A RAKE-Based Iterative Receiver for Space-Time Block-Coded Multipath CDMA

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***Abstract*—A turbo multiuser receiver is proposed for space-time block and channel-coded code division multiple access (CDMA) systems in multipath channels. The proposed receiver consists of a first stage that performs detection, space-time decoding, and multipath combining followed by a second stage that performs the channel decoding. A reduced complexity receiver suitable for systems with large numbers of transmitter antennas is obtained by performing the space-time decoding along each resolvable multipath component and then diversity combining the set of space-time decoded outputs. By exchanging the soft information between the first and second stages, the receiver performance is improved via iteration. Simulation results show that while in some cases a noniterative space-time coded system may have inferior performance compared with a system without space-time coding in a multipath channel, proposed iterative schemes significantly outperform systems without space-time coding, even with only two iterations. Furthermore, the performance loss in the reduced-complexity receiver due to decoupling of interference suppression, space-time decoding, and multipath combining is very small for error rates of practical interest.**

***Index Terms*—Code division multiple access, multiuser detection, RAKE combining, space-time coding, turbo detection.**

I. Introduction

In RECENT years, inspired by the development of turbo coding [1], [2], various types of iterative detection and decoding schemes have been proposed in the literature [3], [4]. These proposals have shown that iterative receivers can offer significant performance improvements over their noniterative counterparts. In [5], a soft interference cancelling turbo receiver was proposed for convolutionally coded, code-division multiple-access (CDMA), and the performance results obtained via simulations showed that near single-user performance is possible with only a few iterations. After the invention of space-time codes and demonstration of their impressive performance gains in single-user channels [6]–[8], application of space-time codes to multiuser systems has been considered in [9]–[13] and references therein. The concept of turbo multiuser detection and decoding of [5] has also been applied to space-time coded, flat-fading CDMA [10], [11] and to space-time coded, space-division multiple-access (SDMA) systems [12], confirming that the same type of performance improvement is possible in the multiple transmit antenna case.

In this paper, we consider the application of space-time coding to frequency-selective CDMA channels. Space-time block coded (STBC) downlink CDMA systems in multipath channels have been considered previously, for example, in [14] and [15]. Space-time coding techniques investigated in, for example [16] and [17] were concerned with frequency-selective channels, but they focused on orthogonal frequency division multiplexing (OFDM) systems and/or did not consider multiple-access interference (MAI) suppression. In this paper, on the other hand, we propose an uplink scheme that combines the concepts of RAKE combining, turbo multiuser detection, and space-time block coding and is well suited for multipath CDMA systems. Specifically, it consists of two stages similar to those of [5] for convolutionally coded CDMA. The first stage consists of detection, space-time decoding and diversity combining, whereas the second stage consists of channel decoding.

The proposed scheme can be considered to be an adaptation of the similar-type of receiver based on the idea of iterative soft interference cancellation and instantaneous minimum mean square error (MMSE) filtering proposed in [12] for space-time block coded SDMA in flat-fading environments to space-time block coded CDMA in frequency-selective fading channels. The interference suppression in [12] is achieved via spatial processing at the receiver. This means that the receiver proposed in [12] requires multiple receiver antennas (in fact, more receiver antennas than the product of the number of simultaneous users and the number of transmit antennas at each user) for its successful operation. In contrast, by replacing the SDMA scheme with a CDMA system, we exploit knowledge of the structure of the multiuser signal in order to suppress the residual MAI and noise present in the soft interference cancelled channel outputs and especially do not rely on the availability of receiver diversity. Thus, although [12] can be considered to be a spatial interference suppression scheme, ours is a temporal technique.

There are some advantages in cancelling MAI based on the multiuser signal structure embedded in the received signal (as in CDMA) rather than on receiver diversity, as was the case in [12]. First and foremost is that to suppress K interferers, a system based purely on spatial processing (i.e. SDMA) requires K receiver antennas, which can be excessive even at a base station. On the other hand, knowledge about the multiuser structure of the received signal is readily available at a base station, and digital signal processing power makes exploiting this knowledge with sophisticated signal processing algorithms to suppress the MAI a viable option. In addition, whenever receiver

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diversity is available, it can easily be incorporated into the proposed CDMA-based iterative receivers in order to further improve performance.

A shortcoming of the turbo receiver in [12] is that the instantaneous MMSE interference suppression filter requires the inversion of a $PN_R \times PN_R$ matrix, assuming a $P \times N_T$ space-time block code is employed by every user, where N_R and N_T are the number of receiver and transmitter antennas, respectively. If we were to adapt the receiver in [12] directly to multipath CDMA environments, it will then also require the instantaneous inversion of a $PN' \times PN'$ matrix, where N' is the length of the modified signature sequence (see Section II). This could be computationally expensive whenever N_T is large since usually, N' can be large, and $P \geq N_T$. For example, even for $N_T = 4$, the rate-1/2 complex STBC proposed in [7] requires $P = 8$, and thus, for large N_T and N', the computational cost could easily become too much of a burden at the receiver. In order to reduce this computational cost, we propose a modified algorithm that performs instantaneous MMSE interference suppression, space-time decoding, and multipath combining separately rather than jointly, as would be the case if we were to use the direct modification of the receiver given in [12] to a multipath CDMA channel. This new algorithm incorporates the decorrelating RAKE (DRAKE) receiver concept for multipath CDMA channels developed in [18] with the iterative soft interference cancellation and MMSE filtering for convolutional coded CDMA of [5] and extends these ideas to the case where the system also employs space-time block coding in addition to the convolutional channel coding.

The modified receiver still consists of two stages, and the channel decoder step in the second stage is still identical to that of [12]. However, the first stage of the modified receiver operates by first performing interference cancellation on the received signal and then applying a bank of linear MMSE filters along each multipath component present in the received signal. Space-time decoding is then performed on each multipath component separately. In doing so, the computational cost is reduced to the inversion of P matrices of size $N' \times N'$. Next, the space-time decoded outputs from each branch are RAKE combined to make the final decision statistic of the first stage. These combined outputs are used to generate the soft outputs of the first stage.

This presentation is organized as follows. In Section II, we present our signal model and the system description. Next, in Section III, we derive a space-time turbo receiver for space-time block coded, multipath CDMA by modifying the receiver given in [12] for an SDMA system. In Section IV, we modify the receiver derived in Section III in order to reduce its complexity by performing interference suppression, space-time decoding, and multipath combining in separate steps. Finally, in Section V, we give performance results obtained via computer simulation of a multipath CDMA system and demonstrate the performance gains possible with the proposed turbo space-time receivers.

II. Signal Model and the System Description

Consider a system with K simultaneous users, each equipped with N_T transmit antennas and a base station consisting of N_R receiver antennas. For the sake of simplicity, we will assume that $N_T = 2$ and $N_R = 1$ throughout this discussion, although generalization to larger numbers of transmit and receive antennas is straightforward. The binary phase shift keying (BPSK) information symbol sequence $\{d_k(i')\}_{i'=0}^{\infty}$ of user k, for $k = 1, \ldots, K$, is first encoded by a convolutional channel encoder. Again, for simplicity, we assume that all users employ the same convolutional code having constraint length ν and rate R_c, although it is easy to accommodate different channel encoders for each user. Let B_1 be the number of information symbols per convolutional channel codeword, including the trellis terminating $\nu - 1$ tail bits. Thus, the channel codeword of user k corresponding to an input information symbol frame of length B_1 has a length of $B_2 = B_1/R_c$ and is denoted by $c_k(j)$ for $j = 0, 1, \ldots, B_2 - 1$.

The channel encoder outputs are next block interleaved by a random interleaver, and these interleaved symbols are input to the space-time encoders. If we denote the interleaver function of user k by $\prod_k$, then the interleaver output can be written as $\{c_k(i)\}_{i=0}^{B_2-1}$, where $i = \prod_k(j)$ for $j = 0, 1, \ldots, B_2 - 1$. The input to the space-time block encoder of user k is the interleaved symbol stream $\{c_k(i)\}_{i=0}^{B_2-1}$. Since $N_T = 2$, we assume that each user employs an Alamouti space-time encoder [6] with space-time block code rate $R_s = 1$. Hence, the length of the STBC is $P = 2$, and during each code block, $M = R_sP = 2$ symbols are transmitted. Thus, if two consecutive input symbols of user k into its space-time block encoder are denoted by $c_k(i)$ and $c_k(i+1)$, then according to the Alamouti scheme, during the first symbol period, the symbols $c_k(i)$ and $c_k(i+1)$ are transmitted simultaneously from the first and second antennas, respectively. During the second symbol period, the symbols $-c_k^*(i+1)$ and $c_k^*(i)$ are transmitted from the first and second antennas, respectively, where * denotes complex conjugation.

The space-time encoder output of each user is next modulated by the user's spreading sequence and transmitted simultaneously from two transmit antennas. Note that the two symbols transmitted from the two antennas corresponding to a particular user are modulated by the same spreading sequence, i.e., each user is assigned only one spreading code. The spreading waveform $s_k(t)$ of user k is assumed to be of the form

$$s_k(t) = \sum_{q=0}^{N-1} s_k(q)\varphi(t - qT_c) \tag{1}$$

where N denotes the processing gain of the CDMA system, and $\mathbf{s}_k = [s_k(0), \ldots, s_k(N-1)]^T$ denotes the kth user's spreading code. In (1), T_c denotes the chip period, $s_k(q) \in \{(+1/\sqrt{N}), (-1/\sqrt{N})\}$, and $\varphi(t)$ is the normalized chip waveform.

The continuous-time received signal can be written as

$$r(t) = \sum_{k=1}^{K}\sum_{n=1}^{N_T} x_{k,n}(t) \star f_{k,n}(t) + n(t) \tag{2}$$

where $x_{k,n}(t)$ and $f_{k,n}(t)$ denote the transmitted signal and channel impulse response, respectively, corresponding to antenna n of user k, $\star$ denotes the convolution operation, and $n(t)$ is a complex white Gaussian noise process with zero mean and variance 1/2 per dimension. The transmitted signal $x_{k,n}(t)$ in (2) is given by

$$x_{k,n}(t) = \sum_{i=0}^{B_2-1} b_{k,n}(i) s_k(t-iT) \tag{3}$$

where $b_{k,n}(i)$ denotes the transmitted symbol of user k on antenna n during the transmission interval $[iT,(i+1)T]$.

The channel impulse response $f_{k,n}(t)$ can be written as

$$f_{k,n}(t) = \sqrt{\frac{\rho_k}{N_T}} \sum_{l=1}^{L} h_{k,n,l} \delta\left(t-(l-1)T_c\right) \tag{4}$$

where L is the number of paths, $h_{k,n,l}$ denotes the fading coefficient of the lth path in the channel between the receiver and the transmit antenna n of user k, $\delta(t)$ denotes the Dirac-delta function, and ρ_k is the average received signal-to-noise-ratio (SNR) of user k per path. We have normalized this SNR so that it does not depend on the number of transmit antennas employed at the transmit end. This ensures that the average transmit power at the transmitter is fixed, regardless of the number of transmit antennas.

Note that in order to minimize the notational complexity, we have assumed that the number of paths in all the channels between transmit antennas of each user and the base station is constant and equal to L, that the multipath delays are multiples of the chip interval, and that all users have the same delays. Again, it is straightforward to modify the above model in order to let the number of multipaths depend on the transmit antenna and user index. In addition, in a practical implementation, it is not difficult to include different values for the multipath delays at the expense of some extra notational complexity. However, here, we are concerned with demonstrating the general technique of the proposed scheme, and thus, we adhere to the simplest possible model, keeping only the essential elements of the channel impulse response.

Next, we assume that the maximum delay spread of the channel is $(L-1)T_c$ and define the symbol period T as $T=(N+L-1)T_c$. In that case, if we assume symbol-synchronous user transmissions, then all the delayed replicas of the transmitted signals will be received at the base station during the same symbol period, resulting in no intersymbol interference. It should be noted that this is a reasonable approximation for channels with delay spreads on the order of few chip intervals. This is equivalent to the assumption that the actual spreading waveform is obtained by appending $L-1$ zeros to the tail of the spreading code $\mathbf{s}_k$. Let $N'=N+L-1$ be the modified signature sequence length.

The signal $r(t)$ is first chip matched filtered and then sampled at the chip rate. The resulting observables corresponding to the ith symbol period are given, for $q=0,1,\ldots,N'-1$, by

$$r(i,q) = \int_0^{\infty} r(t)\varphi(t-iT-qTc)dt.$$

These observables can be collected to form a vector of length N' as

$$\mathbf{r}(i) = \sum_{k=1}^{K} \sqrt{\frac{\rho_k}{N_T}} \mathbf{S}_k \left[\mathbf{h}_{k,1} b_{k,1}(i) + \mathbf{h}_{k,2} b_{k,2}(i)\right] + \mathbf{n}(i) \tag{5}$$

where $\mathbf{n}(i)$ is a complex Gaussian random noise vector such that $\mathbf{n}(i) \sim \mathcal{N}(\mathbf{0}, \mathbf{I}_{N'})$, the $N' \times L$ matrix $\mathbf{S}_k$ is defined, for $k=1,2,\ldots,K$, as

$$\mathbf{S}_k = \begin{bmatrix} s_k(0) & 0 & \dots & 0 \\ s_k(1) & s_k(0) & \dots & 0 \\ \vdots & \vdots & \dots & \vdots \\ s_k(N-1) & s_k(N-2) & \dots & s_k(N-L) \\ 0 & s_k(N-1) & \dots & s_k(N-L+1) \\ \vdots & \vdots & \dots & \vdots \\ 0 & 0 & \dots & s_k(N-1) \end{bmatrix}$$

and the L-vector of fading coefficients of the different paths between the transmit antenna n of user k and the base station is given by $\mathbf{h}_{k,n} = [h_{k,n,1}, h_{k,n,2}, \ldots, h_{k,n,L}]^T$. It should be noted that if we were to let different paths of different users have different values of delays, then we may still write a similar equation for the chip-matched filter output. However, in that case, the columns of the matrix $\mathbf{S}_k$ corresponding to different users will have different initial offsets. In addition, we may eliminate the constraint of the multipath delays being integral multiples of the chip interval and include the effect of fractional correlation of the chip waveforms in the fading coefficient $h_{k,n,l}$. Thus, our model is in fact general enough to absorb these generalizations and, yet, simple enough to avoid unnecessary notational complexity in the current discussion.

Defining the $L \times N_T$ matrix $\mathbf{H}_k$ of fading coefficients as

$$\mathbf{H}_{1,k} = [\mathbf{h}_{k,1} \quad \mathbf{h}_{k,2}] = \begin{bmatrix} h_{k,1,1} & h_{k,2,1} \\ h_{k,1,2} & h_{k,2,2} \\ \vdots & \vdots \\ h_{k,1,L} & h_{k,2,L} \end{bmatrix}$$

and the N_T-length transmit symbol vector of user k at time instant i as $\mathbf{b}_k(i) = [b_{k,1}(i), b_{k,2}(i)]^T$, we may write the chip-rate sampled output (5) corresponding to the symbol time i compactly as

$$\mathbf{r}(i) = \sum_{k=1}^{K} \sqrt{\frac{\rho_k}{N_T}} \mathbf{S}_k \mathbf{H}_{1,k} \mathbf{b}_k(i) + \mathbf{n}(i) \tag{6}$$

$$= \mathcal{H}_1 \mathbf{b}(i) + \mathbf{n}(i) \tag{7}$$

where, in (7), we have defined the KN_T-vector $\mathbf{b}(i)$ of transmit symbols of all users from all antennas as $\mathbf{b}(i) = [\mathbf{b}_1(i)^T, \ldots, \mathbf{b}_K(i)^T]^T$ and the $N' \times KN_T$ matrix $\mathcal{H}_1$ as

$$\mathcal{H}_1 = \left[\sqrt{\frac{\rho_1}{N_T}} \mathbf{S}_1 \mathbf{H}_{1,1} \quad \ldots \quad \sqrt{\frac{\rho_K}{N_T}} \mathbf{S}_K \mathbf{H}_{1,K}\right]. \tag{8}$$

The first stage of the receiver processes received chip matched signal vectors in blocks of size $P=2$, corresponding to the space-time codeword length. Then, the two consecutive received signals during the mth received space-time code block correspond to the two consecutive symbols $c_k(i)$ and $c_k(i+1)$ of user k, for $m=0,1,2,\ldots$. Thus, without loss of generality, from here on, we will assume that the symbol index i is of the form $i=mP$ for some $m=0,1,2,\ldots$ and simply refer to the two received chip-matched signals during the mth block as $\mathbf{r}(i)$ and $\mathbf{r}(i+1)$ (i.e. by assuming that $i=mP$, for $m=0,1,2,\ldots$, we may refer to the mth space-time code block simply as the

ith block without causing any confusion). In this case, the two transmit symbol vectors $\mathbf{b}_k(i)$ and $\mathbf{b}_k(i+1)$ corresponding to the symbol times i and $i+1$ are given by

$$\mathbf{b}_k(i) = [c_k(i), c_k(i+1)]^T \tag{9}$$

and

$$\mathbf{b}_k(i+1) = [-c_k^*(i+1), c_k^*(i)]^T. \tag{10}$$

Suppose that the receiver generates a set of P decision statistics $\mathbf{y}^1(i), \mathbf{y}^2(i), \ldots, \mathbf{y}^P(i)$ corresponding to the P received signals $\mathbf{r}(i), \mathbf{r}(i+1), \ldots, \mathbf{r}(i+P-1)$ during the ith space-time code block. In the case of $P=2$, these are defined as $\mathbf{y}^1(i) = \mathbf{r}(i)$ and $\mathbf{y}^2(i) = \mathbf{r}^*(i+1)$. Hence

$$\mathbf{y}^1(i) = \sum_{k=1}^{K} \sqrt{\frac{\rho_k}{N_T}} \mathbf{S}_k \mathbf{H}_k \mathbf{b}_k(i) + \mathbf{n}_1(i) \tag{11}$$

and

$$\begin{aligned} \mathbf{y}^2(i) &= \sum_{k=1}^{K} \sqrt{\frac{\rho_k}{N_T}} \mathbf{S}_k \mathbf{H}_{2,k} \mathbf{b}_k(i) + \mathbf{n}_2(i) && (12) \\ &= \mathcal{H}_2 \mathbf{b}(i) + \mathbf{n}_2(i) && (13) \end{aligned}$$

where

$$\mathbf{H}_{2,k} = \begin{bmatrix} \mathbf{h}_{k,2}^* & -\mathbf{h}_{k,1}^* \end{bmatrix} = \begin{bmatrix} h_{k,2,1}^* & -h_{k,1,1}^* \\ h_{k,2,2}^* & -h_{k,1,2}^* \\ \vdots & \vdots \\ h_{k,2,L}^* & -h_{k,1,L}^* \end{bmatrix} \tag{14}$$

$\mathcal{H}_2 = [\sqrt{(\rho_1/N_T)}\mathbf{S}_1\mathbf{H}_{2,1} \ldots \sqrt{(\rho_K/N_T)}\mathbf{S}_K\mathbf{H}_{2,K}]$, and the two noise vectors $\mathbf{n}_1(i) = \mathbf{n}(i)$ and $\mathbf{n}_2(i) = \mathbf{n}^*(i+1)$ are independent $\mathcal{N}(\mathbf{0}, \mathbf{I}_{N'})$ vectors. Note also that we always have the property that the lth row of $\mathbf{H}_{1,k}$ is orthogonal to the lth row of $\mathbf{H}_{2,k}$, i.e., $\boldsymbol{\alpha}_{1,k,l}^H \boldsymbol{\alpha}_{2,k,l} = 0$ for $l = 1, \ldots, L$ and $k = 1, \ldots, K$, where $\boldsymbol{\alpha}_{1,k,l}^H$ and $\boldsymbol{\alpha}_{2,k,l}^H$ denote the lth row of $\mathbf{H}_{1,k}$ and $\mathbf{H}_{2,k}$, respectively.

III. Joint Space-Time MMSE-Based Turbo Receiver for Space-Time Block Coded Multipath CDMA

In this section, we generalize the turbo receiver for SDMA proposed in [12] for a space-time block coded, multipath CDMA channel. In the next section, we modify this joint space-time MMSE-based receiver in order to lower the computational complexity at the expense of slightly degraded performance.

The first stage of the iterative receiver performs soft detection of all the user symbols from all transmit antennas. Essentially, it treats each transmitter of a particular user as a separate *virtual* user. The second stage of the receiver consists of a bank of K single-user channel decoders. In the first stage of the receiver, we assume that *a priori* information about the code symbols $c_{k'}(i)$ for $k' = 1, \ldots, K$ and $i = 0, \ldots, B_2 - 1$ is available. Generally, this *a priori* information comes from the second stage of the receiver at the previous iteration, as we will see below. For the initial iteration, we may assume a uniform distribution for these symbols. For the following discussion, we assume that we are interested in detecting the particular user k.

For a $P \times N_T$ STBC, we may collect all the received signals corresponding to a space-time block codeword in to an PN'-vector $\mathbf{y}(i) = [(\mathbf{y}^1(i))^T, \ldots, (\mathbf{y}^P(i))^T]^T$, where $\mathbf{y}^p(i)$'s are defined appropriately by generalizing (11) and (12) for a particular space-time block code. Then, we may write this combined signal as

$$\mathbf{y}(i) = \mathcal{H}\mathbf{b}(i) + \mathbf{m}(i) \tag{15}$$

where $\mathbf{m}(i)$ is an PN'-vector of independent, zero-mean AWGN components, and $\mathcal{H}$ is a $PN' \times KN_T$ (in general, $PN' \times KM$, but the modification is straightforward) matrix. With the 2 × 2 STBC, from (11) and (12), we have that

$$\mathbf{y}(i) = \sum_{k=1}^{K} \sqrt{\frac{\rho_k}{N_T}} \begin{bmatrix} \mathbf{S}_k \mathbf{H}_{1,k} \\ \mathbf{S}_k \mathbf{H}_{2,k} \end{bmatrix} \mathbf{b}_k(i) + \begin{bmatrix} \mathbf{n}_1(i) \\ \mathbf{n}_2(i) \end{bmatrix}. \tag{16}$$

Hence, in this case, $\mathbf{m} = [(\mathbf{n}_1(i))^T (\mathbf{n}_2(i))^T]^T$, and $\mathcal{H} = [\mathcal{H}_1^T \mathcal{H}_2^T]^T$.

The receiver uses *a priori* information to make soft estimates of all the users' symbols corresponding to the received frame. These soft estimates are used to reconstruct the interference caused by all the other transmissions to the signal from the nth antenna of user k (there are $KN_T - 1$ such interferers), and then, the first stage of the receiver performs an interference cancellation step by subtracting out this MAI from the received signal.

A. Interference Cancellation

The soft interference canceller at the first stage of the proposed receiver is similar to what is proposed in [5] for convolutionally coded CDMA and in [10] and [11] for space-time trellis-coded multiple access systems. Suppose that at the first stage of the receiver, we have available *a priori* log likelihood ratios (llrs) $\lambda_2[k,i]^p$ of all users' transmitted symbols. Note that subscript 2 and superscript p indicate that these *a priori* log likelihood ratios were in fact generated by the second stage of the receiver (i.e. the single-user channel decoders) at the previous iteration. In general, the log likelihood ratio $\Lambda[k,i]$, for $k = 1, \ldots, K$ and $i = 0, \ldots, B_2 - 1$, is defined as

$$\Lambda[k,i] = \log \frac{\mathrm{P}[c_k(i) = +1]}{\mathrm{P}[c_k(i) = -1]}. \tag{17}$$

Using the *a priori* log likelihood ratios $\lambda_2[k,i]^p$, the interference-cancelling first stage of the receiver computes soft estimates of the transmitted symbol vectors of all the users. In fact, from (17), we have that

$$\mathrm{P}[c_k(i) = j] = \begin{cases} \frac{\exp(\lambda_2[k,i]^p)}{1+\exp(\lambda_2[k,i]^p)}, & \text{if } j = +1 \\ \frac{1}{1+\exp(\lambda_2[k,i]^p)}, & \text{if } j = -1 \\ 0, & \text{otherwise} \end{cases}.$$

Thus, the soft estimates of the user transmit symbols are given by

$$\hat{c}_k(i) = E\{c_k(i)\} = \tanh\left(\frac{\lambda_2[k,i]^p}{2}\right) \tag{18}$$

for $k = 1, \ldots, K$ and $i = 0, \ldots, B_2 - 1$. These soft estimates $\hat{c}_k(i)$ can then be mapped to the transmit symbol estimates $\hat{b}_{k,n}(i)$ for $n = 1, \ldots, N_T$.

The interference cancelled signal corresponding to the kth user's symbol on antenna n is then obtained by subtracting out a soft estimate of MAI:

$$\mathbf{y}_{k,n}(i) = \mathbf{y}(i) - \mathcal{H}\hat{\mathbf{b}}_{k,n}(i) \tag{19}$$

where $\hat{\mathbf{b}}_{k,n}(i) = \hat{\mathbf{b}}(i) - \hat{b}_{k,n}(i)\mathbf{e}_{k,n}$, $\mathbf{e}_{k,n}$ is an $N_T K$-vector of all zeros except for the single unity element at the $((k-1)N_T + n)$th position, $\hat{b}_{k,n}(i)$ is the soft estimate of the kth user's symbol on the nth transmit antenna, and $\hat{\mathbf{b}}(i) = [\hat{\mathbf{b}}_1(i)^T, \ldots, \hat{\mathbf{b}}_K(i)^T]^T = [\hat{b}_{1,1}(i), \ldots, \hat{b}_{1,N_T}(i), \hat{b}_{2,1}(i) \ldots, \hat{b}_{K,N_T}(i)]^T$ is the KN_T-vector of soft estimates of all the user symbols on all the antennas. From (15) and (19), we have

$$\mathbf{y}_{k,n}(i) = \mathcal{H}\left(\mathbf{b}(i) - \hat{\mathbf{b}}_{k,n}(i)\right) + \mathbf{m}(i). \tag{20}$$

B. Linear MMSE Filtering

Next, similar to [5] and [12], an instantaneous MMSE filter is applied to the interference canceller output in order to further suppress the residual MAI and noise. Unlike the MMSE-filter employed in [5], this filter also suppresses the self-interference caused by the $N_T - 1$ other antennas of the same user and is a space-time filter, as in [12]. However, compared with [12], this space-time filter also combines the signal energy from different paths in order to maximize the output signal-to-interference-plus-noise ratio (SINR).

The linear MMSE filter for the kth user's symbol on the nth transmit antenna is chosen so that the MSE between the filter output $\gamma_{k,n}(i) = \boldsymbol{\theta}_{k,n}(i)^H \mathbf{y}_{k,n}(i)$ and $b_{k,n}(i)$ is minimized:

$$\boldsymbol{\theta}_{k,n}(i) = \arg\min_{\boldsymbol{\theta}} \mathrm{E}\left\{\left\|\boldsymbol{\theta}^H \mathbf{y}_{k,n}(i) - b_{k,n}(i)\right\|^2\right\}. \tag{21}$$

Solving the optimization problem (21), we can show that the required instantaneous MMSE filter is

$$\boldsymbol{\theta}_{k,n}(i) = (\mathcal{H}\mathbf{Q}_{k,n}\mathcal{H}^H + \mathbf{I}_{PN'})^{-1}\mathcal{H}\mathbf{e}_{k,n} \tag{22}$$

where the covariance matrix $\mathbf{Q}_{k,n} = \mathrm{Cov}[\mathbf{b}(i) - \hat{\mathbf{b}}_{k,n}(i)]$ is given by

$$\begin{aligned}\mathbf{Q}_{k,n} = \mathrm{diag}\Bigg\{&1 - \left|\hat{b}_{1,1}(i)\right|^2, \ldots, 1 - \left|\hat{b}_{1,N_T}(i)\right|^2, \ldots,\\ &1 - \left|\hat{b}_{1,n-1}(i)\right|^2, 1, 1 - \left|\hat{b}_{1,n+1}(i)\right|^2, \ldots, 1 - \left|\hat{b}_{K,N_T}(i)\right|^2\Bigg\}.\end{aligned}$$

It has been shown in [19] that the residual MAI plus the background receiver noise at the output of a linear MMSE multiuser detector can be well modeled as being Gaussian. It is reasonable to expect the same property to hold in this situation, despite the extra soft interference cancellation step. The Gaussian assumption for the interference-plus-noise at the MMSE filter outputs in similar types of soft interference cancellation receivers was previously employed in [5] and [10]–[12], and the results reported in those works suggest that it is indeed a reasonable approximation. Thus, we may model the MMSE filter output $\gamma_{k,n}(i)$ with the following Gaussian model:

$$\gamma_{k,n}(i) = \mu_{k,n}(i)b_{k,n}(i) + \eta_{k,n}(i) \tag{23}$$

where $\mu_{k,n}(i) = \mathrm{E}\{\gamma_{k,n}(i)b_{k,n}(i)^*\}$, and $\nu^2_{k,n}(i) = \mathrm{Var}\{\gamma_{k,n}(i)\} = \mu_{k,n}(i) - \mu^2_{k,n}(i)$. It can be shown that

$$\mu_{k,n}(i) = \mathbf{e}^H_{k,n}\mathcal{H}^H(\mathcal{H}\mathbf{Q}_{\mathbf{k},\mathbf{n}}\mathcal{H}^H + \mathbf{I})^H\mathcal{H}\mathbf{e}_{k,n}. \tag{24}$$

The model (23) is used to compute the soft outputs from the first stage of the receiver. In fact, it can easily be shown that the *a posteriori* llr corresponding to the kth user's signal on the nth transmit antenna $b_{k,n}(i)$ is as in (25), shown at the bottom of the page. From (25), we immediately see that the required soft output is the extrinsic information term $\lambda_1[k,n,i]$, and from (23) and (24), we may compute it as

$$\lambda_1[k,n,i] = 2\frac{\mathrm{Re}\left\{\gamma_{k,n}(i)\mu^*_{k,n}(i)\right\}}{\nu^2_{k,n}(i)}. \tag{26}$$

The set of soft outputs $\{\lambda_1[k,n,i]\}$ are next associated with the corresponding transmit symbols to generate the soft output $\{\lambda_1[k,i]\}$ of the first stage for $k = 1, \ldots, K$ and $i = 0, \ldots, B_2 - 1$. The set of outputs $\{\lambda_1[k,i]\}$ are next deinterleaved and passed on to the second stage of the receiver.

C. Channel Decoding

The second stage of the iterative receiver is identical to the second stage of the turbo receiver proposed in [5] for convolutionally coded CDMA. Specifically, it consists of a bank of K independent soft-in-soft-out (SISO) single-user channel decoders corresponding to the K users in the channel. The input to the kth user's individual channel decoder is the deinterleaved log likelihood ratio information $\lambda_1[k,j]$ for $j = 0, 1, \ldots, B_2 - 1$ from the first stage. Using these inputs and the trellis structure of the convolutional channel code, the kth user's SISO channel decoder updates the *a posteriori* log likelihood ratios of the output symbols $c_k(j)$ from the kth channel encoder, for $j = 0, \ldots, B_2 - 1$. The extrinsic portion of the updated log likelihood ratio at the output of the channel decoder is taken to be the soft output $\lambda_2[k,j]$ from the second stage, for $k = 1, \ldots, K$

$$\begin{aligned}\Lambda_1[k,n,i] &= \log\frac{\mathrm{P}\left[b_{k,n}(i) = +1|\mathbf{y}(i), \{\lambda_2[k',i]^p\}_{i=0}^{B_2-1} \text{ for } k' = 1, \ldots, K\right]}{\mathrm{P}\left[b_{k,n}(i) = -1|\mathbf{y}(i), \{\lambda_2[k,i]^p\}_{i=0}^{B_2-1} \text{ for } k' = 1, \ldots, K\right]}\\ &= \log\frac{\mathrm{P}\left[\gamma_{k,n}(i)|b_{k,n}(i) = +1\right]}{\mathrm{P}\left[\gamma_{k,n}(i)|b_{k,n}(i) = -1\right]} + \log\frac{\mathrm{P}\left[b_{k,n}(i) = +1|\{\lambda_2[k',i]^p\}_{i=0}^{B_2-1} \text{ for } k' = 1, \ldots, K\right]}{\mathrm{P}\left[b_{k,n}(i) = -1|\{\lambda_2[k',i]^p\}_{i=0}^{B_2-1} \text{ for } k' = 1, \ldots, K\right]}\\ &= \lambda_1[k,n,i] + \lambda_2[k,n,i]^p.\end{aligned} \tag{25}$$

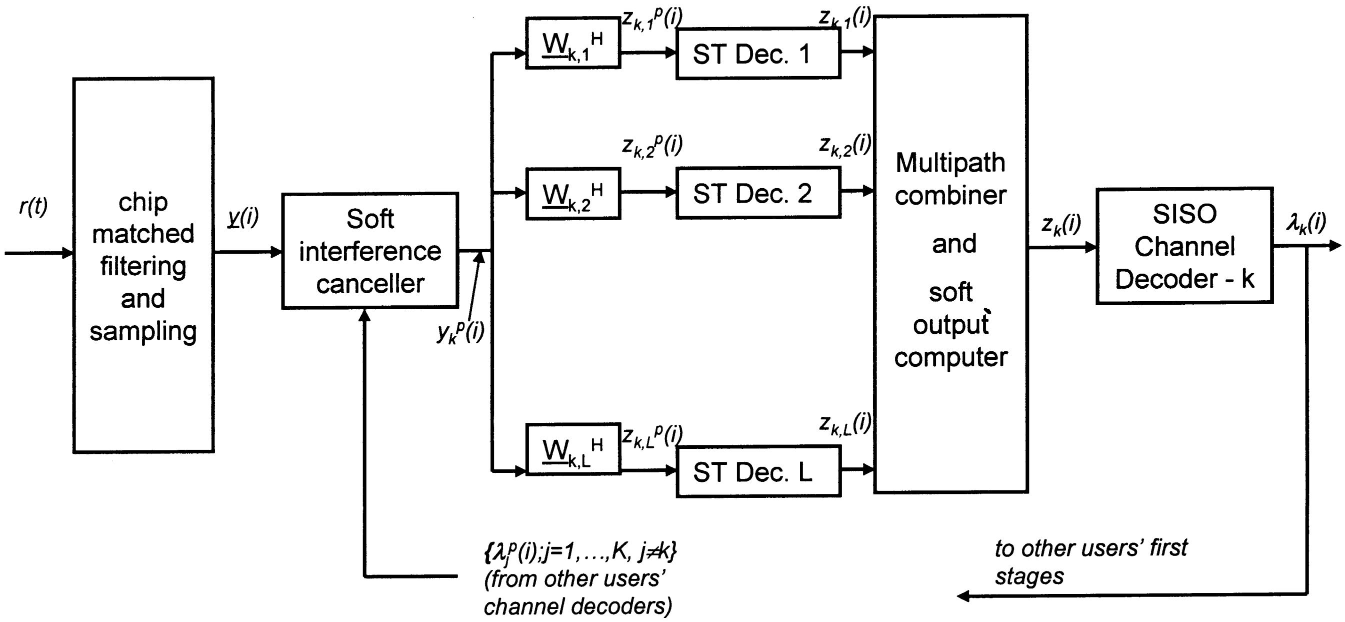


Fig. 1. Decorrelating RAKE-based iterative space-time receiver.

and $j = 0, 1, \ldots, B_2 - 1$. These soft outputs are next interleaved and fed back into the first stage of the turbo receiver to be used as the *a priori* log likelihood ratio $\lambda_2[k, i]^p$, for $k = 1, \ldots, K$ and $i = 0, 1, \ldots, B_2 - 1$, in the next iteration. The operation of each of the individual SISO channel decoders and the computation of the extrinsic log likelihood ratios are the same as that given in [5], and thus, we do not repeat them here.

This iterative process continues until a prespecified number of iterations are performed or until an acceptable level of performance is achieved. In the final iteration, each channel decoder at the second stage of the receiver outputs hard decisions on the information bits of its corresponding user.

As we mentioned earlier, one of the shortcomings of this iterative algorithm is its complexity. A direct implementation of the algorithm would be dominated by the complexity of the $PN' \times PN'$ matrix inversion required in (22). In the following section, we introduce a modification for the above receiver that will reduce the required complexity to the complexity of inverting a set of P matrices of size $N' \times N'$.

IV. RAKE-BASED TURBO RECEIVER FOR SPACE-TIME BLOCK-CODED CDMA

In this section, we develop the structure of the modified two-stage receiver for multipath CDMA systems. This may be considered as a combination of the DRAKE receiver developed for multipath channels in [18] and the iterative interference cancelling MMSE receiver for convolutional coded CDMA given in [5].

The modified receiver achieves complexity reduction by performing interference suppression, space-time decoding, and multipath combining separately in the first stage, as shown in Fig. 1. As before, we assume that *a priori* information about the code symbols $c_{k'}(i)$ for $k' = 1, \ldots, K$ and $i = 0, \ldots, B_2 - 1$ is available at the first stage of the receiver and that we are interested in detecting the particular user k. The receiver uses *a priori* information to make soft estimates of all the users' symbols corresponding to the received frame, and these soft estimates are used to reconstruct the interference caused by all the other users to user k. The interference cancelling step then subtracts out this MAI from the received signal.

The next step of the receiver consists of K banks of linear filters, each consisting of L branch filters each matched to a different multipath component of a particular user. The filter coefficients for each branch filter, in the filterbank corresponding to the kth user are chosen to minimize the error between the interference cancelled received signal and the fading modulated transmit symbol vector corresponding to the particular multipath channel of user k. We design these filters so that the structure of the space-time code embedded in the received signal is preserved at the output of each branch filter.

Next, the receiver performs space-time decoding on each branch filter output, and the final step in the first stage consists of maximal ratio combining of the space-time decoded outputs from the branch filters. The diversity combined outputs are used to compute *a posteriori* information about the channel symbols. These are deinterleaved and passed on to the second stage of the receiver as soft inputs.

Note that the second stage of the modified receiver stays exactly the same as before consisting of a bank of K SISO single-user channel decoders. The updated soft information from the second stage is interleaved and fed back to the first stage of the receiver to be used as the *a priori* information in the next iteration. As before, the iterative process continues until a prespecified number of iterations are performed, and in the final iteration, each channel decoder outputs hard decisions on the information bits of its corresponding user.

The details of the first stage of the modified receiver are given in the following sections.

A. Interference Cancellation

Interference cancellation is performed separately for each $\mathbf{y}^p(i)$ for $p = 1, \ldots, P$. The soft estimates of the user transmitted symbols are formed as before, and these are used to subtract out the MAI from other users. The interference cancelled signal corresponding to the kth user is obtained, for $p = 1, \ldots, P$, as

$$\mathbf{y}_k^p(i) = \mathbf{y}^p(i) - \sum_{k' \neq k}^{K} \sqrt{\frac{\rho_{k'}}{N_T}} \mathbf{S}_{k'} \mathbf{H}_{p,k'} \hat{\mathbf{b}}_{k'}(i) \tag{27}$$

where $\hat{\mathbf{b}}_{k'}(i) = [\hat{c}_{k'}(i), \hat{c}_{k'}(i+1)]^T$. From (11) and (27), we have

$$\mathbf{y}_k^p(i) = \sqrt{\frac{\rho_k}{N_T}} \mathbf{S}_k \mathbf{H}_{p,k} \mathbf{b}_k(i) + \mathbf{v}_k^p(i) \tag{28}$$

where, in (28), the noise term $\mathbf{v}_k^p(i)$ represents the total interference-plus-noise at the output of the soft interference canceller:

$$\mathbf{v}_k^p(i) = \sum_{k' \neq k}^{K} \sqrt{\frac{\rho_{k'}}{N_T}} \mathbf{S}_{k'} \mathbf{H}_{p,k'} \tilde{\mathbf{b}}_{k'}(i) + \mathbf{n}_p(i) \tag{29}$$

where we have introduced the notation $\tilde{\mathbf{b}}_{k'}(i) = \mathbf{b}_{k'}(i) - \hat{\mathbf{b}}_{k'}(i)$.

B. Linear MMSE Filtering

The next stage of the proposed receiver for user k consists of a bank of L linear MMSE filters with weights $\mathbf{w}_{p,k,l}$ for $l = 1, \ldots, L$ and $p = 1, \ldots, P$, i.e. the output of the lth branch filter corresponding to the kth user is given by

$$z_{k,l}^p(i) = \mathbf{w}_{p,k,l}^H \mathbf{y}_k^p(i). \tag{30}$$

The weights $\mathbf{w}_{k,l}$ are chosen so that the MSE between the filter output and the fading-modulated space-time coded transmit symbols of the user k along path l is minimized, i.e.,

$$\mathbf{w}_{p,k,l} = \arg\min_{\mathbf{w}} \mathrm{E}\left\{ \left\| z_{k,l}^p(i) - \boldsymbol{\alpha}_{p,k,l}^H \mathbf{b}_k(i) \right\|^2 \right\} \tag{31}$$

where, as before, we have denoted the lth row of the matrix $\mathbf{H}_{p,k}$ by $\boldsymbol{\alpha}_{p,k,l}^H$. In order to preserve the structure of the space-time block code at the end of each branch filter, we also impose the constraint

$$\mathbf{S}_k^H \mathbf{w}_{p,k,l} = \mathbf{e}_l \tag{32}$$

where $\mathbf{e}_l$ is a unit vector of length L, i.e., an L-vector having all zeros except a single one at the lth position.

Now, define, for $p = 1, 2, \ldots, P$

$$\mathbf{R}_{y,p,k} = \mathrm{E}\left\{ \mathbf{y}_k^p(i) \mathbf{y}_k^p(i)^H \right\} \tag{33}$$

and

$$\mathbf{P}_{p,k} = \mathrm{E}\left\{ \mathbf{y}_k^p(i) \mathbf{b}_k(i)^H \right\}. \tag{34}$$

Combining (31) and (32) and using the above notation, the branch filter design problem reduces to the minimization of the following unconstrained cost function:

$$\begin{aligned} J_{p,k,l}(\mathbf{w}) = &\mathbf{w}^H \mathbf{R}_{y,p,k} \mathbf{w} - \mathbf{w}^H \mathbf{P}_{p,k} \boldsymbol{\alpha}_{p,k,l} - \boldsymbol{\alpha}_{p,k,l}^H \mathbf{P}_{p,k}^H \mathbf{w} \\ &+ \boldsymbol{\alpha}_{p,k,l}^H \boldsymbol{\alpha}_{p,k,l}^H + \left(\mathbf{w}^H \mathbf{S}_k - \mathbf{e}_l^H \right) \boldsymbol{\mu} \end{aligned} \tag{35}$$

where $\boldsymbol{\mu}$ is a vector of Lagrange multipliers. Here, we have assumed that due to the interleaving operation performed after the channel encoding, the symbol stream $c_{k'}(i)$ into the space-time encoder can be considered to be a sequence of independent, identically distributed (iid) BPSK symbols, and thus, $\mathrm{E}\{\mathbf{b}_{k'}(i) \mathbf{b}_{k'}(i)^H\} = \mathbf{I}_{N_T}$, for $k' = 1, \ldots, K$. Combining this with the natural assumption that the data streams of different users are independent, we can show that $\mathbf{P}_{p,k} = \sqrt{(\rho_k / N_T)} \mathbf{S}_k \mathbf{H}_{p,k}$. Setting the gradient of the cost function $J_{p,k,l}(\mathbf{w})$ to zero and using the constraint (32), we obtain the required branch filter for the kth user's lth branch to be

$$\mathbf{w}_{p,k,l} = \mathbf{R}_{y,p,k}^{-1} \mathbf{S}_k \left(\mathbf{S}_k^H \mathbf{R}_{y,p,k}^{-1} \mathbf{S}_k \right)^{-1} \mathbf{e}_l. \tag{36}$$

Thus, if we denote by $\mathbf{W}_{p,k}$ the $N' \times L$ matrix whose lth column is the lth branch filter $\mathbf{w}_{p,k,l}$ of user k, then

$$\mathbf{W}_{p,k} = \mathbf{R}_{y,p,k}^{-1} \mathbf{S}_k \left(\mathbf{S}_k^H \mathbf{R}_{y,p,k}^{-1} \mathbf{S}_k \right)^{-1} \tag{37}$$

and the output from the L branch filters, corresponding to the desired user k, is given by the vector $\mathbf{z}_k^p(i) = \mathbf{W}_{p,k}^H \mathbf{y}_k^p(i)$.

Let us introduce the following matrices:

$$\begin{aligned} \mathbf{V}_{k'}(i) &= E\left\{ \left(\mathbf{b}_{k'}(i) - \hat{\mathbf{b}}_{k'}(i) \right) \left(\mathbf{b}_{k'}(i) - \hat{\mathbf{b}}_{k'}(i) \right)^H \right\} \\ &= \mathbf{I}_2 - \hat{\mathbf{V}}_{k'}(i) \end{aligned} \tag{38}$$

and

$$\mathbf{R}_{y,p} = \sum_{k'=1}^{K} \frac{\rho_{k'}}{N_T} \mathbf{S}_{k'} \mathbf{H}_{p,k'} \mathbf{V}_{k'}(i) \mathbf{H}_{p,k'}^H \mathbf{S}_{k'}^H + \mathbf{I}_{N'} \tag{39}$$

where $\hat{\mathbf{V}}_k(i) = \mathrm{diag}[\hat{c}_k(i)^2, \hat{c}_k(i+1)^2]$, with $\hat{c}_k(i)$ given by (18).

Then, from (28), (33), and (39), it is easily seen that

$$\mathbf{R}_{y,p,k} = \mathbf{R}_{y,p} + \frac{\rho_k}{N_T} \mathbf{S}_k \mathbf{H}_{p,k} \hat{\mathbf{V}}_k(i) \mathbf{H}_{p,k}^H \mathbf{S}_k^H. \tag{40}$$

Substituting (40) into (37) and then applying the matrix inversion lemma [20], we can show that

$$\mathbf{W}_{p,k} = \mathbf{R}_{y,p}^{-1} \mathbf{S}_k \left(\mathbf{S}_k^H \mathbf{R}_{y,p}^{-1} \mathbf{S}_k \right)^{-1}. \tag{41}$$

Note that the computational complexity of this filter is dominated by the inversion of the $N' \times N'$ matrix $\mathbf{R}_{y,p}$, which can be a considerable reduction compared with the inversion of $PN' \times PN'$ matrix, as required in the algorithm described in previous section. In addition, observe from (41) that this inversion needs to be performed only once for all users.

C. Branch-Wise Space-Time Decoding

It is easily seen from (28) and (36) that the output of the lth branch filter $z_{k,l}^p(i)$, for $p = 1, \ldots, P$, is of the form

$$z_{k,l}^p(i) = \sqrt{\frac{\rho_k}{N_T}} \boldsymbol{\alpha}_{p,k,l}^H \mathbf{b}_k(i) + \eta_{k,l}^p(i) \tag{42}$$

where we have denoted the residual interference-plus-noise at the output of the lth branch filter of user k by

$$\eta_{k,l}^p(i) = \mathbf{w}_{p,k,l}^H \mathbf{v}_k^p(i). \tag{43}$$

Continuing with our discussion of the decoding of user k in a 2×2 STBC system, the space-time decoding is performed on the L branch filter outputs $z_{k,l}^p(i)$ for $l = 1, \ldots, L$ and $p = 1, \ldots, P$. Observing from (42) that the space-time code property is preserved at the output of each branch filter, we may form the vector

$$\mathbf{z}_{k,l}(i) = \begin{bmatrix} z_{k,l}^1(i) \\ z_{k,l}^2(i) \end{bmatrix} = \sqrt{\frac{\rho_k}{N_T}} \mathbf{H}_{k,l} \mathbf{b}_k(i) + \boldsymbol{\eta}_{k,l}(i)$$

where we have defined the 2×2 matrix ($P \times N_T$, in general) $\mathbf{H}_{k,l} = [\boldsymbol{\alpha}_{1,k,l}^H; \tilde{\boldsymbol{\alpha}}_{2,k,l}^H]$ and the 2-vector (P-vector, in general) of noise $\boldsymbol{\eta}_{k,l}(i) = [\eta_{k,l}^1(i)\eta_{k,l}^2(i)]^T$. Note that the matrix $\mathbf{H}_{k,l}$ satisfies the property

$$\mathbf{H}_{k,l}^H \mathbf{H}_{k,l} = |\boldsymbol{\alpha}_{1,k,l}|^2 \mathbf{I}_2 = \left(|h_{k,1,l}|^2 + |h_{k,2,l}|^2\right) \mathbf{I}_2. \quad (44)$$

Thus, we still have the advantage of simple linear decoding of the space-time block code along each branch filter as

$$\tilde{\mathbf{z}}_{k,l}(i) = \mathbf{H}_{k,l}^H \mathbf{z}_{k,l}(i) = \sqrt{\frac{\rho_k}{N_T}} |\boldsymbol{\alpha}_{1,k,l}|^2 \mathbf{b}_k(i) + \tilde{\boldsymbol{\eta}}_{k,l}(i)$$

where we have let $\tilde{\boldsymbol{\eta}}_{k,l}(i) = \mathbf{H}_{k,l}^H \boldsymbol{\eta}_{k,l}(i)$.

Note that $\eta_{k,l}^1(i)$ and $\eta_{k,l}^2(i)$ are no longer independent due to the MAI component included in them, resulting in correlated components in the noise vector $\tilde{\boldsymbol{\eta}}_{k,l}(i)$. Thus, it is no longer optimal to decode the components of $\tilde{\mathbf{z}}_{k,l}(i)$ separately, as is done in a single-user channel [6]. However, in order to keep the receiver complexity to a minimum, we will ignore these correlations in the noise components and decode $\tilde{\mathbf{z}}_{k,l}(i)$ component-wise, as in a single-user channel, i.e. we will assume that the space-time decoder output decouples the effect of $c_k(i)$ and $c_k(i+1)$. This assumption will be correct if the interference cancellation at the previous stage was perfect, thus completely removing the MAI component in $\mathbf{v}_k(i)$. This, in turn will be achieved if the *a priori* information into the first stage $\lambda_2[k,i]^p$ is perfect. In our proposed receiver, we would expect this to happen at least after a few iterations, resulting in no significant performance loss due to our approximation, yet avoiding unnecessary receiver complexity.

D. Multipath Combining

Finally, the receiver combines the space-time decoder outputs from all branches in order to form the final decision variable of the first stage of the receiver. Due to the above assumption of decoupled space-time decoder outputs, the decision variable for $c_k(i)$ must take into account only the L-vector of branch outputs defined as

$$\tilde{\mathbf{z}}_k^1(i) = \begin{bmatrix} \tilde{z}_{k,1,1}(i) \\ \tilde{z}_{k,2,1}(i) \\ \vdots \\ \tilde{z}_{k,L,1}(i) \end{bmatrix} = \sqrt{\frac{\rho_k}{N_T}} \mathbf{g}_k c_k(i) + \mathbf{u}_{k,1}(i)$$

where $\tilde{z}_{k,l,m}(i)$ and $\tilde{\eta}_{k,l,m}(i)$ denote the mth component of the vectors $\tilde{\mathbf{z}}_{k,l}(i)$ and $\tilde{\boldsymbol{\eta}}_{k,l}(i)$, respectively. Similarly, the decision variable for $c_k(i+1)$ needs only to be based on the vector of outputs

$$\tilde{\mathbf{z}}_k^2(i) = \begin{bmatrix} \tilde{z}_{k,1,2}(i) \\ \tilde{z}_{k,2,2}(i) \\ \vdots \\ \tilde{z}_{k,L,2}(i) \end{bmatrix} = \sqrt{\frac{\rho_k}{N_T}} \mathbf{g}_k c_k(i+1) + \mathbf{u}_{k,2}(i)$$

where we have introduced the notation $\mathbf{g}_k = [|\boldsymbol{\alpha}_{1,k,1}|^2, \ldots, |\boldsymbol{\alpha}_{1,k,L}|^2]^T$ and $\mathbf{u}_{k,n}(i) = [\tilde{\eta}_{k,1,n}(i), \ldots, \tilde{\eta}_{k,L,n}(i)]^T$ for $n = 1, 2$. It should be noted that the noise vectors $\mathbf{u}_{k,n}(i)$, for $n = 1$, 2, are vectors of correlated noise components.

These space-time decoded output vectors $\tilde{\mathbf{z}}_k^1(i)$ and $\tilde{\mathbf{z}}_k^2(i)$ are next multipath combined to form the final decision variables corresponding to $c_k(i)$ and $c_k(i+1)$, respectively. Using maximal ratio combining of the multipath components, the diversity combined outputs become

$$\hat{z}_k(i) = \mathbf{g}_k^H \tilde{\mathbf{z}}_k^1(i) \quad (45)$$

and

$$\hat{z}_k(i+1) = \mathbf{g}_k^H \tilde{\mathbf{z}}_k^2(i). \quad (46)$$

E. Soft Output Computation at the End of the First Stage

From (43), we have that

$$E\left\{\eta_{k,l}^p(i)\right\} = \mathbf{w}_{p,k,l}^H E\left\{\mathbf{v}_k^p(i)\right\} \quad (47)$$

and

$$\mathrm{Var}\left(\eta_{k,l}^p(i)\right) = \mathbf{w}_{p,k,l}^H \mathbf{R}_{v,p,k} \mathbf{w}_{p,k,l} \quad (48)$$

where we have introduced the notation

$$\mathbf{R}_{v,p,k} = E\left\{\mathbf{v}_k^p(i) \mathbf{v}_k^p(i)^H\right\}. \quad (49)$$

From the definition of $\mathbf{v}_k^p(i)$ in (29), it is easy to show that $E\{\mathbf{v}_k^p(i)\} = \mathbf{0}$. Hence, from (47), we have that

$$E\left\{\eta_{k,l}(i)\right\} = \mathbf{0}. \quad (50)$$

In addition, we can show that

$$\mathbf{R}_{v,p,k} = \mathbf{R}_{y,p} - \frac{\rho_k}{N_T} \mathbf{S}_k \mathbf{H}_{p,k} \mathbf{V}_k(i) \mathbf{H}_{p,k}^H \mathbf{S}_k^H \quad (51)$$

where $\mathbf{V}_{k'}(i)$ is defined in (38).

Substituting (36) into (48) and then using (40) and (51), we get the variance of the interference suppressed output to be

$$\mathrm{Var}\left(\eta_{k,l}^p(i)\right) = \mathbf{e}_l^H \left[\left(\mathbf{S}_k^H \mathbf{R}_{y,p}^{-1} \mathbf{S}_k\right)^{-1} - \mathbf{H}_{p,k} \mathbf{V}_k(i) \mathbf{H}_{p,k}^H\right] \mathbf{e}_l. \quad (52)$$

Next, we again make the customary assumption that the interference-plus-noise term $\eta_{k,l}^p(i)$ at each MMSE filter output is Gaussian. Combining this assumption with the results in (48) and (50), we conclude that $\eta_{k,l}^p(i) \sim \mathcal{N}(0, \mathrm{Var}(\eta_{k,l}^p(i)))$, where $\mathrm{Var}(\eta_{k,l}^p(i))$ is given by (52). Due to our simplifying assumption that $\eta_{k,l}^1(i)$ and $\eta_{k,l}^2(i)$ are independent, it then follows that $E\{\boldsymbol{\eta}_{k,l}(i)\boldsymbol{\eta}_{k,l}^H(i)\} = \mathrm{diag}[\mathrm{Var}(\eta_{k,l}^1(i)), \mathrm{Var}(\eta_{k,l}^2(i))]$. The noise at the space-time decoded branch outputs is then also zero-mean Gaussian with the covariance matrix $E\{\tilde{\boldsymbol{\eta}}_{k,l}(i)\tilde{\boldsymbol{\eta}}_{k,l}^H(i)\} \approx |\boldsymbol{\alpha}_{1,k,l}|^2 \mathrm{diag}[\mathrm{Var}(\eta_{k,l}^1(i)), \mathrm{Var}(\eta_{k,l}^2(i))]$, where the approximation is due to the ignoring of the effects of residual MAI.

Assuming that the noise on each branch filter output is independent (which, strictly speaking, is not true), this then allows us to conclude that the noise vectors $\mathbf{u}_{k,n}(i)$, for $n = 1$, 2, are also zero-mean Gaussian, having $L \times L$ diagonal covariance matrix of the form $\mathbf{R}_{u,k,n} = E\{\mathbf{u}_{k,n}(i)\mathbf{u}_{k,n}(i)^H\} = \mathrm{diag}[|\boldsymbol{\alpha}_{1,k,1}|^2\mathrm{Var}\{\eta^n_{k,1}(i)\}, \ldots, |\boldsymbol{\alpha}_{1,k,L}|^2\mathrm{Var}\{\eta^n_{k,L}(i)\}]$, where $\mathrm{Var}\{\eta^n_{k,l}(i)\}$ is given by (52) with p replaced by n.

Then, from (45), we finally have that the noise term present in the final multipath combined output $\hat{z}_k(i)$ for the kth user is still zero-mean Gaussian, having a variance ν_k^2 equal to

$$\nu_{k,n}^2 = \sum_{l=1}^{L} |\boldsymbol{\alpha}_{1,k,l}|^6 \mathrm{Var}\left\{\eta^n_{k,l}(i)\right\}. \tag{53}$$

As usual, we intend to use the extrinsic part of the *a posteriori* log likelihood ratio of the BPSK symbols as our soft output. As we did earlier in (25), the *a posteriori* log likelihood ratio of the symbol $c_k(i)$ at the end of the first stage can again be separated into two parts as $\Lambda_1[k,i] = \lambda_1[k,i] + \lambda_2[k,i]^p$, where $\lambda_1[k,i]$ is the extrinsic information, which we take as the required soft output. From (45) and (53), $\lambda_1[k,i]$ can be computed as

$$\lambda_1\left[k, i+n-1\right] = 2\sqrt{\frac{\rho_k}{N_T}}\left(\sum_{l=1}^{L}|\boldsymbol{\alpha}_{1,k,l}|^4\right) \times \frac{\mathrm{Re}\left\{\hat{z}_k(i+n-1)\right\}}{\nu_{k,n}^2}, \quad \text{for} \quad n = 1, 2. \tag{54}$$

The set of soft outputs $\{\lambda_1[k', i]\}$ for $k' = 1, \ldots, K$ and $i = 0, \ldots, B_2 - 1$ are next deinterleaved and passed on to the second stage of the receiver.

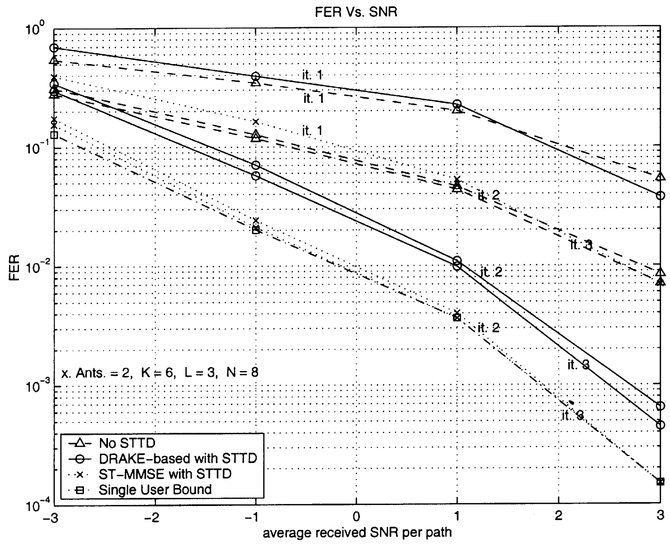
V. Simulation Results

In this section, we simulate a symbol-synchronous multipath CDMA system. All users employ the same $N_T = 2$ space-time block code due to Alamouti [6]. Since we are not concerned with receiver diversity, we set the number of receiver antennas to $N_R = 1$ in all simulations. The fading is assumed to be quasistatic Rayleigh fading, where fading coefficients were assumed to be constant for a frame of 128 information bits and then change independently to a new value.

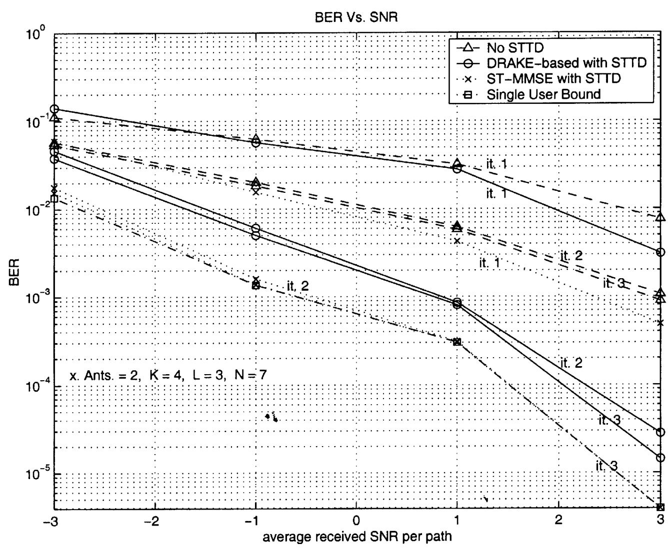
The channel code employed by each user is a constraint length $\nu = 5$, rate-1/2, convolutional code with octal generators (46,72) [21].

First, in Fig. 2, we plot the performance of a system with $K = 6$ users, all employing random spreading codes and having equal average transmit power. The processing gain of this system is assumed to be $N = 8$, and there are $L = 3$ paths per user between each transmitter and receiver antenna pair. Fig. 2(a) and (b) correspond to the FER and BER performance of the proposed receivers, respectively, averaged over different sets of random codes.

Included in the same plots is the performance of a similar system without using space-time coding but still employing the same multipath combining receiver, for the sake of comparison. It is clear from Fig. 2 that in the presence of multipath, the proposed iterative schemes are superior to a system without space-time coding, in terms of both BER and FER. Interestingly, however, we observe that the performance of the DRAKE-based



(a)



(b)

Fig. 2. Performance of the iterative space-time receivers versus SNR (in decibels). $N_T = 2, N_R = 1, K = 6, L = 3$, and $N = 8$. (a) FER. (b) BER.

iterative receiver and the system without STTD after only a single iteration is comparable, and in fact, the system without space-time coding is sometimes slightly better. However, from next iteration onwards, the new scheme outperforms the single antenna system by a considerable margin. The slightly better performance of the single antenna system without iteration may be attributed to the fact that in the presence of no *a priori* information about the interfering symbols, the multiantenna system results in more interference due to more antennas in the presence of multipath. However, the soft information from the first stage of the space-time coded system is much more reliable than the single antenna system due to the diversity advantage. This results in better interference cancellation, which leads to improved performance in the space-time coded system as we perform more iterations. This observation justifies the use of the proposed receiver scheme in space-time block-coded CDMA systems operating in frequency-selective channels.

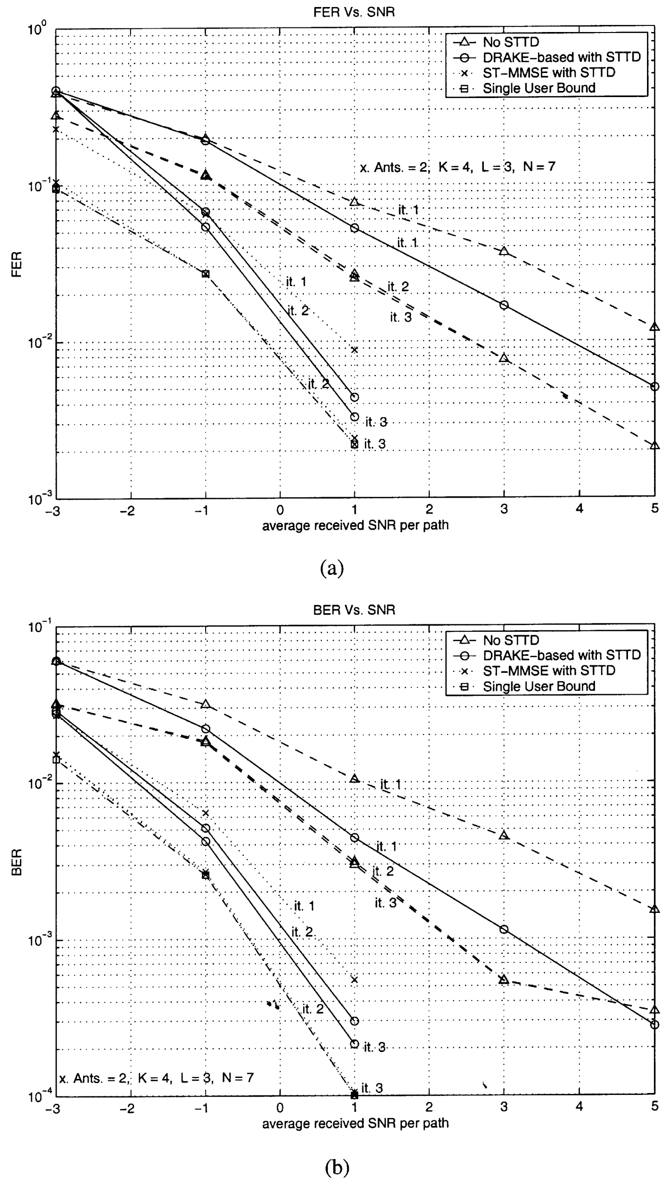


Fig. 3. Performance of the iterative space-time receivers versus SNR (in decibels). $N_T = 2, N_R = 1, K = 4, L = 3$, and $N = 7$. (a) FER. (b) BER.

From Fig. 2, it is also clear that as the SNR per path becomes larger, the performance improvement offered by the proposed receivers over the single antenna system becomes also more pronounced. More importantly, with only a few iterations, the performance of the joint space-time MMSE-based scheme is very close to a single-user system.

On the other hand, comparing the performance of the joint space-time MMSE-based receiver of Section III with the DRAKE-based iterative receiver, we observe that there is a performance penalty due to the reduced complexity. However, the performance loss due to separating the interference suppression and diversity combining seems to be small for error rates of practical interest.

In practice, the spreading codes are usually chosen so that they have special auto- and cross-correlation properties. Gold codes [22] are well known for their low cross-correlation properties. In Fig. 3(a) and (b), we have shown the FER and BER performance of the proposed receivers for a four-user system, where each user is assigned a distinct Gold code of length $N = 7$. The specific codes used are given in Table I [5]. Again, we assume that there are $L = 3$ paths per user between each transmitter and receiver antenna pair. Fig. 3 corresponds to the performance of user 1 in the above system. Again, from these plots, we observe the performance advantage offered by the proposed multiple antenna communications system for a multiple-access channel. As before, the joint space-time MMSE-based receiver achieves almost near single-user performance after only a few iterations, and the DRAKE-based low complexity receiver is also not far from that. As can be seen from Fig. 3, both these schemes also offer a significant gain over a single transmit antenna system. For example, at 10^{-2} FER and 10^{-3} BER, there is more than 2-dB gain achieved by the DRAKE-based scheme over a system without transmit antenna diversity. Moreover, the performance loss against the joint space-time MMSE-based receiver of Section III is only about 0.5 dB at this performance level.

TABLE I
GOLD CODES USED TO PRODUCE FIG. 3

User	Code
1	0 1 0 1 1 1 0
2	1 0 0 1 1 1 0
3	1 0 0 0 0 1 0
4	1 1 0 0 0 0 0

VI. CONCLUSIONS

By generalizing the turbo receiver proposed in [12] for an SDMA system, we have proposed iterative uplink receivers for space-time block coded CDMA systems in multipath channels. The iterative receivers consist of a first stage that performs the interference suppression, multipath combining, and space-time coding followed by a second stage of channel decoding. In order to reduce the complexity of the exact MMSE-based interference cancelling receiver, we have also proposed a modified scheme that can be of use in large transmit antenna systems. The modified scheme performs the interference suppression, space-time decoding, and multipath combining in separate stages. Specifically, after the interference suppression stage, the space-time decoding is performed along each resolvable multipath component, and then, maximal ratio combine the set of space-time decoded outputs. By exchanging the soft information between the first and second stages, the receiver performance is improved with iterations.

Simulation results show that although, in some cases, a noniterative space-time coded system may have inferior performance compared with a system without space-time coding, the proposed iterative receivers significantly outperform systems without space-time coding, even with two iterations. We have also provided an explanation as to why a noniterative receiver may have inferior performance in a multipath environment compared with the performance of a single transmit antenna system. It is also observed that the performance loss due to the modified receiver scheme is very small for error rates of practical interest.

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