A Linear Equalizer for High-Speed APD-Based Integrate-and-Dump Receivers

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***Abstract*— A linear minimum mean-square error (LMMSE) equalizer for avalanche-photodiode (APD) based integrate-and-dump receivers is designed to compensate for intersymbol-interference (ISI) introduced by the APD's random impulse response. Recent work on the characterization of the joint probability distribution of the APD's gain and buildup-time are adopted and utilized to determine the mean and correlation matrix of the APD-based receiver's random impulse response. This LMMSE equalizer is shown to improve the bit-error-rate dramatically at high transmission rates, resulting in an improvement in the receiver sensitivity by approximately 2.3 dB at 15GHz.**

***Index Terms*— Avalanche photodiodes, equalization, random impulse response, bit-error-rate, receiver sensitivity.**

I. Introduction

MANY modern high-speed optical receivers employ a PIN photodiode as a photodetector, combined with an Erbium-doped fiber amplifier (EDFA). Such combination is prevalently adopted due to its high bandwidth, thanks to the fast response time of the PIN detector, and its good signal-to-noise ratio characteristics, which is attributable to the high gains that an EDFA can provide. However, optical amplifiers are very costly and bulky, as they require a laser pump as well as several meters of coiled fiber to support the optical amplification [1]. In such systems, linear equalization is often used to compensate for intersymbol-interference (ISI) resulting from fiber-dispersion induced pulse broadening [1], [2]. However, an avalanche photodiode (APD) offers a cost-effective alternative to the PIN-EDFA combination as it is capable of amplifying the photocurrent internally, without the need for optical preamplification. Unfortunately, this internal optoelectronic gain, which is referred to avalanche carrier multiplication, is accompanied by the so-called avalanche-buildup time, during which the cascade of impact ionizations is realized. This results in a relatively slow response time for the APD (compared to a PIN) and introduces additional ISI. Although high-speed APDs are available which are suitable for transmission speeds up to 10 Gbps, their utility at higher speeds would result in significant ISI, which must be compensated for in the receiver.

Even though equalization techniques [2] have been brought out to compensate for fiber-dispersion-induced ISI, they are not suitable for compensating ISI introduced by the APD as the impulse response function of an APD is highly random, both in its duration (representing the avalanche-buildup time) and in its area (representing the multiplication factor, i.e., gain). This makes ISI compensation for APDs distinct from that designed for dispersion-induced ISI. In this work, we develop a linear minimum mean-square error (LMMSE) equalizer for an APD-based integrate-and-dump receiver in on-off keying (OOK) optical communication systems. The APD-based integrate-and-dump receiver is viewed as a random linear channel, since the APD's impulse-response function is a stochastic process. In contrast to the deterministic channels, the equalization of this random channel has to take into account the statistical properties of the random channel's impulse response. Even though the adopted LMMSE equalization scheme is simple, the required correlations of the random channel's output and the cross-correlation between the random channel's input and output involve characterizing the complex statistical properties of the shot noise [3] generated by the APD's impulse response function. In our earlier work [4], the joint statistics of the APD's gain and buildup-time was determined for the first time, and the random-duration (RD) rectangular model was proposed to accurately characterize the mean and autocorrelation of the APD's impulse response function. This facilitates deriving the required statistics in the equalization design. In this work, the RD rectangular model for the APD's impulse response function is utilized to design a finite-tap LMMSE equalizer [5]. To emphasize the importance of using the exact statistics of the APD's impulse response, we also calculated another set of equalizer coefficients by assuming a deterministic-duration (DD) exponential model [4], which ignores the statistical correlation between the APD's gain and buildup time.

Let I_n, $n \in \mathbb{Z}$, denote the input binary sequence with bit duration T_b. The optical intensity is assumed ϕ for a "1" bit and 0 for a "0" bit. Let R_n, $n \in \mathbb{Z}$, denote the raw receiver output (i.e., prior to any decision). Each optical input I_n contributes a term $A_0^{(n)} I_n$ to the present output R_n and an ISI term $A_k^{(n)} I_n$ to future outputs R_{n+k}, $k = 1, 2, \ldots$. Mathematically, $A_k^{(n)} = \int_{(n+k)T_b}^{(n+k+1)T_b} \sum I(t-\tau_i)dt$, where $I(t)$ is the APD's random impulse response function, τ_i's are the photons' arrival times, and $nT_b \le \tau_i < (n+1)T_b$. If we assume the ISI affects up to $L-1$ future receiver outputs, then the receiver output can be written as

$$R_n = A_0^{(n)} I_n + A_1^{(n-1)} I_{n-1} + \ldots + A_{L-1}^{(n-L+1)} I_{n-L+1} + J_n, \quad (1)$$

where J_n is a zero-mean Gaussian Johnson noise, independent

Manuscript received June 13, 2005. The associate editor coordinating the review of this letter and approving it for publication was Prof. Maode Ma. This work was supported by the National Science Foundation through awards ECS-0334813 and ECS-0196569.

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Digital Object Identifier 10.1109/LCOMM.2005.12005.

of the data sequence $\{I_k\}$ and the impulse response of the APD. In the above equation, the sum $A_1^{(n-1)}I_{n-1} + A_1^{(n-2)}I_{n-2} + \ldots + A_{L-1}^{(n-L+1)}I_{n-L+1}$ is referred to as the random ISI. Additionally, the signal $H(n) \triangleq A_0^{(0)}\delta(n) + A_1^{(0)}\delta(n-1) + \ldots + A_{L-1}^{(0)}\delta(n-L+1)$ is identified as the random impulse response of the receiver. We can therefore think of the APD-based receiver as a stochastic finite-impulse-response (FIR) filter. Note that $A_p^{(n-k)}$ and $A_q^{(n-k)}$ are statistically correlated since they are ISI terms induced by the same photons in the optical pulse I_{n-k}. On the other hand, $A_p^{(n-k)}$ and $A_q^{(n-l)}$ are independent as long as $k \neq l$ since they arise from different photons. Since the mean and correlations of this APD-based receiver's impulse response are time invariant, we ignore the superscript elsewhere.

Next, we design an equalizer that is a linear combination of current observation R_n and the past observations to best restore the original transmitted signal I_n:

$$\hat{I}_n = b_0 R_n + b_1 R_{n-1} + \ldots + b_{M-1}R_{n-M+1}, \tag{2}$$

where the b_is, $i = 0, 1, \ldots, L-1$, are the tap coefficients of the linear equalizer, $\hat{I}_n$ is the ISI compensated estimate of I_n, and M is the number of taps in the equalizer.

II. Equalizer Design

A. Stochastic Model for the APD's Impulse Response Function

Conventionally, the DD exponential model is used in most analysis, which assumes the APD's impulse response function $I(t)$ taking the form $I_{\rm DD}(t) = Gbe^{-b_{\rm 3dB}t}$, where G is APD's random gain, and $b_{\rm 3dB}$ is its 3dB bandwidth. The buildup-time is completely determined by its random gain in this DD exponential model, so the temporal statistical correlation in $I(t)$ is omitted. In contrast, in the RD rectangular model [4], the APD's impulse-response function assumes the form $I_{\rm RD}(t) = GT^{-1}[u(t) - u(T-t)]$, where T is the random duration of the impulse response function, and $u(t)$ is the unit-step function. The joint probability distribution of G and T is also calculated in [4].

With the availability of the RD rectangular model, the mean and autocorrelation function of the APD's impulse response are approximated by [4]

$$\mathsf{E}[I_{\rm RD}(t)] = \mathsf{E}[G]b_{\rm se}e^{-b_{\rm se}t}, \tag{3}$$

$$R_{\rm RD}(\mu,\nu) = \mathsf{E}[G^2/T]b_{\rm se}e^{-b_{\rm se}(\mu\vee\nu)}, \tag{4}$$

where $b_{\rm se}$ is the shot-noise equivalent bandwidth [4], and $\mu \vee \nu$ is the maximum of μ and ν. For the DD exponential model, the mean and autocorrelation function are expressed as $\mathsf{E}[I_{\rm DD}(t)] = \mathsf{E}[G]b_{\rm 3dB}e^{-b_{\rm 3dB}t}$ and $R_{\rm DD}(\mu,\nu) = \mathsf{E}[G^2]b_{\rm 3dB}^2e^{-b_{\rm 3dB}(\mu+\nu)}$ [4].

B. Statistics of Receiver's Impulse Response

By using standard techniques for filtered Poisson processes [3], the mean and the correlation matrix of the receiver's impulse response $H(n)$, denoted by a_p and $R_A(p,q)$, respectively, can be obtained using

$$a_p = \phi \iint_{\mathcal{D}_1} \mathsf{E}[I_{\rm RD}(t-\tau)]\mathcal{I}_{[0,T_b]}(\tau)d\tau dt,$$

$$R_A(p,q) = \phi \iiint_{\mathcal{D}_2} R_{\rm RD}(\mu-\xi,\nu-\xi)\mathcal{I}_{[0,T_b]}(\xi)d\xi d\mu d\nu,$$

where $p, q = 0, 1, \ldots, L-1$, $\mathcal{I}_{[0,T_b]}(\xi)$ is the indicator function of the interval $[0, T_b]$, $\mathcal{D}_1 = \{(t,\tau) \in \mathbb{R}^2 : pT_b \leq t \leq (p+1)T_b, 0 \leq \tau \leq t\}$ and $\mathcal{D}_2 = \{(\mu,\nu,\tau) \in \mathbb{R}^3 : pT_b \leq \mu \leq (p+1)T_b, qT_b \leq \nu \leq (q+1)T_b, 0 \leq \tau \leq (\mu \wedge \nu)\}$. By applying (3), (4), we obtain the mean and autocorrelation functions of the receiver's impulse response:

$$\begin{aligned}
a_0 &= \frac{\phi}{b_{se}}\langle G\rangle\big(b_{se}T_b - 1 + e^{-b_{se}T_b}\big),\\
a_p &= \frac{\phi}{b_{se}}\langle G\rangle(e^{b_{se}T_b} - 1)(e^{-pb_{se}T_b} - e^{-(p+1)b_{se}T_b}),\\
R_A(0,0) &= a_0^2 + \frac{2\phi}{b_{se}^2}\langle\frac{G^2}{T}\rangle\big(b_{se}T_b - 2 + 2e^{-b_{se}T_b}\\
&\quad + b_{se}T_be^{-b_{se}T_b}\big),\\
R_A(p,0) &= a_na_0 + \frac{\phi}{b_{se}^2}\langle\frac{G^2}{T}\rangle\big(e^{-pb_{se}T_b} - e^{-(p+1)b_{se}T_b}\big)\\
&\quad \big(e^{b_{se}T_b} - 1 - b_{se}T_b\big),\\
R_A(p,p) &= a_p^2 + \frac{2\phi}{b_{se}^2}\langle\frac{G^2}{T}\rangle\big(e^{b_{se}T_b} - 1\big)\big(1 - e^{-b_{se}T_b}\\
&\quad - b_{se}T_be^{-b_{se}T_b}\big)e^{-pb_{se}T_b},\\
R_A(p,q) &= a_pa_q + \frac{\phi T_b}{b_{se}}\langle\frac{G^2}{T}\rangle\big(e^{b_{se}T_b} - 1\big)\big(e^{-pb_{se}T_b}\\
&\quad - e^{-(p+1)b_{se}T_b}\big).
\end{aligned}$$

The indices in the above equations range in $1 \leq q < p$ and $1 \leq p \leq L-1$, the rest of the elements in correlation matrix R_A are obtained by symmetry. The results for the DD exponential mode is obtained similarly; we omit the details here.

C. The LMMSE Equalizer

The desired LMMSE equalizer in (2) results in the minimum mean square error $\mathsf{E}[(I_n - \hat{I}_n)^2]$ among all the linear combinations of past observations. By the orthogonal principle [5], $\hat{I}_n$ should satisfy $\mathsf{E}[(I_n - \hat{I}_n)R_{n-k}] = 0$ for $k = 0, 1, \ldots, M-1$. Define

$$\mathrm{R_{IR}} \triangleq \left[\begin{array}{cccc} \mathsf{E}[I_nR_n] & \mathsf{E}[I_nR_{n-1}] & \ldots & \mathsf{E}[I_nR_{n-M+1}] \end{array}\right]^T,$$

$$\mathrm{R_R} \triangleq \left[\begin{array}{ccc} \mathsf{E}[R_nR_n] & \mathsf{E}[R_{n-1}R_n] & \ldots \\ \mathsf{E}[R_nR_{n-1}] & \mathsf{E}[R_{n-1}R_{n-1}] & \ldots \\ \ldots & \ldots & \ldots \end{array}\right]_{M\times M},$$

and $\mathrm{B} \triangleq \left[\begin{array}{cccc} b_0 & b_1 & \ldots & b_{M-1} \end{array}\right]^T$. We now invoke the orthogonal principle to obtain $\mathrm{R_{IR}} = \mathrm{R_R}\mathrm{B}$. Moreover, by (1) and the independence of the J_k's, it follows that $\mathrm{R_I}\bar{\mathrm{a}} = \mathrm{R_R}\mathrm{B}$, where $\bar{\mathrm{a}} \triangleq \left[\begin{array}{cccc} a_0 & a_1 & \ldots & a_{L-1} \end{array}\right]^T$ and

$$\mathrm{R_I} \triangleq \left[\begin{array}{ccc} \mathsf{E}[I_nI_n] & \mathsf{E}[I_nI_{n-1}] & \ldots \\ \mathsf{E}[I_nI_{n-1}] & \mathsf{E}[I_nI_{n-2}] & \ldots \\ \ldots & \ldots & \ldots \end{array}\right]_{M\times L}.$$

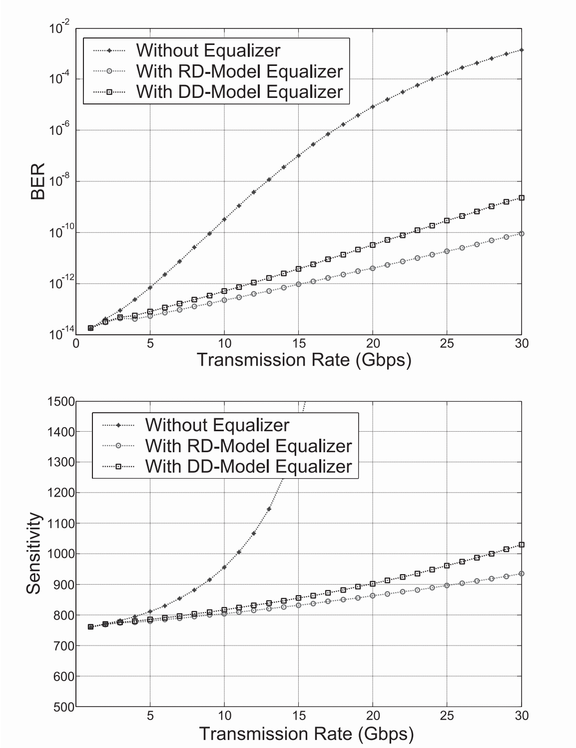

Fig. 1: BER and receiver sensitivity as a function of the transmission rate.

Thus, the tap coefficients of the LMMSE equalizer are given by $\mathrm{B} = \mathrm{R_R}^{-1}\mathrm{R_I}\bar{\mathrm{a}}$.

Each element $\mathsf{E}[R_k R_l]$ in the correlation matrix $\mathrm{R_R}$ can be calculated by substituting for R_k and R_l from (1); therefore, $\mathsf{E}[R_k R_l]$ can be expressed in terms of $\bar{\mathrm{a}}$, $\mathrm{R_I}$ and R_A. Note that $\mathrm{R_R}$ and $\mathrm{R_I}$ are time invariant if we assume the receiver input is wide sense stationary. It is stressed that R_A accounts for the channel's random nature and makes the correlation matrix distinct from those of the regular equalization problems.

III. Performance Evaluation

We calculate the bit-error rate (BER) of the receiver using a Gaussian approximation [6] for the receiver output with the correct mean and correlation matrix of the L receiver outputs used by the equalizer.

As an example, consider a GaAs APD that has average gain of 10 and a 100-nm multiplication region. The parameters of this APD are $\mathsf{E}[G] = 10$, $\mathsf{E}[G^2/T] = 27.06$ THz, $b_{\mathrm{se}} = 34.9$ GHz and $b_{3\mathrm{dB}} = 27.4$ GHz (taken from [4]). The top figure in Fig. 1 depicts the calculated BER as a function of transmission rate for the receiver without equalizer, with the RD-model equalizer, and with the DD-model equalizer. The ISI length parameter L varies depending on the transmission rate, and the number of taps of the equalizer, M, is always set to twice of ISI length, i.e., $M = 2L$. The correctly modeled RD equalizer yields the least BER while the incorrectly modeled DD equalizer still improves BER. For instance, the BERs at 20 GHz transmission rate are 8×10^{-6}, 3.25×10^{-11} and 4.0×10^{-12} for the receiver without any equalizer, with the DD-model equalizer and with the RD-model equalizer, respectively.

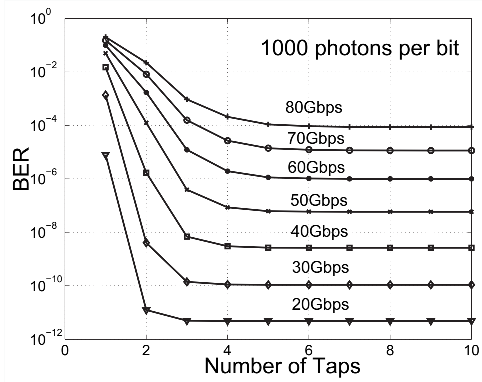

Fig. 2: BER of the receiver with the RD-model equalizer as a function of the number of taps, M, for various level of input optical signal intensity.

The bottom figure in Fig. 1 shows the receiver sensitivity (defined as the number of photons in per bit resulting in a BER of 10^{-9}) as a function of the transmission rate. As the sensitivity for the receiver without the equalizer diverges at about 20 GHz, the sensitivity increases much more modestly (approximately linearly at a small slope) for the equalized receivers. While both equalizers reduce the sensitivity significantly, especially at high-speeds, the performance of the equalizer obtained from the DD-exponential model deteriorates much faster at high transmission speeds than that of the equalizer obtained from the more accurate RD-rectangular model. This establishes the importance of using a complete statistical model for the APD's impulse response in deriving an equalizer for high-speed receivers.

Finally, Fig. 2 shows the BER as a function of M at various transmission rates. It is observed that 3-tap equalizer is able to achieve significant ISI cancelation at 20GHz transmission rate. 5 taps are efficient even at 80GHz transmission rate.

IV. Conclusion

The LMMSE-equalized APD-based receiver exhibits a significant performance improvement, especially at high transmission rates. It is observed that this improvement can be achieved with a 3-6 taps equalizer. This work reveals that the slowness of the APD can be compensated for using simple equalization. Therefore, the equalized APD-based receiver has the potential to be a cost-effective alternative to the PIN-EDFA combination in high-speed optical communications.

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