Data Synchronization for Throughput Maximization in Distributed Transmit Beamforming

Alireza Ghasempour[1] and Sudharman K. Jayaweera[1,2]
[1]Communications and Information Sciences Lab (CISL), Department of Electrical and Computer Engineering,
University of New Mexico, Albuquerque, NM 87131.
[2]Bluecom Systems and Consulting LLC, Albuquerque, NM.

***Abstract*—In distributed transmit beamforming, two, or more, spatially separated communications nodes act as elements of an antenna array to beamform common data to a destination node. Spatially separated cooperating nodes synchronize their carrier frequencies and control their transmission phases so that at the destination node the received signals combine constructively. An important example is cooperative communications from a cluster of small satellites to a ground station. The focus of this paper is on optimizing the number of packets that each cooperative node should send to others during each data sharing time interval in order to maximize the data throughput during distributed transmit beamforming stage. The problem is formulated as an optimization problem and a novel heuristic method is proposed to obtain the optimum solutions, as an alternative to the exhaustive search with high computational complexity. Simulation results show that the proposed heuristic method has excellent performance compared to the exhaustive search but with very low computational complexity.**

***Index Terms*—Carrier frequency synchronization, carrier phase synchronization, data sharing, data synchronization, distributed transmit beamforming, exhaustive search, heuristic method, timing synchronization.**

I. Introduction

Distributed transmit beamforming is a cooperative communications technique in which two, or more, spatially separated communications nodes act as elements of an antenna array to beamform common data to a destination node [1]-[2]. In this paper, we assume that source nodes are satellites in specific orbits with known velocities and locations and the destination node is a satellite ground station. This can be an important application scenario for distributed transmit beamforming given the increasing popularity of using clusters of smaller satellites as an alternative to expensive large satellites.

We divide the distributed transmit beamforming into two stages: data sharing and beamforming. During the data sharing stage, cooperating nodes share their data with others to achieve data synchronization. The presumption is that if distance among the cooperating nodes are sufficiently shorter than the distance between cooperating nodes and the destination node, energy required for data sharing may relatively be small compared to the energy required for data transmission to the destination node. Otherwise, the use of distributed transmit beamforming may not be justifiable. In general, it may not be possible to neglect the time needed for data sharing which depends on the dynamics of the node constellation. Perhaps the simplest approach to data synchronization is uniform time sharing in which each node uses an equal duration of time to broadcast its data to all other nodes that can receive its data. But, this may not be optimal when there is only a limited time available for data sharing. Indeed, allocated times do not necessarily need to be equal if the performance objective is to maximize the data transmitted to the destination node during the beamforming stage. Thus, the motivation for this paper is optimizing data synchronization among nodes to achieve maximum throughput (transmitted data) during distributed transmit beamforming. We formulate this as an optimization problem and find the optimum number of packets that each node should send to others during each data sharing time interval.

If there is an M number of cooperative nodes in distributed transmit beamforming, this can provide an M-fold increase in the received signal amplitude. Assume that all cooperative nodes have the same message signal $m(n)$ with $E[|m(n)|^2] = 1$. Cooperative node i multiplies $m(n)$ by a complex weight $w_i = |w_i|e^{j\angle w_i}$ to make the transmitted signal $s_i(n) = w_i m(n)$. Under a normalized sum-power constraint at the cooperative nodes, we have $\sum_{i=1}^{M} |w_i|^2 \leq 1$. Assume that the channel fading is frequency-flat and slow, so that the complex-valued channel gains $h_i = |h_i|e^{j\angle h_i}$ can assumed to be constant over several symbol periods. We assume that the $|h_i|$'s are normalized so that $E[|h_i|^2] = 1$. If $z(n) \sim \mathcal{CN}(0, \sigma^2)$ is a circularly-symmetric complex white Gaussian noise with zero mean and variance σ^2, then, the received signal at the destination node is $r(n) = (\sum_{i=1}^{M} h_i w_i)m(n) + z(n)$. The received signal amplitude is maximized by choosing $w_i = h_i^*$ [3]. Assume that all h_i are the same, i.e., $h_i = h$. Then, we have $r(n) = M|h|^2 m(n) + z(n)$ which shows M-fold increase in the received signal amplitude.

Key challenges to achieving this potential benefit include distributed timing, carrier frequency and phase synchronization. Various timing synchronization techniques have been proposed in literature for different applications. Since individual nodes have their own local oscillators, their carrier frequencies can vary with respect to the nominal frequency. These carrier frequency variations may cause the mis-alignment in the phases of the received signals leading to their destructive or constructive combining at the destination node. Thus, distributed carrier frequency synchronization is critical to eliminate, or at least minimize, frequency offsets.

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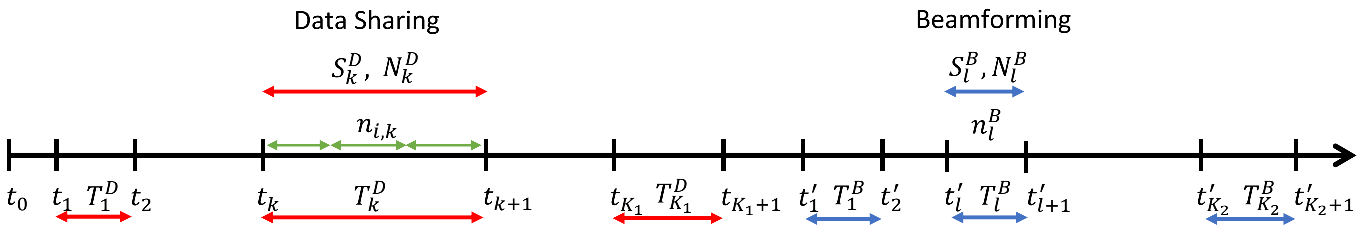

Fig. 1. The general timing scenario during data sharing and beamforming stages in a distributed transmit beamforming application

The rest of the paper is organized as follows. In section II, we formulate the data sharing problem as an optimization problem to find the optimum number of packets that each cooperative node should send to others during each data sharing time interval in order to maximize the transmitted data during the distributed beamforming stage. Section III discuses two methods for solving the proposed optimization problem. Section IV presents simulation results and analysis. The conclusions are given in section V.

II. Data Sharing

Our objective is to find the optimum number of packets that each cooperative node should send to others during each data sharing time interval so that the total throughput during the distributed transmit beamforming stage is maximized. We consider a general timing scenario for data sharing and beamforming (see Fig. 1) and formulate the problem as an optimization problem subject to a set of constraints.

As Fig. 1 shows, assume that there are K_1 number of time intervals in data sharing stage and K_2 number of time intervals in beamforming stage. T_k^D is the k-th data-sharing time interval where $k = 1, \ldots, K_1$ and T_l^B is the l-th distributed transmit beamforming time interval where $l = 1, \ldots, K_2$. Set S_k^D denotes the indices of available cooperative nodes during the k-th data sharing time interval T_k^D. Set S_l^B denotes the indices of the cooperative nodes during beamforming interval T_l^B whose cooperative transmissions can provide enough beamforming gain to satisfy quality of service requirements at the destination node (e.g., maximum tolerable bit error rate or minimum received SNR). Without loss of generality, we may assume that, S_k^D is necessarily distinct from S_{k+1}^D, since otherwise we may combine them to create a single interval. The same is true with sets S_l^B. We assume that cooperative nodes share their data but they don't act as relays during data sharing stage.

We assume that each cooperative node i (for $i = 1, \ldots, M$) has enough packets (or theoretically unlimited number of packets) during each data sharing time interval. Note that this can easily be justified since in most practical applications, we have a large amount of data but limited resources. Let N_k^D denote the total number of packets that the cooperative nodes in set S_k^D can share with each other during the k-th data sharing time interval T_k^D. We assume that each cooperative node i has N_i number of packets at the beginning of the data sharing stage which is enough to share during whole this stage. Let $N_{i,k}$ show the number of packets that node i has at the beginning of the k-th data sharing time interval T_k^D with $N_{i,1} = N_i$ and $N_{i,k} \geq N_k^D$. Let us denote the number of packets that cooperative node i sends to others during the k-th data sharing interval by $n_{i,k}$ where $0 \leq n_{i,k} \leq N_k^D$ and $\sum_i n_{i,k} = N_k^D$ for $k = 1, \ldots, K_1$, $i \in S_k^D$, and $n_{i,k}$ is an integer. Note that, if $i \notin S_k^D$ then $n_{i,k} = 0$. Then, $N_{i,k+1}$ must be updated as follows: $N_{i,k+1} = N_{i,k} - n_{i,k}$ where $k = 1, \ldots, K_1 - 1$. Our goal is to find the optimum values of $n_{i,k}$ so that the total number of transmitted packets during beamforming stage will be maximized.

We assume that the message bandwidth (B), packet rate (R), and maximum transmitted power (P_T) of all cooperative nodes are the same. Also, we assume that the message bandwidth (B) is much less than the coherence bandwidth of the channel (B_c) between cooperative nodes and the destination node, i.e, $B \ll B_c$ so that the channel can assumed to be frequency-flat. The channels exhibit slow fading, i.e., the complex channel gain vector $\boldsymbol{h} = [h_1, \ldots, h_M]^T$ is constant during several symbol periods (corresponding to one or more packets) where $h_i = |h_i| e^{j\angle h_i}$ is the complex channel gain between cooperative node i and the destination node.

The total number of packets that cooperative node i sent to cooperative node j during the data sharing stage can be written as $n_{i\to j} = \sum_{k=1}^{K_1} n_{i,k} I_k(j)$ where $i, j = 1, \ldots, M$, $i \neq j$, and

$$I_k(j) = \begin{cases} 1 & \text{if } j \in S_k^D \\ 0 & \text{if } j \notin S_k^D. \end{cases} \qquad (1)$$

The number of transmitted packets to the destination node during each beamforming time interval T_l^B, denoted by n_l^B, for $l = 1, \cdots, K_2$, is determined by two factors:

1) N_l^B: The maximum number of packets that cooperative nodes in S_l^B, for $l = 1, \cdots, K_2$, can send to the destination node during time interval T_l^B (given by network topology and dynamics).
2) n_l^c: The maximum available shared packets among cooperative nodes S_l^B during time interval T_l^B.

We are now in a position to formulate the data synchronization problem for distributed transmit beamforming as the following optimization problem:

$$maximize \quad \sum_{l=1}^{K_2} n_l^B, \qquad (2)$$

where

$$n_l^B = \min(N_l^B, n_l^c), \quad l = 1, \ldots, K_2, \tag{3}$$

$$n_l^c = \sum_{i \in S_l^B} \min_{j \in S_l^B} n_{i,j,l}^S, l = 1, \ldots, K_2, \tag{4}$$

$$n_{i,j,l}^S = \begin{cases} n_{i \to j} & \text{if } l = 1 \\ n_{i,j,l-1}^S - n_{l-1}^B & \text{if } l = 2, \ldots, K_2 \end{cases}, \tag{5}$$

subject to

$$0 \le n_{i,k} \le N_k^D, \sum_{i \in S_k^D} n_{i,k} = N_k^D, k = 1, \ldots, K_1. \tag{6}$$

III. Solution methods

The optimization problem (2)-(6) is a nonlinear optimization problem for which there is no apparent direct closed-form solution. Therefore, in the following we first resort to exhaustive search.

A. Exhaustive search

The exhaustive search can be implemented by checking, for $k = 1, \cdots, K_1$, all possible integer values of $n_{i,k}$ from 0 to N_k^D subject to $\sum_i n_{i,k} = N_k^D$ and finding their optimum values so that (2) would be maximized. For example, if $S_k^D = \{1, 2\}$ and $N_k^D = 5$, then for $n_{1,k}$ and $n_{2,k}$ we must check all possible integer values from 0 to 5 such that $n_{1,k} + n_{2,k} = 5$. In this case, the possible values of $n_{1,k}$ and $n_{2,k}$ are (0,5), (1,4), (2,3), (3,2), (4,1), and (5,0). In each beamforming time interval T_l^B, the exhaustive search uses (4) to find the maximum shared data between each node and other cooperative nodes and then uses (3) to calculate n_l^B.

In general, $N = N_k^D$ and $L = |S_k^D|$ be the cardinality of S_k^D. In each time interval T_k^D, exhaustive search has to check all possible integer combinations of $n_{i,k}$'s such that $\sum_i n_{i,k} = N$ for $i \in S_k^D$. For example, if $S_k^D = \{1, 2\}$, $L = 2$, and $N = 10$, exhaustive search has to check all possible integer values (from 0 to 10) for $n_{1,k}$ and $n_{2,k}$ such that $(n_{1,k} + n_{2,k}) = 10$ which is 11 distinct pairs of values, i.e., (0,10), (1,9), ..., (10,0). If $N_L(N)$ indicates the total number of distinct L-tuples of integer values for L number of $n_{i,k}$ whose sum is N, then, for $L > 4$, it is hard to find a closed form expression for $N_L(N)$. We can, however, show that approximately in the time interval T_k^D, computational complexity of exhaustive search is in the order of $O(N^{L-1})$. Hence, if we have K_1 number of time intervals and in each time interval T_k^D, we have $N_{i,k} \ge N_k^D$, then, computational complexity of exhaustive search will be on the order of $O(\prod_{k=1}^{K_1} {N_k}^{(L-1)})$ where $N_k = N_k^D$ and $L = |S_k^D|$.

In general, using exhaustive search to find the optimum solution may not be desirable (due to its very high computational complexity) unless the number of cooperative nodes is relatively small. We may instead use meta-heuristic algorithms such as genetic algorithm or a heuristic method. We can, however, use the exhaustive search results as a reference for assessing the performance of the proposed heuristic method to solve the problem in (2)-(6).

B. Heuristic method

In our proposed heuristic method, first, we use the following two steps to remove those unnecessary sets S_k^D and S_l^B which don't influence the problem formulation, data sharing optimization, and throughput maximization:

1) Drop any S_l^B which is not the subset of any set or sets in the data sharing time intervals since cooperative nodes in that S_l^B will not be able to have common data to beamform based on the sets in the data sharing time intervals.
2) Drop any S_k^D none of whose subsets is a given set in then beamforming stage since the shared data between cooperative nodes in S_k^D cannot be transmitted to the destination node during beamforming stage.

After removing these redundant sets, the data sharing optimization and throughout maximization can be divided to two steps:

1) Since cooperative nodes can in general be mobile, we start from T_1^D and then go to T_2^D and so on. In each data sharing time interval T_k^D (starting from T_1^D), we must select a subset or subsets of cooperative nodes (from set S_k^D) based on related sets S_l^B and N_l^B. We divide each time interval T_k^D to several disjoint subintervals based on the number of selected subsets. Now, assume that we have K_1' number of subintervals during data sharing stage ($T_{k'}^D$ for $k' = 1, \ldots, K_1'$). Correspondingly, we define $S_{k'}^D$, $N_{k'}^D$, $N_{i,k'}$, and $n_{i,k'}$. For example, if $S_k^D = \{1, 2, 3\}$, $S_i^B = \{1, 2\}$, and $S_j^B = \{1, 2, 3\}$, then we divide the k-th data sharing time interval to two subintervals $S_{k'}^D = \{1, 2\}$ and $S_{k'+1}^D = \{1, 2, 3\}$.
The values of $N_{k'}^D$ and $N_{k'+1}^D$ have to satisfy the following constraints:
$$N_{k'}^D + N_{k'+1}^D \le N_k^D, \tag{7}$$
$$N_{k'}^D \le N_i^B, \tag{8}$$
$$N_{k'+1}^D \le N_j^B. \tag{9}$$
We impose the fairness criterion, $N_{k'}^D / N_{k'+1}^D = N_i^B / N_j^B$. Therefore, $N_{k'}^D$ and $N_{k'+1}^D$ can be calculated based on the values of N_k^D, N_i^B, and N_j^B as follows:
   a) If $N_i^B + N_j^B \le N_k^D$, we have $N_{k'}^D = N_i^B$ and $N_{k'+1}^D = N_j^B$.
   b) If $N_i^B + N_j^B > N_k^D$, we have
$$N_{k'}^D = \left\lceil N_k^D \left(\frac{N_i^B}{N_i^B + N_j^B} \right) \right\rceil, \tag{10}$$
$$N_{k'+1}^D = \left\lfloor N_k^D \left(\frac{N_j^B}{N_i^B + N_j^B} \right) \right\rfloor, \tag{11}$$
   where $\lceil . \rceil$ and $\lfloor . \rfloor$ functions map a real number to the greatest preceding or the least succeeding integer number, respectively.

We follow a similar procedure as above when we have more than two subsets to find the values of $N_{k'}^D$'s. After finding $N_{k'}^D$ for each subinterval, we update corresponding N_i^B by subtracting $N_{k'}^D$ from that N_i^B, i.e., $N_i^B = N_i^B - N_{k'}^D$ so that in the next subintervals, we try to provide as many packets as possible for beamforming time intervals based on the remaining N_i^B's.

2) The optimum values of $n_{i,k'}$ are computed based on $S_{k'}^D$, $N_{i,k'}$, and $N_{k'}^D$ such that a fairness criterion over cooperative nodes is maintained (i.e., the number of transmitted packets by source nodes should be proportional to their available data). Since all cooperative nodes have the same E_p and they have enough packets to share in each data sharing time interval, it turns out that $n_{i,k'} = (N_{i,k'} N_{k'}^D)/\sum_{i \in S_{k'}^D} N_{i,k'}$.

IV. Simulation Results and Analyses

To evaluate the performance of the exhaustive search and the proposed heuristic method to solve the data synchronization problem (2)-(6) for distributed beamforming, we consider 2 scenarios which include multiple cooperating nodes and one destination node.

In scenario 1, we assume that there are 4 cooperative nodes (with $N_i = 40$ packets for $i = 1,2,3,4$) and a single destination node. The corresponding timing schedule is shown in Fig. 2. The values of parameters are given in Table I. This scenario has one optimum and fair solution with the maximum throughput of 32 packets. The optimal non-zero values of $n_{i,k}$ (in packets) are as follows: $n_{3,1} = 11$, $n_{4,1} = 11$, $n_{1,2} = 5$, and $n_{2,2} = 5$.

In scenario 2, we assume that there are 3 cooperative nodes (with $N_i = 100$ packets for $i = 1,2,3$) and a single destination node. The timing schedule for the scenario 2 is shown in Fig. 3. The values of parameters are given in Table I. This scenario has one fair and optimum solution with the maximum throughput of 80 packets. The optimum non-zero values of $n_{i,k}$ (in packets) corresponding to this solution are as follows: $n_{1,1} = 5$, $n_{2,1} = 5$, $n_{1,2} = 10$, $n_{3,2} = 10$, $n_{2,3} = 5$, $n_{3,3} = 5$, $n_{1,4} = 15$, $n_{2,4} = 10$, and $n_{3,4} = 15$.

Table II shows the total number of transmitted packets during distributed transmit beamforming in the above two scenarios by exhaustive search and the proposed heuristic method. As it is seen, exhaustive search has the best performance (i.e., highest number of transmitted packets) with the highest computational complexity (since it checks all valid integer values of $n_{i,k}$). The heuristic method has a very low computational complexity while it has an excellent performance compared to the exhaustive search.

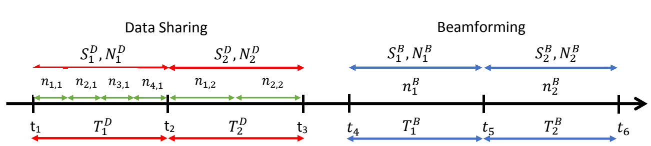

Fig. 2. Timing schedule for 4 nodes in the 1st scenario

TABLE I
THE PARAMETER VALUES FOR THE TWO SCENARIOS.

Parameter	Value of scenario 1	Value of scenario 2
M	4	3
K_1, K_2	2, 2	4, 4
N_1^D, N_2^D	22, 10 Packets	10, 20 Packets
N_3^D, N_4^D	N/A, N/A	20, 40 Packets
S_1^D, S_2^D	$\{1,2,3,4\}, \{1,2\}$	$\{1,2\}, \{1,3\}$
S_3^D, S_4^D	N/A, N/A	$\{2,3\}, \{1,2,3\}$
N_1^B, N_2^B	10, 100 Packets	10, 30 Packets
N_3^B, N_4^B	N/A, N/A	10, 30 Packets
S_1^B, S_2^B	$\{1,2\}, \{3,4\}$	$\{1,2\}, \{1,3\}$
S_3^B, S_4^B	N/A, N/A	$\{2,3\}, \{1,2,3\}$

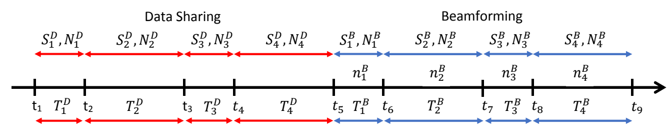

Fig. 3. Timing schedule for 3 nodes in the 2nd scenario

V. Conclusion

In this paper, we considered the problem of data synchronization among cooperative nodes in order to maximize the transmitted data throughput during distributed transmit beamforming. We formulated this as an optimization problem whose exhaustive search solution appears to be computationally too demanding. As an alternative, we proposed a novel heuristic method. Simulation results show that the proposed heuristic method has a very low computational complexity while it has an excellent performance compared to the exhaustive search.

VI. Acknowledgment

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TABLE II
THE TOTAL NUMBER OF TRANSMITTED PACKETS IN TWO SCENARIOS BY THE EXHAUSTIVE SEARCH AND THE PROPOSED HEURISTIC METHOD

	Scenario 1	Scenario 2
Exhaustive	32	80
Heuristic	30	80