# Bayesian Data Fusion for Asynchronous DS-CDMA Sensor Networks in Rayleigh Fading

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***Abstract*— In this paper, we propose asynchronous non-orthogonal communication between distributed sensors and a data fusion center via asynchronous direct-sequence code-division multiple access (DS-CDMA). Furthermore, we evaluate the performance of such a system in the presence of local-sensor and channel errors due to multiple-access interference (MAI) and noise. We derive the optimal Bayesian receiver for this system operating over both a simple AWGN channel and a Rayleigh-fading channel, assuming full knowledge of the channel fading coefficients. The optimal receiver is shown to have exponential complexity as a function of the number of sensors. A set of low-complexity partitioned receivers are then proposed and analyzed in relation to the optimal one. These partitioned receivers are based on well-known linear and nonlinear multiuser detectors for DS-CDMA. The synchronous DS-CDMA case was studied in [1], and it is shown here that the degradation induced by moving to an asynchronous model is relatively small for the optimal receiver. Moreover, we provide compelling evidence that the quality of the local-sensor decisions is the limiting factor in fusion performance for fading channels with high signal-to-noise ratio.**

## I. Introduction

Much of the current literature on low-power wireless sensor networks assumes fully-orthogonal sensor communication and in many cases assumes no errors in communication between the sensors and the fusion center. However, in some applications involving dense, low-power, distributed wireless sensor networks, it may be more effective to employ non-orthogonal multi-sensor communication. Direct-sequence code-division multiple-access (DS-CDMA) can be a good candidate for this situation and similar spread-spectrum techniques have already been considered for wireless sensor networks in [2], [3].

In most cases, overall system efficiency is a major concern. The transmission of only the local decisions made at the nodes of a wireless sensor network to a fusion center, as opposed to directly sending the sensor observations themselves, can save significant resources in terms of communication bandwidth and sensor power. The treatment of such problems dates back at least two decades [4]–[6]. In particular, it has been shown that in a Bayesian approach, the solution to the distributed detection problem is a set of likelihood-ratio–based decision rules at the local sensors, potentially involving coupled thresholds. The data fusion problem for such a distributed detection system can also be formulated as a Bayesian hypothesis testing problem. The optimal Bayesian data fusion rule was derived in [6] assuming error-free reception of the local decisions at the fusion center.

In this paper, we consider Bayesian data fusion based on distributed local decisions in a coherent asynchronous DS-CDMA sensor network in Rayleigh fading. Although each sensor node may have data to be sent to the fusion center very rarely, due to the large number of nodes in a dense network, multiple nodes may have data at the same time. In such cases, a CDMA based scheme may allow all nodes to access the channel simultaneously and then return quickly to an *idle* or *sleep* mode for energy savings, rather than waiting for a long time in an active mode as in a time-division multiple-access (TDMA) based orthogonal signaling scheme. This ability is further enhanced by allowing for channel access to occur asynchronously. A similar synchronous DS-CDMA scheme was considered in [1].

We begin by deriving the optimal Bayesian data fusion receiver for an asynchronous DS-CDMA–based distributed wireless sensor network, where we seek to optimize the individual-hypothesis decision at a particular time instant. We show that the optimal receiver has computational complexity that is exponential in the number of sensors. In an effort to reduce this complexity and provide practical solutions for low-power sensor networks, we proceed to consider several low-complexity suboptimal fusion receiver structures based on an approach first proposed in [1] that partitions the receivers into two distinct stages. The first stage corresponds to multi-sensor detection of the received data and the second stage handles the data fusion. For the first stage, we consider several well-known multiuser detectors for asynchronous DS-CDMA, namely the one-shot joint-maximum-likelihood (JML) receiver, the linear MMSE receiver and the linear decorrelating receiver. The second stage receives the output from the first stage and performs the data fusion on them.

This paper makes several contributions to the distributed detection and data fusion literature for wireless sensor networks. One of the main contributions is the consideration of asynchronous non-orthogonal DS-CDMA for low-power wireless sensor networks dedicated to distributed detection and data fusion. Although there is a considerable amount

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of work on the subject of distributed detection, only a few consider the effect of channel errors on performance. In particular, to the best of our knowledge, synchronous non-orthogonal DS-CDMA was first considered in this context in [1]. Another major contribution is the formulation of the problem of Bayesian fusion of distributed sensors for such a network and the associated general performance characteristics via numerical simulation. Our current work involves somewhat modified local detector models in order to facilitate the analytical performance analysis.

The rest of this paper is organized as follows: Section II provides an outline of our system model, Section III derives the optimal receiver under the given model, Section IV presents several low-complexity suboptimal partitioned receivers, Section V provides simulations and performance results, and, finally, Section VI gives conclusions and paths to explore in future research.

## II. System Model

For the purposes of our analysis, we consider a wireless sensor network with a total of $K$ nodes, each of which are connected to a data fusion center in a distributed parallel architecture [5]. At regular time intervals, each of the sensors makes a local observation and a corresponding local decision and transmits this binary decision asynchronously via DS-CDMA over a Rayleigh fading channel to the fusion center.

Let $H_0$ and $H_1$ be the null and alternative hypotheses with prior probabilities $P(H_0) = p_0$ and $P(H_1) = p_1$, respectively. We further assume that each sensor is observing an underlying process that can be modeled as a Gaussian-location testing problem. That is, under the two hypotheses, the $k$th local-sensor observation, denoted $z_k$, for $k = 1, 2, \ldots, K$ is distributed as

$$\begin{aligned} H_0: &\quad z_k \sim \mathcal{N}(0, {\sigma_k}^2) \\ H_1: &\quad z_k \sim \mathcal{N}(\mu_k, {\sigma_k}^2) \end{aligned} \tag{1}$$

where $\mathcal{N}(\mu, \sigma^2)$ denotes a Normal (Gaussian) probability distribution with mean $\mu$ and variance $\sigma^2$. Conditioned on the underlying hypothesis, each of the local sensor observations are considered to be jointly independent. Furthermore, each sensor processes its observations independently to arrive at a local decision $u_k \in \{0, 1\}$, where $u_k$ is the local decision of the $k$th sensor and is computed according to

$$u_k = \begin{cases} 1 & \text{if } L(z_k) \geq \tau_k \\ 0 & \text{if } L(z_k) < \tau_k \end{cases} \tag{2}$$

Here, we adopt a Bayesian formulation, where $L(z_k)$ is the local likelihood ratio defined by

$$L(z_k) = \frac{p(z_k \mid H_1)}{p(z_k \mid H_0)}$$

where $p(\cdot \mid H_j)$ is the probability density function under hypothesis $H_j$, $j \in \{0, 1\}$. In (2), $\tau_k$ corresponds to the threshold of the likelihood ratio test at the $k$th sensor. These local sensor thresholds depends on the prior probabilities, $p_0$ and $p_1$, and an assumed cost function [7]. If we assume that $p_0 = p_1 = 1/2$ and that we wish to minimize the total probability of committing an error, than the cost function can be chosen to be uniform, and hence, $\tau_k = \tau = 1$ for all $k = 1, 2, \ldots, K$ [7]. We note here that while an optimal distributed detection scheme may require the joint determination of local thresholds, the above formulation can easily be modified for such a case. In either case, the optimal fusion rule only depends on the quality of the individual local sensor decisions as long as they are independent.

The local decision $u_k$'s are then transmitted asynchronously to the fusion center using DS-CDMA over a Rayleigh channel. Each node employs a signature waveform, $s_k(t)$, normalized to have unit energy. We will assume that the local sensors take a sequence, $z_k(i)$ of observations which correspond to a sequence of true hypotheses, denoted by $H_0(i)$ or $H_1(i)$. Furthermore, as the local decisions are binary-valued, it makes sense to transmit these decisions using binary phase-shift keying (BPSK), where each local decision is mapped as $b_k(i) = 2u_k(i) - 1$, so that $b_k(i) \in \{-1, +1\}$ for each $k$ and each $i$. The resulting antipodal bit stream is then modulated using the corresponding node's signature waveform for transmission to the fusion center. By making hard decisions at the local sensors and transmitting these, as opposed to an approximation of the actual sensor observations, $z_k$'s, the distributed detection and fusion system can greatly reduce the transmission requirements. This can lead to significant energy conservation in a wireless sensor network.

The complex baseband received signal at the fusion center is expressed as

$$r(k) = \sum_{i=0}^{M-1} \sum_{k=1}^{K} A_k b_k(i) s_k(t - iT - \tau_k) + n(t) \tag{3}$$

where $M$ is the number of data symbols per sensor per frame, $T$ is the symbol interval, $n(t)$ is the zero-mean complex additive white-Gaussian noise at the receiver, with variance $\sigma^2 = N_0$ ($N_0/2$ per dimension) and $s_k(t)$ denotes the normalized signature waveform of the $k$th sensor. The coefficients, $A_k$'s, are assumed to be zero-mean complex Gaussian with variance, $\overline{A}_k^2$. Furthermore, each of the $A_k$'s are assumed to remain fixed over the entire duration of the $M$-bit frame (i.e., block-fading model). We also consider only the case where the receiver must make a bit-by-bit decision, i.e., a one-shot model.

For ease of notation in the subsequent analysis, we define the $k$th local-sensor signal-to-noise ratio as $SNR_k^l$, where $SNR_k^l = \frac{{\mu_k}^2}{{\sigma_k}^2}$, and the average channel SNR of the $k$th sensor as $SNR_k^{ch} = \frac{\overline{A}_k^2}{\sigma^2}$.

## III. Optimal Fusion Receiver for an Asynchronous DS-CDMA Wireless Sensor Network

This Bayesian fusion problem can be formulated as one of deciding between $H_0(i)$ and $H_1(i)$ based on the observed receiver waveform, $r(t)$, in order to minimize a cost function. It is easily shown that a sufficient statistic for this fusion problem is given by the output of a bank of $K$ matched filters,

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each of which are matched to a particular sensor's signature waveform. This is similar to optimal multiuser detection for DS-CDMA described in [8]. The vector of matched filter outputs for the $i$th bit of the frame, $\mathbf{y}[i] = (y_1, y_2, \ldots, y_K)^T$ can be shown to be given by [8]

$$\mathbf{y}[i] = \mathbf{R_1}^T\mathbf{A}\mathbf{b}[i+1] + \mathbf{R_0}\mathbf{A}\mathbf{b}[i] + \mathbf{R_1}\mathbf{A}\mathbf{b}[i-1] + \mathbf{n}[i] \quad (4)$$

where $\mathbf{R_0}$ is the $K \times K$ cross-correlation matrix of the various signature sequences, $\mathbf{R_1}$ is a strict upper-triangular matrix of the partial cross-correlations among signature waveforms due to asynchronicity, $\mathbf{A}$ is the diagonal $K \times K$ matrix of fading coefficients, and $\mathbf{n}$ is the $K$-dimensional vector of zero-mean correlated Gaussian noise. Though this noise is correlated among the sensors and in time, Verdú showed in [8] that there exist matrices $\mathbf{F_0}$, lower triangular, and $\mathbf{F_1}$, strictly upper triangular, such that $\mathbf{R_0} = \mathbf{F_0}^T\mathbf{F_0} + \mathbf{F_1}^T\mathbf{F_1}$, and $\mathbf{R_1} = \mathbf{F_0}^T\mathbf{F_1}$. Via a whitening filter at the receiver, the following set of equivalent matched-filter outputs results

$$\overline{\mathbf{y}}[i] = \mathbf{F_0}\mathbf{A}\mathbf{b}[i] + \mathbf{F_1}\mathbf{A}\mathbf{b}[i-1] + \overline{\mathbf{n}}[i] \quad (5)$$

where $\overline{\mathbf{n}} \sim \mathcal{N}(0, \sigma^2\mathbf{I})$. This equivalent system given in (5) will provide for an easier formulation of the optimal Bayesian receiver, and so we will focus on it for the remainder of this paper.

Now, a particular vector $\mathbf{b}[i]$ contains the local decisions of each of the sensors based on the underlying true hypothesis at time-instant, $i$. We denote this true hypothesis by $H(i)$. Now, $H(i)$ enters in only through $\mathbf{b}[i]$; thus, by examining (5) we see that the $2K \times 1$ vector

$$\tilde{\mathbf{y}}[i] = \begin{pmatrix} \overline{\mathbf{y}}[i] \\ \overline{\mathbf{y}}[i+1] \end{pmatrix} \quad (6)$$

contains all available information regarding $H(i)$.

The data fusion problem for this coherent DS-CDMA wireless sensor network can now be viewed as a binary-hypothesis–testing problem based on the observation vector, $\tilde{\mathbf{y}}$, and the fading coefficient matrix, $\mathbf{A}$. Thus, the optimal Bayesian fusion rule is given by the likelihood ratio test

$$L(\tilde{\mathbf{y}}) = \frac{p(\tilde{\mathbf{y}} \mid H_1, \mathbf{A})}{p(\tilde{\mathbf{y}} \mid H_0, \mathbf{A})} \underset{H_0}{\overset{H_1}{\gtrless}} \tau_F$$

where $\tau_F$ corresponds to the threshold at the fusion center. This threshold depends on the prior probabilities, $p_0$ and $p_1$, as well as the cost function. We now introduce some further notation that will greatly simplify the expression for the optimal Bayesian fusion rule. First, we denote the false-alarm probability at the $k$th sensor as, $P_{F_k}$, and the corresponding miss probability as $P_{M_k}$. It is straightforward to verify [7] that

$$\begin{aligned} P_{F_k} &= \mathcal{Q}\left(\frac{\tau_k'}{\sigma_k}\right) \quad \text{and} \\ P_{M_k} &= 1 - \mathcal{Q}\left(\frac{\tau_k' - \mu_k}{\sigma_k}\right) \end{aligned} \quad (7)$$

where $\mathcal{Q}(\cdot)$ denotes the Gaussian tail distribution and $\tau_k' = \frac{\sigma_k{}^2}{\mu_k}\log(\tau_k) + \frac{\mu_k}{2}$.

Now, assuming, as we have, that the local sensor decisions are independent, we can calculate the conditional probabilities $p(\mathbf{b} \mid H_j)$, for $j = 0, 1$ as

$$p(\mathbf{b} \mid H_j) = \prod_{k=1}^{K} p(b_k \mid H_j)$$

where

$$p(b_k \mid H_1) = \begin{cases} 1 - P_{M_k} & \text{if } b_k = +1 \\ P_{M_k} & \text{if } b_k = -1 \end{cases}$$

and

$$p(b_k \mid H_0) = \begin{cases} P_{F_k} & \text{if } b_k = +1 \\ 1 - P_{F_k} & \text{if } b_k = -1 \end{cases} .$$

For simplicity of notation, let $\tilde{\mathbf{y}}_i \equiv \tilde{\mathbf{y}}[i]$, $\overline{\mathbf{y}}_i \equiv \overline{\mathbf{y}}[i]$, $\mathbf{b}_i \equiv \mathbf{b}[i]$, $\mathbf{b}_{i-1} \equiv \mathbf{b}[i-1]$, $\mathbf{b}_{i+1} \equiv \mathbf{b}[i+1]$. Thus, for the $i$th time instant, our likelihood-ratio test (LRT) becomes

$$L(\tilde{\mathbf{y}}_i) = \frac{p(\tilde{\mathbf{y}}_i \mid H_1(i), \mathbf{A})}{p(\tilde{\mathbf{y}}_i \mid H_0(i), \mathbf{A})}. \quad (8)$$

As we shall see, for purposes of the derivation, it is sufficient to focus our attention on the numerator of (8). First we rewrite the numerator as

$$\begin{aligned} p(\tilde{\mathbf{y}}_i \mid H_1(i), \mathbf{A}) = \sum_{\mathbf{b}_{i-1}} \sum_{\mathbf{b}_i} \sum_{\mathbf{b}_{i+1}} \big(&p(\tilde{\mathbf{y}}_i \mid \mathbf{A}, \mathbf{b}_i, \mathbf{b}_{i-1}, \mathbf{b}_{i+1}) \times \\ &p(\mathbf{b}_i \mid H_1(i)) p(\mathbf{b}_{i-1} \mid H_1(i)) \times \\ &p(\mathbf{b}_{i+1} \mid H_1(i))\big) \end{aligned} \quad (9)$$

where the summations are taken over all $\mathbf{b}_{i-1} \in \{-1, +1\}^K$, $\mathbf{b}_i \in \{-1, +1\}^K$, and $\mathbf{b}_{i+1} \in \{-1, +1\}^K$, respectively.

Now, since the local decision at time $i-1$ is assumed to be independent of that at time $i$, it is evident that the vector $\mathbf{b}_{i-1}$ must be independent of the true hypothesis at time $i$, which leads to the conclusion that $p(\mathbf{b}_{i-1} \mid H_j(i)) = p(\mathbf{b}_{i-1})$ for $j = 0, 1$. Clearly, by independence of the $b_k$'s and the theorem of total probabilities [9]

$$p(\mathbf{b}_{i-1}) = \prod_{k=1}^{K} \left(p(b_k \mid H_1)p(H_1) + p(b_k \mid H_0)p(H_0)\right) \quad (10)$$

where $\mathbf{b}_{i-1} = (b_1, b_2, \ldots, b_K)$ in (10). This same argument also holds for $\mathbf{b}_{i+1}$for the same reasons. Furthermore, it is obvious from the system model that

$$p(\tilde{\mathbf{y}}_i \mid \mathbf{A}, \mathbf{b}_i, \mathbf{b}_{i-1}, \mathbf{b}_{i+1}) = \frac{1}{(2\pi\sigma^2)^{\frac{K}{2}}} \exp\left(\frac{-(Q_i + Q_{i+1})}{2\sigma^2}\right) \quad (11)$$

where

$$Q_i = (\overline{\mathbf{y}}_i - \mathbf{F_0}\mathbf{A}\mathbf{b}_i - \mathbf{F_1}\mathbf{A}\mathbf{b}_{i-1})^H(\overline{\mathbf{y}}_i - \mathbf{F_0}\mathbf{A}\mathbf{b}_i - \mathbf{F_1}\mathbf{A}\mathbf{b}_{i-1}) \quad (12)$$

and $Q_i$ is a real-valued scalar for the $i$th bit of a frame. We further define the set of functions given by

$$\begin{aligned} f_m(\tilde{\mathbf{y}}_i, \mathbf{b}_{i-1}, \mathbf{b}_i, \mathbf{b}_{i+1}) = \exp&\left(\frac{-(Q_i + Q_{i+1})}{2\sigma^2}\right) \times \\ &p(\mathbf{b}_i \mid H_m(i))p(\mathbf{b}_{i-1})p(\mathbf{b}_{i+1}) \end{aligned} \quad (13)$$

where $m \in \{0, 1\}$.

At this stage of the development, combining (9), (10), (11), and (13) gives a straightforward final construction of the likelihood ratio test. $L(\tilde{\mathbf{y}}_i)$ can now be expressed as

$$L(\tilde{\mathbf{y}}_i) = \frac{\sum_{\mathbf{b}_{i-1}} \sum_{\mathbf{b}_i} \sum_{\mathbf{b}_{i+1}} f_1(\tilde{\mathbf{y}}_i, \mathbf{b}_{i-1}, \mathbf{b}_i, \mathbf{b}_{i+1})}{\sum_{\mathbf{b}_{i-1}} \sum_{\mathbf{b}_i} \sum_{\mathbf{b}_{i+1}} f_0(\tilde{\mathbf{y}}_i, \mathbf{b}_{i-1}, \mathbf{b}_i, \mathbf{b}_{i+1})} \quad .$$

Note that one can "simplify" this by factoring out the like terms of the numerator and denominator, but the corresponding expression is a bit more cumbersome. In any case, the optimal fusion decisions are then simply

$$\delta_{\mathrm{opt}}(\tilde{\mathbf{y}}_i) = \begin{cases} 1 & \text{if } L(\tilde{\mathbf{y}}_i) \geq \tau_F \\ 0 & \text{if } L(\tilde{\mathbf{y}}_i) < \tau_F \end{cases} \quad . \tag{14}$$

Finally, for equal *a priori* probabilities, $p_0$ and $p_1$, in order to minimize the probability of error at the fusion center, the threshold for the fusion LRT should be chosen as $\tau_F = 1$ [7].

## IV. Low-Complexity Partitioned Receivers

From the preceding section, we see that the optimal fusion decision requires $\mathcal{O}(4^K)$ multiplications and additions in order to arrive at its global decision. In an effort to try to reduce this complexity, in this section, we present a set of lower-complexity partitioned sub-optimal receiver structures analogous to those first proposed in [1] and evaluate their performance relative to the optimal fusion rule. The basic approach used here is to separate the detection of the binary-valued sensor data from the fusion decision. For the detection of the sensor data, we apply several approaches to multiuser detection from the literature. The demodulated estimates of the $b_k$'s are then fed to a second stage that performs the data fusion. This stage assumes that it receives the true local decisions. Thus, in effect, we are making hard decisions in the first stage, and these are fed to the fusion center in the second stage. As with convolutional decoding, there is a performance loss associated with transitioning from the use of soft decisions (i.e., the optimal fusion rule) to that of hard decisions (i.e., the suboptimal approaches of this section). On the other hand, a considerable reduction in complexity is attained.

We begin by addressing the first stage of the partitioned receiver. In what follows, we will consider several well-known multiuser detectors for asynchronous DS-CDMA as the first stage of the partitioned receivers, namely: the one-shot joint-maximum-likelihood (JML) detector, the MMSE detector, and the decorrelating detector.

The one-shot JML detector estimates the symbol vector, $\mathbf{b}_i$, by the following estimate. This estimate maximized the joint likelihood function of $\mathbf{b}_i$ for a one-shot model, however, it is important to note that unlike in the synchronous case, this is *not* the optimal multiuser detector for an asynchronous DS-CDMA system [8].

$$\hat{\mathbf{b}}^{(JML)} = \underset{\mathbf{b}_i, \mathbf{b}_{i-1} \in \{-1,+1\}^K}{\arg\max} \; Q_i \tag{15}$$

where $Q$ is defined as in (12).

Another conventional multiuser detector is the linear decorrelating detector. This receiver is designed to output a scaled version of the transmitted signal under the condition in which there is no noise present and is most easily described in the $z$-transform domain. Referring back to (5), we can rewrite this in the $z$-transform domain as

$$\overline{\mathbf{y}}(z) = (\mathbf{F_0} + \mathbf{F_1} z^{-1}) \mathbf{A} \mathbf{b}(z) + \overline{\mathbf{n}}(z) \quad .$$

If we let $\mathbf{G}(z) = \left(\mathbf{F_0} + \mathbf{F_1} z^{-1}\right)$, then under the condition in which there is no noise, i.e., $\overline{\mathbf{n}}(z) = \mathbf{0}$, the function that will produce a scaled version of the transmitted signal is simply

$$\hat{\mathbf{b}}^{(decorr)} = \mathrm{sgn}\left\{\mathrm{IZT}\left\{\mathbf{A}^H \mathbf{G}^{-1}(z) \overline{\mathbf{y}}(z)\right\}\right\} \tag{16}$$

where $\mathrm{IZT}\{\cdot\}$ denotes the inverse $z$-transform. One must be careful that the inverse, $\mathbf{G}^{-1}(z)$, exists and is stable, and these conditions are given in [8].

The final multiuser detector that we wish to evaluate for the first stage of the low-complexity partitioned receivers is the linear minimum mean-square error (MMSE) receiver. This receiver seeks to minimize the expected value of the square error between the transmitted signal and a linear combination of the matched-filter outputs. In the case in which no noise is present, the MMSE detector degenerates to the decorrelating detector, and thus we would expect a similar structure. In fact, the estimates of the asynchronous MMSE multiuser detector are

$$\hat{\mathbf{b}}^{(MMSE)} = \mathrm{sgn}\left\{\mathrm{IZT}\left\{\mathbf{A}^H \left(\mathbf{G}(z) + \sigma^2 (\mathbf{A}^H \mathbf{A})^{-1}\right)^{-1} \overline{\mathbf{y}}(z)\right\}\right\} \tag{17}$$

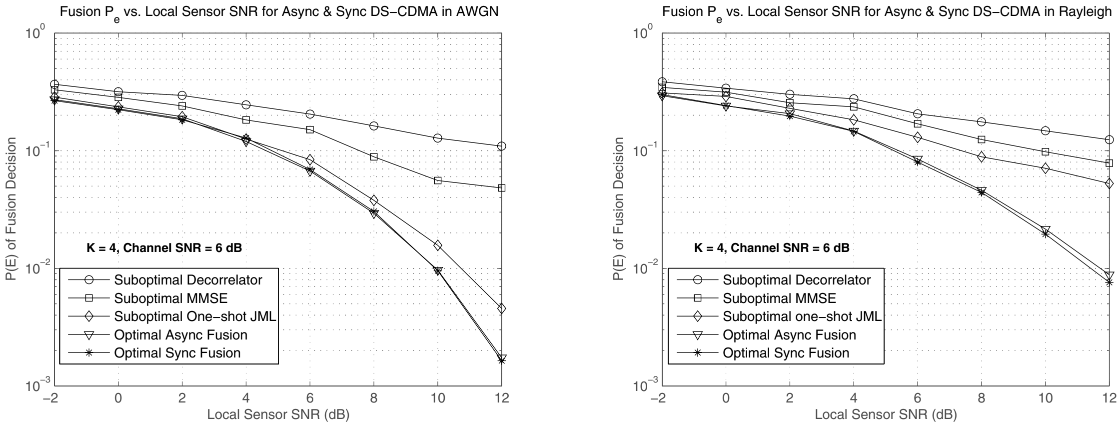
It turns out that $\left(\mathbf{G}(z) + \sigma^2 (\mathbf{A}^H \mathbf{A})^{-1}\right)^{-1}$ always exists and is always stable [8]. In addition, the MMSE receiver achieves better performance than the decorrelating detector for lower channel SNRs. These two properties make it an attractive choice.

Now that we have addressed some possible choices for the first stage of the partitioned receiver, we turn our attention to the second stage. The second stage of the receiver performs the actual data fusion using the estimates from the first stage. The second stage assumes that the estimated binary-valued vector that it receives are the true local decisions of the sensors and, furthermore, that the estimates are independent of one another. Strictly speaking, neither one of these assumptions are true; however, our primary intent is to develop low-complexity receivers that give reasonable performance, and these assumptions allow for this (as we shall see in the following section). For the purposes or our development, we will use the estimates of the one-shot JML, however, the second stage is identical no matter how the first stage develops its estimates. Therefore, under the assumptions of independence and perfect detection, the likelihood ratio test is

$$L(\hat{\mathbf{b}}^{(JML)}) = \prod_{k=1}^{K} \frac{p(\hat{b}_k^{(JML)} \mid H_1)}{p(\hat{b}_k^{(JML)} \mid H_0)} \quad . \tag{18}$$

Furthermore, it is clear from the preceding development that

$$p(\hat{b}_k^{(JML)} \mid H_1) = \begin{cases} 1 - P_{M_k} & \text{if } \hat{b}_k^{(JML)} = +1 \\ P_{M_k} & \text{if } \hat{b}_k^{(JML)} = -1 \end{cases}$$




(a) (b)

Fig. 1. Fusion Probability of Error vs. Local Sensor SNR, for a fixed channel $SNR_k^{ch} = 6$dB for all $k$. (a) AWGN channel, (b) Rayleigh channel.

and

$$p(\hat{b}_k^{(JML)} \mid H_0) = \begin{cases} P_{F_k} & \text{if } \hat{b}_k^{(JML)} = +1 \\ 1 - P_{F_k} & \text{if } \hat{b}_k^{(JML)} = -1 \end{cases} .$$

The false-alarm and miss probabilities are calculated as before in (7). Under the stated assumptions, and for equal prior probabilities and uniform cost function, the optimal Bayesian fusion rule is then simply

$$\delta_{(JML)} = \begin{cases} 1 & \text{if } L(\hat{\mathbf{b}}^{(JML)}) \geq 1 \\ 0 & \text{if } L(\hat{\mathbf{b}}^{(JML)}) < 1 \end{cases} . \qquad (19)$$

As previously stated, the second stage described above is identical for all of the first-stage detectors examined. That is, the estimated bits from the first stage are fed to the likelihood ratio in (18), which in turn is used by (19) to arrive at the decision. The complexity of these partitioned receivers is determined by the first stage, as the second stage can be implemented as a simple table lookup [1]. Note that the complexity of the one-shot JML receiver is exponential in $K$, like the optimal fusion rule of the previous section, and that of the decorrelating and MMSE detectors is effectively $\mathcal{O}(K^3)$, as they both result in a multiple of $K$ vector-matrix operations that each involve a $K \times K$ matrix.

## V. Simulation Results

In this section, we evaluate the performance of the optimal fusion receiver and the low-complexity partitioned receivers of the previous sections in a coherent asynchronous DS-CDMA system. This analysis is performed via computer simulation, as the analytical analysis of such a system seems intractable. For these simulations, we assume a four-node sensor network using non-orthogonal signature sequences, employing BPSK, and transmitting in both AWGN and Rayleigh-fading channels.

The assumed correlations among the users are represented by $\mathbf{R_0}$ and $\mathbf{R_1}$ of the previous development, and are taken to be

$$\mathbf{R_0} = \begin{pmatrix} 1 & 0.7 & 0.7 & 0.7 \\ 0.7 & 1 & 0.7 & 0.7 \\ 0.7 & 0.7 & 1 & 0.7 \\ 0.7 & 0.7 & 0.7 & 1 \end{pmatrix}$$

and

$$\mathbf{R_1} = \begin{pmatrix} 0 & 0.1807 & 0.1704 & 0.2236 \\ 0 & 0 & 0.1916 & 0.2246 \\ 0 & 0 & 0 & 0.2220 \\ 0 & 0 & 0 & 0 \end{pmatrix} .$$

We further assume that the null and alternative hypotheses are equally likely and that the Bayesian cost function is uniform. Under these conditions, the probability of error at the output of the fusion receiver is minimized, and thus, this probability of error will be our metric of performance. Figure 1 shows the performance of the various receivers as a function of the local SNR when the channel SNR is fixed at 6dB. In Figure 1(a) we show the performance in AWGN for the asynchronous receivers. We also show, for reference, the performance of the optimal fusion receiver for the synchronous case (i.e., when $\mathbf{R_1} = \mathbf{0}$) studied in [1]. From this we note that the synchronous and asynchronous performance are essentially identical, which is essentially a function of the fixed channel SNR selected and the partial cross-correlations (i.e., the $\mathbf{R_1}$ matrix) of the present bits with the previous bits transmitted. Also, of all of the low-complexity partitioned receivers, the best performing is the one-shot JML, followed by the MMSE and decorrelating detectors, both of which exhibit a floor at higher SNRs due to MAI.

In the Rayleigh case of Figure 1(b), we see a very slight degradation in the optimal performance in an asynchronous system from that of the synchronous system of [1]. Even so, out to a local-sensor SNR of 12dB, the loss is less than 1dB. Furthermore, the one-shot JML still results in the best performance for the low-complexity receivers, again with the MMSE and decorrelating detectors lagging in performance. However, the performance gain achieved by the one-shot JML

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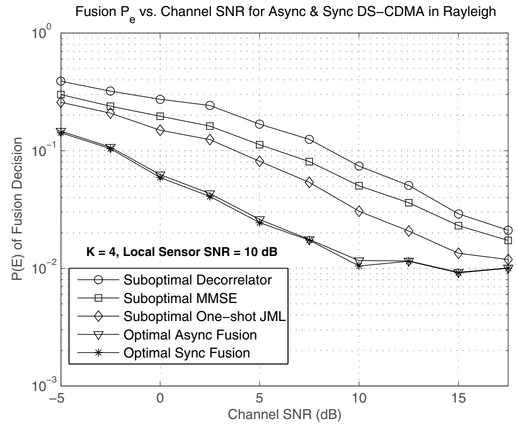


Fig. 2. Fusion Probability of Error vs. Channel SNR, for a fixed local $SNR_k^l = 10$dB for all $k$.

over the two linear detectors is not as pronounced in the Rayleigh case.

We also observe from Figure 1 that for a fixed channel SNR, the optimal asynchronous performance is monotonic as a function of the local SNR. The sizable difference in performance between the optimal and suboptimal approaches in the asynchronous case, leads one to conjecture that better low-complexity partitioned receivers can be developed.

Figure 2 depicts the performance of the optimal Bayesian fusion rule and the low-complexity partitioned receivers as a function of the channel SNR, $SNR_k^{ch}$. For this simulation, the local SNR has been fixed at 10dB for each of the nodes. Here we note that the optimal asynchronous performance is very slightly worse than the synchronous performance found in [1]. All of the asynchronous receivers, including the optimal receiver, exhibit a performance floor at high channel SNR. This is due to the fact that the errors introduced by channel noise, fading, and asynchronicity are overshadowed by the quality (or lack thereof) of the local decisions. Hence, the local-sensor SNR becomes the limiting factor on overall performance. We also see that there is a considerable separation between the suboptimal partitioned receivers and the optimal fusion receiver. These receivers also converge to the optimal for very high channel SNR, as the number of errors introduced by the channel becomes negligible with respect to the errors induced at the local sensors. In terms of a tradeoff between complexity and performance, the MMSE provides the best tradeoff, particularly for high channel SNR, however, the performance difference is significant for lower channel SNR. This leads one to conclude that it may be fruitful to search for better low-complexity receivers.

## VI. Conclusions and Future Work

In this paper, we considered the problem of optimal Bayesian data fusion using a one-shot model for a wireless sensor network transmitting via asynchronous DS-CDMA in a Rayleigh channel. We have derived the optimal one-shot receiver for this model and have shown that the complexity is exponential in the number of nodes in the network. Furthermore, the performance of this optimal fusion rule is not substantially different from that of a synchronous model, as long as the partial correlations among sensors is only of moderate magnitude. We have also proposed a set of one-shot low-complexity partitioned receivers based on various multiuser detector structures for DS-CDMA, though work remains to develop ones that more closely approximate the optimal. Moreover, for higher channel SNR, the performance of each of the systems under study is limited by the quality of the local-sensor decisions, as measured through the local-sensor SNR. Future work will include the analytical performance of both optimal and proposed low-complexity partitioned receivers, in addition to the exploration of low-complexity receivers that attain even better performance than those examined herein.

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