Decision-Feedback and Transversal Equalizion for High-Speed APD-Based Receivers

Peng Sun and Majeed M. Mayat
Department of Electrical and Computer Engineering, University of New Mexico,
Albuquerque, New Mexico 87131
Email: pengsun@ece.unm.edu, hayat@ece.unm.edu

Abstract

Transversal and decision-feedback equalizers are designed for avalanche-photodiode (APD) based integrate-and-dump receivers to compensate for the intersymbol-interference induced by the APD's random avalanche-buildup time. The designs are based on the rigorous calculation of the mean and the correlation matrix of the receiver's impulse response, which is the random sequence of the integrator's outputs, as a function of the bit index, when the receiver is triggered by a single optical pulse. These statistics are determined by utilizing a recently developed analytical model for the joint probability distribution of the gain and the buildup time of thin APDs. It is shown that these equalizers can reduce the bit-error-rate significantly at high transmission rates.

OCIS codes: (040.5160) Photodetectors; (060.4510) Optical communications.

I. Introduction

Many modern high-speed optical receivers employ a PIN photodiode as a photodetector, combined with an Erbium-doped fiber amplifier (EDFA). Such combination is prevalently adopted due to its high bandwidth, thanks to the fast response time of the PIN detector, and its good signal-to-noise ratio characteristics, which is attributable to the high gains that an EDFA can provide. However, optical amplifiers are very costly and bulky, as they require a laser pump as well as several meters of coiled fiber to support the optical amplification [1]. In such systems, linear equalization is often used to compensate for intersymbol-interference (ISI) resulting from fiber-dispersion induced pulse broadening [1], [2]. However, an avalanche photodiode (APD) offers a cost-effective alternative to the PIN-EDFA combination as it is capable of amplifying the photocurrent internally, without the need for optical preamplification. Unfortunately, this internal optoelectronic gain, which is referred to avalanche carrier multiplication, is accompanied by the so-called avalanche-buildup time, during which the cascade of impact ionizations is realized. This results in a relatively slow response time for the APD (compared to a PIN) and introduces additional ISI. Although high-speed APDs are available which are suitable for transmission speeds up to 10 Gbps, their utility at higher speeds would result in significant ISI, which must be compensated for in the receiver.

Even though equalization techniques [2] have been brought out to compensate for fiber-dispersion-induced ISI, they are not suitable for compensating ISI introduced by the APD as the impulse response function of an APD is highly random, both in its duration (representing the avalanche-buildup time) and in its area (representing the multiplication factor, i.e., gain). This makes ISI compensation for APDs distinct from that designed for dispersion-induced ISI. A linear minimum mean-square error (LMMSE) equalizer for an APD-based integrate-and-dump receiver in on-off keying (OOK) optical communication systems has been developed in [3] which brings significant improvement in terms of bit-error rate (BER) and receiver sensitivity. In [3], the estimate of the input binary signal is obtained by a linear combination of the current and previous receiver's outputs; and the tap coefficients are optimized by the minimum mean square error (MMSE) criterion. In this work, the transversal equalizer (TE) is designed, which is a linear combination of the a finite number of previous and future receiver's outputs; the decision feedback equalization (DFE) is designed too, which consists of an feedforward section and a section that feeds back the previous decisions of the receiver. The coefficients of these equalizers are also optimized by the MMSE criterion. However, because of ISI, the future outputs of the receiver do contain valuable information about the current input signal. The TE does use this added information, and its performance, therefore, would be superior to that of the LMMSE equalizer reported in [3]. Moreover, the DFE additionally uses past decisions in the feedback section, and would therefore yield yet better performance. The APD-based integrate-and-dump receiver is viewed as a random linear channel, since the APD's impulse-response function is a stochastic process. In contrast to the deterministic channels, the equalization of this random channel has to take into account the statistical properties of the random channel's impulse response. The required correlations of the random channel's output and the cross-correlation between the random channel's input and output involve characterizing the complex statistical properties of the shot noise [4] generated by the APD's impulse response function. In our earlier work [5], the joint statistics of the APD's gain and buildup-time was determined for the first time, and the random-duration (RD) rectangular model was proposed to accurately characterize the mean and autocorrelation of the APD's impulse response function. In this work, the RD rectangular model for the APD's impulse response function is utilized to numerically obtain the mean and the autocorrelation function of the photocurrent.

II. Receiver-Output Model

Let I_n, $n \in \mathbb{Z}$, denote the input binary sequence with bit duration T_b. The optical intensity is assumed ϕ for a "1" bit and 0 for a "0" bit. Let R_n, $n \in \mathbb{Z}$, denote the raw receiver output (i.e., prior to any decision). Each optical input I_n contributes a term $A_0^{(n)} I_n$ to the present output R_n and an ISI term $A_k^{(n)} I_n$ to future outputs R_{n+k}, $k = 1, 2, \ldots$. Mathematically, $A_k^{(n)} = \int_{(n+k)T_b}^{(n+k+1)T_b} \sum I(t - \tau_i) dt$, where $I(t)$ is the APD's random impulse response function, τ_i's are the photons' arrival times, and $nT_b \leq \tau_i < (n+1)T_b$. If we assume that the ISI affects up to $L-1$ future receiver outputs, then the receiver output can be written as

$$R_n = A_0^{(n)} I_n + A_1^{(n-1)} I_{n-1} + \ldots + A_{L-1}^{(n-L+1)} I_{n-L+1} + J_n, \tag{1}$$

where J_n is a zero-mean Gaussian Johnson noise, independent of the data sequence $\{I_k\}$ and the impulse response of the APD. In the above equation, the sum $A_1^{(n-1)} I_{n-1} + A_1^{(n-2)} I_{n-2} + \ldots + A_{L-1}^{(n-L+1)} I_{n-L+1}$ is referred to as the random ISI. Additionally, the signal $H(n) \triangleq A_0^{(0)} \delta(n) + A_1^{(0)} \delta(n-1) + \ldots + A_{L-1}^{(0)} \delta(n - L + 1)$ is identified as the random impulse response of the receiver. We can therefore think of the APD-based receiver as a stochastic finite-impulse-response (FIR) filter. Note that $A_p^{(n-k)}$ and $A_q^{(n-k)}$ are statistically correlated since they are ISI terms induced by the same photons in the optical pulse I_{n-k}. On the other hand, $A_p^{(n-k)}$ and $A_q^{(n-l)}$ are independent as long as $k \neq l$ since they arise from different photons. Since the mean and correlations of this APD-based receiver's impulse response are time invariant, we ignore the superscript elsewhere.

In [3], the equalizer is a linear combination of current observation R_n and the past observations to best restore the original transmitted signal I_n: $\hat{I}_n = b_0 R_n + b_1 R_{n-1} + \ldots + b_{M-1} R_{n-M+1}$, where the b_is, $i = 0, 1, \ldots, L-1$, are the tap coefficients. In this work, the estimates obtained by TE and DFE, denoted by I_{TE} and I_{DFE} respectively, are in the forms of

$$I_{\mathrm{TE}} = c_1 R_{n-1} + \ldots + c_{M_c} R_{n-M_c} + b_0 R_n + b_1 R_{n+1} + \ldots + b_{M_b-1} R_{n-M_b+1}, \tag{2}$$

$$I_{\mathrm{DFE}} = c_1 \tilde{I}_{n-1} + \ldots + c_{M_c} \tilde{I}_{n-M_c} + b_0 R_n + b_1 R_{n+1} + \ldots + b_{M_b-1} R_{n-M_b+1}, \tag{3}$$

where $\tilde{I}_{n-k}$ is the previous decision, M_b future observations of the APD's output are used in both TE and DFE, and M_c past observations or previous decisions are used TE or DFE.

III. Equalizer Design

Statistics of Receiver's Impulse Response: In [5], the joint probability distribution of the APD's random gain G and the duration T, denoted by $f_{G,T}(g,t)$ is determined numerically. Also a random duration (RD) rectangular model is proposed which assumes the APD's random impulse response function takes the form of $I(G,T,\tau) = GT^{-1}[u(\tau) - u(T - \tau)]$. With the availability of the RD rectangular model, and by using standard techniques for filtered Poisson processes [4], the mean and the correlation matrix of the receiver's impulse response $H(n)$, denoted by a_p and $R_A(p,q)$, respectively, can be obtained using

$$\mathsf{E}[I(\tau)] = \mathsf{E}[I(G,T,\tau)] = \sum_g \int_t I(g,t,\tau) f_{G,T}(g,t) dt,$$

$$\mathrm{R}_I(u,v) = \mathsf{E}[I(g,t,u) I(g,t,v)] = \sum_g \int_t I(g,t,u) I(g,t,v) f_{G,T}(g,t) dt,$$

$$a_p = \phi \iint_{\mathcal{D}_1} \mathsf{E}[I(t-\tau)] \mathcal{I}_{[0,T_b]}(\tau) d\tau dt,$$

$$R_A(p,q) = \phi \iiint_{\mathcal{D}_2} \mathrm{R}_I(\mu - \xi, \nu - \xi) \mathcal{I}_{[0,T_b]}(\xi) d\xi d\mu d\nu,$$

where $\mathsf{E}[A]$ denotes the expectation of A, $p, q = 0, 1, \ldots, L-1$, $\mathcal{I}_{[0,T_b]}(\xi)$ is the indicator function of the interval $[0, T_b]$, $\mathcal{D}_1 = \{(t,\tau) \in \mathbb{R}^2 : pT_b \leq t \leq (p+1)T_b, 0 \leq \tau \leq t\}$ and $\mathcal{D}_2 = \{(\mu,\nu,\tau) \in \mathbb{R}^3 : pT_b \leq \mu \leq (p+1)T_b, qT_b \leq \nu \leq (q+1)T_b, 0 \leq \tau \leq (\mu \wedge \nu)\}$.

Design of TE and DFE equalizers: TE and DFE are optimized according to a MMSE criterion; hence, the orthogonality principle [6] applies here to obtain the optimal coefficients. For the TE, I_{TE} should satisfy $\mathsf{E}[(I_n - I_{\mathrm{TE}}) R_{n+k}] = 0$ for $k = -M_c, \ldots, M-1$, this results in the following linear equation:

$$\begin{bmatrix} \overline{I_n R_{n+M_b-1}} \\ \vdots \\ \overline{I_n R_{n-M_c}} \end{bmatrix} = \begin{bmatrix} \overline{R_{n+M-b-1} R_{n+M_b-1}} & \ldots & \overline{R_{n+M_b-1} R_{n-M_c}} \\ \vdots & \vdots & \vdots \\ \overline{R_{n-M_c} R_{n+M_b-1}} & \ldots & \overline{R_{n-M_c} R_{n-M_c}} \end{bmatrix} \begin{bmatrix} \dot{b} \\ \dot{c} \end{bmatrix}, \tag{4}$$

where $\overline{AB}$ is the expectation of AB, i.e., $\overline{AB} = \mathsf{E}[AB]$; $[\dot{b}^T, \dot{c}^T]^T = [b_{M_b-1}, \ldots, b_0, c1, \ldots, cM_c]$. The taps $[b, c]$ is obtained by solving the above linear equation. Each element $\overline{R_k R_l}$ in the correlation matrix can be calculated by substituting for R_k and R_l from (1); therefore, $\overline{R_k R_l}$ can be expressed in terms of the correlation of input binary signal $\overline{I_p I_q}$, the mean impulse response of the receiver $[a_0, a_1, \ldots, a_{L-1}]$ and its correlation R_A. It is stressed that R_A accounts for the channel's random nature and makes the correlation matrix distinct from those of the regular equalization problems.

The DFE has M_b taps in its feedforward section and M_c taps in its feedback section. It should be observed that this equalizer is *non-linear* because the feedback filter contains previously detected signal $\tilde{I}_{n-k}$. The optimization of the taps is carried on the assumption that the previously detected signals in the feedback section are correct, i.e., we assume $\tilde{I}_{n-k} = I_{n-k}$ in equation (3). This leads to the linear equation:

$$\begin{bmatrix} \overline{I_n R_n} \\ \vdots \\ \overline{I_n R_{n+M_b-1}} \\ \overline{I_n I_{n-1}} \\ \vdots \\ \overline{I_n R_{n-M_c}} \end{bmatrix} = \begin{bmatrix} \overline{R_n R_n} & .. & \overline{R_{n+M_b-1} R_n} & \overline{I_{n-1} R_n} & .. & \overline{I_{n-M_c} R_n} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \overline{R_n R_{n+M_b-1}} & .. & \overline{R_{n+M_b-1} R_{n+M_b-1}} & \overline{I_{n-1} R_{n+M_b-1}} & .. & \overline{I_{n-M_c} R_{n+M_b-1}} \\ \overline{R_n I_{n-1}} & .. & \overline{R_{n+M_b-1} I_{n-1}} & \overline{I_{n-1} I_{n-1}} & .. & \overline{I_{n-M_c} I_{n-1}} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \overline{R_n I_{n-M_c}} & .. & \overline{R_{n+M_b-1} I_{n-M_c}} & \overline{I_{n-1} I_{n-1}} & .. & \overline{I_{n-M_c} I_{n-M_c}} \end{bmatrix} \begin{bmatrix} b_0 \\ \vdots \\ b_{M_b-1} \\ c_1 \\ \vdots \\ c_{M_c} \end{bmatrix}. \quad (5)$$

The DFE's taps are obtained by solving linear equation (5). The DFE is superior to the TE when the effect of decision errors on performance is neglected. In deterministic channel, it is apparent that a considerable gain in performance can be achieved by inclusion of the decision-feedback section, which eliminates the ISI from previously detected signals; in the random channel, the decision-feedback section removes ISI too, despite of the noise residual due to the channel's random impulse response.

IV. Performance Evaluation and Conclusion

We evaluated the performance of the TE and DFE by computing the achieved MMSEs and BERs with these two equalization techniques. The calculation of the BER of the receiver assumes a Gaussian approximation [7] for both the receiver output and the equalizer output. As an example, a GaAs APD is considered, which has average gain of 10 and a 100-nm multiplication region. The 3dB bandwidth is 27.4 GHz (taken from [5]). The ISI length parameter L varies depending on the transmission rate, and the number of taps of the equalizers, M_b, is always set to twice the ISI length, i.e., $M_b = 2L$; and M_c, is always set to the ISI length, i.e., $M_c = L$. The transmission rates are from 1Gbps to 40Gbps, each "1" bit contains 1000 photons, and the Johnson noise parameter is assumed to be 500 noise-counts per bit. The MMSE and BER as a function of the transmission rate are illustrated in Fig. 1. For example, at 40Gbs, the MMSEs for TE and DFE are 0.0071 and 0.0069 respectively; and the BERs are 3.45×10^{-3}, 1.20×10^{-9} and 5.43×10^{-10} for receiver without equalization, with TE and DFE respectively.

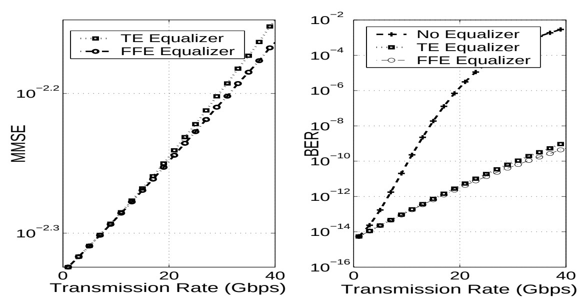

Fig. 1. MMSE and BER of the receiver without equalizer, with TE and DFE as a function of the transmission rate.

An equalized APD-based receiver exhibits a significant performance improvement, especially at high transmission rates. This work reveals that the speed limitations of an APD, brough about by the avalanche-buildup time, can be compensated for using a TE or DFE equalizers. Therefore, the equalized APD-based receiver has the potential to be a cost-effective alternative to the PIN-EDFA combination in high-speed optical communications.

References

[1] G. P. Agrawal, *Fiber-Optic Communication System*. New York: Wiley, 2002.

[2] J. H. Winters and R. D. Gitlin, "Electrical signal processing techniques in long-haul fiber-optic systems," *IEEE Trans. Comm.*, v38, pp. 1439–1453, 1990.

[3] P. Sun and M. M. Hayat, "A Linear Equalizer for High-Speed APD-Based Integrate-and-dump Receivers", *IEEE Comm. Letters*, to appear.

[4] D. R. Snyder and M.I. Miller, *Random Point Processes in Time and Space*. New York: Springer-Verlag, 1991.

[5] P. Sun, M. M. Hayat, B. E. A. Saleh, M. C. Teich, "Statistical correlation of gain and buildup time in APDs and its effects on receiver performance," *IEEE J. Lightwave Technol.*, to appear.

[6] H. Stark and J. W. Woods, *Probability, Random Process, and Estimation Theory For Engineers*. Prentice Hall, 1994.

[7] M. M. Hayat *et all, "Bit-error rates for optical receivers using avalanche photodiodes with dead space," IEEE Trans. Comm., v43, page 99–107, 1995.*