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Limitations in tracking systems

Ivan Lopez HURTADO [1], Chaouki T. ABDALLAH [2], Sudharman K. JAYAWEERA [2]
(1.Department of Engineering, Northern New Mexico College, 921 Paseo de Onate, Espanola, NM 87532, USA;
2.Department of Electrical & Computer Engineering, University of New Mexico, MSC01 1100, Albuquerque, NM 87131-0001, USA)

Abstract: In this paper, we obtain information theoretical conditions for tracking in linear time-invariant control systems. We consider the particular case where the closed loop contains a channel in the feedback loop. The mutual information rate between the feedback signal and the reference input signal is used to quantify information about the reference signal that is available for feedback. This mutual information rate must be maximized in order to improve the tracking performance. The mutual information is shown to be upper bounded by a quantity that depends on the unstable eigenvalues of the plant and on the channel capacity. If the channel capacity reaches a lower limit, the feedback signal becomes completely uncorrelated with the reference signal, rendering feedback useless. We also find a lower bound on the expected squared tracking error in terms of the entropy of a random reference signal. We show a misleading case where the mutual information rate does not predict the expected effect of nonminimum phase zeros. However, mutual information rate helps generalize the concept that there is a tradeoff when tracking and disturbance rejection are simultaneous goals, and a constraint communication channel is present in the feedback loop. Examples and simulations are provided to demonstrate some of the results.

Keywords: Limited channel capacity; Tracking systems; Constraint feedback

1 Introduction

The goal of this article is to find fundamental limitations on feedback tracking systems in terms of information theoretical quantities. This is important since the emerging control applications involve the presence of a constraint communication channel in the feedback loop. Typically, control systems have been understood as signal processing blocks or systems interchanging energy. However, these approaches are not appropriate for the new scenarios. That is why we suggest that an interpretation in terms of information flow may be more suitable for the future design of control algorithms.

Previous related work in references [1∼7] detailed some aspects of performance and limitations of control systems in terms of information theoretic quantities. Specifically, the work in [5] dealt with the tracking issues without a channel in the feedback link, while [1] dealt with disturbance rejection. A result in [5] shows that a necessary condition for efficient tracking is that the information flow from the reference signal to the output should be greater than the information flow between the disturbance and the output. We know that in the absence of noise and without a communication channel in the feedback loop, the mutual information rate (or information rate) between reference signal and the output is infinite. From [3], we know, however, that if the feedback signal is transmitted by means of a finite capacity channel, the mutual information rate is upper bounded by $C_f - \sum_{i=1} \max\{0, \log_2(|\lambda_i(A)|)\}$.

Following the same approach of [8], we expect that the parameters of the plant and feedback channel capacity C_f will be related and that there will be a trade-off between these parameters. If by some reason this upper bound happens to be zero, then we reach a fundamental limitation where no information of the reference signal is available for feedback. This means that the two signals are independent, therefore, uncorrelated, and this is exactly the condition that implies that tracking is impossible. In other words, the feedback signal does not provide any useful information for the reference to be tracked.

We note that the condition for a nonzero mutual information rate between the reference and the feedback signal is a necessary condition for tracking but not a sufficient one. A large mutual information rate between the reference signal and the feedback signal does not necessarily imply that tracking is possible (it only implies that the signals are highly correlated). This is expected because even in the case of a perfect infinite capacity channel, the tracking issue requires additional conditions to be satisfied.

The results in this work do not intend to be applied in the design of a new control algorithm. These results are fundamental limitations in terms of information quantities that any control system designer must be aware of before trying to design a new control system. We note that the results shown in this article are an extension to those in [9].

2 Notation

We present next the notation used in the rest of this article.

- Let $\boldsymbol{x}^k = \{\boldsymbol{x}(1), \boldsymbol{x}(2), \cdots, \boldsymbol{x}(k)\}$ and $\boldsymbol{y}^k = \{\boldsymbol{y}(1), \boldsymbol{y}(2), \cdots, \boldsymbol{y}(k)\}$ be sets of observations of stochastic processes $\boldsymbol{x}$ and $\boldsymbol{y}$. We follow the notation in [10] where bold letters represent stochastic processes.

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- Let $\boldsymbol{x}(k)$ be a time sample of the stochastic process $\boldsymbol{x}$.
- Let $\boldsymbol{x}_j$ be the "jth" state component. For example, if $\boldsymbol{x}$ has dimension $n = 3$, then $\boldsymbol{x}_j$ will denote any of the state components $\boldsymbol{x}_1$, $\boldsymbol{x}_2$, or $\boldsymbol{x}_3$.
- Let $\boldsymbol{x}_J$ denote the set of state components, $\boldsymbol{x}_j$, such that $j \in J$. For example, if $J = \{1, 3\}$, then $\boldsymbol{x}_J$ is the set $\{\boldsymbol{x}_1, \boldsymbol{x}_3\}$.
- Let $|\cdot|$ denote the absolute value and $\mathbf{det}(\cdot)$ denotes the absolute value of the determinant of a matrix.

We also define the blocks in Fig.1:

- C is the controller, which does not have any constraints (it could be time-invariant, nonlinear, etc.).
- P is the plant to be controlled and is assumed to be discrete, linear, time-invariant, with state-space realization:

$$\boldsymbol{x}(k+1) = A\boldsymbol{x}(k) + B\boldsymbol{u}(k), \tag{1}$$
$$\boldsymbol{y}(k) = \mathcal{C}\boldsymbol{x}(k). \tag{2}$$

- E is the encoder assumed to be a causal operator well defined in the input alphabet of the channel.
- D is the decoder assumed to be well defined and conserving equimemory with the encoder.
- The channel block is any type of communication channel with finite capacity.
- c is the channel noise.

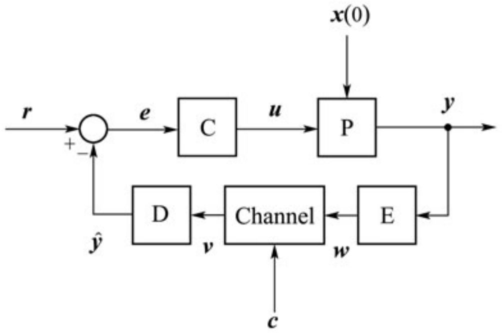

Fig. 1 Closed-loop system with communication channel in feedback link.

3 Information theory preliminaries

Before proceeding, we enumerate some well-known information-theoretical properties that will be very useful later on.

Proposition 1 Assume that $\boldsymbol{z}$, $\boldsymbol{w}$, $\boldsymbol{u} \in \mathbb{R}$ are random variables, and $f(\boldsymbol{z})$, $g(\boldsymbol{z})$ are real functions. All of the following may be found in several references, as references [8, 11, 12].

a) $h(\boldsymbol{z} - \boldsymbol{w}) \leqslant h(\boldsymbol{z})$ with equality if $\boldsymbol{z}$ and $\boldsymbol{w}$ are independent.

b) Let $\boldsymbol{z}$ have mean μ and covariance $\mathrm{Cov}\{\boldsymbol{z}^n\}$. Then,

$$h(\boldsymbol{z}^n) \leqslant \frac{1}{2}\log_2\left((2\pi e)^n \mathbf{det}(\mathrm{Cov}\{\boldsymbol{z}^n\})\right)$$

with equality if $\boldsymbol{z}$ has a multivariate normal distribution.

c) $h(a\boldsymbol{z}) = h(\boldsymbol{z}) + \log_2(|a|)$ for nonzero constant a.

d) $h(A\boldsymbol{z}) = h(\boldsymbol{z}) + \log_2(\mathbf{det}(A))$ for nonsingular A matrix.

e) $h(\boldsymbol{z}|\boldsymbol{w}) = h(\boldsymbol{z} - g(\boldsymbol{w})|\boldsymbol{w})$.

f) $I(\boldsymbol{z}; \boldsymbol{w}) = I(\boldsymbol{w}; \boldsymbol{z}) \geqslant 0$.

g) $I(\boldsymbol{z}; \boldsymbol{w}) \geqslant I(g(\boldsymbol{z}); f(\boldsymbol{w}))$.

h) $I(\boldsymbol{z}; \boldsymbol{w}|\boldsymbol{u}) = I((\boldsymbol{u}, \boldsymbol{z}); \boldsymbol{w}) - I(\boldsymbol{u}; \boldsymbol{w}) = h(\boldsymbol{z}|\boldsymbol{u}) - h(\boldsymbol{z}|\boldsymbol{w}, \boldsymbol{u}) = h(\boldsymbol{w}|\boldsymbol{u}) - h(\boldsymbol{w}|\boldsymbol{z}, \boldsymbol{u})$.

i) For any random variable $\boldsymbol{z}$ and estimate $\hat{\boldsymbol{z}}$: $E\{(\boldsymbol{z} - \hat{\boldsymbol{z}})^2\} \geqslant \frac{1}{2\pi e}2^{2h(\boldsymbol{z})}$, with equality if and only if $\boldsymbol{z}$ is Gaussian, and $\hat{\boldsymbol{z}}$ is the mean of $\boldsymbol{z}$.

j) The variance of the error in the estimate $\hat{\boldsymbol{z}}$ of $\boldsymbol{z}$ given the infinite past is lower bounded as $\sigma^2_\infty(\boldsymbol{z}) = \lim\limits_{k\to\infty} E\{(\boldsymbol{z} - \hat{\boldsymbol{z}})^2(k)|(\boldsymbol{z} - \hat{\boldsymbol{z}})(k-1)\} \geqslant \frac{1}{2\pi e}2^{2h_\infty(\boldsymbol{z})}$ with equality if $\boldsymbol{z}$ is Gaussian.

k) If z is an asymptotically stationary process, then

$$h_\infty(z) \leqslant \frac{1}{4\pi}\int_{-\pi}^{\pi}\log_2\left(2\pi e\widehat{\varPhi}_z(\omega)\right)\mathrm{d}\omega,$$

where $\widehat{\varPhi}_z$ is the asymptotic power spectral density of z and equality holds if, in addition, z is Gaussian autoregressive.

4 Signal analysis

The functional dependencies among the signals involved in the closed-loop shown in Fig.1 are as follows:

$$\begin{aligned}
&\boldsymbol{y}(k) = f_1(\boldsymbol{r}^{k-1}, \boldsymbol{c}^{k-1}, \boldsymbol{x}(0)),\\
&\boldsymbol{e}(k) = f_2(\boldsymbol{r}^k, \hat{\boldsymbol{y}}^k) = \boldsymbol{r}(k) - \hat{\boldsymbol{y}}(k),\\
&\boldsymbol{u}(k) = f_3(\boldsymbol{e}^k),\\
&\hat{\boldsymbol{y}}(k) = f_4(\boldsymbol{y}^k, \boldsymbol{c}^k).
\end{aligned}$$

5 Assumptions

The matrix A in block P in Fig.1 is assumed to be diagonal with only unstable eigenvalues ($|\lambda_i(A)| > 1$), and therefore, A^k is invertible $\forall k$. We assume that A has unstable eigenvalues since it is the worst case. The general case in which A has stable eigenvalues is discussed in [9], and the nondiagonal case was discussed in [13]. Since we are considering the tracking problem, the control law is a function of the error $\boldsymbol{e}^k = \boldsymbol{r}^k - \hat{\boldsymbol{y}}^k$, $\boldsymbol{u}(k) = f_3(\boldsymbol{e}^k)$. We note for now that the output is an n-dimensional vector, but this will be relaxed later on. In our setup, f_3 is not limited to be a linear or time-invariant control law. We note that the solution of the difference equation (1) may be written as $\boldsymbol{x}(k) = A^k\boldsymbol{x}(0) + \sum\limits_{i=0}^{k-1}A^{k-i-1}Bf_3(\boldsymbol{e}^i)$. If $\mathcal{C} = \mathcal{I}$, then from the tracking error, defined by $\boldsymbol{\epsilon}(k) = \boldsymbol{r}(k) - \boldsymbol{y}(k)$, we have

$$\begin{aligned}
\boldsymbol{r}(k) - \boldsymbol{\epsilon}(k) &= \boldsymbol{y}(k) = \boldsymbol{x}(k)\\
&= A^k\boldsymbol{x}(0) + \sum_{i=0}^{k-1}A^{k-i-1}Bf_3(\boldsymbol{e}^i).
\end{aligned} \tag{3}$$

We rearrange the terms as

$$\begin{aligned}
&\boldsymbol{x}(0) + A^{-k}\sum_{i=0}^{k-1}A^{k-i-1}Bf_3(\boldsymbol{e}^i)\\
&= -A^{-k}(\boldsymbol{\epsilon}(k) - \boldsymbol{r}(k)).
\end{aligned} \tag{4}$$

In a tracking problem, we do not necessarily assume that the state is bounded, since for unbounded reference signals, the state may grow unbounded. Instead, we assume that the closed-loop is such that the error is bounded, i.e.,

$$E\{\boldsymbol{\epsilon}^{\mathrm{T}}\boldsymbol{\epsilon}\} < \infty.$$

Since this implies that $\boldsymbol{\epsilon}$ is a second-order process, the mean $E\{\boldsymbol{\epsilon}\}$ and the covariance $\mathrm{Cov}\{\boldsymbol{\epsilon}\} = E\{(\boldsymbol{\epsilon} +$

$E\{\boldsymbol{\epsilon}\})(\boldsymbol{\epsilon}+E\{\boldsymbol{\epsilon}\})^{\mathrm{T}}\}$ must be finite. For bounded reference signals, the condition $E\{\boldsymbol{\epsilon}^T\boldsymbol{\epsilon}\}<\infty$ guarantees stability since by the triangle inequality [10], we know that

$$\sqrt{E\{\boldsymbol{x}^2(k)\}}\leqslant\sqrt{E\{\boldsymbol{r}^2(k)\}}+\sqrt{E\{\boldsymbol{\epsilon}^2(k)\}}. \quad (5)$$

Since the two terms on the right side of equation (5) are finite, we also get that $\sqrt{E\{\boldsymbol{x}^2(k)\}}<\infty$, and therefore, the system remains stable.

6 Auxiliary results

The next three results were proven in [9] and will be used later on.

Lemma 1 Consider the closed-loop system in Fig.1, where the plant is a DTLI system described by equations (1) and (2), with $\mathcal{C}=\mathcal{I}$, and A diagonal in equation (2). If $E\{\boldsymbol{x}_{\boldsymbol{P}_j}(k)\boldsymbol{x}^{\mathrm{T}}_{\boldsymbol{P}_j}(k)\}<\infty$, then

$$\lim_{k\to\infty}\frac{I(\boldsymbol{x}_{\boldsymbol{P}_j}(0);\boldsymbol{e}^k|\boldsymbol{r}^k,\boldsymbol{x}_j(0))}{k}\geqslant\sum_{i\neq j}\log_2(|\lambda_i(A)|).$$

Lemma 2 Consider the closed-loop system in Fig.1, where the plant is a DLTI system described by equations (1) and (2), $\mathcal{C}=\mathcal{I}$. If $E\{\boldsymbol{\epsilon}(k)\boldsymbol{\epsilon}^{\mathrm{T}}(k)\}<\infty$, then

$$\lim_{k\to\infty}\frac{I(\boldsymbol{x}(0);\boldsymbol{e}^k|\boldsymbol{r}^k)}{k}\geqslant\sum_{i}\log_2(|\lambda_i(A)|).$$

Lemma 3 Consider the closed-loop system given in Fig.1, where the plant is a DLTI system described by equation (1) and $\boldsymbol{y}=q\boldsymbol{x}_j$ for some $j\in\{1,\cdots,n\}$, q is a nonzero constant. If $E\{\boldsymbol{\epsilon}(k)\boldsymbol{\epsilon}^{\mathrm{T}}(k)\}<\infty$ and $E\{\boldsymbol{x}_{\bar{Y}}(k)\boldsymbol{x}^{\mathrm{T}}_{\bar{Y}}(k)\}<\infty$, then

$$\lim_{k\to\infty}\frac{I(\boldsymbol{x}(0);\boldsymbol{e}^k|\boldsymbol{r}^k)}{k}\geqslant\sum_{i}\log_2(|\lambda_i(A)|).$$

7 Results

Using the results in Section 6, we find limitations on tracking systems that are imposed by the presence of a finite capacity channel. We consider the expression $I(\boldsymbol{r}^k;\hat{\boldsymbol{y}}^k)$ instead of $I(\boldsymbol{r}^k;\boldsymbol{y}^k)$. Although $I(\boldsymbol{r}^k;\boldsymbol{y}^k)$ provides the actual information between the output and the reference signals, the former is easier to calculate than the later. The mutual information $I(\boldsymbol{r}^k;\hat{\boldsymbol{y}}^k)$ represents the information between the transmitted feedback, i.e., $\hat{\boldsymbol{y}}^k$, and the reference signal. If this mutual information happens to be zero, all information contained in the feedback signal about the reference signal will be lost, and the error $\boldsymbol{e}$ used to generate the control signal will be useless. In fact, $I(\boldsymbol{r};\hat{\boldsymbol{y}})$ measures the usefulness of feedback. By the properties of mutual information, we have

$$I((\boldsymbol{r}^k,\boldsymbol{x}(0));\hat{\boldsymbol{y}}^k)=I(\boldsymbol{r}^k;\hat{\boldsymbol{y}}^k)+I(\boldsymbol{x}(0);\hat{\boldsymbol{y}}^k|\boldsymbol{r}^k). \quad (6)$$

From the definition of mutual information, Proposition 1 e), and from the fact that $\boldsymbol{e}^k=\boldsymbol{r}^k-\hat{\boldsymbol{y}}^k$, we have

$$\begin{aligned}I(\boldsymbol{x}(0);\hat{\boldsymbol{y}}^k|\boldsymbol{r}^k)&=h(\hat{\boldsymbol{y}}^k|\boldsymbol{r}^k)-h(\hat{\boldsymbol{y}}^k|\boldsymbol{x}(0),\boldsymbol{r}^k)\\&=h(\boldsymbol{e}^k|\boldsymbol{r}^k)-h(\boldsymbol{e}^k|\boldsymbol{x}(0),\boldsymbol{r}^k)\\&=I(\boldsymbol{x}(0);\boldsymbol{e}^k|\boldsymbol{r}^k). \quad (7)\end{aligned}$$

From equations (6) and (7), we have

$$I((\boldsymbol{r}^k,\boldsymbol{x}(0));\hat{\boldsymbol{y}}^k)=I(\boldsymbol{r}^k;\hat{\boldsymbol{y}}^k)+I(\boldsymbol{x}(0);\boldsymbol{e}^k|\boldsymbol{r}^k). \quad (8)$$

From equation (8) and knowing that

$$kC_f\geqslant I((\boldsymbol{r}^k,\boldsymbol{x}(0));\hat{\boldsymbol{y}}^k),$$

we obtain

$$I(\boldsymbol{r}^k;\hat{\boldsymbol{y}}^k)\leqslant kC_f-I(\boldsymbol{x}(0);\boldsymbol{e}^k|\boldsymbol{r}^k). \quad (9)$$

By Lemma 2 and dividing equation (9) by k and taking the limit as $k\to\infty$, we finally have

$$I_\infty(\boldsymbol{r};\hat{\boldsymbol{y}})\leqslant C_f-\sum_i\log_2(|\lambda_i(A)|).$$

We summarize this result in the following lemma:

Lemma 4 Consider the closed-loop system given in Fig.1, where the plant is a DLTI system described by equations (1) and (2), which is a feedback capacity C_f in the channel. If $E\{\boldsymbol{\epsilon}(k)\boldsymbol{\epsilon}(k)^{\mathrm{T}}\}<\infty$, then

$$I_\infty(\boldsymbol{r};\hat{\boldsymbol{y}})\leqslant C_f-\sum_i\log_2(|\lambda_i(A)|).$$

We note from Lemma 4 that if the channel does not have a minimum capacity of $\sum_i\log_2(|\lambda_i(A)|)$, the feedback signal does not provide any information of the reference signal. Lemma 2 is one of the main contributions of this work. We note that Lemma 3 is needed when the output is only one of the state components and not the whole state.

7.1 Limitations on reference signals

The results of the previous sections deal with the idea of bounding the error signal, $\boldsymbol{\epsilon}(k)=\boldsymbol{r}(k)-\boldsymbol{y}(k)$. However, it is well known that given a plant and a particular controller, there will be limitations on the type of signals that may be tracked. We show next that a tracking system may be thought of as a channel where the reference signal is the input message, the closed-loop is a feedback channel (with the encoder-decoder embedded), and the system output is the received message. Under this scenario, good message estimation is synonymous with good tracking. We consider $\boldsymbol{\epsilon}=\boldsymbol{r}-\boldsymbol{y}$ as the error estimate of the message. Note from Proposition 1 i) that

$$E\{(\boldsymbol{r}-\boldsymbol{y})^2\}\geqslant\frac{1}{2\pi e}2^{2h(\boldsymbol{r})}.$$

This inequality captures the idea that the greater the entropy of the reference signal, the larger the error signal, $\boldsymbol{\epsilon}$. Moreover, since $E\{(\boldsymbol{r}-\boldsymbol{y})^2\}$ is a nonnegative number, we note that the error between the output and the reference cannot reach zero unless the reference signal is deterministic ($h(\boldsymbol{r})=-\infty$). In other words, perfect tracking is not possible, and tracking gets worse for high entropy reference signals regardless of the type or quality of the channel and the controller. Moreover, the following result holds regardless of the plant. Let us consider that the expected value of $(\boldsymbol{\epsilon}^k)^2$ given the entire past $\boldsymbol{\epsilon}_0^{k-1}$ as k tends to infinity, given by

$$\sigma^2_\infty(\boldsymbol{r})=\lim_{k\to\infty}E\{\boldsymbol{\epsilon}^2(k)|\boldsymbol{\epsilon}(k-1)\}.$$

From information theory, the entropy rate makes the variance $\sigma^2_\infty(\boldsymbol{r}):\sigma^2_\infty(\boldsymbol{r})\geqslant\frac{1}{2\pi e}2^{2h_\infty(\boldsymbol{r})}$ as the lower bounds. We then obtain the following lemma.

Lemma 5 Consider the closed-loop system given in Fig.1, where the plant is a DLTI system described by equations (1) and (2). Then, the best estimator $\boldsymbol{y}$ for $\boldsymbol{r}$ is bounded as

$$E\{(\boldsymbol{r}-\boldsymbol{y})^2\} \geqslant \frac{1}{2\pi e}2^{2h(\boldsymbol{r})}. \tag{10}$$

Moreover, the variance of the best reference estimator, $\sigma^2_\infty(\boldsymbol{r})$, is bounded from below as follows:

$$\sigma^2_\infty(\boldsymbol{r}) \geqslant \frac{1}{2\pi e}2^{2h_\infty(\boldsymbol{r})}. \tag{11}$$

8 Examples

The results derived so far are necessary conditions but not sufficient ones because the quantity $I_\infty(\boldsymbol{r};\hat{\boldsymbol{y}})$ implies correlation of signals and not necessarily that $\boldsymbol{y}$ is tracking $\boldsymbol{r}$. The following examples capture how conservative the results of this work are.

8.1 Example 1: Erasure channel

We consider the tracking problem shown in Fig.1 for the reference signal, $\boldsymbol{r}(k)$. The reference signal is assumed to be a white Gaussian sequence, with zero-mean and with $\sigma_r^2 = 1$. We consider a memoryless erasure channel, as shown in Fig.2 in the feedback link with limited rate and a probability of receiving the state measurement of $p_\gamma = 0.70479$. The probability of dropping a packet is therefore $1 - p_\gamma$. We consider a two-part encoder-decoder scheme: first, the encoder converts the real state-vector measured, $\boldsymbol{x}(k)$, to its binary form, truncates the binary representation to its R most significant bits, encapsulates the bits in a packet, and sends the packet through the channel. If the packet is not dropped, the decoder on the receiver site receives the packet, extracts the bits, and converts them to its real number representation. If the receiver does not receive a packet, the decoder will assume that a zero was sent, and the controller does not apply any control signal. In [14], it is shown that for the scalar case, this scheme guarantees that the error between the actual measurement signal and the decoded signal, $\boldsymbol{\epsilon}(k) = \boldsymbol{x}(k) - \bar{\boldsymbol{x}}(k)$, is bounded and that the feedback channel capacity $C_f = \log_2(a)/p_\gamma$ is achieved. The scheme also assumes that the decoder knows exactly the operation of the encoder and that both have access to the control signal. Consider the following plant:

$$\begin{aligned}
&\boldsymbol{x}(k+1) = 4.33\boldsymbol{x}(k) + \boldsymbol{u}(k),\\
&\boldsymbol{y}(k) = \boldsymbol{x}(k),\\
&\boldsymbol{u}(k) = 4.33(\boldsymbol{r}(k) - \bar{\boldsymbol{y}}(k)).
\end{aligned}$$

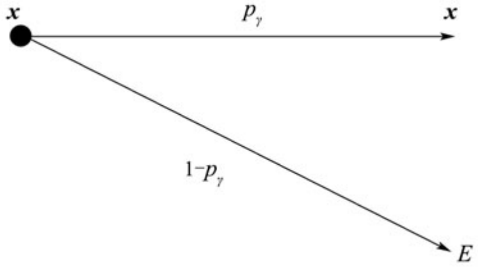

Fig. 2 Erasure channel scheme.

One limitation of our result is that it is given in terms of the mutual information rate, which is difficult to compute for this type of problems. However, we know that it imposes a limit to guarantee that $E\{\boldsymbol{\epsilon}(k)\boldsymbol{\epsilon}^{\mathrm{T}}(k)\} < \infty$. In order to explore what happens to $E\{\boldsymbol{\epsilon}(k)\boldsymbol{\epsilon}^{\mathrm{T}}(k)\}$, we plot the power spectrum of $\boldsymbol{\epsilon}$, $S_{\boldsymbol{\epsilon\epsilon}}(\omega)$, whose enclosed area from $[-\pi,\pi]$ is equivalent to the squared output average of $\boldsymbol{\epsilon}$, i.e.,

$$E\{\boldsymbol{\epsilon}^2\} = \int_{-\pi}^{\pi} S_{\boldsymbol{\epsilon\epsilon}}(\omega)\mathrm{d}\omega.$$

According to Lemma 4, the minimum feedback channel capacity for stabilization needed is 3 bits/timestep. The power spectrum density is shown in Fig.3, where we notice that the power spectrum is bounded, and therefore, $E\{\boldsymbol{\epsilon}^2(k)\}$ is finite. If, instead of using 3 bits/timestep, we use 2 bits/timestep, we obtain the new power spectrum of the error in Fig.4.

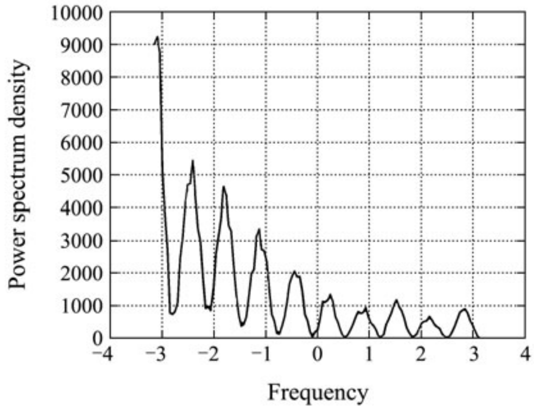

Fig. 3 Example with erasure channel and bit rate of 3 bits/timestep.

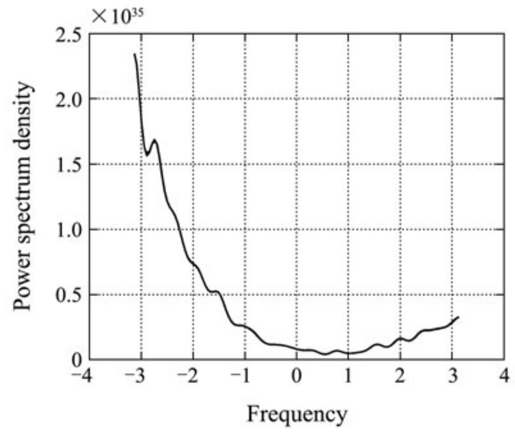

Fig. 4 Example with erasure channel and bit rate of 2 bits/timestep.

Note that the power spectrum is becoming unbounded and so the area below the curve, i.e., $E\{\boldsymbol{\epsilon}^2(k)\}$ is no longer finite.

8.2 Example 2: AWGN channel

We consider the problem of tracking (see Fig.1) a reference signal, $\boldsymbol{r}(k)$, which is assumed to be a white Gaussian sequence with zero-mean and $\sigma_r^2 = 5000$. We consider a memoryless AWGN channel (Fig.5) in the feedback link with feedback channel capacity, $C_f = (1/2)\log_2(1+P/\Phi)$, where Φ is the noise variance, and P is the power constraint such that $E\{\hat{\boldsymbol{y}}^2\} \leqslant P$. The variance Φ is varied in the range $[1000; 200000]$, i.e, the SNR from the reference signal to the noise signal changes between 0.025 and 5. Let the plant be:

$$\begin{cases}
\boldsymbol{x}(k+1) = 2\boldsymbol{x}(k) + \boldsymbol{u}(k),\\
\boldsymbol{y}(k) = \boldsymbol{x}(k),\\
\boldsymbol{u}(k) = 2(\boldsymbol{r}(k) - \hat{\boldsymbol{y}}(k)).
\end{cases}$$

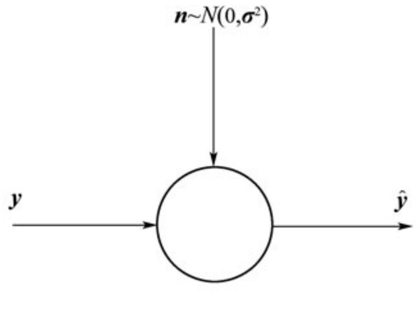

Fig. 5 AWGN channel scheme.

In this example, we can actually measure the mutual information rate between the reference and the feedback signal for different SNR values and monitor the upperbound $C_f - \log_2(a)$ given in Lemma 4. We use previous results from [12] to measure the mutual information rate, $I_\infty(\boldsymbol{r};\hat{\boldsymbol{y}})$, and results from [15] to design a controller. Since the system is linear and all inputs are white Gaussian processes, the output $\hat{\boldsymbol{y}}$ is also a Gaussian process. From [12], we know that if $\boldsymbol{r}$ and $\hat{\boldsymbol{y}}$ are two jointly Gaussian stationary processes, with spectral densities $\Phi_r(\omega)$ and $\Phi_{\hat{\boldsymbol{y}}}(\omega)$, and if we define $w = \begin{bmatrix}\boldsymbol{r}\\ \hat{\boldsymbol{y}}\end{bmatrix}$, with spectral density $\Phi_w(\omega)$, the mutual information rate of $\boldsymbol{r}$ and $\hat{\boldsymbol{y}}$ is given by

$$I_\infty(\boldsymbol{r};\hat{\boldsymbol{y}}) = \frac{1}{4\pi}\int_{-\pi}^{\pi}\frac{\det(\Phi_r(\omega))\det(\Phi_{\hat{y}}(\omega))}{\det(\Phi_w(\omega))}\mathrm{d}\omega. \quad (12)$$

Fig.6 illustrates that we obtain the expected result. The mutual information rate tends to zero for low SNR, and for this particular case, it reaches its upper bound, i.e., $C_f - \log_2(a)$, for high SNR. We see that this upper bound never reaches a value of zero (actually, for a SNR of 0, its value is 0.61 bits/time). We conclude, however, that the bound for good tracking, as measured by $I_\infty(\boldsymbol{r};\hat{\boldsymbol{y}})$, is higher than the one for stabilization.

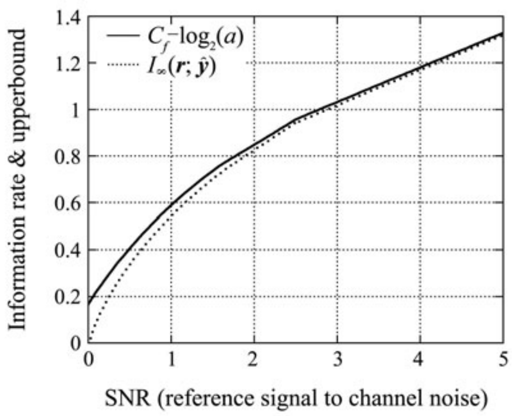

Fig. 6 Example with AWGN channel for different SNR levels.

9 A misleading case: nonminimum phase zeros

The mutual information rate $I_\infty(\boldsymbol{r},\hat{\boldsymbol{y}})$ between the reference signal $\boldsymbol{r}(k)$ and the feedback signal $\hat{\boldsymbol{y}}(k)$ has been our performance measure in the previous sections of this article. Although it may be adequate to determine the relationship between channel capacity, unstable poles, and the possibility of achieving tracking, $I_\infty(\boldsymbol{r},\hat{\boldsymbol{y}})$ is limited in predicting other important properties.

In order to illustrate the limitations of $I_\infty(\boldsymbol{r};\hat{\boldsymbol{y}})$, we choose an AWGW channel. Let us consider the same LTI plant $P(z)$ as before, and let us restrict the controller to a linear time-invariant controller $C(z)$. We assume that the open loop transfer function is given by

$$C(z)P(z) = \gamma\frac{\prod\limits_{i=1}^{n_z}(z-z_i)}{\prod\limits_{i=1}^{n_p}(z-p_i)}.$$

Since we consider an AWGN channel and if we assume $\boldsymbol{r}(k)$ to be a Gaussian signal, $\boldsymbol{r}(k)$ and $\hat{\boldsymbol{y}}(k)$ are jointly Gaussian, we can then evaluate the mutual information rate exactly using equation (12). We start with the following relation $\hat{\boldsymbol{y}} = T(\mathrm{e}^{i\omega})\boldsymbol{r} + S(\mathrm{e}^{i\omega})\boldsymbol{n}$, where $T(\mathrm{e}^{i\omega})$ is the complementary sensitivity function, and $S(\mathrm{e}^{i\omega})$ is the sensitivity function. Let $w = \begin{bmatrix}\boldsymbol{r}\\ \hat{\boldsymbol{y}}\end{bmatrix}$, then $\Phi_w = \Phi_r\Phi_{\hat{y}} - \Phi_{r\hat{y}}\Phi_{\hat{y}r}$ so that

$$\Phi_{\hat{y}} = |T|^2\Phi_r + |S|^2\Phi_n,$$
$$\Phi_w = \Phi_r\Phi_n|S|^2.$$

Substituting these relations in equation (12), we obtain

$$\begin{aligned}&I_\infty(\boldsymbol{r};\hat{\boldsymbol{y}})\\ &= \frac{1}{4\pi}\int_{-\pi}^{\pi}\log_2\Big(\frac{|T(\mathrm{e}^{iw}|^2\Phi_r + |S(\mathrm{e}^{iw})|^2\Phi_n}{\Phi_n|S(\mathrm{e}^{iw})|^2}\Big)\mathrm{d}\omega\\ &= \frac{1}{4\pi}\int_{-\pi}^{\pi}\log_2\Big(\frac{\Phi_r}{\Phi_n}|C(\mathrm{e}^{iw})P(\mathrm{e}^{iw})|^2+1\Big)\mathrm{d}\omega, \quad (13)\end{aligned}$$

where

$$|C(z)P(z)|^2 = \Bigg|\gamma\frac{\prod\limits_{i=1}^{n_z}(z-z_{pi})}{\prod\limits_{i=1}^{n_p}(z-p_{pi})}\Bigg|^2.$$

Now, from equation (13) and using the properties of logarithms, we have

$$\begin{aligned}I_\infty(\boldsymbol{r};\hat{\boldsymbol{y}}) &= \frac{1}{4\pi}\int_{-\pi}^{\pi}\log_2\Big(\frac{\Phi_r}{\Phi_n}|C(\mathrm{e}^{iw})P(\mathrm{e}^{iw})|^2+1\Big)\mathrm{d}\omega\\ &\geqslant \frac{1}{4\pi}\int_{-\pi}^{\pi}\log_2\Big(\frac{\Phi_r}{\Phi_n}|C(\mathrm{e}^{iw})P(\mathrm{e}^{iw})|^2\Big)\mathrm{d}\omega\\ &= \frac{1}{4\pi}\int_{-\pi}^{\pi}\log_2\Big(\frac{\Phi_r}{\Phi_n}\Big)\mathrm{d}\omega\\ &\quad+\frac{1}{4\pi}\int_{-\pi}^{\pi}\log_2\Big(|C(\mathrm{e}^{iw})P(\mathrm{e}^{iw})|^2\Big)\mathrm{d}\omega\\ &= \log_2(|\gamma|) + \frac{1}{4\pi}\int_{-\pi}^{\pi}\log_2\Big(\frac{\Phi_r}{\Phi_n}\Big)\mathrm{d}\omega\\ &\quad+\frac{1}{4\pi}\Big(\sum_{i=1}^{n_z}\int_{-\pi}^{\pi}\log_2|z-z_i|^2\mathrm{d}\omega\\ &\quad-\sum_{i=1}^{n_p}\int_{-\pi}^{\pi}\log_2|z-p_i|^2\mathrm{d}\omega\Big).\end{aligned}$$

From complex variable Calculus, we have the following result:

$$\int_{-\pi}^{\pi}\log_2|z-p|^2\mathrm{d}\omega = \begin{cases}0, & \text{if } |p|\leqslant 1,\\ 2\pi\log_2(p^2), & \text{if } |p|>1.\end{cases}$$

Finally, we obtain the following lower bound for the mutual information rate:

$$I_\infty(\boldsymbol{r};\hat{\boldsymbol{y}}) > \log_2(|\gamma|) + \frac{1}{4\pi}\int_{-\pi}^{\pi}\log_2\left(\frac{\Phi_r}{\Phi_n}\right)\mathrm{d}\omega + \sum_{i=1}^{n_z}\log_2(|z_i|) - \sum_{i=1}^{n_p}\log_2(|p_i|).$$

We note that the right-hand side contains a signal-to-noise ratio term, a gain term, a term that corresponds to the unstable open loop poles, and one that corresponds to the open-loop unstable zeros. We note first as expected that the greater the signal-to-noise ratio is, the greater the mutual information rate between the reference and the output signal. Second, we note that the unstable open-loop poles decrease the mutual information rate. Finally, we note that the nonminimum phase zero term increases the mutual information rate. This is unexpected since we know from control theory that the presence of nonminimum phase zeros decreases the performance of a tracking systems; therefore, it seems that we reach a contradiction.

We have another interpretation to this issue. Since the unstable poles decreases the information flow, the presence of the unstable zeros can help to cancel this effect (with perfect zero-pole cancelation). From control theory, we now that this is not an option if we want to preserve internal stability. However, this issue was not consider in the analysis, i.e., the only analysis of the mutual information rate is not enough when designing a tracking feedback system and we see that it could be misleading.

10 Tracking under the presence of disturbances

10.1 Upper bound of the information flow in the presence of disturbance

Let us suppose that a disturbance is present at the sensor and that the disturbance $\boldsymbol{d}^k$ is independent of $\boldsymbol{x}(0)$ and of $\boldsymbol{r}^k$. The new diagram is shown in Fig.7.

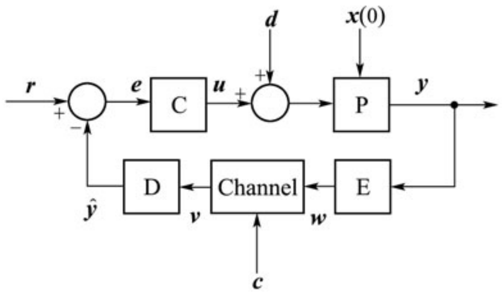

Fig. 7 Closed-loop system with additive disturbance.

We try next to find conditions for tracking. We first redefine the feedback capacity in this new setup. Recall that the feedback capacity is the quantity C_f that satisfies

$$\sup_{k\in\mathbb{N}_+}\frac{I((\boldsymbol{r}^k,\boldsymbol{d}^k,\boldsymbol{x}(0));\hat{\boldsymbol{y}}^k)}{k} \leqslant C_f.$$

If we expand the quantity $I((\boldsymbol{r}^k,\boldsymbol{d}^k,\boldsymbol{x}(0));\hat{\boldsymbol{y}}^k)$, by Proposition 1 h), we obtain

$$I((\boldsymbol{r}^k,\boldsymbol{d}^k,\boldsymbol{x}(0));\hat{\boldsymbol{y}}^k) = I(\boldsymbol{r}^k;\hat{\boldsymbol{y}}^k) + I(\boldsymbol{x}(0);\hat{\boldsymbol{y}}^k|\boldsymbol{r}^k) + I(\boldsymbol{d}^k;\hat{\boldsymbol{y}}^k|\boldsymbol{x}(0),\boldsymbol{r}^k). \quad (14)$$

Let us focus on $I(\boldsymbol{d}^k;\hat{\boldsymbol{y}}^k|\boldsymbol{x}(0),\boldsymbol{r}^k)$ to obtain

$$\begin{aligned} & I(\boldsymbol{d}^k;\hat{\boldsymbol{y}}^k|\boldsymbol{x}(0),\boldsymbol{r}^k) \\ &= h(\boldsymbol{d}^k|\boldsymbol{x}(0),\boldsymbol{r}^k) - h(\boldsymbol{d}^k|\boldsymbol{x}(0),\boldsymbol{r}^k,\hat{\boldsymbol{y}}^k); \\ &= h(\boldsymbol{d}^k) - h(\boldsymbol{d}^k|\boldsymbol{x}(0),\boldsymbol{r}^k,\hat{\boldsymbol{y}}^k); \quad (15) \\ &\geqslant h(\boldsymbol{d}^k) - h(\boldsymbol{d}^k|\hat{\boldsymbol{y}}^k); \quad (16) \\ &= I(\boldsymbol{d}^k;\hat{\boldsymbol{y}}^k); \quad (17) \end{aligned}$$

where equations (15) and (16) are due to Proposition 1, and equation (17) results form the mutual information definition. We showed in equation (7) that $I(\boldsymbol{x}(0);\hat{\boldsymbol{y}}^k|\boldsymbol{r}^k) = I(\boldsymbol{x}(0);\boldsymbol{e}^k|\boldsymbol{r}^k)$.

If we revisit Lemma 2's proof, we see that the lemma holds even with disturbances. Therefore, $I(\boldsymbol{x}(0);\boldsymbol{e}^k|\boldsymbol{r}^k) \geqslant k\sum_i \log_2(|\lambda_i(A)|)$. Moreover, from the definition of feedback capacity, we know that

$$kC_f \geqslant I((\boldsymbol{r}^k,\boldsymbol{d}^k,\boldsymbol{x}(0));\hat{\boldsymbol{y}}^k),$$

then, from equation (14), we obtain

$$kC_f - k\sum_i \log_2(|\lambda_i(A)|) \geqslant I(\boldsymbol{r}^k;\hat{\boldsymbol{y}}^k) + I(\boldsymbol{d}^k;\hat{\boldsymbol{y}}^k).$$

If we divide by k and take the limit as $k\to\infty$, we finally have $I_\infty(\boldsymbol{r};\hat{\boldsymbol{y}}) + I_\infty(\boldsymbol{d};\hat{\boldsymbol{y}}) \leqslant C_f - \sum_i \log_2(|\lambda_i(A)|)$. This result may be summarized in the following theorem:

Theorem 1 Consider the closed-loop system given in Fig.1, where the plant is a DLTI system described by equations (1) and (2), a feedback capacity C_f in the channel. If $E\{\boldsymbol{\epsilon}(k)\boldsymbol{\epsilon}(k)^{\mathrm{T}}\} < \infty$, then

$$I_\infty(\boldsymbol{r};\hat{\boldsymbol{y}}) + I_\infty(\boldsymbol{d};\hat{\boldsymbol{y}}) \leqslant C_f - \sum_i \log_2(|\lambda_i(A)|).$$

From this result, we can see that if $I_\infty(\boldsymbol{d};\hat{\boldsymbol{y}})$ is large enough, compared with $C_f - \sum_i \log_2(|\lambda_i(A)|)$, no useful information about the reference would appear in the feedback, since the inequality may also be interpreted as $I_\infty(\boldsymbol{r};\hat{\boldsymbol{y}}) \leqslant C_f - \sum_i \log_2(|\lambda_i(A)|) - I_\infty(\boldsymbol{d};\hat{\boldsymbol{y}})$. Similarly, $I_\infty(\boldsymbol{d};\hat{\boldsymbol{y}}) \leqslant C_f - \sum_i \log_2(|\lambda_i(A)|) - I_\infty(\boldsymbol{r};\hat{\boldsymbol{y}})$. If $I_\infty(\boldsymbol{r};\hat{\boldsymbol{y}})$ is large enough, compared with $C_f - \sum_i \log_2(|\lambda_i(A)|)$, no useful information about the disturbance would appear in the feedback.

10.2 Disturbance rejection and tracking tradeoff

The previous section is concluded with Theorem 1. The goal of this section is to interpret Theorem 1 in the frequency domain. For this purpose, we assume that the following conditions hold:

- The signals $\boldsymbol{r}$ and $\boldsymbol{d}$ are Gaussian.
- The signals $\boldsymbol{r}$ and $\boldsymbol{e}$ are jointly asymptotically stationary.
- The signals $\boldsymbol{d}$ and $\boldsymbol{e}$ are jointly asymptotically stationary.

These conditions are needed to replace the stochastic processes by their corresponding asymptotic power spectra.

Next, we start with the definition of the mutual information between $\boldsymbol{r}$ and $\hat{\boldsymbol{y}}$:

$$\begin{aligned} I(\boldsymbol{r}^k;\hat{\boldsymbol{y}}^k) &= h(\boldsymbol{r}^k) - h(\boldsymbol{r}^k|\hat{\boldsymbol{y}}^k); \\ &= h(\boldsymbol{r}^k) - h(\boldsymbol{e}^k|\hat{\boldsymbol{y}}^k); \quad (18) \end{aligned}$$

$$\geqslant h(\boldsymbol{r}^k) - h(\boldsymbol{e}^k); \tag{19}$$

where equation (18) is due to the fact that $\boldsymbol{e} = \boldsymbol{r} - \hat{\boldsymbol{y}}$ and Proposition 1 e), whereas equation (19) is due to Proposition 1 a). If we divide by k and let $k \to \infty$, we obtain

$$\begin{aligned} I_\infty(\boldsymbol{r};\hat{\boldsymbol{y}}) &\geqslant h_\infty(\boldsymbol{r}) - h_\infty(\boldsymbol{e}) \\ &\geqslant \frac{1}{4\pi}\int_{-\pi}^{\pi}\log_2\left(2\pi e\widehat{\Phi}_r\right)\mathrm{d}\omega \\ &\quad -\frac{1}{4\pi}\int_{-\pi}^{\pi}\log_2\left(2\pi e\widehat{\Phi}_e\right)\mathrm{d}\omega; \qquad (20) \\ &= \frac{1}{4\pi}\int_{-\pi}^{\pi}\log_2\left(\frac{\widehat{\Phi}_r}{\widehat{\Phi}_e}\right)\mathrm{d}\omega; \qquad (21) \end{aligned}$$

where equation (20) is due to Proposition 1 k). Changing the sign in inequality (21), we get $-I_\infty(\boldsymbol{r};\hat{\boldsymbol{y}}) \leqslant \frac{1}{4\pi}\int_{-\pi}^{\pi}\log_2\left(\frac{\widehat{\Phi}_e}{\widehat{\Phi}_r}\right)\mathrm{d}\omega$. Then, using the inequality of Theorem 1, we obtain

$$I_\infty(\boldsymbol{d};\hat{\boldsymbol{y}}) \leqslant C_f - \sum\log(\lambda) + \frac{1}{4\pi}\int_{-\pi}^{\pi}\log_2\left(\frac{\widehat{\Phi}_e}{\widehat{\Phi}_r}\right)\mathrm{d}\omega. \tag{22}$$

In Fig.8, we group together the blocks enclosed within the dashed line (block named $\boldsymbol{K}$). By doing so, we obtain the same block diagram that was exposed in [8]. We notice that some of the internal signals are labeled differently: in Fig.8, the signals $\boldsymbol{u}$ and $\hat{\boldsymbol{u}}$ correspond to signals $\boldsymbol{z}$ and $\boldsymbol{e}$ in [8], respectively. According to [8], we know that $I_\infty(\boldsymbol{d};\hat{\boldsymbol{u}})$ is related to a disturbance rejection measure as follows:

$$\frac{1}{2\pi}\int_{-\pi}^{\pi}\min\{0,\log_2(S_{\hat{u},d}(\omega))\}\mathrm{d}\omega \geqslant -I_\infty(\boldsymbol{d};\boldsymbol{u}),$$

where $S_{\hat{u},d}(\omega) = \sqrt{\frac{\widehat{\Phi}_{\hat{u}}}{\widehat{\Phi}_d}}$. We note that the smaller the term $\frac{1}{2\pi}\int_{-\pi}^{\pi}\min\{0,\log_2(S_{\hat{u},d}(\omega))\}\mathrm{d}\omega$ is, the better is the disturbance rejection. From Proposition 1 g), we know that $I_\infty(\boldsymbol{d};\boldsymbol{u}) \leqslant I_\infty(\boldsymbol{d};\hat{\boldsymbol{y}})$. Substituting this expression in equation (22), we obtain

$$\begin{aligned} &\frac{1}{2\pi}\int_{-\pi}^{\pi}\min\{0,\log_2(S_{\hat{u},d}(\omega))\}\mathrm{d}\omega \\ &\geqslant \sum_{\lambda(A)}\log_2(\lambda) - C_f - \frac{1}{4\pi}\int_{-\pi}^{\pi}\log_2\left(\frac{\widehat{\Phi}_e}{\widehat{\Phi}_r}\right)\mathrm{d}\omega. \end{aligned}$$

We summarize this result in the following theorem.

Theorem 2 Consider the closed-loop system given in Fig.1, where the plant is a linear system described by equations (1) and (2), which is a feedback capacity C_f in the channel. If $E\{\boldsymbol{\epsilon}(k)\boldsymbol{\epsilon}(k)^{\mathrm{T}}\} < \infty$, $\boldsymbol{r}$ and $\boldsymbol{d}$ are Gaussian signals, $\boldsymbol{r}$ and $\boldsymbol{e}$ are jointly asymptotically stationary, and $\boldsymbol{d}$ and $\boldsymbol{e}$ are jointly asymptotically stationary, then

$$\begin{aligned} &\frac{1}{2\pi}\int_{-\pi}^{\pi}\min\{0,\log_2(S_{\hat{u},d}(\omega))\}\mathrm{d}\omega \\ &\geqslant \sum_{\lambda(A)}\log_2(\lambda) - C_f - \frac{1}{4\pi}\int_{-\pi}^{\pi}\log_2\left(\frac{\widehat{\Phi}_e}{\widehat{\Phi}_r}\right)\mathrm{d}\omega, \qquad (23) \end{aligned}$$

where $S_{\hat{u},d}(\omega) = \sqrt{\frac{\widehat{\Phi}_{\hat{u}}}{\widehat{\Phi}_d}}$ is a sensitivity-like function, $\widehat{\Phi}_{\hat{u}}$, $\widehat{\Phi}_d$, $\widehat{\Phi}_e$, and $\widehat{\Phi}_r$ are the asymptotical power spectrum densities of the signals $\hat{\boldsymbol{u}}$, $\boldsymbol{d}$, $\boldsymbol{e}$, and $\boldsymbol{r}$, respectively.

We therefore observe that good tracking, formally defined as being $\widehat{\Phi}_e$ near zero, implies $\log_2\left(\frac{\widehat{\Phi}_e}{\widehat{\Phi}_r}\right)$ to be negative and the whole integral term in inequality (23) positive. Therefore, the lower bound will be larger than the one where no tracking is required. In other words, if we improve tracking performance, we loose the information between the disturbance $\boldsymbol{d}$ and the feedback signal $\hat{\boldsymbol{y}}$, and the disturbance can no longer be rejected.

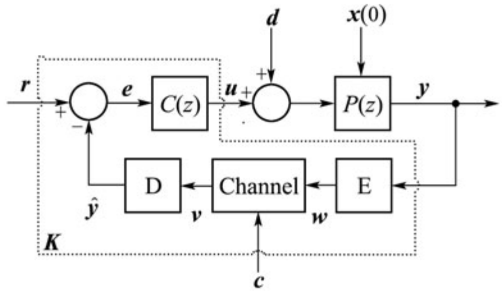

Fig. 8 Tracking closed-loop.

11 Conclusions

This work has provided information theoretic conditions for tracking control systems. Our results are in terms of the mutual information rate between the feedback signal and the reference signal, the channel capacity, and the unstable eigenvalues of the DLTI system. We also obtained a lower bound for the maximum achievable accuracy for a tracking system, even in the absence of a channel. This bound is in terms of the entropy of the reference signal. These results were verified with several examples and simulations.

We also reported some limitations of the mutual information rate approach. In particular, we analyzed the case where nonminimum phase zeros, counterintuitively, increase the mutual information rate instead of decreasing it, as expected from the control theory.

Finally, we analyzed the case where both good tracking and good disturbance rejection are required at the same time. We noted that the finite-capacity channel imposes a tradeoff between the two objectives. This limitation was interpreted in the frequency domain.

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Ivan Lopez HURTADO received the B.S. degree in Industrial Physics Engineering with Summa Cum Laude Honors and M.S. degree in Automation with Summa Cum Laude Honors from Monterrey Institute of Technology (Mexico) in 1995 and 1998, respectively. He received a Ph.D. degree in Electrical Engineering from the University of New Mexico in 2008. He is currently an assistant professor at the Department of Engineering at Northern New Mexico College, Espanola, NM. Dr. Lopez is a member of IEEE, Sigma Xi and Phi Kappa Phi. He has served as a guest editor of the International Journal of Robust and Nonlinear and Adaptive Control. His current research interests include information theory of networked-control systems, control under communication constraints and genetic programming applications. E-mail: ilopez@nnmc.edu.

Chaouki T. ABDALLAH started his college education at the Ecole Suprieure dIngnieurs de Beyrouth - Universit Saint-Joseph in Beirut, Lebanon, but finished his undergraduate studies at Youngstown State University, with a Bachelor's of Engineering degree in Electrical Engineering in 1981. He then obtained his M.S. and Ph.D. degrees in Electrical Engineering from GA Tech in 1982, and 1988, respectively. He joined the Electrical and Computer Engineering Department at the University of New Mexico where he is currently a professor and the chair. Professor Abdallah conducts research and teaches courses in the general area of systems theory with focus on control and communications systems. His research has been funded by national funding agencies, national laboratories, and by various companies. He has also been active in designing and implementing various international graduate programs with Latin American and European countries. He has published 7 books, and more than 300 peer-reviewed papers. E-mail: chaouki@ece.unm.edu.

Sudharman K. JAYAWEERA received the B.E. degree in Electrical and Electronic Engineering with First Class Honors from the University of Melbourne, Australia, in 1997 and M.A. and Ph.D. degrees in Electrical Engineering from Princeton University in 2001 and 2003, respectively. He is currently an assistant professor in Electrical Engineering at the Department of Electrical and Computer Engineering at University of New Mexico, Albuquerque, NM. From 2003-2006, he was an assistant professor in Electrical Engineering at the ECE Department of Wichita State University. Dr. Jayaweera is a senior member of IEEE and currently serves as an associate editor of EURASIP Journal on Advances in Signal Processing. Among his honors are two best paper awards at the IEEE international conferences and a faculty fellowship at the AFRL. His current research interests include cooperative and cognitive communications, information theory of networked-control systems, statistical signal processing and wireless sensor networks. E-mail: jayaweera@ece.unm.edu.