Optimal Power Scheduling for Data Fusion in Inhomogeneous Wireless Sensor Networks

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Abstract

We consider the problem of optimal power scheduling for the decentralized detection of a deterministic signal in an inhomogeneous wireless sensor network. The observation noise at local sensors is assumed to be independently and identically distributed (i.i.d.) and at the local sensors each node performs analog-relay amplifier processing for its observation independently. The communication between the local sensors and the fusion center is assumed to be through an inhomogeneous channel. The optimal power scheduling scheme suggests that the sensors with poor observation quality and channels should be inactive in order to save total power expenditure of the system. For the remaining active sensors the optimal transmit power is determined jointly by the individual channel gains, total number of active sensors, local observation signal to noise ratio (SNR) and the required probability of error at the fusion center. We show that the optimal scheme can provide significant system power savings compared to the uniform power allocation scheme when the number of sensors in the system is large.

1 Introduction

Wireless sensor networks (WSN) are ideal for a wide variety of applications such as environmental monitoring, smart factory instrumentation, intelligent transportation and military surveillance [6] because of their low implementation cost, agility and robustness to sensor failures . Low power sensors are manufactured using low cost miniature sensing technologies together with widely available computing resources necessary to handle complex data. The use of wireless sensor networks in video surveillance systems is presented in [5]. It shows that such system takes the advantage of the redundant information coming from multiple sensors monitoring the same target and this information is fused together to obtain a more accurate estimate. They also showed that multisensor surveillance system outerperforms the single sensor system in object tracking. The use of WSN in surveillance systems is becoming popular due to its enhanced monitoring and control capabilities in large environments.

Typically, in a wireless sensor network (WSN) the sensor nodes are equipped with small batteries. The replacement of batteries may not be convenient due to cost of the equipment and difficulties in accessibility. This leads to power constrained wireless communication between sensor nodes and the fusion center. The optimal data fusion in a WSN is limited by system power, channel properties and other communication constraints such as available bandwidth. In [2], [3], the fusion performance of total power constrained sensor systems are considered. The problem of distributed detection under communication constraints has been considered in [1], [4]. Detection problems with a constraint on the expected cost arising from a measurement have been addressed by various authors. In [1], they considered the detection problem with constraints on the expected cost arising from transmission (from sensor nodes to a fusion node) and measurement (at each sensor node) to take into account some of the system level cost constraints in a sensor network.

In this paper we consider the optimal power allocation for decision fusion in an inhomogeneous sensor network. This work is motivated by [6], which considered the optimal power scheduling for decentralized estimation. We consider a WSN consisting of a fusion center and a large number of spatially separated sensors. The local sensor nodes collect observations, computes a local message and transmit it to the fusion center. The local observation noise is assumed to be independent and identically distributed. We also assume that each node performs analog-relay amplifier processing on its own observation. The locally processed data is sent by each node to the fusion center over a noisy wireless channel to make a final decision. The channel is assumed to be inhomogeneous. As we will show, according to the optimal scheme that conserve power of the whole system, the sensors with poor observation quality and bad channels become inactive while the rest of the sensors transmit locally processed data. We will show that when local signal-to-noise ratio is high a very small number of sensors should be active to achieve the required fusion error performance while a relatively large number of sensors should be active when the local signal-to-noise ratio is low. But still it has better performance in terms of system energy consumption than that of the uniform power allocation.

The remainder of this paper is organized as follows: Section 2 formulates the fusion problem. In Section 3 the optimal fusion performance is analyzed. Optimal power allocation scheme is given in Section 4. Section 5 presents the performance results and finally concluding remarks are given in Section 6.

2 Fusion Problem Formulation

We consider a binary hypothesis testing problem in an n-node distributed wireless sensor network. The k-th sensor observation under each hypothesis is given by,

$$\begin{aligned} H_0 : z_k &= v_k;\ k = 1, 2, ..., n \\ H_1 : z_k &= x_k + v_k;\ k = 1, 2, ..., n\ , \end{aligned} \tag{1}$$

where v_k is the observation noise and x_k is the signal to be detected. In vector notation (1) becomes, $\mathbf{z} = \mathbf{x} + \mathbf{v}$, where $\mathbf{v}$ is a zero mean Gaussian n-vector of observation noise samples with covariance matrix $\mathbf{\Sigma_v}$. We assume that v_k's are independently and identically distributed (i.i.d) so that $\mathbf{v} \sim \mathcal{N}(0, \sigma_v^2 \mathbf{I})$. We consider the detection of a deterministic signal so that $x_k = m$ for all k. Let us define the local observation quality SNR at each node as $\gamma_0 = \frac{m^2}{\sigma_v^2}$. The prior probabilities of the two hypotheses, H_1 and H_0 are denoted by $P(H_1) = \pi_1$ and $P(H_0) = \pi_0$, respectively.

In a distributed network, each node precesses its own observation to produce a local decision $u_k(z_k)$ and sends it to the fusion sensor. Here we assume that amplify and relay local processing is used at each node, according to which each node retransmits an amplified version of its own observation to the fusion center. This class of sensors was shown to perform well when the observations at the sensor nodes are corrupted by additive noise [3]. Then, the local decisions sent to the fusion center are ,

$$u_k = g_k z_k;\ k = 1, 2, ...n\ ,$$

where g_k is the relay amplifier gain at node k. The received signal at the fusion center is

$$r_k = g_k z_k + w_k; k = 1, 2, ...n\ ,$$

where channel noise $w_k \sim N(0, \sigma_k^2)$ is assumed to be independent but not identically distributed across the sensor nodes. Under each hypothesis, the received signal r_k is given by

$$\begin{aligned} H_0\ : r_k &= n_k;\ k = 1, 2, ..., n \\ H_1\ : r_k &= g_k x_k + n_k;\ k = 1, 2, ..., n \end{aligned}$$

where $n_k = g_k v_k + w_k$ and $n_k \sim N(0, g_k^2 \sigma_v^2 + \sigma_k^2)$. In vector notation, $\mathbf{r} = \mathbf{A}\mathbf{x} + \mathbf{n}$ where $\mathbf{A} = \text{diag}(g_1, g_2, ..., g_n)$. The detection problem at the fusion center can be formulated as,

$$\begin{aligned} H_0 &: \mathbf{r} \sim p_0(\mathbf{r}) = N(0, \mathbf{\Sigma}_n) \\ H_1 &: \mathbf{r} \sim p_1(\mathbf{r}) = N(\mathbf{Am}, \mathbf{\Sigma}_n) \end{aligned} \tag{2}$$

where $\mathbf{\Sigma}_n = \text{diag}(g_1^2\sigma_v^2 + \sigma_1^2, g_2^2\sigma_v^2 + \sigma_2^2, \cdots, g_n^2\sigma_v^2 + \sigma_n^2)$.

The log-likelihood ratio (LLR) for the detection problem (2) can be written as,

$$T(\mathbf{r}) = \mathbf{m}^T \mathbf{A} \Sigma_n^{-1} \mathbf{r} - \frac{1}{2} \mathbf{m}^T \mathbf{A} \Sigma_n^{-1} \mathbf{Am} \tag{3}$$

$$= m \sum_{k=1}^{n} \frac{g_k}{g_k^2 \sigma_v^2 + \sigma_k^2} r_k - \frac{m^2}{2} \sum_{k=1}^{n} \frac{g_k^2}{g_k^2 \sigma_v^2 + \sigma_k^2} \tag{4}$$

It is well known that optimal fusion tests should be threshold tests on the above LLR. Thus the optimal Bayesian decision rule at the fusion center is given by,

$$\delta(\mathbf{r}) = \begin{cases} 1 & \text{if } T(\mathbf{r}) > \log \tau \\ 0 & \text{if } T(\mathbf{r}) < \log \tau, \end{cases} \tag{5}$$

where τ is the threshold given by $\tau = \frac{\pi_1}{\pi_0}$ (assuming minimum probability of error Bayesian fusion). Substituting (4) in (5) leads to,

$$\delta(\mathbf{r}) = \begin{cases} 1 & \text{if } \bar{r} > \tau' \\ 0 & \text{if } \bar{r} < \tau' \end{cases}$$

where $\bar{r} = m \sum_{k=1}^{n} \frac{g_k}{g_k^2 \sigma_v^2 + \sigma_k^2} r_k$ and we have let $\tau' = \log \tau + \frac{m^2}{2} d^2$ with $d^2 = \sum_{k=1}^{n} \frac{g_k^2}{g_k^2 \sigma_v^2 + \sigma_k^2}$.

3 Analysis of Optimal Fusion Performance

Note that $\bar{r}$ is distributed as $\bar{r} \sim N(0, m^2 d^2)$ and $\bar{r} \sim N(m^2 d^2, m^2 d^2)$ under H_0 and H_1, respectively. Hence the false alarm probability at the fusion center is,

$$\begin{aligned} P_f &= P(\bar{r} > \tau' / H_0) = Q\left(\frac{\tau'}{md}\right) \\ &= Q\left(\frac{\log \tau}{md} + \frac{md}{2}\right)\ . \end{aligned} \tag{6}$$

where Q-function is defined by

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-\frac{\zeta^2}{2}} d\zeta.$$

Similarly, the probability of detection is given by,

$$P_D \;=\; P(\bar{r} > \tau^{'}/H_1) = Q\left(\frac{\log\tau}{md} - \frac{md}{2}\right). \quad (7)$$

The probability of error at the fusion center for a Bayesian detector is then,

$$\begin{aligned} P(E) &= P_f\pi_0 + (1-P_D)\pi_1, \\ &= \frac{1}{2}(1 + P_f - P_D)\,, \end{aligned} \quad (8)$$

where prior probabilities are assumed to be equal. Substituting (6) and (7) in (8) gives,

$$P(E) = Q\left(\frac{md}{2}\right)\,, \quad (9)$$

where, as before, $d^2 = \sum_{k=1}^{n} \frac{g_k^2}{g_k^2\sigma_v^2 + \sigma_k^2}$ and we have used the fact that $\tau = 1$ for equal priors.

It is interesting to note that,

$$\begin{aligned} \lim_{g_k^2 \to \infty} d^2 &= \lim_{g_k^2 \to \infty} \sum_{k=1}^{n} \frac{g_k^2}{g_k^2\sigma_v^2 + \sigma_k^2} \\ &= \frac{n}{\sigma_v^2}\,. \end{aligned} \quad (10)$$

From (9) and (10) it can be seen that the probability of fusion error for infinite local amplifier gain has a performance floor.

$$P(E) = Q\left(\frac{\sqrt{n\gamma_0}}{2}\right)\,. \quad (11)$$

Therefore, for a fixed n, the probability of fusion error is limited by the observation quality at local sensor nodes.

4 Optimal Power Allocation

In the following, we derive an optimal power allocation scheme that minimizes the total power spent by the whole sensor network. In contrast, in a uniform power allocation scheme each sensor node transmits locally processed data to the fusion center with equal power irrespective of the quality of local observation and channel. In this case, the nodes having small local SNR and poor channels waste power since their contribution to the decision at the fusion center may be negligible. In the optimal power allocation scheme, on the other hand, sensor nodes transmit locally processed data depending on the local SNR and the channel quality. The optimal power allocation problem can be formulated as

$$\min_{g_k \geq 0} \sum_{k=1}^{n} g_k^2\,,$$

such that

$$P(E) \leq P_e, \quad (12)$$

where P_e is a given threshold of required fusion error probability. Using (9) in (12), it is easy to see that the constraint (12) can equivalently be written as

$$q \;\leq\; d, \quad (13)$$

where we have defined $q = \frac{2}{m}Q^{-1}(P_e)$. From (13) we can write, $q^2 - d^2 \leq 0$ since q and d are positive.

Therefore the optimal power allocation problem can be rewritten as,

$$\min_{g_k \geq 0} \quad \sum_{k=1}^{n} g_k^2,$$

such that

$$q^2 - \sum_{k=1}^{n} \frac{g_k^2}{g_k^2\sigma_v^2 + \sigma_k^2} \leq 0,$$

and

$$g_k \geq 0 \;\text{ for } k = 1, 2, ..., n\,.$$

The Lagrangian for the above problem is,

$$G(L, \lambda_0, \mu_k) = \sum_{k=1}^{n} g_k^2 + \lambda_0\left[q^2 - \sum_{k=1}^{n} \frac{g_k^2}{g_k^2\sigma_v^2 + \sigma_k^2}\right] + \sum_{k=1}^{n} \mu_k(-g_k).$$

where $\lambda_0 \geq 0$ and $\mu_k \geq 0$ for $k = 1, 2, .., n$. Then, the KKT conditions are given by,

$$2g_k - \lambda_0 \frac{2g_k\sigma_k^2}{g_k^2\sigma_v^2 + \sigma_k^2}^2 - \mu_k = 0 \;\text{ for }\; k = 1, 2, ..., n, \quad (14)$$

$$\lambda_0\left[q^2 - \sum_{k=1}^{n} \frac{g_k^2}{g_k^2\sigma_v^2 + \sigma_k^2}\right] = 0, \quad (15)$$

$$\mu_k g_k = 0; \quad \text{for} \quad k = 1, 2, ...n,$$

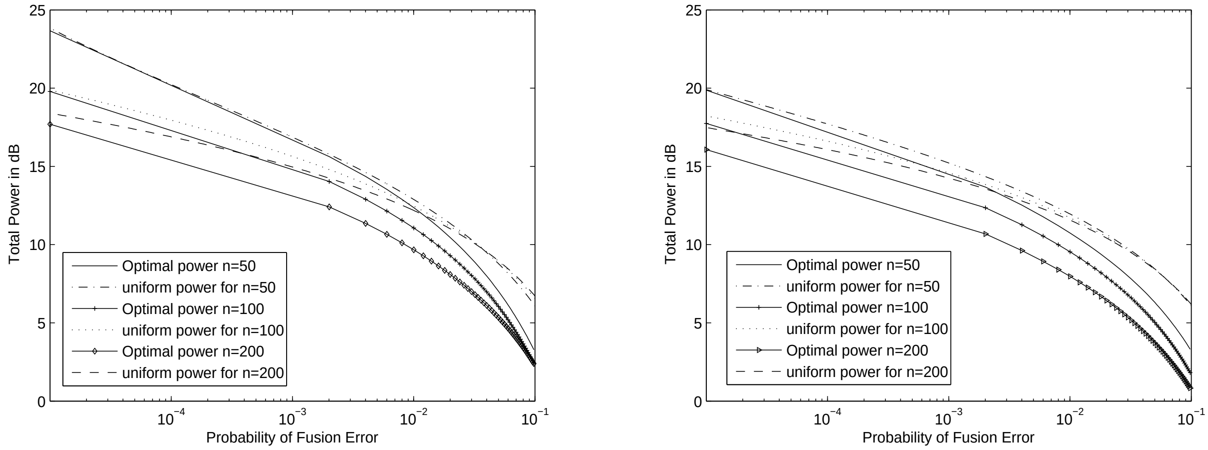
$$q^2 - \sum_{k=1}^{n} \frac{g_k^2}{g_k^2\sigma_v^2 + \sigma_k^2} \leq 0,$$

and

$$g_k \geq 0; \quad k = 1, 2, ...n\,. \quad (16)$$

In order to find a solution that satisfies above KKT conditions, let us assume $\lambda_0 \neq 0$ and $\mu_k = 0$ for $k = 1, 2, \cdots, n$. Then, for $g_k \neq 0$, from (14),

$$g_k^2 = \frac{\sigma_k^2}{\sigma_v^2}\left(\frac{\sqrt{\lambda_0}}{\sigma_k} - 1\right)\,, \quad (17)$$



(a) (b)

Figure 1. Total Power Vs Probability of Fusion Error (a) $\gamma_0 = 3dB$ **(b)** $\gamma_0 = 5dB$ **.**

Let us define the set Φ such that $\Phi = \{k; g_k \neq 0\}$. From (14), (15) and (17) we then have

$$\sqrt{\lambda_0} = \frac{\sum_{k\in\Phi} \sigma_k}{|\Phi| - q^2\sigma_v^2}, \tag{18}$$

where $|\Phi|$ denotes the cardinality of set Φ.

Let us define a function $f(.)$ as below:

$$f(k) = \frac{\sigma_k(k - q^2\sigma_{\underset{v}{2}})}{\sum_{j=1}^{k} \sigma_j}, \quad 1 \leq k \leq n$$

Suppose that, without loss of generality, $\sigma_1 < \sigma_2 < \cdots < \sigma_n$. Then it can be shown that (see Appendix), we can find a unique K_1 such that $f(K_1) < 1$ and $f(K_1 + 1) \geq 1$ for $1 \leq K_1 \leq n$. The value of K_1 can be found by searching the maximum integer k such that $f(k) < 1$ where $k = 1, \cdots, n$. Then (18) is given by

$$\sqrt{\lambda_0} = \frac{\sum_{k=1}^{K_1} \sigma_k}{K_1 - q^2\sigma_v^2} = \frac{\sigma_{K_1}}{f(K_1)} \, . \tag{19}$$

From (17) and (19)

$$g_k^2 = \frac{\sigma_k^2}{\sigma_v^2}\left(\frac{\sigma_{K_1}}{\sigma_k f(K_1)} - 1\right) , \tag{20}$$

which satisfies the KKT condition (16) if $q^2\sigma_v^2 < K_1$.

Suppose $q^2\sigma_v^2 > K_1$, and assume $\lambda_0 = 0$ and $\mu_k \neq 0$ for $k = 1, 2, \cdots, n$. From the KKT conditions it can be seen that there is no non-trivial solution for g_k whenever $\mu_k \neq 0$ for $k = 1, 2, \cdots, n$.

Thus the solution to the optimal power allocation problem can be given as,

$$g_k^2 = \begin{cases} \frac{\sigma_k^2}{\sigma_v^2}\left[\frac{\sum_{j=1}^{K_1}\sigma_j}{\sigma_k(K_1 - q^2\sigma_v^2)} - 1\right] & \text{if } f(k) - 1 < 0 \\ & \text{and } q^2\sigma_v^2 < K_1 \\ 0 & ; \text{ if } f(k) - 1 > 0 \\ \text{infeasible} & ; \text{if } q^2\sigma_v^2 > K_1 \end{cases} . \tag{21}$$

Since there is a feasible optimal solution only when $q^2\sigma_v^2 < K_1$, i.e. $\gamma_0 > \frac{4}{K_1}(Q^{-1}(P_e))^2$, it implies that we can not achieve probability of errors below $Q\left(\frac{\sqrt{K_1\gamma_0}}{2}\right)$ which is consistent with (11).

5 Performance Results

In this section we employ an example wireless sensor network to illustrate performance gains possible with the derived optimal power allocation scheme. The wireless channel between local nodes and the fusion center is assumed to be inhomogeneous and we assume channel noise variances, σ_k^2' s are drawn from a Rayleigh distribution with the probability distribution function $p(x) = \frac{x}{\sigma^2}e^{-x^2/2\sigma^2}$, $x \geq 0$. In all our numerical results we have assumed that the mean of the Rayleigh distribution is unity.

Note that our performance measure is the total network power expenditure defined as

$$P_{optimal} = \sum_{k=1}^{K_1} g_k^2 \, ,$$

where g_k^2's are given by (21) for the optimal scheme. On the other hand in the case of uniform power allocation, the

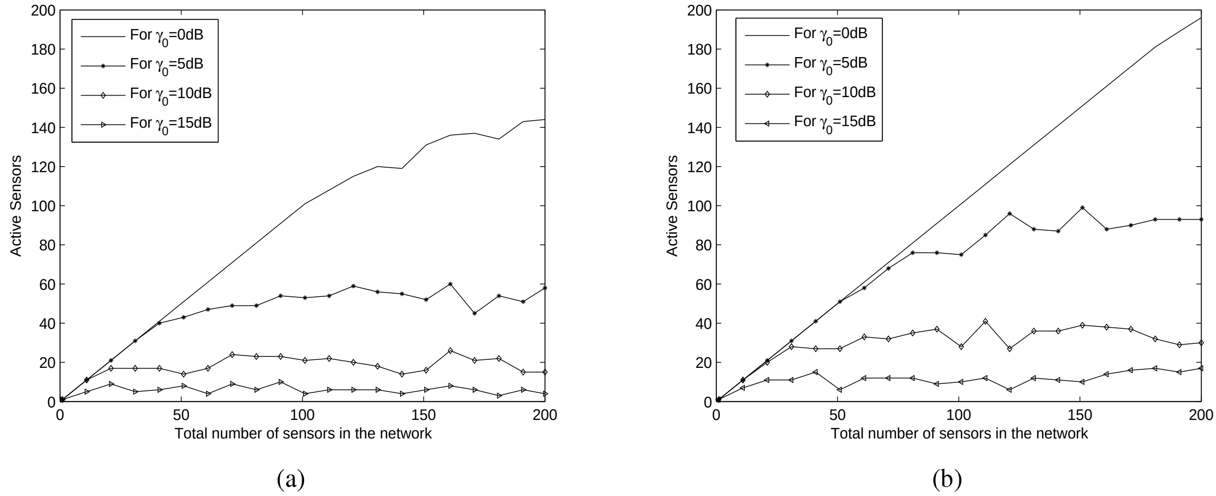


Figure 2. Number of Active sensors for the same local SNR (a) Fusion Error Probability = 10^{-3} (b) Fusion Error Probability = 10^{-5} .

total power is given by,

$$P_{uniform} = ng^2 ,$$

where g^2 is given by

$$\sum_{k=1}^{n} \frac{g^2}{g^2\sigma_v^2 + \sigma_k^2} = q^2 .$$

Fig. 1 shows the performance characteristics for local sensor signal-to-noise-ratio $\gamma_0 = 3dB$ and $\gamma_0 = 5dB$. It can be seen that when the number of sensors are increased then the energy savings due to proposed optimal scheme are more significant compared to uniform power allocation. When the local signal-to-noise ratio increases it can be seen that the total power of the system decreases since very high amplification power is not required when the observation quality is good. When the required fusion error probability is not significantly low, it can be seen that the gain of the optimal power allocation scheme over the uniform allocation scheme is high. According to Fig. 2, we can see that when the required fusion error probability is high, only a small number of sensors is active for the same local SNR saving the total power spent by the optimal scheme. Fig. 2 also shows that it is enough to activate only a small number of sensors when the observation quality of the local sensor nodes is high to achieve the same fusion error probability.

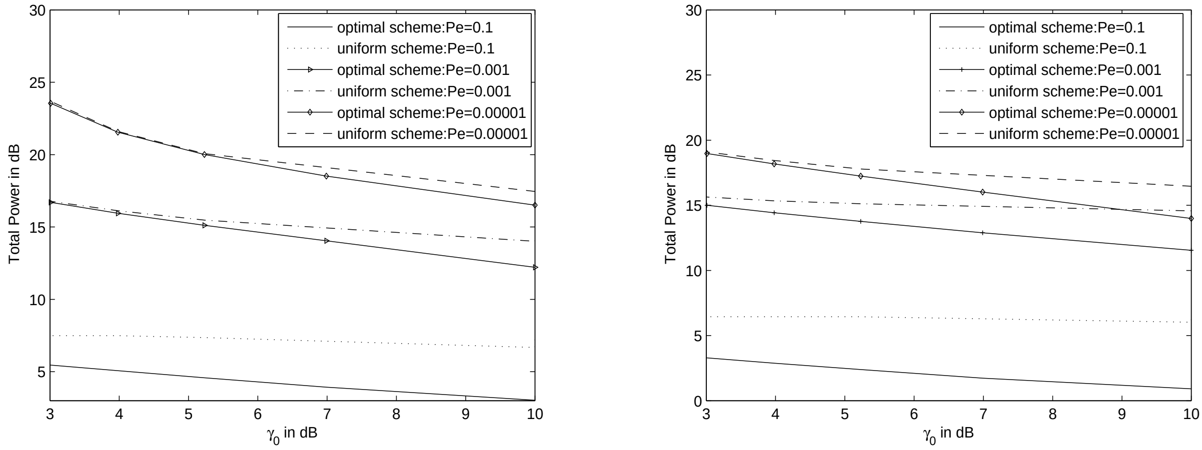
In Fig. 3, the variation of the total power of the system with local SNR is shown for $n = 50$ and $n = 100$ for the same required fusion error probability. It can be seen that when the local SNR increases the optimal power allocation scheme significantly outerperforms the uniform power allocation scheme. From Figs. 1 and 3, we see that when the number of sensors in the system is large, then the total power of the system decreases. This is because when the number of sensor nodes in the system increases it is not necessary for all the sensors to transmit data with a large power level since a large number of replicas of the same observation contributes to the fusion decision.

6 Conclusion

In this paper we derived the optimal power scheduling scheme for data fusion in an inhomogeneous wireless sensor networks. We showed that according to the optimal algorithm the sensors with poor observation and channel quality must be inactive to save total power spent by the system. Moreover, when the observation quality is very good it is sufficient to operate a very small number of sensors out of the total available nodes in the network keeping others shut off. From numerical results we also observed that the optimal scheme provides significant total energy savings over the uniform power allocation when the number of sensors in the system is large, or the local observation quality is good.

Acknowledgement

This research was supported by Kansas National Science Foundation (NSF) EPSCOR program under the First Award grant KUCR # NSF32241.



(a) (b)

Figure 3. Total Power Vs. Local SNR (a) n=50 (b) n=100 .

Appendix

In this Appendix, we show the existence of a unique K_1, where $1 \leq K_1 \leq n$ is such that $f(K_1) < 1$ and $f(K_1 + 1) \geq 1$ where

$$f(k) = \frac{\sigma_k(k - q^2\sigma_v^2)}{\sum_{j=1}^{k} \sigma_j}, \quad 1 \leq k \leq n \ ,$$

and we have assumed $\sigma_1 < \sigma_2 <$ .... $< \sigma_n$.

When $k = 1$,

$$f(1) = \frac{\sigma_1(1 - q^2\sigma_v^2)}{\sigma_1} < 1$$

So, $f(k) > 1$ is not possible for all $k = 1, 2, \cdots, n$. Therefore there are two possibilities:

- $f(k) < 1$ for all $1 \leq k \leq n$: In this case we set $K_1 = n$.
- There exists a unique K_1 such that $f(K_1) < 1$ and $f(K_1 + 1) \geq 1$, where $1 \leq K_1 \leq n$:

The uniqueness of K_1 implies that for any $k \geq K_1 + 1$, we should have that $f(k) \geq 1$. This can be proved by showing that if $f(k) \geq 1$, then $f(k+1) \geq 1$. When $f(k) \geq 1$, it implies that

$$\begin{aligned} f(k+1) &= \frac{\sigma_{k+1}(k + 1 - q^2\sigma_v^2)}{\sum_{j=1}^{k+1} \sigma_j} \\ &= \frac{\sigma_k(k - q^2\sigma_v^2) + \sigma_{k+1}}{\sum_{j=1}^{k} \sigma_j + \sigma_{k+1}} \\ &+ \frac{(\sigma_{k+1} - \sigma_k)(k - q^2\sigma_v^2)}{\sum_{j=1}^{k} \sigma_j + \sigma_{k+1}} \end{aligned} \qquad (22)$$

The second term of the (22) is positive since we have assumed that $\sigma_{k+1} > \sigma_k$ and $k - q^2\sigma_v^2 > 0$ for $f(k) \geq 1$. Hence

$$\begin{aligned} f(k+1) &> \frac{\sigma_k(k - q^2\sigma_v^2) + \sigma_{k+1}}{\sum_{j=1}^{k} \sigma_j + \sigma_{k+1}} \\ &> 1 \end{aligned}$$

as required.

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