Correlation Between Gain and Buildup-Time Fluctuations in Ultrafast Avalanche Photodiodes and Its Effect on Receiver Sensitivity

P. Sun pengsun@ece.unm.edu, M. M. Hayat, *Senior Member, IEEE* hayat@ece.unm.edu, J. C. Campbell, *Fellow, IEEE*, B. E. A. Saleh, *Fellow, IEEE*, and M. C. Teich, *Fellow, IEEE*

Abstract: The joint statistics of the random gain and random buildup time in ultrafast avalanche photodiodes is determined for the first time. It is shown that the strong correlation present adversely affects intersymbol-interference noise and receiver sensitivity at high transmission rates well beyond previously-known limits.

Avalanche photodiodes (APDs) are widely-used photodetectors in many high-speed optical receivers including those deployed in 10-Gbps/channel lightwave systems. The popularity of APDs is due to their ability to provide high internal optoelectronic gains despite their simplicity (being junction devices). The high gain that an APD provides translates into improved receiver sensitivity as the gain combats the Johnson noise in the pre-amplifier stage of an optical receiver. This, however, comes at the tolerable expense of an increase in shot noise by a factor called the APD's excess–noise factor, *F*, which is a measure of the gain uncertainty. However, the APD's avalanche buildup time, which is the duration of the APD's single-photo-excited impulse response and causes intersymbol-interference (ISI), is a more serious concern and it has been the limiting factor prohibiting their use in 40-Gbps systems. From a device perspective, advances in APD technology, such as the development of APDs that may feature evanescent light coupling into very thin multiplication regions (where both carrier transit time and avalanche buildup time are significantly reduced without much sacrifice in coupling efficiency) may be a promising path for bringing APDs to the 40-Gbps ranges. On the other hand, to know the exact device requirements for specified system performance requires yet to be developed exact theoretical models for assessing the receiver performance at high speeds, where factors such as ISI and the stochastic nature of the APD's impulse response play critical roles. In this paper we present recent advances in the analysis of APD-based receivers at high transmission speeds that are expected to impact the way we translate system requirements (e.g., those for next-generation lightwave systems) to device requirements.

The APD's impulse response is a stochastic process, with a random duration (viz., avalanche buildup time) and a random area (viz., avalanche multiplication, or gain). Moreover, the random gain and random buildup time are stochastically correlated. One manifestation of this correlation is the APD's gain-bandwidth product, which is traditionally calculated as the product between the mean gain and average bandwidth (which is related to the average buildup time, or equivalently the average impulse response function). However, this correlation carries to a higher level, which has been ignored to this date, since the uncertainties in the gain and the buildup time are also correlated. In particular, at transmission rates near the APD's bandwidth, it is plausible to suspect that the statistical correlation between the random gain and the random avalanche buildup time may play a complex role in receiver performance. To thoroughly assess the extent of the role played by this stochastic correlation, it is required that we (1) characterize the joint probability distribution function (PDF) of the buildup time and the gain; and (2) launch a receiver sensitivity analysis that includes the effects of such stochastic correlation. In the present paper we report on our work on both of these accounts.

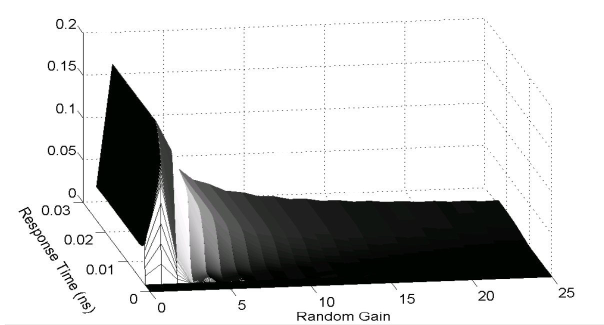

Fig. 1. The PDF $F_{G,T}(m,t)$ of the random gain *G* and the buildup time *T* for a GaAs APD with a 160-nm multiplication layer. The applied electric field is taken as 5.47 x103 kV/cm, yielding an average gain of $<G>=10.46$.

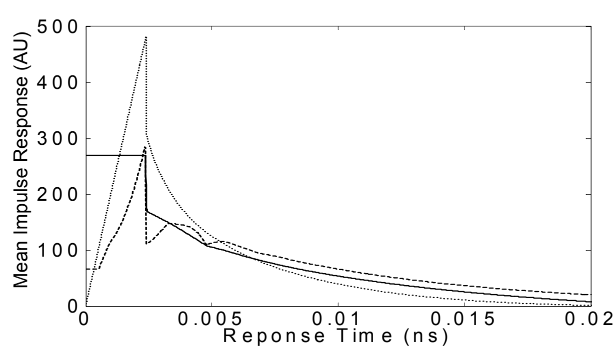

Fig. 2. The mean impulse response function *i*(*t*) calculated from (i) the randomly-parameterized rectangular model (solid curve); (ii) parameterized triangular model (dotted curve); and (iii) the exact model [1] (dashed curve). Note that the rectangular model slightly underestimates the bandwidth while the triangular model overestimates it.

Although the marginal probability distribution of the gain and response time have been individually determined in prior work [1-3], the joint PDF of the APD gain and the buildup time (denoted by G and T, respectively), has not been available to date. Here, we report the results of a recurrence theory, comprising a pair of coupled nonlinear integral equations, whose solution characterizes the joint PDF of the random variables G and T, denoted by $F_{G,T}(m,t)=\mathrm{P}\{G=m,\ T\leq t\}$, $m=1,2,3,\ldots$, $t\geq 0$. Figure 1 shows $F_{G,T}(m,t)$ calculated for a homojunction GaAs APD with a 160nm multiplication layer. The electron and hole saturation velocities are assumed to be $0.67\text{x}10^7$ cm/s and the non-localized ionization coefficients for electrons and holes and their respective ionization threshold energies are taken form Saleh *et al.* [3], which were extracted according to the dead-space multiplication theory. With the knowledge of the joint PDF of the APD gain and response time, the computation of any statistic of the APD's impulse response function can be done by approximating the random impulse response function by a function of time that is parameterized by the random quantities G and T. For example, we may use a rectangular pulse with the duration given by the random response time T and a random height G/T, such that the total area of the pulse is the random gain G. Another approximation would be a triangular function whose peak is at the transit time of the injected electron upon reaching the end of the multiplication region and its duration and area are T and G, respectively. Using the above randomly parameterized models in conjunction with the knowledge of $F_{G,T}(m,t)$, we can accurately estimate the mean impulse response, denoted by $i(t)$, where in calculating the average, an ensemble average is taken jointly over G and T. Figure 2 depicts $i(t)$ obtained using both the rectangular and triangular parametric models in conjunction with the calculated PDF shown in Fig. 1. Indeed, the area under $i(t)$ is 10.48, which is in good agreement to the average gain (which is calculated independently using Ref. [3]). The comparison with the theoretical prediction of the APD mean impulse response by [1] is also presented on this plot. The real value of the parametric model of the APD's random impulse response is that it allows us to calculate higher-order statistics easily, as done next to evaluate the receiver's signal-to-noise ratio (SNR) and receiver sensitivity.

We first compute the correlation coefficient between G and T and we show the presence of strong correlation between them. Figure 3 (lower curve) shows the correlation coefficient as a function of the multiplication width, while holding the mean average gain fixed at 10. The correlation coefficient is approximately 0.9.

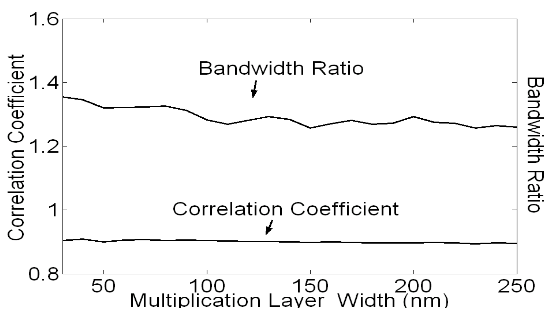

Fig. 3. The ratio of effective bandwidth to 3-dB bandwidth and correlation coefficients versus the multiplication layer width of GaAs APDs with average gain $<G>=10$, the width ranges from 30nm to 250nm.

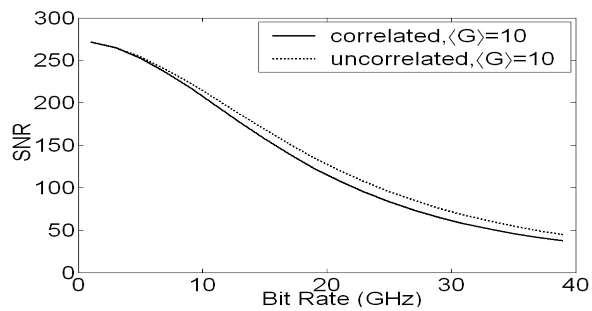

Fig. 4. SNR as a function of the transmission speed. Solid curve corresponds to the randomly-parameterized model (where G and T are correlated) for the impulse response. Dashed curve corresponds to the traditional fixed-shape model (where G and T are assumed statistically uncorrelated).

Traditionally, SNR calculations implicitly assume that the impulse response takes the deterministic shape in which case the variance of the APD-bandwidth-limited of the photocurrent is proportional to $2F<G>^2B_c$, where B_c is conventional 3-dB bandwidth governed by the shape of the mean impulse response. Such deterministic-shape assumption of the impulse response is clearly inconsistent with the strong correlation between the buildup time and the gain. Thus, to effectively capture such a correlation and have a more accurate estimate of the shot-noise variance, we must directly calculate the variance of the photocurrent (which turns out to be proportional to the area under $<I^2(t)>$, the mean of the square of the impulse response) while using a model for the APD's random impulse response that captures both its random area and duration. This allows us to introduce an effective bandwidth, B_{eff}, which we would need to use in place of B_c in the traditional formula $2Fg^2B_c$ in order to obtain the correct variance. This leads to the definition $B_{eff}=\int<I^2(t)>dt\ /\ 2<G>^2F$. We emphasize that B_{eff} includes the effects of the correlation in the shape (or duration) *and* gain (area) of APD's random impulse response. With the randomly parameterized rectangular model for the stochastic impulse response that was described earlier, the expression for B_{eff} can by simplified to $B_{eff}=<G^2/T>/2<G>^2F$. Figure 3 (upper curve) shows the ratio of effective bandwidth to the 3-dB bandwidth (B_c calculated from the mean impulse response of rectangular model) versus the width of the multiplication layer for devices of average gain $<G>=10$. Notably, the effective bandwidth exceeds the 3dB bandwidth by 30% approximately. *Thus, our calculations indicate that the bandwidth that should be used in calculating the shot-noise variance is significantly underestimated if the statistical correlation in the buildup time and gain of the APD are ignored*. We next see the ramifications of this fact for high-speed communication.

At high operational speeds in digital on-off-keying transmission, ISI plays an important role in the SNR. Photons in a stream of optical pulse generate a stream of random impulse responses. These overlap to produce the random photoelectric current. In an integrate-and-dump receiver, the photocurrent is fed into a bit-integrator, which yields the integral of the photocurrent synchronously over each bit of duration T_b. We computed the SNR and the receiver sensitivity of an integrate-and-dump receiver output in two cases: In one case we assumed that the shape of the random impulse response function of the APD is deterministic as $I(t)=Gbe^{-bt}$, where we set $b=2\pi B_c$. This case corresponds to the traditional way of calculating the performance where the statistical correlation between the buildup time and gain are ignored. In this case, calculations yield the SNR_{un} expression shown on the left:

$$SNR_{un} = \eta \frac{2\phi T_b}{F}\left(\frac{2\pi T_b B_c - 1 + \exp(-2\pi T_b B_c)}{2\pi T_b B_c}\right); \qquad SNR_c = \eta \frac{2\phi T_b}{F}\left(\frac{4T_b B_{eff} - 1 + \exp(-4T_b B_{eff})}{4T_b B_{eff}}\right)$$

Where $\phi=P/h\nu$ and P are respectively the photon flux and received optical power in a "1" bit, and η is the APD's quantum efficiency. In the second case, we use the randomly parameterized (by G and T) rectangular model for $I(t)$, which does account for the buildup-time gain correlation. After algebraic manipulations, we obtain the SNR_c expression on the right. (The Johnson noise is not shown in these expressions.) In both of the above SNR expressions, the first fraction is the SNR for an instantaneous detector, in which, of course, ISI absent; the second fraction represents the factor due to ISI. Recall that the buildup-time-gain statistical correlation effect is incorporated in the latter SNR formula through B_{eff}. For simplicity, the two SNR formulas are derived assuming that all the bits sent are ones, which leads to maximum ISI scenario. The SNRs for the general case can be also obtained; however, the expressions become too lengthy and we therefore omit them. However, we computed the SNR for the general case for a GaAs APD with 100nm multiplication layer with an average gain 10 and a 3dB bandwidth of B_c=29GHz (similar results can also obtained for InP APDs). The average number of photons absorbed by the APD is taken to be 300 photons per bit and the Johnson-noise parameter (viz., equivalent number of noise counts per bit) is σ_N=500 noise counts per bit. Figure 4 shows the SNR as function of the transmission speed. We see that the correlation between the random buildup time and the random gain has a detrimental effect on the SNR. Moreover, the adverse effect becomes more severe as the transmission rate increases. Figure 5 depicts the bit-error probability (BER) as a function of the transmission speed, which is consistent with the SNR plots. The receiver sensitivity, defined as the minimum number of photons per bit necessary to produce a BER of 10^{-9}, is shown in Fig. 6. We observe that at high transmission speeds, the sensitivity increases as expected due to ISI. For the 29 GHz APD considered, ignoring the statistical correlation results in overestimating the sensitivity performance by 9.4 % at transmission speed of 10 Gbps, 16.9% at 15 Gbps and 38.4% at 20 Gbps. Thus, the statistical correlation between build-time and the gain contributes adversely to ISI and the receiver performance, especially at high transmission rates.

In conclusion, the statistical correlation between the random gain and the random avalanche buildup time in APDs is determined for the first time and its effect on the receiver performance is established. This correlation is shown to adversely affect inter-symbol-interference noise and receiver sensitivity beyond what is predicted by conventional receiver-performance analyses that do not account for such statistical correlation. Notably, this new effect becomes progressively more significant as the transmission rate increases. This project is supported by the National Science Foundation under grants ECS-0334813 and ECS-0196569.

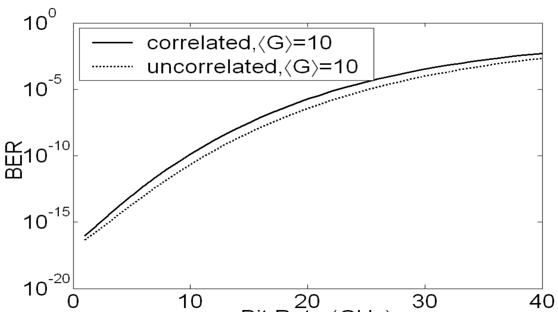

Fig. 5. Bit error rate as a function of the transmission speed. The solid curve corresponds to the case when the statistical correlation between the random build-up time and the random gain is accounted for through the parametric rectangular model for the impulse response. The dashed curve corresponds to the case when the correlation is ignored. The mean gain is 10.

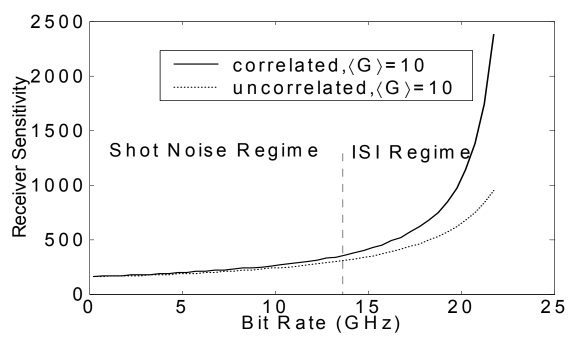

Fig. 6. The receiver sensitivity as a function of the transmission speed. All system parameters are the same as those used in Fig. 5. Note that ignoring the statistical correlation between the random buildup time and gain (as shown by the dotted curve) results in overestimating the receiver performance (i.e., underestimating the receiver sensitivity).

[1] M. M. Hayat and B. E. A. Saleh, *J. Lightwave Technol.*, vol. 10, no. 10, pp. 1415-1425, 1992.

[2] M. M. Hayat and G. Dong, *IEEE Trans. Electron Devices*, vol. 47, no. 6, pp. 1273-1279, June 2000.

[3] M. A. Saleh, *et al.*, *IEEE Trans. Electron Devices*, vol. 48, pp. 2722-2731, 2001