)}(1 - Pe_{jj'}^{(cs1)})Pe_{-+,j} + Pe_{j'j}^{(cs1)}Pe_{jj'}^{(cs1)}Pe_{--,j},$$

where $Pe_{s_1 s_2, j}$, for $s_1, s_2 \epsilon \{+1, -1\}$ and $j \epsilon \{1, 2\}$ are given by

$$Pe_{s_1 s_2, j} = \frac{1}{2} Q\left(\frac{\hat{B}_j(s_1, s_2) + \tilde{B}_j(s_1, s_2)}{\sigma_T(s_1, s_2)}\right) + \frac{1}{2} Q\left(\frac{\hat{B}_j(s_1, s_2) - \tilde{B}_j(s_1, s_2)}{\sigma_T(s_1, s_2)}\right). \tag{9}$$

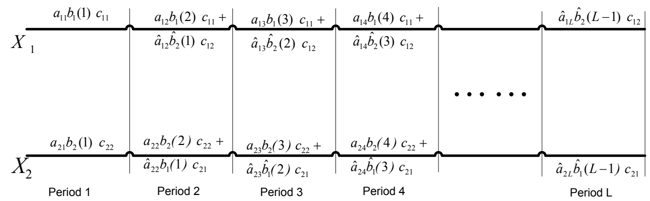

Figure 2: Proposed cooperative scheme (CS2)

2.3. Proposed cooperative communication scheme (CS2):

The proposed scheme uses two new spreading sequences for relaying of cooperative symbols. Consequently, we need four different codes, namely c_{11}, c_{22}, c_{12} and c_{21}, as opposed to only two in the CS1 scheme above. The difference in this scheme, compared to CS1, is that in every symbol period a new symbol is introduced. When $L_c = 1$ the sequence of symbols transmitted by node j is $X_j = \left\{ a_{j,1} b_j(1) c_{jj}, \quad a_{j,2} b_j(2) c_{jj} + \hat{a}_{j,2} \hat{b}_{j'}(1) c_{jj'}, \quad \hat{a}_{j,3} \hat{b}_{j'}(2) c_{jj'} \right\}$. The relationship between cooperative symbol periods and the total number of symbol periods in a block is given by $L = L_c + 2$. In general, the power constraint for the node j becomes $a_{j,1}^2 + L_c(a_{j,2}^2 + \hat{a}_{j,2}^2) + \hat{a}_{j,3}^2 \leq LP$. In an L-length block, the error probabilities for the symbols $i = 1$ and $i = L - 1$ (Note that there are $L - 1$ symbols in a block of L transmission periods) are the same and different from that of the rest. In the following we derive the error probability of symbol $b_j^{(i)}$, for $i \epsilon \{1, \ldots, L-1\} \setminus \{1, L-1\}$, and $j = 1, 2$. The received signal at node $j' \neq j$ after matched filtering is given in matrix form as $\underline{y}_{j'}^{(cs2)}(i) = \mathbf{R}\mathbf{A}_{j'}^{(cs2)}(i)\underline{\tilde{b}}^{(cs2)}(i) + \underline{n}(i)$ where

$$\underline{y}_{j'}^{(cs2)}(i) = \left(\begin{array}{c} y_{j',jj}^{(cs2)}(i) \\ y_{j',jj'}^{(cs2)}(i) \end{array} \right), \mathbf{A}_{j'}^{(cs2)}(i) = \left(\begin{array}{cc} h_{j'j}a_{j,i} & 0 \\ 0 & h_{j'j}\hat{a}_{j,i} \end{array} \right),$$

$$\underline{\tilde{b}}^{(cs2)}(i) = \left(\begin{array}{c} b_j(i) \\ \hat{b}_{j'}(i-1) \end{array} \right),$$

and $y_{j,j'k}^{(cs2)}(i)$ is the output of the matched filter at node j matched to $c_{j'k}$. After LMMSE filtering the detection of symbol i of user j is obtained as $\hat{b}_j(i) = sgn([\mathbf{M}_{j'}^{(cs2)}\underline{y}_{j'}^{(cs2)}(i)]_j)$ where $\mathbf{M}_{j'}^{(cs2)} = (\mathbf{R} + \sigma_{j'}^2(\mathbf{A}_{j'}^{(cs2)}(i))^{-2})^{-1}$. The resultant error probability is

$$Pe_{j'j}^{(cs2)} = \frac{1}{2}\left(Q\left(\frac{\tilde{B}_j + \tilde{B}_{j'}}{\nu_j} \right) + Q\left(\frac{\tilde{B}_j - \tilde{B}_{j'}}{\nu_j} \right) \right) \tag{10}$$

where $\tilde{B}_k = [\mathbf{A}_{j'}^{(cs2)}]_{kk}[\mathbf{M}_{j'}^{(cs2)}\mathbf{R}]_{jk}, k\epsilon\{j,j'\}$ and $\nu_j^2 = \sigma_{j'}^2[\mathbf{M}_{j'}^{(cs2)}\mathbf{R}\mathbf{M}_{j'}^{(cs2)}]_{jj}$.

The signal received at the BS during period i after matched filtering is given by $\underline{y}_0^{(cs2)}(i) = \mathbf{R}^{(cs2)}\mathbf{A}_0^{(cs2)}(i)\underline{b}^{(cs2)}(i) + \underline{n}(i)$ where $\mathbf{A}_0^{(cs2)}(i) = diag[h_{0j}a_{j,i}, h_{0j}\hat{a}_{j',i}, h_{0j'}a_{j',i}, h_{0j'}\hat{a}_{j',i}]$,

$$\underline{y}_0^{(cs2)}(i) = \left(\begin{array}{c} y_{0,jj}^{(cs2)}(i) \\ y_{0,jj'}^{(cs2)}(i) \\ y_{0,j'j'}^{(cs2)}(i) \\ y_{0,j'j}^{(cs2)}(i) \end{array} \right), \underline{b}^{(cs2)}(i) = \left(\begin{array}{c} b_j(i) \\ \hat{b}_{j'}(i-1) \\ b_{j'}(i) \\ \hat{b}_j(i-1) \end{array} \right),$$

$$\mathbf{R}^{(cs2)} = \left(\begin{array}{cccc} 1 & \rho & \rho & \rho \\ \rho & 1 & \rho & \rho \\ \rho & \rho & 1 & \rho \\ \rho & \rho & \rho & 1 \end{array} \right).$$

After LMMSE filtering the received signal vector at the BS, we get $\underline{z}_{0j}^{(cs2)}(i) = \mathbf{M}_0^{(cs2)}\underline{y}_0^{(cs2)}(i)$ where $\mathbf{M}_0^{(cs2)} = (\mathbf{R}^{(cs2)} + \sigma_0^2(\mathbf{A}_0^{(cs2)})^{-2})^{-1}$. To detect $b_j^{(i)}$, we linearly combine $[\underline{z}_{0j}^{(cs2)}(i)]_1$ and $[\underline{z}_{0j}^{(cs2)}(i+1)]_4$ as $\hat{\hat{b}}_j(i) = sgn(\underline{v}_\lambda^T[[\underline{z}_{0j}^{(cs2)}(i)]_1 \; [\underline{z}_{0j}^{(cs2)}(i+1)]_4]^T)$ where $\underline{v}_\lambda = [\tilde{B}_{1,1,1} \; \tilde{B}_{4,4,4}]^T$ and $\tilde{B}_{k,l,m} = [\mathbf{A}_0^{(cs2)}]_{kk}[\mathbf{M}_0^{(cs2)}\mathbf{R}^{(cs2)}]_{lm}$, for $k,m\epsilon\{1,2,3,4\}$ and $l\epsilon\{1,4\}$. Note that strictly speaking the residual interference plus noise component in $\underline{v}_\lambda^T[[\underline{z}_{0j}^{(cs2)}(i)]_1 \; [\underline{z}_{0j}^{(cs2)}(i+1)]_4]^T$ may not be Gaussian. However, in the following we use a version of the central limit theorem for sums of independent but non-identical random variables (Lyapunov's central limit theorem [13], [14]) to approximate it as being Gaussian (Proof is omitted here due to space). With the Gaussian assumption on residual interference the probability of error becomes,

$$\begin{aligned} Pe_{0j}^{(cs2)} &= (1 - Pe_{j'j}^{(cs2)})(1 - Pe_{jj'}^{(cs2)})Q\left(\frac{\tilde{B}_{1,1,1}^2 + \lambda\tilde{B}_{4,4,4}^2}{\sigma_+}\right) \\ &+ Pe_{j'j}^{(cs2)}(1 - Pe_{jj'}^{(cs2)})Q\left(\frac{\tilde{B}_{1,1,1}^2 - \lambda\tilde{B}_{4,4,4}^2}{\sigma_+}\right) + \\ &(1 - Pe_{j'j}^{(cs2)})Pe_{jj'}^{(cs2)}Q\left(\frac{\tilde{B}_{1,1,1}^2 + \lambda\tilde{B}_{4,4,4}^2}{\sigma_-}\right) + \\ &Pe_{j'j}^{(cs2)}Pe_{jj'}^{(cs2)}Q\left(\frac{\tilde{B}_{1,1,1}^2 - \lambda\tilde{B}_{4,4,4}^2}{\sigma_-}\right), \end{aligned}$$

where, for $s\epsilon\{+,-\}$, we have let

$$\begin{aligned} \sigma_s^2 &= \sigma_0^2([\mathbf{M}_0^{(cs2)}\mathbf{R}^{(cs2)}\mathbf{M}_0^{(cs2)}]_{11}\tilde{B}_{1,1,1}^2 + \lambda^2[\mathbf{M}_0^{(cs2)}\mathbf{R}^{(cs2)}\mathbf{M}_0^{(cs2)}]_{44}\tilde{B}_{4,4,4}^2) + \\ &(\tilde{B}_{1,1,1}\tilde{B}_{4,1,4})^2 + \lambda^2(\tilde{B}_{4,4,4}\tilde{B}_{1,4,1})^2 + \lambda^2(\tilde{B}_{4,4,4}\tilde{B}_{3,4,3})^2 + \\ &(\tilde{B}_{1,1,1}\tilde{B}_{3,1,3} + s\lambda\tilde{B}_{4,4,4}\tilde{B}_{2,4,2})^2. \end{aligned}$$

3. Performance analysis

3.1. Achievable average throughput

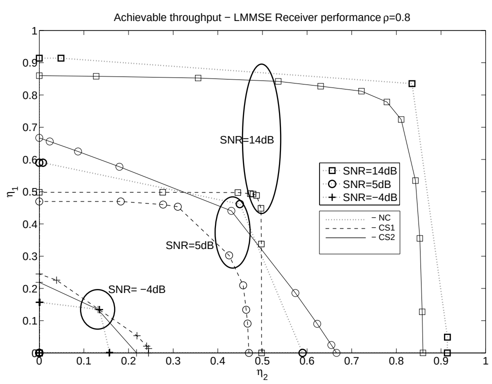

Figure 3: Throughput variation with SNR for $\rho = 0.8$ with the LMMSE receiver

Similar to [4], we define throughput η as the capacity of a binary symmetric channel (BSC), with bit error rate (BER) of the receiver as the transition probability, multiplied by an asymptotic spectral efficiency factor μ: $\eta = \mu(1 - H(p))$, where $H(p) = -p\log_2(p) - (1-p)\log_2(1-p)$ is the binary entropy function, p is the receiver BER and μ (in bits-per-channel-use) is defined as $\mu = r/W$, where r is the rate of transmission (in bits/symbol) and W is the average number of channel uses per symbol. It is easy to show that for the three schemes of interest, we have that $\mu_{nc} = 1, \mu_{cs1} = \frac{L_c}{L} = \frac{1}{2}$ and $\mu_{cs2} = \frac{L-1}{L}$. By substituting these we obtain the throughput of the three cooperative schemes discussed above. The achievable throughputs of the three schemes for $\rho = 0.8$ with the LMMSE multiuser detector is shown in Fig. 3. From Fig. 3 it is seen that in low SNR region, none of the schemes has a considerable gain over the others in the symmetric rate situation. As SNR increases we see that the performance of CS1 is severely limited due to the loss in multiplexing. However, the proposed scheme continues to perform as well as the direct transmission scheme. For asymptotically high SNR's the CS1 throughput reaches its upper bound of 0.5 while CS2 throughput reaches its upper bound of 0.875 (NC throughput is bounded by the maximum limit of 1). Consequently, we see that NC performs better than both CS2 and CS1 in terms of throughput for high SNR values. This, of course, is due to the dominance of smaller μ factors corresponding to cooperative schemes, as compared to that of direct transmission, in determining the throughput η in the high SNR region. Therefore, from Fig. 3 it is evident that the SNR range, below which cooperation is useful in terms of throughput, has been significantly increased with the proposed scheme CS2 even with non orthogonal signalling. Moreover, for non-symmetric rates, the CS1 scheme performs slightly better in low SNR. However, for medium SNR values CS2 outperforms others and even for large SNR's its performance is only slightly inferior to that of direct transmission.

3.2. Outage throughput

However, throughput itself is not a fair criterion for comparing the performance of communication systems as it does not take channel outage into consideration. Consequently, it is also of interest to investigate the reliability of these schemes. A suitable criterion for this is the outage throughput, η_ϵ, which we define via $\max_{\eta_\epsilon} Pr(\eta \leq \eta_\epsilon) \leq \epsilon$.

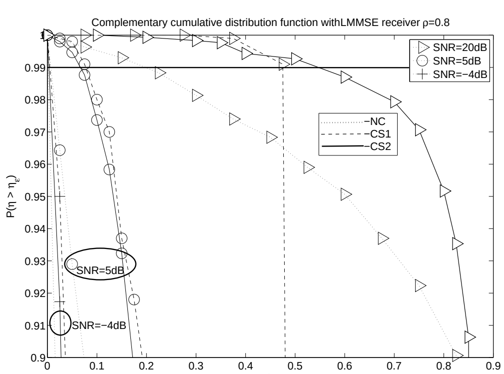

Figure 4: Throughput CCDF for $\rho = 0.8$ with the LMMSE receiver

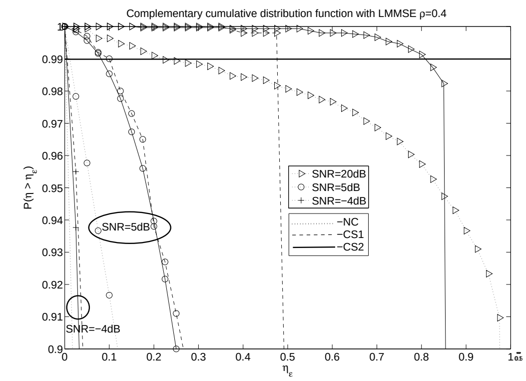

Figure 5: Throughput CCDF for $\rho = 0.4$ with the LMMSE receiver

Figure 4 depicts the throughput complementary-cumulative-distribution-function (CCDF) of the proposed (CS2) and CS1 cooperative communication schemes and direct transmission with the LMMSE receiver for $\rho = 0.8$. From Fig. 4 we observe that unless the required outage probability is very low CS2 outperforms CS1 in high SNR region and have almost the same performance in the low SNR region. For example, even when $P(\eta > \eta_\epsilon) = 0.99$, CS2 outperforms CS1 for high SNR and no significant loss is observed in the low SNR region. This shows that the proposed scheme can be used over a large range of SNR values compared to the direct transmission and the scheme CS1 (for small enough outages both schemes outperform the direct transmission). From Fig. 4 we notice that for very low required outage probabilities CS1 scheme performs better than CS2. However, the gain in CS1 in this region over CS2 is very small. On the other hand, as outage probability is increased there is a significant gain in CS2 over CS1 (for example, observe $P(\eta > \eta_\epsilon) = 0.9$).

As can be seen from Fig. 5 when ρ is reduced, the claims made for the case of $\rho = 0.8$ becomes more profound. From Fig. 5 we see that the outage throughput gain of CS1 scheme over CS2 scheme for very low outage probabilities becomes almost diminished for $\rho = 0.4$. Moreover, from Figs. 5 and 4 we notice that the performance of direct transmission is significantly better than both schemes for high outages and high SNR values. However as either SNR or outage requirement is increased (low outage) its performance severely degrades.

4. Conclusion

In this paper we proposed a new cooperative communication scheme and analyzed, along with two reference protocols, under non-orthogonal signalling and the LMMSE receiver. In terms of achievable throughput we observed that the proposed scheme increases, compared to the CS1 reference scheme, the SNR range over which the loss compared to NC is small leading to a scheme that is effective over a wide range of SNR values. We also showed that the proposed scheme offers significant gains over the other two schemes in high SNR values for not too small outage probabilities, in terms of outage throughput. It was observed that this gain becomes more significant as ρ decreases. In the very low SNR region the loss that CS2 incurs compared to that of CS1 in terms of outage throughput is shown to be negligible. In addition, direct transmission performance is found to be very poor unless both the allowed outage and the SNR are very high.

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