Effect of Dead Space on Gain and Noise in Si and GaAs Avalanche Photodiodes

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Abstract—We investigate the effect of dead space on the mean gain, the excess noise factor, and the avalanche breakdown voltage for Si and GaAs avalanche photodiodes (APD's) with nonuniform carrier ionization coefficients. The dead space, which is a function of the electric field and position within the multiplication region of the APD, is the minimum distance that a newly generated carrier must travel in order to acquire sufficient energy to become capable of causing impact ionization. Recurrence relations in the form of coupled linear integral equations are derived to characterize the underlying avalanche multiplication process. Numerical solutions to the integral equations are obtained and the mean gain and the excess noise factor are computed. The presence of dead space results in a reduction of both the mean gain and the excess noise factor. This can have a beneficial effect on the performance of the detector when used in optical receivers with photon noise and circuit noise. Dead space also serves to increase the applied bias voltage at which avalanche breakdown occurs.

I. INTRODUCTION

MULTIPLICATION noise due to the random fluctuations of the gain in APD’s determines the performance of fiber-optic communication systems that use APD’s as optical detectors [1]. The simplest measure of this multiplication noise is the excess noise factor. Dead space, which is the distance required by a carrier to acquire sufficient energy to impact ionize, has been shown to reduce both the mean gain and the excess noise factor for APD’s with uniform electric fields [2], [3]. This concurrent reduction has been shown to have a beneficial effect on the performance of the optical receiver when the mean gain is such that the signal exceeds the electronic circuit noise [2], [3]. Okuto and Crowell [4] calculated numerically the mean gain in a double-carrier multiplication (DCM) APD with dead space. Later, Saleh et al. [2] used the theory of age-dependent branching processes [5] to treat the single-carrier multiplication dead-space model. They obtained analytical expressions for the mean gain and the excess noise factor, and determined numerically the mean and standard deviation of the impulse response function of the APD as a function of time following a photoexcitation. Marsland [6] attempted to extend the classical McIntyre theory [7] for the mean gain and excess noise factor of a conventional APD to an APD with dead space. However, the formulation is incomplete; it takes into account the dead space associated only with the initial carrier pair and does not extend it to the subsequent carriers. Hayat et al. [3] formulated a recurrence method and determined the mean gain and the excess noise factor in a DCM APD with dead space. This theory, however, assumed uniform electric fields.

In this paper, we extend the theory developed in [3] to allow the analysis of DCM APDs with nonuniform electric fields. This will allow the ionization coefficients and the dead space to be dependent on the position in the multiplication region of the APD. Modifications to the recurrence equations derived in [3] are developed for the first and second moments of the total number of electrons and holes generated in the multiplication region. These random quantities are related to the random gain in a deterministic way.

The theory is applied to two APD’s: Si which almost operates in the single-carrier multiplication mode, and GaAs which operates in the double-carrier multiplication mode. Numerical solutions to the recurrence equations are obtained and the excess noise factor is plotted as a function of the mean gain. We have found that in each case, the presence of dead space results in a reduction of both the mean gain and the excess noise factor in a manner which has a beneficial effect on the performance of the optical receiver. These results resemble the ones obtained for the case of uniform electric field [3]. Furthermore, dead space serves to increase the applied bias voltage at which avalanche breakdown occurs.

II. MODEL

Consider an APD multiplication region extending from \( x = 0 \) to \( x = W \) and assume that the electron and hole ionization coefficients \( \alpha(x) \) and \( \beta(x) \) are position dependent. The avalanche multiplication process is initiated by the injection of an electron into the multiplication region at \( x = 0 \). This electron travels in the \( x \) direction with a fixed velocity \( v_e \). Within a distance \( d_e(0) \), called the electron dead space at location 0, the electron cannot ionize, but thereafter it may ionize at location \( x \in [d_e(0), W] \) with probability density (per unit length) \( \alpha(x) \). Upon ionization, an electron-hole pair is generated, so that the original electron is replaced by two newly generated electrons.
and a newly generated hole. The two electrons behave identically and independently. Similar to their parent electron, each electron travels without ionizing a distance $d_e(x)$, after which it may ionize at location $y \in [x - d_e(x), W]$ with probability density $\alpha(y)$. The electron is then replaced by two newly generated electrons and a newly generated hole. The hole, generated at location $x$, travels in the $-x$ direction with a fixed velocity $v_h$. Within a distance $d_h(x)$, called the hole dead space at location $x$, the location cannot ionize, but thereafter it may ionize at location $y \in [0, x - d_h(x)]$ with probability density $\beta(y)$. Upon ionization, an electron-hole pair is generated, so that the original hole is replaced by two newly generated holes and a newly generated electron. The electrons and holes repeat the process independently as they travel through the multiplication region, and so on. When an electron reaches the right edge of the multiplication region at $x = W$, its role ends. Similarly, a hole ceases to ionize when it reaches the left edge at $x = 0$. If the process terminates, it does so when all possible carriers have reached the edges of the multiplication region.

Suppose that an electron is generated at location $x$ within the multiplication region. Let $X_e(x)$, called the electron life span at $x$, be the random distance at which impact ionization occurs. Similarly, let $X_h(x)$, called the hole life span at $x$, be the random distance that a hole, generated at location $x$, travels before it ionizes. The family of random variables $X_e(x)$ and $X_h(x)$ are assumed to be statistically independent for all $x \in [0, W]$ and to have probability density functions (p.d.f.'s) $h_e(x, \eta)$ and $h_h(x, \eta)$, respectively. These p.d.f.'s take the following form:

$$
\begin{align*}
    h_e(x, \eta) &= \begin{cases} 
        0 & \eta < d_e(x) \\
        \alpha(\eta + x) \exp \left[ -\int_{d_e(x)}^{\eta} \alpha(x + \sigma) \, d\sigma \right] & d_e(x) \leq \eta \leq W - x
    \end{cases} \\
\end{align*}
$$

and

$$
\begin{align*}
    h_h(x, \eta) &= \begin{cases} 
        0 & \eta < d_h(x) \\
        \beta(x - \eta) \exp \left[ -\int_{d_h(x)}^{\eta} \beta(x - \sigma) \, d\sigma \right] & d_h(x) \leq \eta \leq x
    \end{cases}
\end{align*}
$$

Since our analysis is restricted to the multiplication region, the value of the p.d.f.’s $h_h$ and $h_e$ outside the ranges given in (1) is not relevant. In the special case when the electron and hole ionization coefficients are not position dependent (i.e., $\alpha(x) = \alpha$ and $\beta(x) = \beta$) the expressions for the p.d.f.’s in (1) reduce to the expressions presented in our earlier work [3, (1)].

The total number of electron-hole pairs generated within the multiplication region, including the original electron which initiated the entire multiplication process at $x = 0$, is a random number $G$ that constitutes the gain of the device. Our objective is to determine the statistics of $G$.

### III. Mean Gain and Excess Noise Factor

We will extend the theory developed in [3] to allow the analysis of DCM APD’s with nonuniform electric fields. Then we will determine the statistical properties of the gain. Consider an electron and a hole at location $x$ within the multiplication region, assume that the electron is responsible for the production of a random sum $Z(x)$ of electrons and holes, including the initiating electron itself. Similarly, $Y(x)$ is the random number of all electrons and holes produced by the hole and its offspring, including the hole itself. For the case of electron injection, the gain of the device is then given by the relation

$$
G = \frac{1}{2} (Z(0) + 1).
$$

Once the statistical properties of $Z(x)$ and $Y(x)$ are determined, the statistics of $G$ can be inferred from (2).

The mean and the second moment of $Z(x)$ and $Y(x)$ are defined as follows:

$$
\begin{align*}
    z(x) &= \langle Z(x) \rangle \\
    y(x) &= \langle Y(x) \rangle
\end{align*}
$$

and

$$
\begin{align*}
    z_2(x) &= \langle Z^2(x) \rangle \\
    y_2(x) &= \langle Y^2(x) \rangle
\end{align*}
$$

where the bracket denotes ensemble average. A set of recurrence equations, in the form of coupled linear integral equations [3, (10), (11), (18), and (19)] has been derived to govern the quantities in (3) in the case of uniform electric field. These equations contain the p.d.f.’s $h_e$ and $h_h$ of the electron and hole life spans, respectively. To incorporate the position dependence of the electron and hole life spans we substitute the expressions in (1) for the p.d.f.’s in the recurrence equations to obtain a modified version of the recurrence equations. For the means, these equations become:

$$
\begin{align*}
    z(x) &= [1 - h_e(x, W - x)] + \int_{x}^{W} [2z(\xi) + y(\xi)] \\
    &\quad \cdot h_e(x, \xi - x) \, d\xi
\end{align*}
$$

and

$$
\begin{align*}
    y(x) &= [1 - h_h(x, x)] + \int_{0}^{x} [2y(\xi) + z(\xi)] \\
    &\quad \cdot h_h(x, x - \xi) \, d\xi
\end{align*}
$$

where

$$
H_e(x, \eta) = \int_{-\infty}^{\eta} h_e(x, \xi) \, d\xi
$$
and

\[ H_0(x, \eta) = \int_{-\infty}^{x} h_0(x, \xi) \, d\xi \]

are the distribution functions of the electron and hole life spans at location \( x \), respectively.

Equations (4) and (5) are valid only for \( 0 < x < W \).

At \( x = W \),

\[ z(W) = 1 \quad (6a) \]

and at \( x = 0 \)

\[ y(0) = 1. \quad (6b) \]

The coupled linear integral equations (4) and (5) are the basic equations from which the mean gain \( \langle G \rangle \) will be determined by taking the ensemble average of both sides of (2). Thus the mean gain is

\[ \langle G \rangle = \frac{1}{2} (z(0) + 1). \quad (7) \]

Similarly, for the second moments, the modified recurrence equations are

\[ z_2(x) = [1 - H_e(x, W - x)] + \int_0^x [2z_2(\xi) + y_2(\xi) + 4z(\xi)y(\xi) + 2z^2(\xi)] h_e(x, x - \xi) \, d\xi, \quad (8) \]

and

\[ y_2(x) = [1 - H_h(x, x)] + \int_0^x [2y_2(\xi) + z_2(\xi) + 4z(\xi)y(\xi) + 2y^2(\xi)] h_h(x, x - \xi) \, d\xi. \quad (9) \]

Equations (8) and (9) are valid only for \( 0 < x < W \).

At \( x = W \),

\[ z_2(W) = 1, \quad (10a) \]

and at \( x = 0 \),

\[ y_2(0) = 1. \quad (10b) \]

Equations (8) and (9) require knowledge of the first moments \( z(x) \) and \( y(x) \), which must be computed separately by the use of (4) and (5).

The excess noise factor \( F \) is related to \( z(0) \) and \( z_2(0) \) by

\[ F = \frac{\langle G^2 \rangle}{\langle G \rangle^2} = \frac{z(0) + 2z(0) + 1}{z(0) + 1}. \quad (11) \]

IV. APPLICATIONS

We have used the foregoing equations to compute the mean gain and the excess noise factor in Si and GaAs. We use a conventional \( p^+ \)-n diode structure. The carrier concentrations in the \( p^+ \) and \( n \) layers are \( N_p = 1.0 \times 10^{17} \) cm\(^{-3} \) and \( N_n = 5.0 \times 10^{15} \) cm\(^{-3} \), respectively. The electrons within the depletion region \( 0 \leq x \leq W \) travel in the positive \( x \) direction and the \( p^+ \)-n junction is at \( x = 0 \).

A. Field Distribution

The electric field profile, shown in Fig. 1(a), is obtained by solving Poisson’s equation and is given by

\[ E(x) = E_M - \frac{qN_D}{\varepsilon_r} x \quad (0 \leq x \leq W) \quad (12) \]

where \( E_M \) is the maximum electric field intensity, \( q \) the electron charge, and \( \varepsilon_r \) is the relative dielectric constant.

The depletion region width of the device can be expressed in terms of the applied reverse bias voltage \( V_R \) and the built-in potential \( V_{bi} \) by use of the relation

\[ V_{bi} + V_R = -\int_0^W E(x) \, dx \quad (13) \]

and the boundary condition

\[ E(W) = 0. \]

This gives the depletion layer width

\[ W = \sqrt{\frac{2\varepsilon_r}{q}} \left( \frac{N_A + N_D}{N_A N_D} \right) (V_{bi} + V_R). \quad (14) \]

For example, when \( V_R = 85 \) V, \( W = 4.64 \mu m \) as illustrated in Fig. 1(a).

B. Dead Space

The minimum distance that a newly generated carrier must travel in order to acquire sufficient kinetic energy before ionization is possible is called the carrier dead space. This kinetic energy is taken to be equal to the ionization threshold energy of the carrier \( E_{th} \) for the electron and \( E_{th} \) for the hole \( [4] \). The numerical value of the dead spaces, \( d_e(x) \) and \( d_h(x) \), can therefore be obtained using the relations

\[ q \cdot \int_{x-d_e(x)}^{x+d_e(x)} E(x') \, dx' = E_{th} \quad (15) \]

and

\[ q \cdot \int_{x-d_h(x)}^{x+d_h(x)} E(x') \, dx' = E_{th}. \quad (16) \]

From these equations, we obtain the expressions

\[ d_e(x) = (W - x) - \sqrt{(W - x)^2 - \frac{2E_{th}}{c}} \quad (17) \]

and

\[ d_h(x) = (x - W) + \sqrt{(W - x)^2 + \frac{2E_{th}}{c}} \quad (18) \]

where \( c = qN_D/\varepsilon_r \).

As the electric field intensity decreases with \( x \), a carrier must travel a longer distance before becoming capable of ionizing. The dead spaces \( d_e \) and \( d_h \), therefore, increase with \( x \) until \( x \) reaches a critical value at which the total
acquired kinetic energy can no longer equate the threshold ionization energy. The behavior of $d_e$ and $d_h$ is shown in Fig. 1(b).

C. Ionization Coefficients

The equations representing the electron and hole ionization coefficients, $\alpha(x)$ and $\beta(x)$, respectively, used in our model [as in (1)] are the expressions Okuto and Crowell (OC) [4], [8] used in their dead space theory. OC proposed analytical expressions for $\alpha(x)$ and $\beta(x)$ in terms of the dead spaces $d_e(x)$ and $d_h(x)$, the optical phonon energy $E_{opt}$, the optical-phonon mean free path $\lambda_{opt}$, and the ionization energies $E_{ie}$ and $E_{ih}$:

$$\alpha(x) = \frac{1}{d_e(x)} \left( -1 + \exp\left( \left( \frac{(d_e(x))/\lambda_{opt}}{2} \right)^2 \right) + \left\{ 0.217 \left( \frac{E_{ie}}{E_{opt}} \right)^{1.14} \right\}^{1/2} \right)$$

and

$$\beta(x) = \frac{1}{d_h(x)} \left( -1 + \exp\left( \left( \frac{(d_h(x))/\lambda_{opt}}{2} \right)^2 \right) + \left\{ 0.217 \left( \frac{E_{ih}}{E_{opt}} \right)^{1.14} \right\}^{1/2} \right)$$

The above equations are derived under the assumption of a constant electric field profile; nonetheless, OC [8] argued that the values of $\alpha(x)$ and $\beta(x)$ given in (19) and (20) should not be significantly affected as long the electric field does not change radically over a distance $d_e(x)$ or $d_h(x)$. A typical profile for $\alpha(x)$ and $\beta(x)$ is illustrated in Fig. 1(c).

D. Performance Factor

To assess the effect of dead space on the performance of communication systems consider a binary system receiving a photon flux $\phi$ (photons per second). Assuming Poisson photon statistics, the signal-to-noise ratio (SNR) of the total charge accumulation in the detection circuit in a time interval $T$ is given by [1]

$$\text{SNR} = \frac{\phi T \langle G \rangle}{\sigma^2 T}$$

where $\sigma = i/qT$, and $i$ is the rms current of the circuit noise. Thus $\phi T$ is the mean number of photons collected and $\sigma$ is the rms circuit noise charge flow in the time interval $T$ (units of number of electrons). The quantum efficiency of the APD is assumed to be unity. Since the SNR for an ideal photon-noise limited receiver ($\sigma = 0$, $F = 1$) is $\phi T$, the performance factor

$$P = \frac{\langle G \rangle^2}{\langle G \rangle^2 F + \sigma^2}$$

represents the SNR reduction caused by the combination
of gain fluctuations and circuit noise. The importance of the role played by dead space is governed by the circuit-noise parameter $\sigma^2/\phi T$.

E. Results

We have used the recurrence equations obtained in Section III to determine the mean gain and excess noise factor of a Si and a GaAs APD. The mean gain and the avalanche breakdown voltage for the Si and GaAs APD’s are also determined. The numerical computations are carried out as follows. 1) $z(x)$ and $y(x)$ are set to zero everywhere in the interval $0 \leq x \leq W$, with the two exceptions $z(W) = y(0) = 1$, in accordance with (6). 2) Equation (5) is discretized, using a suitable mesh size, and then used to generate estimates of $y(x)$ in the interval $0 \leq x \leq W$. 3) Using this estimate of $y(x)$ in the discrete version of (4), an estimate of $z(x)$ is generated in the interval $0 \leq x \leq W$. 4) An improved estimate of $y(x)$ is obtained by substituting in the discrete version of (5) the previously calculated estimate of $z(x)$. 5) Steps 3) and 4) are repeated until convergence is achieved. The mean gain $\langle G \rangle$ is computed by use of (7).

In a similar way, the second moments $z'_2(x)$ and $y'_2(x)$ have been computed numerically using discrete versions of (8) and (9). With the aid of (11), the excess noise factor $F$ has been computed. The validity of the numerical results have been verified by comparison with results obtained by use of known expressions in the special case of no dead space [7] and with the special case of uniform electric field [3].

The mean gain $\langle G \rangle$ as a function of the applied bias voltage $V_R$ is depicted for the Si and GaAs APDs in Fig. 2. The plots are compared with the plots obtained using the theory with no dead space [7]. In the latter case, the electron and hole ionization coefficients are taken to be the inverse of the electron and hole mean scattering distances, respectively, as discussed in detail by OC [4]. These coefficients, denoted by $\alpha'(x)$ and $\beta'(x)$, are given by [4]

$$\alpha'(x) = 1/d_e(x) \exp \left\{ \frac{(d_e(x)/\lambda_e)^2}{0.217(E_{ih}/E_r)^{1.14}} \right\}^{1/2} - \left\{ 0.217(E_{ih}/E_r)^{1.14} \right\}^{1/2}$$

and

$$\beta'(x) = 1/d_h(x) \exp \left\{ \frac{(d_h(x)/\lambda_h)^2}{0.217(E_{ih}/E_r)^{1.14}} \right\}^{1/2} - \left\{ 0.217(E_{ih}/E_r)^{1.14} \right\}^{1/2}$$

Note that $\alpha'(x)$ and $\beta'(x)$ are different from $\alpha(x)$ and $\beta(x)$ used in the dead space model. It is apparent that in both devices, the breakdown condition occurs at a higher $V_R$ as a result of dead space. This reduction of $\langle G \rangle$ results in higher values for the avalanche breakdown threshold voltage $V_R$.

Fig. 2. Mean gain $\langle G \rangle$ as a function of applied bias voltage $V_R$ for the ideal case (solid) and for the dead space case (dashed) in Si and GaAs APD’s.

Fig. 3. The excess noise factor $F$ as a function of the mean gain $\langle G \rangle$ for the ideal case (solid line) and for the dead space case (dashed line). (a) Si. (b) GaAs.

The effect of dead space on the excess noise factor $F$ for Si and GaAs is shown by plotting $F$ as a function of $\langle G \rangle$ as illustrated in Fig. 3. It is seen that dead space reduces $F$. This effect is more significant for GaAs. This result resembles the result obtained for the case of uniform electric fields for which the reduction in $F$ due to dead space was more evident for APD’s operating in the DCM mode [3]. This is probably a result of Si being essentially a single-carrier multiplication material, while GaAs operates in a double-carrier multiplication mode. For GaAs, the values of $\alpha$ and $\beta$ are comparable in magnitude where in Si $\alpha$ is an order of magnitude greater than $\beta$. 
Dead space also affects the performance factor. For small circuit noise, \( \sigma^2 \phi T \ll \langle G \rangle^2 F \) and the performance factor \( P \approx 1/F \), so that the performance is enhanced by the presence of dead space. On the other hand, for large circuit noise, \( \sigma^2 \phi T \gg \langle G \rangle^2 F \) and \( P \approx \langle G \rangle^2 \), so that dead space has a performance degradation effect. However, the mean gain can usually be increased by simply increasing the voltage applied to the device. The effect of dead space on \( P \) as a function of \( \langle G \rangle \) is depicted in Fig. 4 for a fixed value of the circuit noise parameter \( \sigma^2 \phi T = 1000 \).

V. CONCLUSION

The conventional model of avalanche multiplication assumes that the probability that a carrier causes an impact ionization is independent of the carrier’s own history. This assumption allows a carrier to impact ionize immediately after its generation. From a physical point of view, however, this is not possible. A newly generated carrier must first travel a dead space to gain sufficient kinetic energy for impact ionization. Once a carrier is able to ionize, it may ionize with a probability density (per unit length) which depends on the location at which ionization takes place. The dead space itself is dependent on the location at which the carrier is generated. To accommodate for the effect of dead space on the statistics of the gain, a theory has been formulated which takes into account the carrier’s age. We presented a theory, based on recurrence equations, which allow us to compute the mean gain and the excess noise factor.

We have found that the presence of dead space results in a reduction of both the mean gain and the excess noise factor in both Si and GaAs APD’s. The excess noise factor versus the mean gain curve is also lowered in comparison to the ideal case in which there is no dead space. Furthermore, dead space serves to increase the applied bias voltage at which avalanche breakdown occurs.

Under certain conditions, the dead space has a beneficial effect on the performance of optical receivers. Nonetheless, the applied bias voltage must be increased in order to compensate for the reduction in the mean gain. It may therefore be advantageous to select materials for which the dead space is large.

References


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