Fairness in Sequential Estimation: A Cooperative Game Theoretic Solution for WSNs

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***Abstract*—In this paper, the problem of sequential estimation in a wireless sensor network (WSN) is formulated in a cooperative game theoretic framework. This framework allows addressing the issue of fair resource allocation for sequential estimation task at the Fusion center (FC) in a WSN as a solution of a cooperative game. We propose a simple game theoretic solution to the problem of power allocation for sensor nodes that are subjected to power constraints. Simulation results show that the proposed solution has the same performance trend (in terms of the estimator quality at the FC) as when all nodes transmit at their maximum powers, while our proposed solution leads to an overall improvement of the network lifetime.**

I. Introduction

Collaborative information processing has a key role to play in efficient handling of large volumes of data collected by mobile sensor nodes in a sensor network. Node collaboration requires inter sensor communications. Payoff and cost of collaboration can be modeled, respectively, as the improved quality of processed outputs and power or bandwidth. Thus there needs to be a trade-off between performance and cost of collaborative information processing.

While collaborative information processing for resource-constrained sensor networks has been explored over the last several years [1], [2], still there is a lack of a formal analytical framework for designing collaborative information processing that allows such tradeoffs systematically. In this paper, we show that cooperative game theoretic concepts can be applied in approaching such collaborative signal processing problems in distributed sensor networks. In contrast to non-cooperative game theory, where individual decision makers compete with each other to achieve their goals of maximizing individual payoffs, cooperative game theory allows competing players (or nodes) to form coalitions so as to efficiently achieve their individual goals.

Since sensor networks are prone to premature failure due to limited battery lives of the nodes, power management has tremendous importance in successful operation of WSNs. The optimal power scheduling for distributed estimation in power/energy-constrained WSNs were discussed in [3], [4]. Power management in wireless sensor networks using a non-cooperative game theoretic approach was addressed in [5], [6]. While there is much literature on sequential estimation in WSNs [7], [8], very few attempts have been made to exploit the rich collection of cooperative game theory in power/energy-constrained WSNs tasked with estimating a Phenomenon of Interest (PoI). For example, in [9], a novel concept of incompletely cooperative game theory was used to simultaneously achieve energy conservation and throughput for WSNs. On the other hand, [10], [11] used cooperative game theory for channel/bandwidth allocation problem. In this paper, we use tools from cooperative game theory to develop a formal framework for solving the problem of sequential estimation under fair allocation of power among collaborating sensor nodes. Specifically, in this paper, we use Nash bargaining (NB) [12] based solution concepts to achieve a justifiable fair power allocation among nodes with power constraints.

The remainder of this paper is organized as follows: Section II presents the assumed sensor network model for the sequential estimation problem. Section III introduces the bargaining problem and summarizes the Nash bargaining solution (NBS). In Sections IV and V, we introduce two fair power allocation algorithms based on the NBS. Section VI evaluates the performance of the proposed algorithms via simulations. Finally, Section VII concludes the paper by summarizing our results.

II. Sensor Network Model for Sequential Estimation

We consider a sensor network consisting of a Fusion Center (FC) and N nodes tasked with estimating a non-random parameter θ sequentially. We consider the FC itself as a node with its own estimate of the parameter θ and is denoted as node-$\{0\}$. The set of other nodes are denoted as $\mathcal{N} = \{1, 2, \cdots, N\}$. Only a *quasi-static network* is considered here, in which node locations as well as fading coefficients can be assumed fixed for a certain period of time, whereas from block to block they could be varying arbitrarily. Local estimator at node i is denoted by $\hat{\theta}_i$, for $i = 0, 1, 2, \cdots, N$. All local estimators are assumed to be unbiased and their respective variances denoted by V_i. Under the quasi-static assumption, we may assume that at the beginning of each block, the FC has access to the quality of estimates at other nodes as given by V_i's for $i \in \mathcal{N}$. It is assumed that the FC updates its estimator sequentially by combining its own observation with the noise-corrupted estimators of the other selected nodes received over noisy communication links. We consider the case when the FC does not want the distributed nodes to transmit at their maximum powers, while achieving a target estimator quality using a few number of nodes, and/or less total consumed power possible. For simplicity, AWGN channel with quasi-static fading is considered. Signal received at the FC from node $j \epsilon \mathcal{N}$ can be expressed as

$$x_j = \sqrt{\alpha_j}\hat{\theta}_j + w_j, \tag{1}$$

where w_j is the zero-mean receiver noise with variance $\frac{\nu_j^2}{P|h_j|^2}$, $\alpha_j = \frac{P_j}{P}$ is the ratio of the transmit power P_j of node j to the maximum allowed transmit power P, and h_j is the fading coefficient between the communication link from node j to the FC. For N nodes, the data vector available at the FC can be written as

$$\mathbf{X}_0 = \begin{pmatrix} 1 \\ \sqrt{\alpha_1} \\ \vdots \\ \sqrt{\alpha_N} \end{pmatrix} \theta + \begin{pmatrix} \tilde{\theta}_0 \\ \sqrt{\alpha_1}\tilde{\theta}_1 \\ \vdots \\ \sqrt{\alpha_N}\tilde{\theta}_N \end{pmatrix} + \begin{pmatrix} 0 \\ w_1 \\ \vdots \\ w_N \end{pmatrix}, \tag{2}$$

so that $\mathbf{X}_0 = \mathbf{a}_0\theta + \tilde{\mathbf{\Theta}}_0 + \mathbf{w}_0$, where $\tilde{\theta}_i = \hat{\theta}_i - \theta$ is zero-mean with variance V_i for all i, $\mathbf{a}_0 = [1, \sqrt{\alpha_1}, \cdots, \sqrt{\alpha_N}]^T$, $\tilde{\mathbf{\Theta}}_0 = [\tilde{\theta}_0, \sqrt{\alpha_1}\tilde{\theta}_1, \cdots, \sqrt{\alpha_N}\tilde{\theta}_N]^T$ and $\mathbf{w}_0 = [0, w_1, \cdots, w_N]^T$. Let us denote by $\Sigma^0 = \Sigma^0_\theta + \Sigma^0_w$ the covariance matrix of $\mathbf{X}_0$, where Σ^0_θ and Σ^0_w are the covariance matrices of $\tilde{\mathbf{\Theta}}_0$ and $\mathbf{w}_0$ respectively. Thus, the optimal BLUE estimator formed at the FC is given by $\hat{\theta}^1_0 = \frac{\mathbf{a}_0^T[\Sigma^0]^{-1}\mathbf{X}_0}{\mathbf{a}_0^T[\Sigma^0]^{-1}\mathbf{a}_0}$ with the updated estimator variance $V^1_0 = \left(\mathbf{a}_0^T\left[\Sigma^0\right]^{-1}\mathbf{a}_0\right)^{-1}$.

III. Basics of Nash Bargaining Solution

In this section, we briefly introduce the concept of Nash bargaining solution (NBS) [12] and then apply it to achieve a fair allocation of power among nodes in our sensor network.

The bargaining problem in a cooperative game can be described as follows [12]: Let $\mathcal{N} = \{1, 2, \cdots, N\}$ be the set of players, and let $\mathbf{S}$ be a closed and convex subset of $\mathcal{R}^N$ representing the set of feasible payoff allocations that the players can get if they all cooperate. Let u^i_{min} be the minimum expected payoff for the i-th player, below which it will not cooperate. Suppose $\{u_i \in \mathbf{S} | u_i \geq u^i_{min}, \forall i \in \mathcal{N}\}$ is a nonempty bounded set. Define $\mathbf{u}_{min} = (u^1_{min}, \ldots, u^N_{min})$; then the pair $(\mathbf{S}, \mathbf{u}_{min})$ is called the N-person bargaining problem. NBS provides a unique and fair Pareto optimal point under the conditions given in [12] (we omit them due to space limitation).

Theorem 1: (**Existence and Uniqueness of NBS**): There is a unique function $\psi(\mathbf{S}, \mathbf{u}_{min})$ that satisfies all the axioms in [12], and it satisfies

$$\psi(\mathbf{S}, \mathbf{u}_{min}) \in \arg \max_{\bar{\mathbf{u}} \in \mathbf{S}, \bar{u}_i \geq u^i_{min}, \forall i} \prod_{i=1}^{N} \left(\bar{u}_i - u^i_{min}\right)^{\beta_i}, \tag{3}$$

where β_i is the bargaining weight associated with the payoff of player i. Intuitively, it means how much importance is given to a particular player in the bargaining process.

The cooperative game for the sequential estimation problem can be described as follows: Each player (sensor) has u_i as its objective function, where u_i is non-negative, bounded from above and has a nonempty, closed and convex support. The goal is to maximize all u_i's simultaneously. u^i_{min} is the minimal payoff that player i would obtain if it had not cooperated with other players.

IV. NBS-based Solution Using Algorithm 1

For simplicity of exposition, in this section, we consider the case when all local estimators are uncorrelated and communication is over orthogonal channels. Let us assume that $\nu_i^2 = \sigma^2$ for $i \in \mathcal{N}$. Hence the covariance matrix at the FC can be written for node i and j, as $\Sigma^0 = diag\left(V_0, \alpha_i V_i + \frac{\sigma^2}{P|h_i|^2}, \alpha_j V_j + \frac{\sigma^2}{P|h_j|^2}\right)$.

In this paper, the utility of a node i is defined as the inverse of the quality of its local estimator: $u_i = \frac{1}{V_i}$. Hence, the optimization goal is to determine nodes i and j's transmission powers to the FC, such that the following objective function can be maximized:

$$U = u_0 - u^0_{min} = \frac{\alpha_i}{\alpha_i V_i + \frac{\sigma^2}{P|h_i|^2}} + \frac{\alpha_j}{\alpha_j V_j + \frac{\sigma^2}{P|h_j|^2}}, \tag{4}$$

where u_0 is the utility corresponding to the updated estimator at the FC when node i and j share their estimators with node 0, and u^0_{min} is the minimum possible payoff of node-0 that it would expect from the bargaining process. Thus, the optimization problem is

$$\min_{\alpha_i, \alpha_j} -U \qquad \text{s.t.} \qquad \left\{ \begin{array}{c} -\alpha_i \leq 0, -\alpha_j \leq 0 \\ \alpha_i + \alpha_j - 1 \leq 0 \end{array} \right\}. \tag{5}$$

Note that, we have defined the power constraint to be $\alpha_i + \alpha_j \leq 1$. The assumption is reasonable because we do not want node i and j to transmit at their maximum powers. Thus, we would like to allocate the total power P between nodes i and j in a fair way (in this case, using NBS) for sequentially estimating the parameter θ provided that no node transmits at its maximum power. To that end, we propose the following NBS based Algorithm 1 to solve the power allocation problem. The proposed sequential estimation process is summarized in Algorithm 1 and described in detail in Fig. 1.

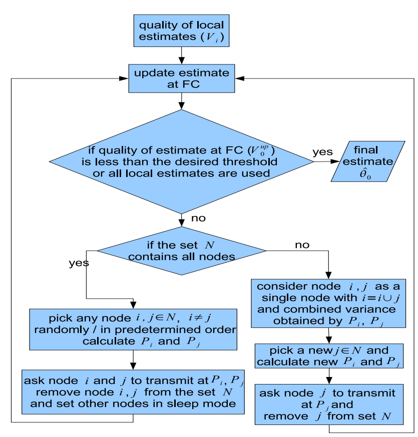

Fig. 1. Sequential estimation process using Algorithm 1.

Since U is concave in α_i, α_j and optimization constraints are linear, the Karush-Kuhn-Tucker (KKT) [13] conditions are

Algorithm 1 NBS based Power Allocation

1. FC picks any two nodes $i, j \in \mathcal{N}$, $i \neq j$ randomly or in a predetermined order.
2. Calculate α_i and α_j from (6). Assign powers P_i and P_j to nodes i and j respectively. Update the estimator variance at FC V_0^{up} and remove i and j from the set $\mathcal{N}$.
3. Consider nodes i and j as a single node with combined variance obtained by using α_i and α_j, and assign $i = i \cup j$. FC picks a new node j from the set $\mathcal{N}$ and calculate new α_i and α_j. Assign power P_j to new node j and update the estimator variance at FC V_0^{up}.
4. Repeat the same procedures from step 3 until $V_o^{UP} \leq \epsilon$ or $\mathcal{N} = \emptyset$, where ϵ is the desired quality of estimate at the FC.

both necessary and sufficient. Solving for α_i and α_j from (5), we get the solution:

$$\left.\begin{aligned} \alpha_i &= \frac{V_j|h_j| + \frac{\sigma^2}{P}\left(\frac{1}{|h_j|} - \frac{1}{|h_i|}\right)}{V_i|h_i| + V_j|h_j|}, \\ \alpha_j &= \frac{V_i|h_i| + \frac{\sigma^2}{P}\left(\frac{1}{|h_i|} - \frac{1}{|h_j|}\right)}{V_i|h_i| + V_j|h_j|}. \end{aligned}\right\} \tag{6}$$

For $|h_i|^2 = |h_j|^2$, $\alpha_i = \frac{V_j}{V_i + V_j}$ and $\alpha_j = \frac{V_i}{V_i + V_j}$. Hence, node with more accurate estimation are allowed to transmit at a higher power than that with less accurate estimation, which intuitively makes sense. Since FC has the knowledge of the quality of estimates at nodes i and j, all the calculations can be done at the FC and it can send control signals to nodes i and j to transmit at powers $P_i = \alpha_i P$ and $P_j = \alpha_j P$ respectively.

V. NBS-based Solution Using Algorithm 2

It is to be noted that for the NBS-based Algorithm 1 above, an explicit analytical solution in the case of correlated observations could not be obtained. This motivated us to propose an NBS-based Algorithm 2 to solve the problem of power allocation for collaborating nodes with correlated observations. We assume that all local estimators are correlated such that $C_{ij} = Cov\{\hat{\theta}_i, \hat{\theta}_j\} = \rho, \forall i, j$, where C_{ij} is the covariance between the random variables $\hat{\theta}_i$ and $\hat{\theta}_j$. The covariance matrix at the FC and at node i can be written respectively as: $\Sigma^0 = \begin{pmatrix} V_0 & \sqrt{\alpha_i}\rho \\ \sqrt{\alpha_i}\rho & \alpha_i V_i + \frac{\sigma^2}{P|h_i|^2} \end{pmatrix}$, and $\Sigma^i = \begin{pmatrix} V_i & \sqrt{\alpha_0}\rho \\ \sqrt{\alpha_0}\rho & \alpha_0 V_0 + \frac{\sigma^2}{P|h_i|^2} \end{pmatrix}$ for $i \in \mathcal{N}$. Hence we have

$$u_0 - u_{min}^0 = (1 \; \sqrt{\alpha_i})[\Sigma^0]^{-1}(1 \; \sqrt{\alpha_i})^T - \frac{1}{V_0}, \tag{7}$$

$$\text{and} \quad u_i - u_{min}^i = (1 \; \sqrt{\alpha_0})[\Sigma^i]^{-1}(1 \; \sqrt{\alpha_0})^T - \frac{1}{V_i}. \tag{8}$$

As a result, the optimization problem becomes:

$$\min_{\alpha_0, \alpha_i} - \prod_{j=0,i} \left(u_j - u_{min}^j\right) \text{ s.t.} \left\{ \begin{array}{c} -\alpha_0 \leq 0, -\alpha_i \leq 0 \\ \alpha_0 + \alpha_i - 1 \leq 0 \end{array} \right\} \tag{9}$$

Since the optimization problem is again convex in α_0, α_i and constraints are linear, the KKT conditions are again both necessary and sufficient. Solving for α_i from (9), the only non-zero solutions that satisfy all the KKT conditions can be obtained as:

$$\alpha_i = \left[\frac{1 + \mathcal{Q} \mp \sqrt{1 + \mathcal{Q}}}{\mathcal{Q}}\right]^+, \tag{10}$$

where $[.]^+$ means only the non-negative bounded values are considered, $\mathcal{Q} = \frac{(V_0 - V_i)(V_0 V_i - \rho^2)}{V_i\left(V_0 \frac{\sigma^2}{P|h_i|^2} + V_0 V_i - \rho^2\right)}$ with $V_0 > 0$, $V_i > 0$ and $0 \leq \rho \leq 1$. It is to be noted that the FC only helps to calculate α_i and it discards the value α_0, as it only transmits control signals. Since all the calculations are done at the FC, the only parameters the FC needs to know are V_i and $\frac{\sigma^2}{P|h_i|^2}$. The details of the sequential estimation process using NBS-based Algorithm 2 is described in Fig. 2.

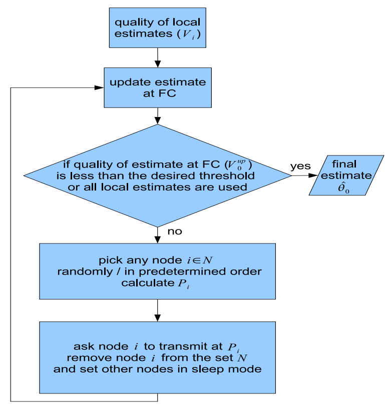

Fig. 2. Sequential estimation process using Algorithm 2.

Algorithm 2 NBS based Power Allocation

1. FC (node-0) picks any node $i \in \mathcal{N}$, randomly or in a predetermined order.
2. Calculate α_i from (10). Assign power P_i to node i. Update the estimator variance at FC V_0^{up} and remove node i from the set $\mathcal{N}$.
3. Repeat the same procedures until $V_o^{up} \leq \epsilon$ or $\mathcal{N} = \emptyset$.

VI. Simulation Results

In this section, we investigate the performance obtained by our proposed algorithms. Parameters used for simulations are: number of distributed nodes $N = 50$, estimator variances $V_i \sim \mathcal{U}[1, 20]$, and $\frac{\sigma^2}{P} = 0.5$. We assume that all channel gains follow Rayleigh distributions with all channel coefficients normalized so that $\mathbb{E}\left\{h^2\right\} = 1$. All simulation results are obtained by averaging over 500 fading realizations.

Figure 3 shows the updated estimator variance V_0^{up} at the FC as a function of number of nodes. As it can be seen from Fig. 3, V_0^{up} monotonically decreases for both algorithms as more nodes are incorporated into the estimation process. The monotonic decrease in variance at the FC center is almost as good as the case when all the nodes transmit at their maximum powers (MPA) with each of the proposed algorithms. By MPA

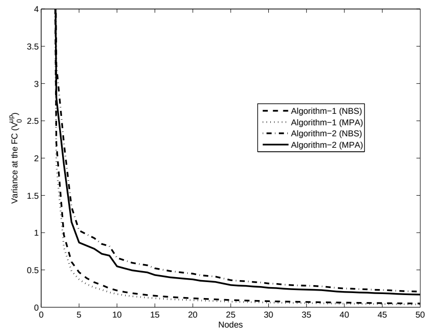

Fig. 3. Updated variance at the FC vs. number of nodes with $\rho = 0$.

(Maximum Power Allocation), we mean to follow all the the steps in each of the proposed algorithms except for setting $\alpha_i = 1, \forall i$. As it can be seen from Fig. 3, Algorithm 1 outperforms Algorithm 2 in terms of the rate of improvement of the estimator quality at the FC. This is because of the formation of Nash product (NP) of the optimization problems in (5) and (9). NP in (5) was obtained by setting $\beta_0 = 1$ and $\beta_i = \beta_j = 0$. Hence, the emphasis is only on maximizing the payoff of node 0 (FC). On the other hand, in (9), we have set $\beta_0 = 1$ and $\beta_i = 1$. As a result, the optimization problem tries to maximize the payoffs of both nodes 0 and i. Since the payoff of a node is defined as the inverse of the quality of estimate, better rate of improvement of the estimator quality is achieved by maximizing the payoff.

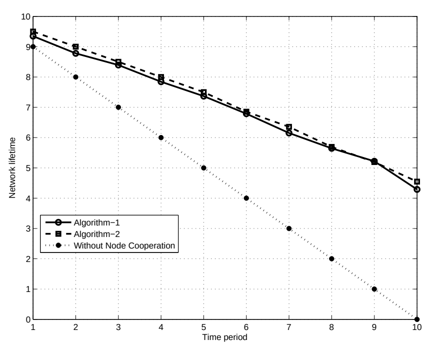

Fig. 4. Network lifetime improvement using Algorithm 1 and 2.

Figure 4 shows network lifetime improvement at the end of each time period or estimation block using the proposed algorithms. Note that, we define the lifetime of a sensor network as the time after which at least one or a certain fraction of sensor nodes run out of their batteries, resulting in a hole within the network. Since FC is equipped with sufficient energy, we are only concerned about the distributed nodes. We assume that each distributed node is provided with limited energy at the beginning of the estimation process. At the end of the j-th time block, the network lifetime can be defined as $T_N^j = \min_i \left(\frac{E_i^{j-1} - P_i^j t_p}{P} \right)$, where E_i^{j-1} is the energy available at the sensor node i at the end of $(j-1)$-th time period, P_i^j is the power to be spent by the node i during the j-th estimation block and t_p is the time each node spends transmitting to the FC. For our simulation, we have used $E_i^0 = 10$ Joules, $t_p = 1$ sec and $P = 1$ watt. As it can be seen from Fig. 4, Algorithm 2 is slightly better than the Algorithm 1 as far as network lifetime improvement is concerned. This is again because of the formation of the optimization problems for Algorithms 1 and 2: the obtained value of $\max_i \alpha_i$ for Algorithm 1 can be higher than that for the Algorithm 2 most of the time, which reduces the lifetime of WSN using Algorithm 1 compared to that with Algorithm 2.

VII. Conclusion

In this paper, a cooperative game-theoretic framework has been proposed to achieve a fair allocation of transmit power for collaborating nodes in a Fusion Center (FC) based wireless sensor network tasked with sequential estimation of a non-random parameter. In particular, we proposed two algorithms based on the concept of Nash bargaining solution (NBS) to arrive at a fair allocation of power for the nodes. Simulation results show that the proposed algorithms sequentially achieve the desired quality of estimate at the FC, and increase the overall network lifetime.

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