Exercise 1
Consider a probability space \((\Omega, \mathcal{F}, P)\) and consider a random variable, \(X\), defined on it. Let \(F_X(x)\) be the distribution function of \(X\) and \(f_X(x)\) be the associated probability density function.
Prove that \(F_X(t) - F_X(t^-) = P\{X = t\}\).

Exercise 2
Continuing Exercise 1, let \(\mathcal{B}\) be the collection of all Borel subsets of \(\mathbb{R}\) and define the mapping \(Q, Q : \mathcal{B} \rightarrow \mathbb{R}\), as follows: \(Q(B) = \int_B f_X(x)dx\), where \(B \in \mathcal{B}\). Show that \(Q\) defines a probability measure on \(\mathcal{B}\).

Exercise 3
Continuing Exercise 1, let \(g : \mathbb{R} \rightarrow \mathbb{R}\) be a continuous function. Define \(Y \triangleq g(X)\) and show that \(Y^{-1}((\neg \infty, r)) \in \mathcal{F}\).

Exercise 4
Continuing Exercise 1, let \(n\) and \(i\) be natural numbers. Show that \(\{\frac{i}{2^n} < X \leq \frac{i+1}{2^n}\}\) is an event.

Exercise 5
Continuing Exercise 1, let \(X\) be defined as follows:
\[X(\omega) = I_E(\omega), E \in \mathcal{F}, \text{where } I_E(\omega) = \begin{cases} 1, & \text{if } \omega \in E, \\ 0, & \text{if } \omega \notin E. \end{cases}\]
Show that \(X\) and \(aX\) are a random variables, where \(a \in \mathbb{R}\).

Exercise 6
Continuing Exercise 1, show that if \(X\) and \(Y\) are two discrete random variables then \(Z \triangleq X + Y\) is also a discrete random variable.

Exercise 7
Continuing Exercise 1, define the discrete random variable, \(X_n(\omega)\), as follows: \(X_n = \sum_{i=1}^{\infty} \frac{i}{2^n} I_{\{\frac{i}{2^n} < X \leq \frac{i+1}{2^n}\}}\). Show that for every \(\omega \in \Omega\), \(X_n(\omega) \uparrow X(\omega)\) as \(n \rightarrow \infty\).