Large System Decision Fusion Performance in Inhomogeneous Sensor Networks

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Abstract

The problem of decision fusion in a large wireless sensor system with many power-constrained distributed nodes is considered. The sensor network is assumed to be inhomogeneous. i.e. the channel statistics are not identical among the sensors. Assuming identical, binary local quantizing schemes at all the nodes, the optimal fusion scheme is derived. The large system performance of this optimal decision fusion detector is next analyzed. The asymptotic fusion performance is derived via the Lindberg-Feller central limit theorem for non-identical large samples. Numerical examples are used to show that the derived large system closed-form fusion error probability expressions provide a very close approximation to the exact finite-size sensor system fusion performance even for a relatively smaller number of nodes.

1 Introduction

Recent advances in microelectronics and wireless communication technology have enabled the low cost production of small sensor devices with wireless transceivers. These devices contain sensing, processing and communication capabilities, that have enabled the idea of wireless sensor networks (WSN). Arguably, WSN is one of the most promising technologies in many diverse applications including, but not limited to, civil surveillance, health care, homeland security, agriculture and industry [2, 6, 9]. In a typical application, a wireless sensor network consists of a large number of distributed nodes, that are linked to a fusion center through a wireless communication channel. The fusion center or the data gathering node, in principle, can have unlimited power and processing capabilities, and performs a decision making procedure, relying on the gathered data. The distributed nodes may have different sensing

modalities and can perform various processing operations such as imaging, seismic or chemical detection and radar.

Power management is considered as a core issue in designing a wireless sensor network since sensor nodes are usually powered by a battery that is impossible, or impractical, to be recharged due to cost and operating environment considerations. For example, the sensor network may have been deployed in a hostile enemy territory by dropping sensor nodes from air. A solution to power conservation at individual nodes is to employ a large number of low-power nodes across the network and combine partial information derived from them at a central fusion center. This is the classical decentralized detection and data/decision fusion with the additional caveat of limited communications resources (transmit power and bandwidth).

In a decentralized sensor network each distributed node derives a partial information about a phenomenon of interest (POI) from its own observation and convey a summary to the fusion center. Relying on the gathered information from distributed nodes, the fusion center makes a final decision by selecting from a possible set of hypotheses. An important problem in this context is the characterization of final fusion error probability performance. A considerable amount of previous work is available on this problem [1,8,14]. However, it is only recently that the problem has been addressed in the specific context of power and bandwidth constrained wireless sensor networks in which channel errors are non-negligible. For example, based on large system techniques [4] and [7] investigated the fusion error performance under communication constraints. However, they were only concerned with amplify-and-relay local processing. In general, however, the local processing can assumed to be a form of quantization at distributed nodes. Only a few results are available on the analysis of final fusion performance with quantized decisions and communication channel impairments. For example, [3] derived an optimum fusion rule in the case of correlated observations. Niu, Chen and Varshney, further assumed a fading channel, and derived a fusion rule that only requires the knowledge of fading channel statistics [10]. A fusion rule that dose not require the knowledge of sensor performance indices was proposed in [11]. The proposed fusion rule used the total number of detections transmitted from local senors to the fusion center as the fusion statistics.

However, none of the above previous work has considered an inhomogeneous network with different communication channel signal-to-noise-ratios (SNR's) among sensors. In a low-power wireless sensor network it is likely that dynamic power control may not be a possibility. Under such situations, it is possible that the received signal powers can vary across the nodes due to their spatial distribution as well as channel fading. In this paper, we address the problem of evaluating optimal decision fusion performance in such an inhomogeneous wireless sensor network with binary quantized local processing.

The remainder of this paper is organized as follows: In Section 2 we describe our assumed sensor system model, formulate the decision fusion problem and derive the optimal fusion detector. Next, in Section 3 we analyze the large system performance of the above optimal fusion detector in a resource-constrained, inhomogeneous wireless sensor network. Numerical examples that validate our asymptotic performance analysis are also provided in Section 3. Finally Section 4 concludes the paper.

2 System model

We consider a binary hypothesis testing problem in an n-node distributed sensor system. The k-th sensor observation is given by

$$H_0: y_k = x_{0,k} + v_k$$

$$H_1: y_k = x_{1,k} + v_k,$$
(1)

where observation noise v_k is assumed to be a sequence of iid (independent and identically distributed) zero-mean Gaussian random variables. We consider the fusion of a deterministic signal, so that $x_{0,k} = -m$ under H_0 and $x_{1,k} = m$ under H_1 for $k = 1, \dots, n$. In vector notation (1) becomes, $\mathbf{y} = \mathbf{x} + \mathbf{v}$, where \mathbf{v} is a zero mean, Gaussian *n*-vector of noise samples with covariance matrix $\Sigma_{\mathbf{v}}$. The *k*-th node applies a local processing scheme to its observation, to generate a message given by $q(y_k)$. In this paper, we assume that the *k*-th node makes a binary decision $q(y_k) \in \{0, 1\}$ with false-alarm and detection probabilities P_{f_k} and P_{d_k} , respectively. These local decisions are transmitted to the fusion center via antipodal signalling. Hence the transmit symbol from node *k* is given by $u_k = 2q(y_k) - 1$ where $u_k \in \{+1, -1\}$.



Figure 1. Distributed sensor network model

In addition, we assume the far field detection as shown in Fig. 1 (the need for this assumption is to be explained later). Assuming orthogonal sensor-to-fusion center communication, the received signal at the fusion center due to the k-th node can be written as $z_k = gu_k + w_k$, where g is the transmit power allocation at the k-th node assumed to be the same at all nodes (i.e. no power control). Note that we consider an inhomogeneous wireless sensor network such that the channel statistics are independent but not identical, i.e. $w_k \sim \mathcal{N}(0, \sigma_k^2)$. In vector notation, we have $\mathbf{z} = \mathbf{Au} + \mathbf{w}$, where $\mathbf{z} = [z_1, \dots, z_n]^T$, $\mathbf{A} = g\mathbf{I}$ and $\mathbf{w} \sim \mathcal{N}(\mathbf{0}, diag(\sigma_1^2, \dots, \sigma_n^2))$.

The optimal procedures at the fusion center should be designed based on the received vector z. It is well-known that for this type of binary hypothesis testing problems, the optimal fusion receivers are the likelihood ratio tests (LRT's) [13]. For the above problem, the likelihood ratio (LR) at the fusion center is given by

$$L(\mathbf{z}) = \prod_{k=1}^{n} \frac{p(z_k|H_1)}{p(z_k|H_0)}.$$
 (2)

We may use the same conditioning approach used in [10] to obtain the conditional densities $p(\mathbf{z}|H_1)$ and $p(\mathbf{z}|H_0)$, as follows. First, note that

$$p(u_k|H_1) = \begin{cases} P_{d_k} & u_k = 1\\ 1 - P_{d_k} & u_k = -1 \end{cases},$$

and

$$p(u_k|H_0) = \begin{cases} P_{f_k} & u_k = 1\\ 1 - P_{f_k} & u_k = -1 \end{cases}$$

where P_{d_k} and P_{f_k} are the detection and false alarm probabilities at the k-th node, respectively. Then, the received signal distribution at the fusion center can be written as:

$$p(z_k|H_1) = \sum_{u_k \in \{+1,-1\}} p(u_k|H_1)p(z_k|u_k)$$
$$= \frac{1}{\sqrt{2\pi\sigma_k}} \left[P_{d_k} e^{-\frac{(z_k-g)^2}{2\sigma_k^2}} + (1-P_{d_k})e^{-\frac{(z_k+g)^2}{2\sigma_k^2}} \right] (3)$$

Using the same approach, we can show that

$$p(z_k|H_0) = \frac{P_{f_k}e^{-\frac{(z_k-g)^2}{2\sigma_k^2}} + (1-P_{f_k})e^{-\frac{(z_k+g)^2}{2\sigma_k^2}}}{\sqrt{2\pi}\sigma_k}.$$
 (4)

Substituting (3) and (4) in (2), and after some simplifications, we obtain

$$L(\mathbf{z}) = \prod_{k=1}^{n} \frac{P_{d_k} e^{\frac{gz_k}{\sigma_k^2}} + (1 - P_{d_k}) e^{-\frac{gz_k}{\sigma_k^2}}}{P_{f_k} e^{\frac{gz_k}{\sigma_k^2}} + (1 - P_{f_k}) e^{-\frac{gz_k}{\sigma_k^2}}}.$$
 (5)

Let us define the sets $S_0 = \{k : z_k < 0\}$, and $S_1 = \{k : z_k > 0\}$. With these definitions, (5) can be rewritten as given in (6) at the top of next page:

In order to obtain a useful characterization of the optimal fusion rule that facilitates performance analysis, we assume identical local detectors at distributed nodes. In this case, all sensor nodes have the same performance such that $P_{d_k} = P_d$ and $P_{f_k} = P_f$, for all $k = 1, \dots, n$. Also, assuming high SNR operation (i.e. $\sigma_k^2 \longrightarrow 0$), the LR in (6) can be simplified as

$$L(\mathbf{z}) = \prod_{k \in S_0} \frac{(1-P_d)}{(1-P_f)} \times \prod_{k \in S_1} \frac{P_d}{P_f}$$

Equivalently, the log-likelihood ratio (LLR) at the fusion center is given by

$$\Gamma = K_1 \log \frac{P_d(1 - P_f)}{P_f(1 - P_d)} + n \log \frac{1 - P_d}{1 - P_f}, \quad (7)$$

where K_1 is defined as

$$K_1 = \sum_{k=1}^n I_{\{z_k: z_k \ge 0\}}(z_k)$$

and I(.) is the indicator function given by

$$I_A(z) = \begin{cases} 1 & z \in A \\ 0 & z \in A \end{cases}$$

From (7) we note that optimal fusion tests compare Γ to the threshold $\log(\tau)$, where τ is a threshold determined by the particular optimality criteria (for example, Bayesian vs. Neyman-Pearson). Since Γ is an affine function of K_1 (which is a function of z), the optimal fusion rule can equivalently be written as

$$\delta_0(\mathbf{z}) = \begin{cases} 1 & & \geq \\ & \text{if } K_1 & \tau' \\ 0 & & < \end{cases}$$
(8)

where τ' is a modified new threshold given by:

$$\tau' = \frac{\log(\tau) - n \log \frac{1 - P_d}{1 - P_f}}{\log \frac{P_d(1 - P_f)}{P_f(1 - P_d)}}.$$

3 Large system asymptotic fusion performance

Let $X_k = I_{\{z_k:z_k \ge 0\}}(z_k)$. Then it is clear that X_k 's, for $k = 1, \dots, n$, are a set of binary random variables that takes values 1 and 0 and the decision variable K_1 in the optimal test (8) can be rewritten as

$$K_1 = \sum_{k=1}^n X_k.$$
 (9)

Note that, under the hypothesis H_i

$$\mathbb{E}\{X_k|H_j\} = p(X_k = 1|H_j) \times 1 + p(X_k = 0|H_j) \times 0$$

= $p(z_k \ge 0|H_j).$ (10)

Similarly, the variance of X_k under hypothesis H_j is

$$\nu_{j,k}^{2} = \mathbb{E}\{X_{k}^{2}|H_{j}\} - \mathbb{E}\{X_{k}|H_{j}\}^{2}$$
$$= p(z_{k} \ge 0|H_{j}) - (p(z_{k} \ge 0|H_{j}))^{2}. \quad (11)$$

As we see from (10) and (11), the first and second order statistics of X_k 's are determined by the probability of a non-negative observation z_k denoted as $p(z_k \ge 0|H_j)$. Let us denote this probability under the hypothesis H_j , for j =0, 1, by $P_{j,k}$. Then we have that

$$P_{1,k} = p(z_k \ge 0|H_1) = \int_0^\infty p(z_k|H_1)dz_k$$

= $\frac{1}{\sqrt{2\pi\sigma_k}} \int_0^\infty \left[P_{d_k} e^{-\frac{(z_k - g)^2}{2\sigma_k^2}} + (1 - P_{d_k}) e^{-\frac{(z_k + g)^2}{2\sigma_k^2}} \right] dz_k$
= $P_d + (1 - 2P_d)Q\left(\frac{g}{\sigma_k}\right).$ (12)

Similarly, it can be shown that

$$P_{0,k} = p(z_k \ge 0|H_0)$$

= $P_f + (1 - 2P_f)Q\left(\frac{g}{\sigma_k}\right).$ (13)

It is clear from (12) and (13) that X_k 's are a set of independent, but not identical, binary random variables due to the inhomogeneous nature of the sensor network. As a result K_1 in (9) is a sum of non-identical random variables. This makes analysis of the fusion performance complicated since it is difficult to characterize the distribution of K_1 . Even in the case of a large sensor system in which $n \rightarrow \infty$, the decision statistic K_1 does not admit a useful density function since convergence in distribution assured in regular central limit theorem does not apply in this case. To get around this problem, we apply a modified version of the central limit theorem (CLT) for non-identical distributions, known as the Lindberg-Feller central limit theorem that requires extra regularity conditions [5, 12]:

$$L(\mathbf{z}) = \prod_{k \in S_0} \frac{P_{d_k} + (1 - P_{d_k})e^{\frac{-2gz_k}{\sigma_k^2}}}{P_{f_k} + (1 - P_{f_k})e^{\frac{-2gz_k}{\sigma_k^2}}} \times \prod_{k \in S_1} \frac{P_{d_k}e^{\frac{2gz_k}{\sigma_k^2}} + (1 - P_{d_k})}{P_{f_k}e^{\frac{2gz_k}{\sigma_k^2}} + (1 - P_{f_k})}.$$
(6)

Figure 2. Fusion center performance as a function of the number of sensors (a) detection probability (b) false-alarm probability.

Theorem 1 Lindberg-Feller Central Limit Theorem Suppose that X_k , for $k = 1, \dots, n$, is a sequence of independent but non-identical random variables with $\mathbb{E}\{X_k\} = \eta_k$ and $Var(X_k) = \nu_k^2$. Further, suppose that following two regularity conditions are held for B_1 and B_2 positive constants:

$$Var(X_k) > B_1, \tag{14}$$

and

$$\mathbb{E}\{|X_k - \mathbb{E}\{X_k\}|^3\} < B_2.$$
 (15)

Then, for large n, the sum $S_n = \frac{1}{n} \sum_{k=1}^n X_k$ converges in distribution to a Gaussian random variable characterized by

$$S_n \quad \underline{i.d.} \quad \mathcal{N}\left(\frac{1}{n}\sum_{k=1}^n \eta_k, \frac{1}{n}\sum_{k=1}^n \nu_k^2\right).$$

(For a proof of the Lindberg-Feller CLT, see [5]). In the Appendix we have shown how above two regularity conditions are satisfied in the case of far-field detection, as assumed in Fig. 1. Thus, in a large sensor network, under the far-field detection assumption at the fusion center, we may apply the Lindberg-Feller CLT to approximate the distribution of the fusion decision statistic under the hypothesis H_j , for j = 0, 1, to be a Gaussian random variable such that

$$K_1 \sim \mathcal{N}\left(\sum_{k=1}^{n} P_{j,k}, \sum_{k=1}^{n} \left(P_{j,k} - P_{j,k}^2\right)\right)$$
 (16)

where $P_{j,k} = p(z_k > 0 | H_j)$ is given by (12) and (13).

The large system distribution of K_1 given in (16) can be conveniently employed to characterize the final fusion performance. In particular, the detection and false alarm probabilities at the fusion center can shown to be given by:

$$P_D = Q\left(\frac{\tau' - \sum_{k=1}^n P_{1,k}}{\sqrt{\sum_{k=1}^n P_{1,k} - P_{1,k}^2}}\right), \quad (17)$$

and

$$P_F = Q\left(\frac{\tau' - \sum_{k=1}^n P_{0,k}}{\sqrt{\sum_{k=1}^n P_{0,k} - P_{0,k}^2}}\right), \quad (18)$$

where $P_{0,k}$ and $P_{1,k}$ are given by (13) and (12). In the special case of equi-probable hypotheses, the minimum achievable probability of fusion error can be written as in

(19), shown at top of next page, where
$$\tau'' = n \frac{\log \frac{1-1}{1-P_d}}{\log \frac{P_d(1-P_f)}{P_f(1-P_d)}}$$

$$P_{e} = \frac{1}{2}Q\left(\frac{\sum_{k=1}^{n}P_{1,k} - \tau''}{\sqrt{\sum_{k=1}^{n}P_{1,k} - P_{1,k}^{2}}}\right) + \frac{1}{2}Q\left(\frac{\tau'' - \sum_{k=1}^{n}P_{0,k}}{\sqrt{\sum_{k=1}^{n}P_{0,k} - P_{0,k}^{2}}}\right)$$
(19)



Figure 3. Fusion center minimum achievable probability of error as a function of the number of sensors

In Figs. 2 and 3 we have shown the fusion center detection, false-alarm and Bayesian error probabilities as a function of the number of nodes n in the sensor network. In all figures we have assumed that local detector performance is characterized by $P_d = 0.85$ and $P_f = 0.1$. Moreover, the channel noise variances σ_k^2 's have assumed to be uniformly distributed in the interval [0.8, 1.8]. The figures clearly show that the above large-sample theory approximation to the optimal fusion performance indeed provides a close approximation to the exact performance (obtained via numerical simulations) even for a relatively smaller number of sensor nodes. In particular, as can be seen from Fig. 3, as n increases, the derived large-sample asymptotic progressively becomes closer to the exact fusion performance.

4 Conclusions

In this paper, we analyzed the decision fusion performance in a large, inhomogeneous, wireless sensor network. Assuming identical, binary quantizers at all distributed nodes we derived the optimal fusion rule for an inhomogeneous sensor network in which channel statistics are not identical among distributed nodes. The main contribution of the paper was to derive the performance of the optimal fusion rule in a large inhomogeneous sensor system via large-sample theory. In particular, we showed that under the far-field detection that can be justified in many realistic wireless sensor networks, the fusion decision statistic converges in distribution to a normal random variable, thereby facilitating the performance analysis. We explicitly derived the large system fusion error probabilities in closed-form and shown them to provide a close approximation to the exact fusion performance, even in a relatively small finite-size sensor system, via numerical examples.

Appendix

Below we show how the two required regularity conditions (14) and (15) for the Lindberg-Feller CLT are met for K_1 when we assume far-field detection at the fusion center. Note that, essentially, the purpose of the two required regularity conditions in the Lindberg-Feller CLT is to guarantee that no one random variable in the sum dominates all the others. Below we list several assumptions on the far filed detection in Fig. 1 that we will need in the sequel:

a. Define s_m and and s_M as the sensor nodes with minimum and maximum distances d_{\min} and d_{\max} , respectively, from the fusion center. Then for any other node s_k with distance d_k from the fusion center

$$\sigma_{s_m}^2 < \sigma_{s_k}^2 < \sigma_{s_M}^2. \tag{20}$$

b. As a result of (20), we also have that

$$Q\left(\frac{g}{\sigma_{s_m}}\right) < Q\left(\frac{g}{\sigma_{s_k}}\right) < Q\left(\frac{g}{\sigma_{s_M}}\right)$$

c. $P_d > 0.5$ (which should be true for any useful detection system) and $P_f < 0.5$.

Note from (11) and (12) that the variance $\nu_{1,k}^2$ of X_k under the hypotheses H_1 is given by:

$$\nu_{1,k}^2 = \left(P_d + \psi(P_d)Q\left(\frac{g}{\sigma_{s_k}}\right)\right) \left(1 - \left(P_d + \psi(P_d)Q\left(\frac{g}{\sigma_{s_k}}\right)\right)\right)$$

where, for brevity, we have defined $\psi(x) = 1 - 2x$. To prove the first condition (14), it is sufficient to show that each of the two factors in the above expression is greater than a positive value. Taking each term alone, we have that

$$P_d + \psi(P_d)Q\left(\frac{g}{\sigma_{s_k}}\right) > P_d + \psi(P_d)Q\left(\frac{g}{\sigma_{s_M}}\right),$$

$$\nu_{1,k}^2 > \left(P_d + (1 - 2P_d)Q\left(\frac{g}{\sigma_{s_M}}\right) \right) \times \left(1 - \left[P_d + (1 - 2P_d)Q\left(\frac{g}{\sigma_{s_m}}\right) \right] \right)$$
(21)

$$\nu_{0,k}^2 > \left(P_f + (1 - 2P_f)Q\left(\frac{g}{\sigma_{s_m}}\right) \right) \times \left(1 - \left[P_f + (1 - 2P_f)Q\left(\frac{g}{\sigma_{s_M}}\right) \right] \right).$$
(22)

$$P_{1,k}(1-P_{1,k}) < \left[P_d + \psi(P_d)Q\left(\frac{g}{\sigma_{s_m}}\right) \right] \left[1 - P_d - \psi(P_d)Q\left(\frac{g}{\sigma_{s_M}}\right) \right]$$
(23)

and

$$1 - \left(P_d + \psi(P_d)Q\left(\frac{g}{\sigma_{s_k}}\right)\right) > 1 - \left(P_d + \psi(P_d)Q\left(\frac{g}{\sigma_{s_m}}\right)\right),$$

where we have used the assumptions (a) and (b).

Therefore, under the hypothesis H_1 , the variance $\nu_{1,k}^2$ of X_k , for $k = 1, \dots, n$, is lower bounded as shown in (21), at the top of the page. Note that since argument of the Q(.)-function $\frac{g}{\sigma_{s_k}}$ is always positive, $Q(\frac{g}{\sigma_{s_k}}) < 0.5$. This ensures that the terms on the left hand side of (21) are positive. The same approach can be used to show that, under the hypothesis H_0 also, the variance of X_k 's are lower bounded as given in (22).

To show that the second regularity condition (15) is held, let us make use of the fact that K_1 has a Binomial distribution (Below we show this under the assumption of hypothesis H_1 explicitly, but the same approach can be used to show its validity under the hypothesis H_0). It is straightforward to show that under the hypotheses H_1

$$\mathbb{E}\left[|X_k - \mathbb{E}[X_k]|^3 | H_1\right] = P_{1,k}(1 - P_{1,k})(P_{1,k}^2 + (1 - P_{1,k})^2) < P_{1,k}(1 - P_{1,k}),$$
(24)9

Hence it is sufficient to show that $P_{1,k}(1 - P_{1,k})$ is less than a positive value under the hypothesis H_1 . Following the same approach as in the proof of (14) leads to (23), at the top of the page, that proves the validity of the condition (15) under H_1 .

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