SENSOR SYSTEM OPTIMIZATION FOR BAYESIAN FUSION OF DISTRIBUTED STOCHASTIC SIGNALS UNDER RESOURCE CONSTRAINTS

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ABSTRACT

Recently there has been a significant interest in distributed detection and data fusion with analog-relay amplifier local processing under a global power constraint [1–3]. In particular, it was shown in [3] that the optimal fusion performance for a distributed stochastic signal detection is achieved by a finite number of sensors. In this paper, we propose a sensor system optimization method based on the Bhattachrya error exponent. In addition to the global power constraint we also consider the case in which the total available bandwidth may also be limited. Assuming an equi-correlated signalling model we derive the error exponents to the Bayesian fusion performance for asymptotically large systems. Again we optimize the sensor system size based on the Bhattacharya error exponent and provide simple rules that are valid for either the low or high observation SNR regimes.

1. INTRODUCTION

Fuelled by various applications of low-power wireless sensor networks, recently there has been a growing interest in design and analysis of distributed detection systems under sensor-to-fusion center communication constraints. In this paper, we consider a sensor system subjected to both finite global bandwidth as well as power constraints such that as the number of nodes in the system increases the available power per node linearly decreases. Distributed detection of a deterministic and a stochastic Gaussian signal in such a network was previously considered in [1–4] and [5], respectively. Though not the optimal, all of them confined the local processing to the special case of analog relay amplifier processing. This greatly facilitates the analysis and can also give useful insight into the performance with more general quantized local decision schemes. Analog relay processing has also shown to perform very well in the presence of additive noise and is well-suited for low-power sensor networks.Thus, in this paper we also consider the case of analog processing at the sensor nodes.

It was shown in [2] and [3], respectively, that the fusion performance of distributed detection of a *deterministic* signal in a global power-constrained sensor system monotonically improves with increasing system size under both orthogonal and non-orthogonal sensor-to-fusion center communications. In other words, it is always better to divide the available total power among as many sensor nodes as possible. In contrast, [5] showed that this is no longer true if the signal to be detected is a stochastic (Gaussian) signal. In particular, assuming orthogonal signalling [5] showed that the Bayesian fusion probability of error is minimized by a finite number of nodes n_0 for any given total power constraint. Beyond the optimal value n_0, any attempt to include more nodes to the system will degrade the performance.

In this paper we extend previous work mentioned above, and in particular the results of [5]. First, we propose a system optimization method to obtain the optimal number of sensor nodes that results in the best Bayesian fusion performance. Second, we generalize the situation considered in [5] by also taking into bandwidth constraints. Assuming that sensor-to-fusion center communication is non-orthogonal we derive the fusion performance under a global system power constraint. We show that the performance is still optimized by a finite number of sensors and obtain simple expressions for the optimal number of sensors that are valid for either the low or high observation SNR regimes.

The remainder of the paper is organized as follows: In Section 2 we present our sensor system model and derive the optimal fusion detector for a stochastic Gaussian signal. In Section 3 we investigate the fusion performance via error exponents, derive expressions for the optimal sensor system size n_0 and provide numerical examples. Section 4 concludes the paper.

2. SYSTEM MODEL AND OPTIMAL FUSION

A binary hypothesis testing problem in an n-node sensor system is assumed. The null and alternative hypotheses are denoted by H_0 and H_1, respectively, having corresponding prior probabilities $P(H_0) = \pi_0$ and $P(H_1) = \pi_1$. Under the alternative hypothesis the observed stochastic process consists of a Gaussian signal, denoted by X_k for $k = 1, \cdots n$, corrupted by additive Gaussian noise. The k-th node observation z_k can be written as

$$\begin{aligned} H_0: \quad z_k &= v_k \\ H_1: \quad z_k &= X_k + v_k, \end{aligned} \tag{1}$$

where the collection of observation noise samples and the collection of desired signal samples are distributed as $\mathbf{v} \sim \mathcal{N}(\mathbf{0}, \Sigma_v)$ and $\mathbf{X} \sim \mathcal{N}(\mathbf{0}, \Sigma_x)$, respectively, and $\mathbf{0}$ denotes the n-vector of all zeros. In this paper, we concentrate on the case in which both v_k's and X_k's are independent and identically distributed (iid) sequences so that $\Sigma_v = \sigma_v^2 \mathbf{I}$ and $\Sigma_x = \sigma_x^2 \mathbf{I}$ where $\mathbf{I}$ is the $n \times n$ identity matrix.

Each local sensor processes its observation z_k independently to generate a local decision $u_k(z_k)$ which are sent to the fusion center over a noisy, bandlimited wireless channel. Let us denote by $\mathbf{r}(u_1(z_1), u_2(z_2), \cdots, u_n(z_n))$ the received signal at the fusion center. The fusion center makes a final decision based on the decision rule $u_0(\mathbf{r})$. In general, the distributed detection and fusion prob-

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lem involves simultaneous optimization for both local and global (i.e. fusion) decision rules $u_1(z_1), u_2(z_2), \cdots, u_n(z_n), u_0(\mathbf{r})$. However, a class of important local processers are the amplify-and-relay schemes in which each node amplifies and retransmits its observation to the fusion center such that

$$u_k = g_k z_k \quad \text{for } k = 1, \cdots n$$

where $g_k > 0$ is the analog relay amplifier gain at the k-th node that depends on the total average power constraint P_0 on the whole sensor system. For simplicity, throughout this paper we assume $g_k = g$ for all k. (The issue of (distributed) power allocation will be considered in a future work). With this assumption, the amplifier gain g is given by $g^2 = \frac{P_0}{n\left(\sigma_v^2 + \frac{\sigma_x^2}{2}\right)}$. Note that, the available power per node linearly decreases as more nodes are introduced into the system. We define the observation quality and channel quality signal-to-noise ratios (SNR) as $\gamma_0 \triangleq \frac{\sigma_x^2}{\sigma_v^2}$ and $\gamma_c \triangleq \frac{P_0}{\sigma_w^2}$, respectively.

A sufficient statistic at the fusion center is given by the n-dimensional, matched filter output that can be written as

$$\mathbf{r} = g\mathbf{R}\mathbf{z} + \mathbf{w} \tag{2}$$

where $\mathbf{w} \sim \mathcal{N}(\mathbf{0}, \sigma_w^2 \mathbf{R})$ is the receiver noise and $\mathbf{R}$ is the $n \times n$, symmetric and normalized received signal correlation matrix in which the (k, k')-th element represents the correlation between the signalling waveforms of nodes k and k'.

Using spectral decomposition of $\mathbf{R}$ it can be shown (we omit the details due to space limitations) that the optimal fusion rule is given by

$$\delta_{opt}(\mathbf{r}) = \left\{ \begin{array}{lll} 1 & & \geq \\ & \text{if} \quad T(\mathbf{r}) & \quad \tau' \\ 0 & & < \end{array} \right. , \tag{3}$$

where $\tau' = 2\log\tau + \sum_{k=1}^{n} \log\left(\frac{g^2(\sigma_x^2+\sigma_v^2)\lambda_k + \sigma_w^2}{g^2\sigma_v^2\lambda_k + \sigma_w^2}\right)$, the decision variable $T(.)$ is the quadratic form $T(\mathbf{r}) = \sum_{k=1}^{n} |y_k|^2$, λ_k's are the eigenvalues of matric $\mathbf{R}$, and $Y_1, \cdots, Y_n$ are a set of independent, zero-mean, Gaussian random variables. The variance $\sigma_{j,k}^2$ of the k-th sample Y_k under H_j can shown to be

$$\sigma_{j,k}^2 = \left\{ \begin{array}{ll} \frac{g^2\sigma_x^2\lambda_k}{g^2(\sigma_x^2+\sigma_v^2)\lambda_k + \sigma_w^2} & \text{if } j = 0 \\ \frac{g^2\sigma_x^2\lambda_k}{g^2\sigma_v^2\lambda_k + \sigma_w^2} & \text{if } j = 1 \end{array} \right. . \tag{4}$$

In this paper we consider Bayesian optimal fusion detectors. Thus our basic performance criteria is the probability of fusion error denoted as P_e. While only in very special circumstances one can evaluate the exact probability of error P_e of (3), even in those cases it may be in terms of special functions that might require numerical computations [5]. As a result, they may not give much insight into the design of decentralized sensor systems. On the other hand, while not exact, error exponents (and the bounds based on them) can be very useful in characterizing the performance of a detection procedure in most situations. The most commonly used bound for Bayesian detection is the Chernoff bound given by (with equal priors) $P_e \leq \frac{1}{2}e^{\mu_C}$ where Chernoff error exponent is defined as

$$\mu_C = \min_{s\in[0,1]} \log \mathbb{E}\left\{\mathcal{L}^s(\mathbf{r})|H_0\right\}. \tag{5}$$

Although somewhat loose than the Chernoff bound a much easier to evaluate is the so-called Bhattacharya upper bound $P_e \leq \frac{1}{2}e^{\mu_B}$ where Bhattacharya error exponent μ_B is defined as [6]

$$\mu_B = \log \mathbb{E}\left\{\mathcal{L}^{\frac{1}{2}}(\mathbf{r})|H_0\right\}. \tag{6}$$

3. FUSION PERFORMANCE AND OPTIMAL SENSOR SYSTEM DESIGN

3.1. Orthogonal Signalling

In this case we have that $\mathbf{R} = \mathbf{I}$ such that the fusion error probability can be determined exactly as was given in [5]. The resulting expression is, however, difficult to optimize over the sensor system size to obtain a useful design rule. As a solution, in this paper we resort to the error exponents. In particular, we have the following:

Proposition 1 *The Chernoff and Bhattacharya error exponents with orthogonal signalling, μ_C^0 and μ_B^0 respectively, corresponding to the Bayesian fusion performance are given by*

$$\mu_C^0 = \frac{n}{2}\left[\log\frac{1+\sigma_1^2}{1+(1-s_0)\sigma_1^2} - s_0\log\left(1+\sigma_1^2\right)\right], \tag{7}$$

and

$$\mu_B^0 = \frac{n}{2}\left[\frac{1}{2}\log\left(1+\sigma_1^2\right) - \log\left(1+\frac{\sigma_1^2}{2}\right)\right], \tag{8}$$

where $\sigma_1^2 = \frac{\gamma_0}{1+\frac{1}{\bar{\gamma}_c}}$, $s_0 = 1 + \frac{1}{\sigma_1^2} - \frac{1}{\log\left(1+\sigma_1^2\right)}$ in (7) and we have defined $\bar{\gamma}_c = \frac{\gamma_c}{n\left(1+\frac{\gamma_0}{2}\right)}$.

It can be shown that in most cases the upper bound to the performance in terms of the Bhattacharya error exponent is close to that with the Chernoff exponent. Due to its particularly simple structure, thus we propose to optimize the sensor systems based on the Bhattacharya error exponent. Note that, numerical examples show that for the range of SNR's that we consider the optimal $n = n_0$ with Bhattacharya error exponent exactly matches that for the exact fusion performance. This is due to the fact that even when the bound is some what loose the behavior of the exact performance and the Bhattachraya bound are almost the same. In particular, we can show that $\lim_{n\longrightarrow\infty} \mu_B = 0$. This indicates that the Bhattacharya upper bound to the error probability agrees with the conclusion $\lim_{n\longrightarrow\infty} P_e = 0.5$ shown in [5]. Following proposition summarizes the solution to the sensor system optimization problem in the case of orthogonal signalling under a global power constraint.

Proposition 2 *The optimal number of nodes n_0 that results in the tightest Bhattacharya upper bound to the fusion error probability in a distributed sensor system subjected to a global power constraint P_0 is given by,*

$$n_0 = \frac{\gamma_c}{\left(\frac{1}{2}+\frac{1}{\gamma_0}\right)}\left(\frac{1}{2x_0} - \frac{1}{\gamma_0}\right) \tag{9}$$

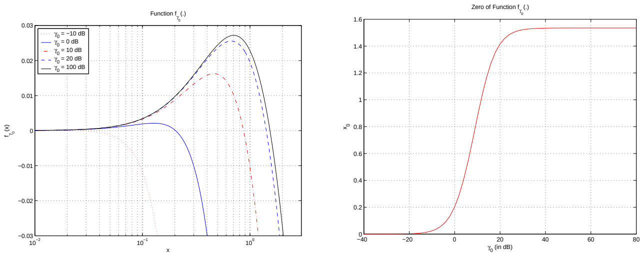
where $x = x_0$ is the unique positive solution to the equation $f_{\gamma_0}(x) = 0$ with

$$f_{\gamma_0}(x) = \log\frac{\sqrt{1+2x}}{1+x} + \left(1 - \frac{2x}{\gamma_0}\right)\frac{x^2}{(1+x)(1+2x)}.$$

The optimal number of sensors for a given total power constraint can be approximated as follows:

$$n_0 \approx \left\{ \begin{array}{ll} \frac{\gamma_c}{\tilde{x}_0} & \textit{if } \gamma_0 \gg 1 \\ \gamma_c & \textit{if } \gamma_0 \ll 1 \end{array} \right. , \tag{10}$$

where $\tilde{x}_0 \approx 1.535$ is the unique zero of the function $\tilde{f}(x) = \log\frac{\sqrt{1+2x}}{1+x} + \frac{x^2}{(1+x)(1+2x)}$.



(a) $f_{\gamma_0}(.)$ (b) Zero of $f_{\gamma_0}(.)$

Fig. 1. Sensor system optimization under a total power constraint. $\gamma_c = 20$ dB.

Proof 1 *Omitted (can be found in [7]).*

The function f_{γ_0} is shown on Fig. 1a as a parameterized plot. It is a well-behaved, smooth function with a unique zero. Moreover, as can be seen from Fig. 1b, for both very small and very large values of γ_0 the zero of f_{γ_0} converges to fixed limits. In Fig. 2 we have shown the optimal number of sensor nodes for distributed detection of a stochastic signal under a total power constraint obtained via the exact solution to the zero of f_{γ_0}. Figure 2 shows that indeed the asymptotic solutions given in (10) provide a good approximation except for a small range of values for the observation SNR γ_0.

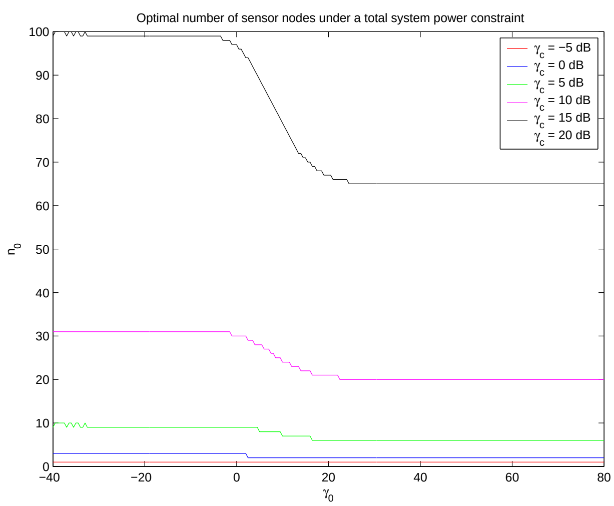


Fig. 2. Optimal sensor system size as a function of observation SNR for a given global power constraint with orthogonal sensor-to-fusion center communication.

3.2. Non-orthogonal Signalling

The validity of orthogonal signalling model may not be justified in a practical system due to various reasons, in particular when the system is subjected to a total bandwidth constraint. A commonly used non-orthogonal signalling model is the equi-correlation model in which correlation between any two different signalling waveforms is assumed to be the same, so that $[\mathbf{R}]_{k,k} = 1$ and $[\mathbf{R}]_{k,k'} = \rho$ for $k \neq k'$ where $|\rho| < 1$ is the common correlation between any pair of received signalling waveforms (in the following we will assume that $0 < \rho < 1$. The analysis for negative ρ follows easily). The eigenvalues of $\mathbf{R}$ can easily be shown to be $\lambda_1 = \lambda_2 = \cdots = \lambda_{n-1} = 1 - \rho \triangleq \lambda_a$ and $\lambda_n = 1 + (n-1)\rho \triangleq \lambda_b$. Note that, this single different eigenvalue makes the closed form analysis of the error probability significantly difficult and requires integration of a pdf involving the confluent hypergeometric function [5]. Let us denote $\sigma_{1,k}^2 = \sigma_{1,a}^2 = \frac{\gamma_0}{1+\frac{n}{(1-\rho)\tilde{\gamma}}}$, for $k = 1, \cdots, n-1$, and $\sigma_{1,n}^2 = \sigma_{1,b}^2 = \frac{\gamma_0}{1+\frac{1}{\left(\frac{1-\rho}{n}+\rho\right)\tilde{\gamma}}}$ under the hypothesis H_1, corresponding to the eigenvalues λ_a and λ_b. The Chernoff and Bhattacharya error exponents in this case is stated in the Proposition 3 at the top of next page.

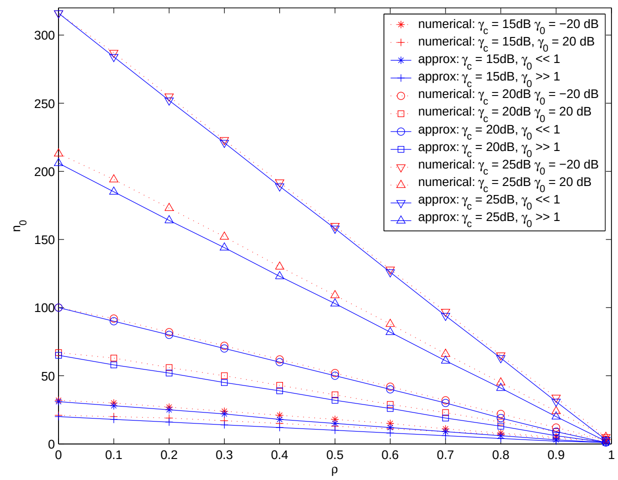


Fig. 3. Optimal sensor system size as a function of signalling correlations for a given global power constraint with non-orthogonal sensor-to-fusion center communication.

A plot of the (tightest) Chernoff and Bhattacharya error exponents shows that, as with individual node power constraints, the performance degrades as ρ increases. Moreover, it can be shown that there is an optimal number of sensor nodes that results in the lowest upper bound for each ρ. As before, we base our sensor system optimization on the Bharracharya error exponent. However, unlike in the case of orthogonal sensor-to-fusion center communication, a direct optimization of μ_B as a function of n does not, in general, yield a closed-form expression.

In Fig. 3 we have shown the numerically obtained optimal $n = n_0$ as a function of ρ for different γ_0 and γ_c values. As can be observed from Fig. 3, for a fixed γ_0 and γ_c, the optimal n_0 decreases as a function of ρ. In fact, an asymptotic expansion shows that optimal n_0 can be approximated as follows:

$$n_0 \approx \begin{cases} \frac{(1-\rho)\gamma_c}{\tilde{x}_0} & \text{if } \gamma_0 \gg 1 \\ (1-\rho)\gamma_c & \text{if } \gamma_0 \ll 1 \end{cases} . \quad (13)$$

where $\tilde{x}_0$ is the unique zero of the function $\tilde{f}(.)$ defined in Proposition 2. In Fig. 3 we have also included the above approximations to optimal n_0. Note that, for a fixed γ_c as ρ increases, the approximations in (13) worsen. It is interesting to note from (13) that $(1-\rho)\gamma_c$ essentially acts as the effective channel SNR. This shows that the effect of non-orthogonal signalling always degrades the final fusion performance. The larger the signalling waveform correlation ρ the more the performance will degrade as one would expect intuitively. Note that, in general, this is not the case in fusion of a deterministic signal with analog local processing as was shown in [8]. With a deterministic signal, at least when the system is perfectly synchronized

Proposition 3 *The Chernoff error exponent for Bayesian fusion performance in distributed stochastic Gaussian signal detection with equi-correlated sensor-to-fusion center signalling is*

$$\mu_C = \frac{n-1}{2}\left[(1-s_0)\log\left(1+\sigma_{1,a}^2\right) - \log\left(1+(1-s_0)\sigma_{1,a}^2\right)\right] + \frac{1}{2}\left[(1-s_0)\log\left(1+\sigma_{1,b}^2\right) - \log\left(1+(1-s_0)\sigma_{1,b}^2\right)\right],$$

where

$$s_0 = \begin{cases} \frac{1}{2}\left[\bar{s}_0\left(1+\sqrt{1+K_1}\right) - \left(1+\frac{1}{\sigma_{1,b}^2}\right)\left(\sqrt{1+K_1}-1\right)\right] & \text{if } \bar{s}_0 \leq 1+\frac{1}{\sigma_{1,b}^2} \\ \frac{1}{2}\left[\bar{s}_0\left(1-\sqrt{1+K_1}\right) + \left(1+\frac{1}{\sigma_{1,b}^2}\right)\left(\sqrt{1+K_1}+1\right)\right] & \text{otherwise} \end{cases}, \quad (11)$$

with $K_1 = \frac{4K_2}{K_3\left(1+\frac{1}{\sigma_{1,b}^2}-\bar{s}_0\right)^2}$, $K_2 = \frac{1}{\sigma_{1,a}^2} - \frac{1}{\sigma_{1,b}^2}$, $K_3 = (n-1)\log\left(1+\sigma_{1,a}^2\right) + \log\left(1+\sigma_{1,b}^2\right)$ *and* $\bar{s}_0 = 1+\frac{1}{\sigma_{1,a}^2} - \frac{n}{K_3}$. *The corresponding Bhattacharya error exponent can be written as*

$$\mu_B = \frac{n-1}{2}\left[\frac{1}{2}\log\left(1+\sigma_{1,a}^2\right) - \log\left(1+\frac{\sigma_{1,a}^2}{2}\right)\right] + \frac{1}{2}\left[\frac{1}{2}\log\left(1+\sigma_{1,b}^2\right) - \log\left(1+\frac{\sigma_{1,b}^2}{2}\right)\right]. \quad (12)$$

the non-zero ρ can in fact improve the fusion performance due to the beam-forming effect.

4. CONCLUSIONS

In this paper, we considered sensor system optimization problem for a distributed detection and fusion system under global power and bandwidth constraints. Assuming analog relay amplifier local processing we derived the error exponents to the fusion performance and proposed a method based on the Bhattacharya error exponent to obtain the optimal sensor system size. We have also shown simple approximations to the exact solutions that are valid for either the low or high observation SNR regimes and provide useful insight.

5. REFERENCES

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