Dead-Space-Based Theory Correctly Predicts Excess Noise Factor for Thin GaAs and AlGaAs Avalanche Photodiodes

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***Abstract*—The conventional McIntyre carrier multiplication theory for avalanche photodiodes (APD's) does not adequately describe the experimental results obtained from APD's with thin multiplication-regions. Using published data for thin GaAs and $Al_{0.2}Ga_{0.8}As$ APD's, collected from multiplication-regions of different widths, we show that incorporating dead-space in the model resolves the discrepancy. The ionization coefficients of enabled carriers that have traveled the dead-space are determined as functions of the electric field, within the confines of a single exponential model for each device, independent of multiplication-region width. The model parameters are determined directly from experimental data. The use of these physically based ionization coefficients in the dead-space multiplication theory, developed earlier by Hayat *et al.* provides excess noise factor versus mean gain curves that accord very closely with those measured for each device, regardless of multiplication-region width. It is verified that the ratio of the dead-space to the multiplication-region width increases, for a fixed mean gain, as the width is reduced. This behavior, too, is in accord with the reduction of the excess noise factor predicted by the dead-space multiplication theory.**

***Index Terms*—AlGaAs, dead space, excess noise factor, GaAs, gain, impact ionization, ionization coefficients, thin avalanche photodiodes.**

I. Introduction

In many optical systems, avalanche photodiodes (APD's) are preferred over p-i-n detectors because of the gain they provide. Unfortunately, this gain is accompanied by excess noise that arises from randomness in the coupled avalanche of the very electrons and holes that give rise to the gain in the first place.

APD noise is most readily characterized by a quantity called the excess noise factor F [1], [2]. A mathematical form for this function was first obtained by McIntyre [3] in a classic paper published some 30 years ago. McIntyre showed that the excess noise factor is solely dependent on the mean gain $\langle G \rangle$ and on the ratio k of the ionization coefficients for holes and electrons. This result is predicated on two assumptions, however: 1) that the avalanche multiplication-region is uniform and 2) that the ability of electrons and holes to effect an impact ionization is not dependent on their past history. In spite of its simplicity, McIntyre's approach has been eminently successful in characterizing the results of a whole host of experiments with conventional thick APD's.

McIntyre's results can be generalized by relaxing either (or both) of the assumptions implicit in his model. Modern bandgap engineering has made it possible to create arbitrary multilayer APD structures with decidedly nonuniform multiplication-regions. In multiquantum-well APD's [4], [5], the carrier multiplication process is constrained to take place at certain preferred locations in the material, which are determined by the externally engineered superlattice. This restriction reduces the randomness in the birth locations, and thereby requires for its description a theory that admits nonuniform multiplication [6]. Clearly, the expression for the excess noise factor will then depend not only on $\langle G \rangle$ and k but also on the detailed structure of the device [6], [7]. Similarly, the effect of past history on the ability of a carrier to impact ionize and create a new carrier pair can also be taken into account, as has been demonstrated by Okuto and Crowell [8] and Hayat *et al.* [9]–[11]. The physical rationale behind the introduction of carrier history is as follows: after each impact ionization an ionizing carrier must travel a sufficient distance (the so-called dead space) to gain enough energy that will enable it to cause another impact ionization.

Both of these modifications typically reduce gain fluctuations because they eliminate some randomness. This phenomenon clearly cannot be accommodated within the McIntyre theory [3], [12]. Okuto and Crowell [8] were the first to account for the dead-space effect by replacing the conventional ionization coefficient with a nonlocal coefficient. They developed a theory for the mean multiplication and numerically calculated the mean gain. Later, Hayat *et al.* [10], [11] formulated a dead-space multiplication theory (DSMT) that determines the mean gain, the excess noise factor, and the probability distribution of the gain. Moreover, the DSMT set forth in [11] assumes a nonuniform electric field and can be used to accommodate arbitrary history-dependent ionization coefficients. It was shown in [10] and [11] that dead-space reduces the excess noise factor, and more importantly, this reduction increases as the ratio of the dead-space to

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the multiplication-region width increases. Subsequently, experiments and Monte Carlo simulation studies alike have demonstrated that there is a concomitant reduction of the excess noise factor as the multiplication-region width is reduced [13]–[17]. It is precisely because of this reduced noise that avalanche photodiodes with thin multiplication-regions have evinced substantial interest in the detector community. Recently, Li *et al.* [18] and Ong *et al.* [19] have shown that in thin APD's the dead-space represents a significant fraction of the multiplication-region width and that dead-space plays an important role in reducing the excess noise factor.

Application of the DSMT [10] requires knowledge of the nonlocal ionization coefficients of enabled carries that have traveled the dead space. Existing models [20], [21] of experimentally determined ionization coefficients are, however, calculated by applying the McIntyre theory [3], and thus are representative of *effective* ionization coefficients. Unfortunately, the ionization coefficients of enabled carriers cannot be linked in a straightforward way to the experimentally determined effective ionization coefficients [22]. However, based on mean-free-path considerations, the reciprocal of the ionization coefficient of enabled carriers is sometimes approximated by the reciprocal effective ionization coefficient less the dead space. In the absence of a complete theory for the ionization coefficients of enabled carriers, Li *et al.* [18] invoked the above mean-free-path considerations to modify the experimentally determined effective ionization coefficients reported in [20] and [23]. The ionization coefficients reported in [23] are based on a semi-analytical model and have not been validated experimentally. Using these approximate ionization coefficients of enabled carriers in conjunction with the DSMT, Li *et al.* [18] predicted the gain-noise characteristics of thin APD's. More recently, McIntyre [24] and Yuan *et al.* [25] used the DSMT developed by Hayat *et al.*[10], [11] to study the excess noise factor and frequency response of thin avalanche photodiodes. However, the formulation presented in [24] does not provide a straightforward way of extracting the ionization coefficients from a set of measurements. Nonetheless, the introduction of certain nontrivial analytical approximations used in calculating ionization coefficients, which depend on the so called history-dependent electric fields, yielded good agreement between the predicted and measured excess noise factors for GaAs APD's of varying thicknesses [25].

One of the main contributions of the current paper is to provide a methodology and procedure for calculating material-specific ionization coefficients of enabled carriers *directly* from gain and noise measurements without having to resort to prior analytical or simulation based models for the ionization coefficients. Specifically, we extract the ionization coefficients by fitting the gain and noise data to the DSMT directly. This is the principal distinction between our work and that reported in [18] where the ionization coefficients are obtained from mean-free-path considerations. Moreover, in contrast to the technique reported in [18], where the ionization threshold energies are taken to be adjustable parameters obtained by fitting the data to the DSMT, our approach utilizes ionization threshold energies available in the literature. Furthermore, the calculated material-specific ionization coefficients of enabled carriers are shown to obey a universal model (framed in terms of the electric field) which is independent of multiplication-region width. Finally, we use the universal model for the ionization coefficients of enabled carriers in conjunction with the DSMT to correctly predict the gain and noise characteristics of GaAs and $Al_{0.2}Ga_{0.8}As$ APD's.

II. Dead-Space Multiplication Theory

We begin this section by reviewing some germane aspects of the dead-space multiplication theory (DSMT). dead-space arises from the fact that band-to-band impact ionization can only occur when an electron or hole has sufficient kinetic energy to cause another electron to make a transition from a valence band state to a conduction band state through their mutual interaction. The smallest possible value for the kinetic energy of the ionizing particle that can achieve this is termed the ionization threshold energy. The minimum distance that a newly generated carrier must travel in order to acquire this threshold energy is called the carrier dead space [8]. In the ballistic model [8], the electron and hole dead spaces are given by

$$d_e = \frac{E_{ie}}{q\mathcal{E}} \tag{1}$$

$$d_h = \frac{E_{ih}}{q\mathcal{E}} \tag{2}$$

respectively, where $\mathcal{E}$ is the uniform electric field in the multiplication-region, E_{ie} and E_{ih} are the ionization threshold energies of the electron and hole, respectively, and q is the charge of an electron.

The gain statistics under uniform and nonuniform electric fields have been developed and reported in [10] and [11]. The theory involves recurrence equations of certain intermediate random variables $Z(x)$ and $Y(x)$. The quantity $Z(x)$ ($Y(x)$) is defined as the overall electron and hole progeny generated by a single parent electron (hole) at the point x in the multiplication-region. The multiplication-region is assumed to extend from $x = 0$ to $x = W$, and the direction of the electric field within the multiplication-region is assumed to be pointing from $x = W$ to $x = 0$. For an electron-injection APD, the random gain of the APD, G, is simply $0.5[Z(0) + Y(0)]$ which can be further reduced to $G = 0.5[Z(0) + 1]$ since $Y(0) = 1$. According to [10, Eqs. (14) and (15)], the averages of $Z(x)$ and $Y(x)$, denoted by $z(x)$ and $y(x)$, respectively, obey the following recurrence equations:

$$z(x) = \left[1 - \int_0^{W-x} h_e(\xi)\,d\xi\right] + \int_x^W [2z(\xi) + y(\xi)]\,h_e(\xi - x)\,d\xi \tag{3}$$

$$y(x) = \left[1 - \int_0^{x} h_h(\xi)\,d\xi\right] + \int_0^x [2y(\xi) + z(\xi)]\,h_h(x - \xi)\,d\xi \tag{4}$$

where $h_e(x)$ and $h_h(x)$ are the probability density functions (pdf's) of the random free-path lengths X_e and X_h of the electron and hole, respectively. If we define the ionization coefficients of *enabled* electrons and holes by α and β, a simple (but reasonable) model for the above pdf's is given by [10]

$$h_e(x) = \alpha e^{-\alpha(x-d_e)} u(x-d_e), \tag{5}$$

$$h_h(x) = \beta e^{-\beta(x-d_h)} u(x-d_h) \tag{6}$$

where $u(x)$ is the unit step function [$u(x) = 1$, if $x \geq 0$, and $u(x) = 0$, otherwise]. Using these relations to obtain solutions to (3) and (4) leads to the mean gain

$$\langle G \rangle = \tfrac{1}{2}\left[z(0)+1\right], \tag{7}$$

while the excess noise factor, defined by $F = \langle G^2 \rangle / \langle G \rangle^2$, is given by

$$F = \frac{z_2(0) + 2z(0) + 1}{[z(0)+1]^2}. \tag{8}$$

Here, $z_2(x) = \langle Z^2(x) \rangle$ and $y_2(x) = \langle Y^2(x) \rangle$ are the second moments of $Z(x)$ and $Y(x)$, respectively, and are governed by the following recurrence relations (these are [10, Eqs. (18) and (19)]):

$$z_2(x) = \left[1 - \int_0^{W-x} h_e(\xi)\, d\xi\right] + \int_x^W [2z_2(\xi) + y_2(\xi) + 4z(\xi)y(\xi) + 2z^2(\xi)]\, h_e(\xi - x)\, d\xi \tag{9}$$

$$y_2(x) = \left[1 - \int_0^{x} h_h(\xi)\, d\xi\right] + \int_0^x [2y_2(\xi) + z_2(\xi) + 4z(\xi)y(\xi) + 2y^2(\xi)]\, h_h(x - \xi)\, d\xi. \tag{10}$$

The recurrence equations (3), (4), (9), and (10) can be solved to estimate F and $\langle G \rangle$ by using a simple iterative numerical recipe (Picard iterations) as outlined in [10].

The above recurrence equations demand knowledge of the electron and hole ionization coefficients of *enabled* carriers. In addition, the calculation of electron and hole dead spaces from (1) and (2) requires knowledge of the ionization threshold energies of the electron and hole (E_{ie} and E_{ih}). By assuming knowledge of the electric field and ionization threshold energies, one can search (by varying α and β and solving for $\langle G \rangle$ and F) for the values of α and β that yield specified gain and excess noise factor.

The dependence of the electron ionization coefficient on the electric field $\mathcal{E}$ is often modeled by the equation

$$\alpha(\mathcal{E}) = A \exp[-(E_c/\mathcal{E})^m] \tag{11}$$

where A, E_c, and m are parameters that are chosen by fitting measured gain-noise data [20], [21]. A similar form is also used for the hole ionization coefficient. After calculating unique pairs of electron and hole ionization coefficients for every pair of experimental gain and excess noise factor (corresponding to a specific electric field $\mathcal{E}$), one can obtain the parameters A, E_c, and m by fitting the ionization coefficients to the model given by (11). In the next section, a search algorithm is outlined that exploits this scheme to estimate the ionization coefficients of enabled carriers.

III. Application of the Dead Space Multiplication Theory to Gain-Noise Data

To demonstrate the applicability of the DSMT to APD's with thin multiplication-regions of various widths, we make use of the GaAs and AlGaAs homojunction APD data reported by Anselm *et al.* [16]. These authors reported gain and excess noise factor measurements for four GaAs APD's with multiplication-region widths of 100, 200, 500, and 800 nm; and for four$Al_{0.2}Ga_{0.8}As$ APD's with widths of 200, 400, 800, and 1600 nm.

A. Estimating the Dead Space

In order to employ the DSMT recurrence equations (3), (4), (9), and (10), it is necessary to determine the electron and hole dead spaces (d_e and d_h) using (1) and (2). We consider the electron and hole ionization threshold energies for GaAs to be 1.7 and 1.4 eV [26], respectively; and for $Al_{0.2}Ga_{0.8}As$ to be 1.84 and 1.94 eV [27], respectively. Calculation of the electric field $\mathcal{E}$ requires knowledge of the total voltage across the junction. Since the above information is not available in [16], we estimate the electric field for each pair of gain-noise data by using the width dependent model for the ionization coefficients provided by Anselm *et al.* [16].[1] By reverse-engineering their model, we are able to determine the electric-field value $\mathcal{E}$ for each gain-noise data point for all devices.

We will call the *effective* hole and electron ionization coefficients modeled by Anselm *et al.* in accordance with McIntyre theory [3], β_a and α_a, respectively, and denote their ratio as k_a. Anselm *et al.* fit the mean gain and the excess noise factor for each device (with specified width) to determine a specific value of k_a by using the conventional expression for the excess noise factor [2]

$$F = k_a \langle G \rangle + (1-k_a)\left(2 - \frac{1}{\langle G \rangle}\right). \tag{12}$$

Hence, for each set $(\langle G \rangle, F)$, k_a is given in [16]. The knowledge of the multiplication-region width W is then used in the conventional expression for the mean gain [2]

$$\langle G \rangle = \frac{1-k_a}{\exp[-(1-k_a)\alpha_a W] - k_a} \tag{13}$$

and the coefficient α_a is determined. Now by equating α_a with the corresponding device specific model for α_a provided in [16],

[1]The parameter values for Anselm's width-dependent model used in this paper (listed in Tables I and II) are taken from a preprint version of [16]. However, there seem to be typographical errors in the tabulated parameter values in the published version of [16].

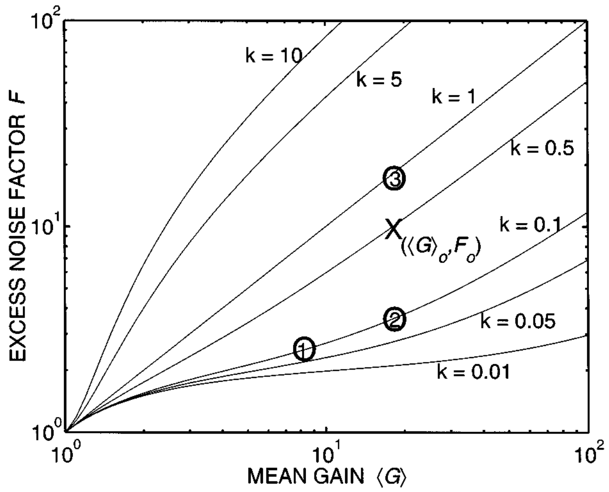

Fig. 1. Excess noise factor (F) versus mean gain ($\langle G\rangle$) characteristics of an APD parameterized by the ratio of the hole coefficient to electron coefficient k, according to the conventional McIntyre theory [3]. The points 1, 2, and 3 illustrate samples of ($\langle G\rangle$, F) pairs generated in the process of finding the ionization coefficients of enabled carriers using the dead-space theory.

we can back-calculate the electric field. [2] The electron and hole dead spaces can then be calculated using (1) and (2).

B. Determining the Universal Model for the Ionization Coefficients of Enabled Carriers

We estimate the ionization coefficients of enabled carriers by a simple search algorithm. To clarify the rationale for the search algorithm, we consider the general behavior of the excess noise factor as a function of the mean gain with the hole to electron ionization ratio k used as a parameter, as shown in Fig. 1. Clearly, for a fixed mean gain, increasing k results in an increase in the excess noise factor. Note that this general behavior is exhibited by both the conventional multiplication theory and the DSMT. It is clear from this general behavior that corresponding to each pair $(\langle G\rangle, F)$, and for a specific material and electric field, there exists a unique value of k. Furthermore, the quantities k, $\langle G\rangle$, and W (along with knowledge of the electron and hole dead spaces) uniquely determine α according to the DSMT. Hence, for each pair $(\langle G\rangle, F)$, a unique pair (α, k) exists and can be determined.

The search algorithm for the unique pair, α and k, is based on exploiting the above one-to-one correspondence between $(\langle G\rangle, F)$ and (α, k). To illustrate the procedure, consider the sample data point $(\langle G\rangle_0, F_0)$ marked by the symbol $\times$ in Fig. 1. The value of α and k are set to α_a and k_a, respectively, to start the search procedure.[3] Substituting α and k in (3), (4), (9), and (10), and numerically solving these equations allow the mean gain and the excess noise factor to be calculated using (7) and (8). Note that the resulting pair $(\langle G\rangle, F)$, shown as point 1 in Fig. 1, underestimates the measured values. We would then increase α until the calculated mean gain matches $\langle G\rangle_0$ (shown as point 2 in Fig. 1). From this point and on, we further adjust k and α concomitantly to increase the excess noise factor, bringing it closer to F_0 while maintaining the mean gain at $\langle G\rangle_0$. In this last step, any increase in k must be accompanied by a reduction in α (and vice versa) to maintain the calculated mean gain at $\langle G\rangle_0$. For example, in Fig. 1, point 2 is still below $(\langle G\rangle_0, F_0)$, and k must therefore be raised (causing the calculated value of F to increase) while adjusting α to a lower value to maintain the mean gain at $\langle G\rangle_0$. This results in point 3 in Fig. 1 where F is now slightly overestimated.

The foregoing adjustments are repeated with progressively finer changes in k and α until $(\langle G\rangle, F) \approx (\langle G\rangle_0, F_0)$. (The relative tolerance used in our calculation for establishing convergence is 0.01.) This completes the procedure for estimating the ionization coefficients of enabled carriers. Although this calculation is highly accurate, it is computationally demanding because of the required numerical solution of the recurrence equations (each data point requires approximately 45 min of CPU time on a SUN SPARC-20 workstation). Higher computational efficiency may be achieved (with less accuracy) using the recently reported analytical approximations of the mean gain and the excess noise factor [28].

For each type of material, GaAs and $Al_{0.2}Ga_{0.8}As$, we are able to find a single set of parameters, A, E_c, and m for the exponential model provided in (11), that fits the estimated ionization coefficients α and β as functions of the electric field, *independent* of multiplication-region width. The applicability of this universal model is confirmed by comparing the experimental gain-noise data with the F versus $\langle G\rangle$ plot that is generated using the ionization coefficients predicted by this width independent model. To plot F as a function of $\langle G\rangle$ for each device, we calculate F and $\langle G\rangle$ for the same range of the electric field $\mathcal{E}$ used in [16] to produce the gain-noise data. For each device, the data points from [16] with the lowest and highest mean gains define the range of electric field used. In Section IV, this theoretical prediction of mean gain and excess noise factor is compared with the experimental data.

IV. RESULTS

The sets of width-independent parameters A, E_c and m that yield the best fitting universal exponential model for the calculated values of α and β using the DSMT are provided in Tables I and II. In addition, Tables I and II also show the sets of *width-dependent* parameters A, E_c, and m reported by Anselm *et al.* [16] as well as those from Bulman *et al.* [20] and from Robbins *et al.* [21] for bulk materials.

Tables III and IV show the relative dead space, defined as the ratio of the dead-space to the multiplication-region width, for the different devices. A thin device requires a higher electric field to provide the same mean gain as a thick device, and the dead-space associated with the thin device is therefore less than that in the thick device. Nevertheless, the relative

[2]This method of calculating the electric field prevents us from incorporating all of the data points reported in [16] since some of the gain-noise data points do not fit well to the curve given by (12). This suggests that the values of k_a provided by Anselm *et al.* result in a marginal fit for these data points so that the corresponding calculated electric-field values are not highly accurate. To minimize such error, we excluded the 1600 nm $Al_{0.2}Ga_{0.8}As$ device from consideration, for which most of the data points did not fit well to the curve given by (12).

[3]Recall that α_a and β_a are the coefficients obtained using the model in [16]. We used the values of α_a and β_a from [16] as starting points, instead of using an arbitrary pair, to save computation time. The derived values of α and k are not dependent on the McIntyre theory [3].

TABLE I
PARAMETERS FOR THE EXPONENTIAL IONIZATION-COEFFICIENT MODEL FOR GaAs

		Units	Ref. [20]	DSMT All Devices	Ref. [16][1] 800 nm	Ref. [16][1] 500 nm	Ref. [16][1] 200 nm	Ref. [16][1] 100 nm
α	A	cm^{-1}	2.99×10^5	5.81×10^6	4.40×10^6	3.90×10^6	2.50×10^6	1.92×10^6
	E_c	V/cm	6.85×10^5	2.17×10^6	1.40×10^6	1.40×10^6	1.40×10^6	1.40×10^6
	m		1.60	0.94	1.20	1.20	1.20	1.20
β	A	cm^{-1}	2.22×10^5	3.17×10^6	22.2×10^5	14.4×10^5	6.20×10^5	3.47×10^5
	E_c	V/cm	6.57×10^5	2.09×10^6	1.15×10^6	1.15×10^6	1.15×10^6	1.15×10^6
	m		1.75	0.95	1.40	1.40	1.40	1.40

TABLE II
PARAMETERS FOR THE EXPONENTIAL IONIZATION-COEFFICIENT MODEL FOR $Al_{0.2}Ga_{0.8}As$

		Units	Ref. [21]	DSMT All Devices	Ref. [16][1] 800 nm	Ref. [16][1] 400 nm	Ref. [16][1] 200 nm
α	A	cm^{-1}	1.09×10^6	5.87×10^6	5.00×10^6	4.20×10^6	3.03×10^6
	E_c	V/cm	1.37×10^6	2.65×10^6	1.81×10^6	1.81×10^6	1.81×10^6
	m		1.30	0.96	1.20	1.20	1.20
β	A	cm^{-1}	6.45×10^5	1.25×10^6	18.1×10^5	9.05×10^5	4.41×10^5
	E_c	V/cm	1.11×10^6	1.85×10^6	1.37×10^6	1.37×10^6	1.37×10^6
	m		1.5	1.03	1.40	1.40	1.40

TABLE III
RELATIVE DEAD SPACE WIDTH FOR FOUR THIN GaAs APD'S PRODUCING COMPARABLE MEAN GAIN. THE LOWER AND UPPER LIMITS OF THE ELECTRIC FIELD PRODUCE THE LOWER AND UPPER LIMITS, RESPECTIVELY, OF THE MEAN GAIN AND THE RELATIVE DEAD SPACE

Multiplication Width (nm)	$\mathcal{E}$ Field ($\times 10^5$ V/cm)	Mean Gain	d_e/W (%)	d_h/W (%)
100	6.3 – 6.8	8 – 29	25 – 27	21 – 22
200	4.7 – 5.0	6 – 30	17 – 18	14 – 15
500	3.5 – 3.7	4 – 28	9.2 – 9.8	7.6 – 8.0
800	3.2 – 3.3	5 – 20	6.4 – 6.6	5.3 – 5.8

dead-space in Tables III and IV is seen to increase as the multiplication-region width is reduced. This behavior is in agreement with the results reported by Li *et al.* [18] and Ong *et al.* [19]. This explains the observed reduction in the excess noise factor in thin devices as a consequence of its dependence on the relative dead-space [10].

TABLE IV
RELATIVE DEAD SPACE WIDTH FOR THREE $Al_{0.2}Ga_{0.8}As$ APD'S PRODUCING COMPARABLE MEAN GAIN. THE LOWER AND UPPER LIMITS OF THE ELECTRIC FIELD PRODUCE THE LOWER AND UPPER LIMITS, RESPECTIVELY, OF THE MEAN GAIN AND THE RELATIVE DEAD SPACE

Multiplication Width (nm)	$\mathcal{E}$ Field ($\times 10^5$ V/cm)	Mean Gain	d_e/W (%)	d_h/W (%)
200	5.9 – 6.2	6 – 20	15 – 16	16 – 17
400	4.8 – 4.9	6 – 20	9.4 – 9.7	9.9 – 10
800	4.0 – 4.2	5 – 21	5.6 – 5.7	5.9 – 6.0

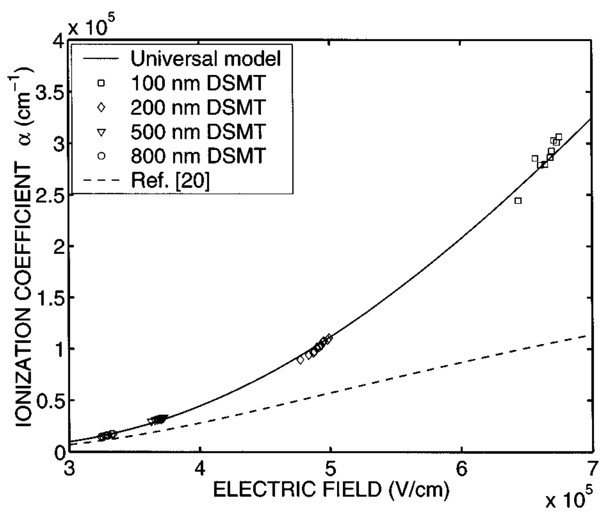

Fig. 2. Electron ionization coefficient (α) of GaAs as a function of the electric field. Symbols represent the ionization coefficients of enabled carriers obtained from the data of four thin GaAs APD's [16] in conjunction with the DSMT while the solid curve represents the universal exponential model for these coefficients. The dashed curve represents the effective ionization coefficients obtained by [20] using the McIntyre theory [3] on bulk GaAs.

The electron and hole ionization coefficients for GaAs are presented in Figs. 2 and 3, whereas those corresponding to $Al_{0.2}Ga_{0.8}As$ are presented in Figs. 4 and 5. It is evident from

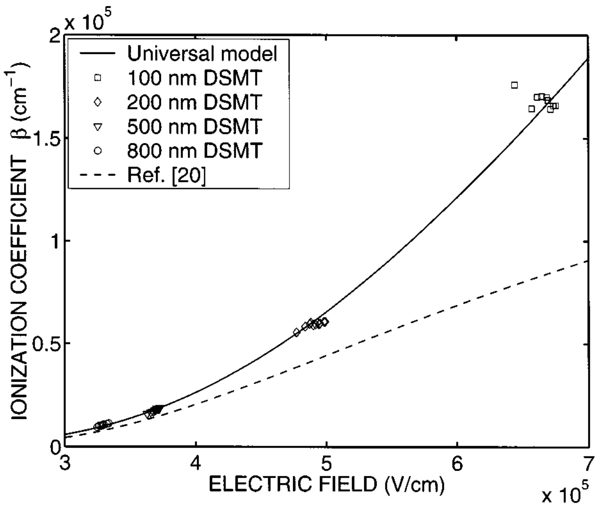

Fig. 3. Hole ionization coefficient (β) of GaAs as a function of the electric field. Plot symbols are the same as in Fig. 2.

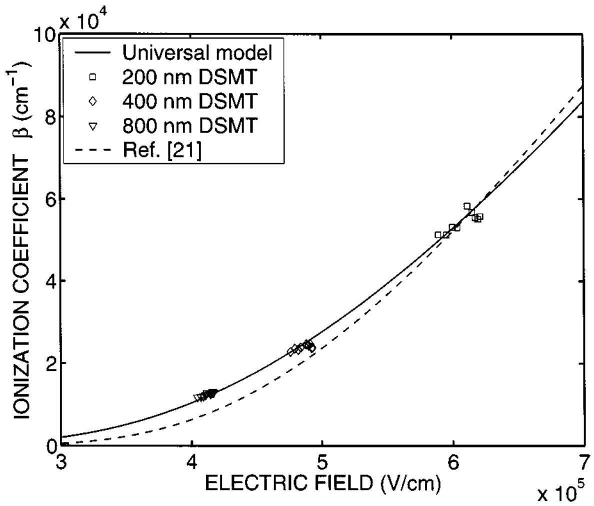

Fig. 5. Hole ionization coefficient (β) of $Al_{0.2}Ga_{0.8}As$ as a function of the electric field. Plot symbols are the same as in Fig. 4.

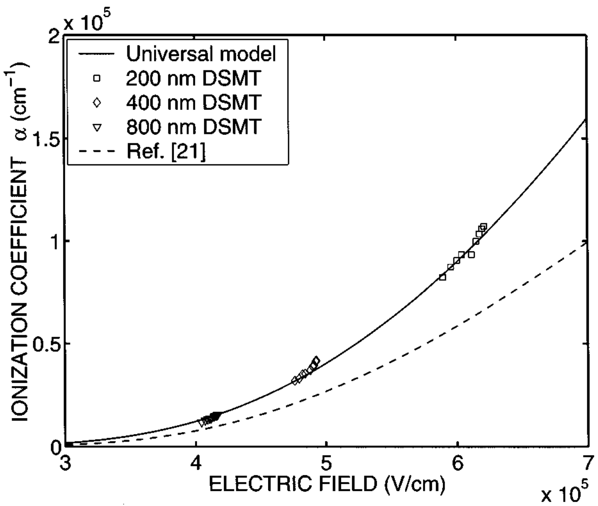

Fig. 4. Electron ionization coefficient (α) of $Al_{0.2}Ga_{0.8}As$ as a function of the electric field. Symbols represent the ionization coefficients of enabled carriers obtained from the data of three thin $Al_{0.2}Ga_{0.8}As$ APD's [16] in conjunction with the DSMT while the solid curve represents the universal exponential model for these coefficients. The dashed curve represents the effective ionization coefficients obtained by [21] using the McIntyre theory [3] on bulk $Al_{0.2}Ga_{0.8}As$.

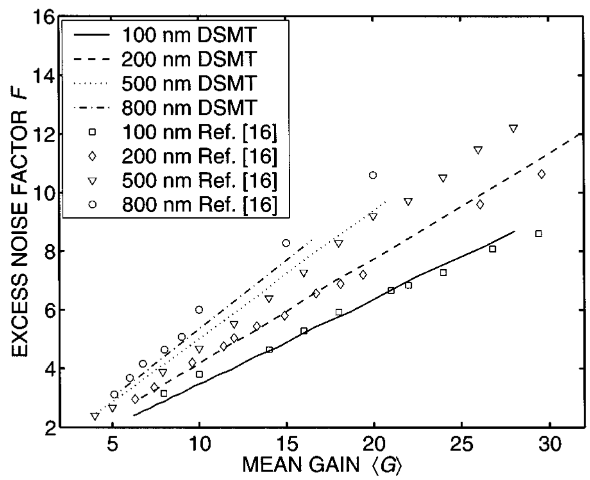

Fig. 6. Comparison of the DSMT-predicted and experimental excess noise factor F and mean gain $\langle G\rangle$ for four thin GaAs APD's with different multiplication-region widths. Symbols represent experimental data (obtained from [16]) and curves represent the predictions using the DSMT.

Figs. 2–5 that for each material, the calculated electron and hole ionization coefficients of enabled carriers, α and β for all devices, with different multiplication-region widths are fit by a single exponential model. This is very much in accord with expectation since physical principles dictate that the ionization coefficient of an enabled carrier should be independent of multiplication-region width. These figures also show that the *effective* ionization coefficients for the bulk materials (obtained from [20] and [21], corresponding to widths of $\sim$3–5 μm) are smaller than the ionization coefficients of enabled carriers in thin APD's, as expected. The hole ionization coefficient of $Al_{0.2}Ga_{0.8}As$ (shown in Fig. 5) is the only exception, where it turns out that the ionization coefficients of enabled carriers are comparable with the bulk coefficients. The cause of this anomaly is not clear. It is important to recall, however that Bulman *et al.* [20] and Robbins *et al.* [21] both calculated the effective ionization coefficient by applying the conventional multiplication theory to the experimental data.

Figs. 6 and 7 show the gain-noise data of Anselm *et al.* [16] along with the DSMT prediction of the gain-noise characteristics for GaAs and $Al_{0.2}Ga_{0.8}As$ APD's, respectively. It can be seen that the predicted gain-noise characteristics, using the DSMT and the universal model for the ionization coefficients, closely approximate the experimental data. However, in comparison to the experimental data, the predicted gain-noise curves exhibit two minor discrepancies: 1) a number of the predicted curves lie slightly below or above the experimental results and 2) some of the mean-gain ranges of the predicted curves are less than the data ranges. (Recall that in predicting the gain-noise curves, the range of values of the electric field used was the same as the range associated with the experiments.) For brevity, we

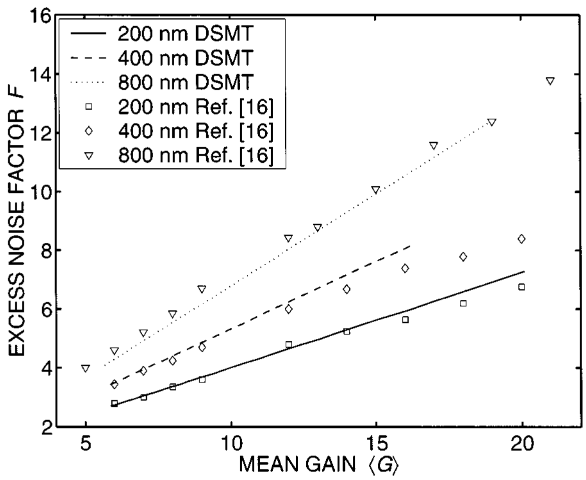

Fig. 7. Comparison of the DSMT-predicted and experimental excess noise factor F and mean gain $\langle G \rangle$ for three thin $Al_{0.2}Ga_{0.8}As$ APD's with different multiplication-region widths. Plot symbols are the same as in Fig. 6.

explain these two types of discrepancies only for the 500 nm GaAs device. In this case, the predicted gain-noise curve lies slightly above the experimental data points and the range of the predicted mean gain is narrower than the range of the experimental mean gain. These discrepancies are both due to small errors in predicting α and β from the universal model. To explain the up-shift of the predicted gain-noise curve, we examine Figs. 2 and 3 and observe that for the 500 nm GaAs device, the predicted values of α (from the solid curve) are slightly lower than the actual values (represented by triangles) while the predicted values of β are slightly higher than the actual values. As a result, the predicted value of k is slightly higher than its actual value. Consequently, the predicted gain-noise curve is above the experimental data in accord with the general dependence of the gain-noise characteristics on the ionization coefficient ratio (as depicted in Fig. 1).

The reduced range in the predicted mean-gain values, on the other hand, is explained as follows. Since the predicted values of α are lower (with an average relative error of 5%) than the corresponding actual values, the predicted values of β must be sufficiently higher than their actual values if the experimental mean gain is to be achieved. However, despite the fact that the predicted values of β are indeed higher (with an average relative error of 2%) than the actual values (as seen from Fig. 3), this increase is not sufficient to fully compensate for the reduction in the predicted mean gain due to the low prediction of α.

We believe that one of the sources of the slight misfit of α and β to the universal model is the error in calculating the electric field which may be attributed to the inaccurate fit of the gain-noise data in [16] to their width-dependent ionization model. Another factor that plays a role in the quality of the fit of α and β to the universal model is the accuracy of the values of the electron and hole ionization threshold energies E_{ie} and E_{ih}. To demonstrate this point, recall that in calculating the dead space, the electron and hole ionization threshold energies for GaAs are taken to be 1.7 and 1.4 eV [26], respectively, whereas for $Al_{0.2}Ga_{0.8}As$, these energies are taken to be 1.84 and 1.94 eV [27], respectively. However, when the value $E_{ih} = 1.63$ eV is used for GaAs (according to [27]) in place of $E_{ih} = 1.4$ eV, the fit of α and β to the universal model is slightly degraded, and the reduction of the predicted gain range for some of the gain-noise devices becomes more pronounced. An accurate estimate of the ionization threshold energies E_{ie} and E_{ih} is, therefore, important for the successful application of the DSMT to modeling the ionization coefficients of enabled carriers. The ionization threshold energies used here are lower than those that have been predicted by the use of high-field carrier transport theory [29]. However, since high-field hole transport theory and experimental data are not available to date [22], we instead used the established results from [26] and [27].

V. Conclusion

The previously developed avalanche multiplication theory incorporating dead-space [10] was employed to correctly predict the gain-noise characteristics of GaAs and $Al_{0.2}Ga_{0.8}As$ thin APD's for a variety of multiplication-region widths. In addition, we provided a methodology and a procedure for calculating the material-specific ionization coefficients of enabled carriers that have traversed the dead space, *directly* from gain and noise measurements, without having to resort to prior analytical or simulation based models for the ionization coefficients. These ionization coefficients are fit in a universal exponential model with parameters that are independent of multiplication-region width.

Together with the dead-space multiplication theory, the model for the ionization coefficients of enabled carriers developed here, quantitatively explains what has long been observed both in experiments and Monte Carlo simulations: dead-space reduces the noise in thin APD's by regularizing the ionization locations and reducing the randomness of the avalanching mechanism. Furthermore, it is observed that although maintaining a constant mean gain in a thinner multiplication-region requires an increase in the electric field, and hence a reduction of the dead space, this effect is overcompensated by the decrease in the multiplication-region width. What is important is that for a fixed mean gain, the relative dead-space (the ratio of the dead-space to the multiplication-region width) increases as the multiplication-region width decreases, as has been previously observed by Li *et al.* [18] and by Ong *et al.* [19]. This observation results in a reduction of the excess noise factor in thin devices in view of the dead-space multiplication theory reported in [10].

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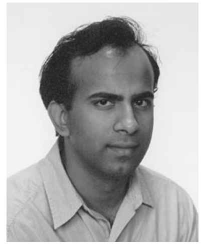

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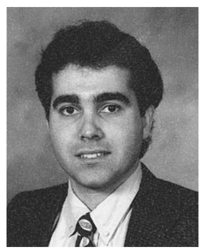

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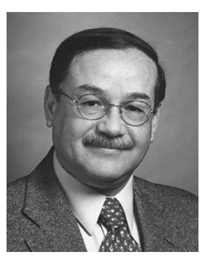

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