

Signal Design for Non-coherent PPM Modulation with Applications to UWB Communications

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Abstract—By deriving the probability of error of a binary pulse-position modulated (PPM), ultra wide-band (UWB) impulse-radio system under non-coherent detection, it is shown that the optimal value for time shift parameter δ for PPM is fundamentally different from that for coherent reception. In particular, the optimal choice of δ is one that results in zero correlation between the two waveforms used in PPM modulation. By examining the performance sensitivity around possible optimal δ solutions for a commonly used UWB mono-pulse, we also show that it is still possible to employ $\delta < T_w$, where T_w is the UWB mono-pulse width, yet achieve optimal and robust performance.

Index Terms—Marcum's Q-function, non-coherent detection, signal design, UWB, ultra wide-band communication.

I. INTRODUCTION

THIS letter addresses the signal design problem for binary pulse-position modulation (PPM) with non-coherent detection. Due to wide use of PPM in ultra wide-band (UWB) communication our results directly leads to the signal design for non-coherent UWB impulse-radio (IR) systems in flat fading. The results in [1] showed that in additive white Gaussian noise (AWGN) with coherent reception, the optimal (in the sense of minimum probability of error) performance is achieved by choosing the modulation time shift parameter δ to minimize the correlation $\rho_w(\delta)$ between the received UWB mono-pulse $w(t)$ and its shifted version $w(t-\delta)$. In particular, the optimal choice is not to use orthogonal signalling (i.e. $\delta \neq T_w$ where T_w is the width of the mono-pulse) waveforms for PPM modulation. The observation that $\delta \neq T_w$ is also important since it allows the use of very small chip period T_c 's in a time-hopping UWB multiple-access system so that more users can be supported.

In this letter, however, we show that in the case of non-coherent UWB the orthogonal signalling is the optimal choice for PPM. Moreover, this is also true for narrow-band (NB) systems. In both cases, we derive exact probability of error expressions for non-coherent, binary PPM modulation. For a commonly used UWB mono-pulse we investigate possible modulation time shift parameter δ values that result in the optimal performance. Our results show that although there are very small (compared to mono-pulse width T_w) δ values that provide the optimal performance, these operating points

can be extremely sensitive to even a small timing mismatch. However, while a safer choice is to use $\delta > T_w$, we also show that there still exist possible optimal choices such that $\delta < T_w$, that can provide robust performance in the presence of timing mismatches.

The remainder of this letter is organized as follows: In Section II we present our PPM-based communication system model. Next, in Section III we present the minimum probability of error, non-coherent detector for a PPM modulated system, derive a closed form expression for the minimum probability of error and obtain the optimal signal design in the case of non-coherent detection. Section IV uses a commonly employed UWB mono-pulse to demonstrate the effect of optimal and sub-optimal modulation time shift parameter choices on the performance of a UWB system and the sensitivity of the performance to timing mismatches. Finally, we conclude in Section V by summarizing our results.

II. SYSTEM MODEL AND DESCRIPTION

We consider a single-user, binary, PPM modulated communication system in a flat fading channel. The received continuous-time signal can be written as

$$r(t) = \sum_i \frac{A(i)}{\sqrt{N_s}} \sum_{j=0}^{N_s-1} w(t - iT_s - jT_f - \delta d(i)) + n(t) \quad (1)$$

where $w(t)$, T_s , N_s , T_f , $A(i)$ and δ are the received UWB mono-pulse (normalized to have unit energy), symbol time, number of mono-pulses per data symbol, pulse repetition period, the fading coefficient at symbol time i and the time-shift parameter for pulse position data modulation, respectively. For analytical reasons, we assume real Gaussian fading coefficients so that $A(i) \sim \mathcal{N}(0, E_b)$ where E_b is the average received energy per bit (while some measurement campaigns have shown fading in UWB systems should be modeled either as log-normal or Nakagami- m distributed [2], the above model of Gaussian fading coefficients has also commonly been used in recent literature [3], [4]). These fading coefficients are independent with respect to time index i . In (1), $n(t)$ is the zero-mean, white Gaussian noise with variance σ^2 and $d(i) \in \{0, 1\}$ is the i -th information bit.

Assuming no inter-symbol interference, the detection of the i -th information bit can be performed based only on the received signal $\{r(t) : iT_s \leq t \leq (i+1)T_s\}$. Without loss of any generality, below we consider the detection problem for $i=0$ case and drop the index i . Note that, if we were to define a transmit waveform $c(t)$ as $c(t) = \frac{1}{\sqrt{N_s}} \sum_{j=0}^{N_s-1} w(t - jT_f)$ then the binary PPM data modulation corresponds to

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transmitting $c(t)$ when $d = 0$ and transmitting $c(t - \delta)$ when $d = 1$ (here we do not consider random time hopping since it does not effect the results obtained for a single-user channel).

Let us define the vector $\mathbf{c}(t)$ as $\mathbf{c}(t) = [c(t), c(t - \delta)]^T$. Then, it is easy to see that a sufficient statistic for detecting information bit d based on the signal $\{r(t) : 0 \leq t \leq T_s\}$ is given by $\mathbf{y} = [y_1, y_2]^T = \int_0^{T_s} r(t)\mathbf{c}(t)dt$.

III. MINIMUM PROBABILITY OF ERROR NON-COHERENT DETECTION FOR PPM AND OPTIMAL SIGNAL DESIGN

We may formulate the non-coherent detection problem based on the decision statistic \mathbf{y} as a binary hypothesis testing problem between H_0 and H_1 where H_0 corresponds to $d = 0$ and H_1 corresponds to $d = 1$. The observations under the two hypotheses can then be written as

$$H_0 : \mathbf{y} = \begin{bmatrix} A \\ A\rho_w \end{bmatrix} + \mathbf{n} \quad \text{and} \quad H_1 : \mathbf{y} = \begin{bmatrix} A\rho_w \\ A \end{bmatrix} + \mathbf{n}. \quad (2)$$

where we have defined $\rho_w = \int_{-\infty}^{\infty} w(t)w(t - \delta)dt$ and the two element noise vector $\mathbf{n} \sim \mathcal{N}(\mathbf{0}, \sigma^2\mathbf{R})$ with (assuming $T_w \ll T_s$) $\mathbf{R} = \begin{bmatrix} 1 & \rho_w \\ \rho_w & 1 \end{bmatrix}$. The non-coherent maximum likelihood detector (which is also the minimum probability of error detector for equally likely information bits) is then given by

$$\hat{d} = \max \left\{ 0, \text{sgn} \left(\mathbf{y}^T (\boldsymbol{\Sigma}_1^{-1} - \boldsymbol{\Sigma}_0^{-1}) \mathbf{y} + \log \frac{|\boldsymbol{\Sigma}_1|}{|\boldsymbol{\Sigma}_0|} \right) \right\}, \quad (3)$$

where $|\boldsymbol{\Sigma}|$ denotes the determinant of matrix $\boldsymbol{\Sigma}$ and we have defined the two covariance matrices of \mathbf{y} under the two hypotheses as $\boldsymbol{\Sigma}_0$ and $\boldsymbol{\Sigma}_1$. It can be shown that

$$\boldsymbol{\Sigma}_0 = \begin{bmatrix} \sigma^2 + E_b & \rho_w(\sigma^2 + E_b) \\ \rho_w(\sigma^2 + E_b) & \sigma^2 + \rho_w^2 E_b \end{bmatrix}, \quad (4)$$

and

$$\boldsymbol{\Sigma}_1 = \begin{bmatrix} \sigma^2 + \rho_w^2 E_b & \rho_w(\sigma^2 + E_b) \\ \rho_w(\sigma^2 + E_b) & \sigma^2 + E_b \end{bmatrix}. \quad (5)$$

Substitution of (4) and (5) in (3) and some simplifications lead to the following equivalent representation of the optimal non-coherent detector:

$$\hat{d} = \max \{0, \text{sgn} (|y_2| - |y_1|)\}. \quad (6)$$

Note that (6) shows that, regardless of the signal waveform correlation ρ_w , the optimal non-coherent detector is just a simple envelope detector. However, the resulting error probability of course depends on the value of ρ_w .

Under the assumption of equally likely information bits we can show that the probability of error of the above detector is

$$P_e = P(|X_2| > |X_1|), \quad (7)$$

where we have defined the two random variables $X_1 = A + n_1$ and $X_2 = \rho_w A + n_2$. If we define a random vector $\mathbf{X} = [X_1, X_2]^T$, then it is easy to see that $\mathbf{X} \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma}_0)$ where $\boldsymbol{\Sigma}_0$ was defined in (4). Then conditioned on $X_1 = x_1$, the distribution of X_2 is of the form of $\mathcal{N}(\rho_w x_1, (1 - \rho_w^2)\sigma^2)$ and thus $P_e = \mathbb{E}_{X_1} \left\{ Q \left(\frac{|X_1| - |\rho_w| X_1}{\sqrt{1 - \rho_w^2} \sigma} \right) + Q \left(\frac{|X_1| + |\rho_w| X_1}{\sqrt{1 - \rho_w^2} \sigma} \right) \right\}$ where $\mathbb{E}_X \{ \cdot \}$ denotes the expectation with respect to X . Since

$X_1 \sim \mathcal{N}(0, \sigma^2 + E_b)$, the probability of error of non-coherent, PPM modulated UWB in flat fading becomes:

$$P_e = \frac{1}{\pi} \left[\arctan \left(\gamma \sqrt{\frac{1 + \rho_w}{1 - \rho_w}} \right) - \arctan \left(\gamma \sqrt{\frac{1 - \rho_w}{1 + \rho_w}} \right) \right]. \quad (8)$$

where we have defined $\gamma = \frac{1}{\sqrt{2(1+\lambda)}}$ and the signal-to-noise ratio λ as $\lambda = \frac{E_b}{\sigma^2}$.

For the above fading model we can easily show that the corresponding coherent detector error probability is given by $P_e^c = \frac{1}{\pi} \arctan((\lambda(1 - \rho_w))^{-1/2})$ which essentially leads to optimizing P_e being equivalent to minimizing ρ_w , so that the optimal value δ^c in the case of coherent reception corresponds to the δ that results in minimum possible value for ρ_w [1]. However, (8) show that the dependence of P_e on the signal correlation ρ_w in non-coherent PPM is fundamentally different from that in coherent detection case and the unique optimal choice that minimizes P_e is $\rho_w^{nc} = 0$. Thus, the best signal design for PPM-UWB with non-coherent detection should use a time shift parameter value δ that results in $\rho_w = 0$. Fig. 1 shows the dependence of P_e of non-coherent PPM in (8) on signal correlation parameter ρ_w .

We can also show that the above result of signal design for PPM systems also holds in the case of narrowband carrier modulated systems¹. To this end, we can treat the signal in (1) as the received baseband signal in a narrowband carrier modulated system by modeling $n(t)$ as a zero-mean, complex white Gaussian noise with variance $\tilde{\sigma}^2$ (denoted as $\mathcal{N}_c(0, \tilde{\sigma}^2)$) and the fading coefficients as $A(i) \sim \mathcal{N}_c(0, E_b)$ where E_b is average received energy per bit. The non-coherent detector will still be given by (6) and the probability of error P_e^{NB} evaluation will be of the form of (7). However, now conditioned on $X_1 = x_1$, $|X_2|$ is Rician distributed. Hence, we have that $P_e^{NB} = \mathbb{E}_{X_1} \left\{ Q \left(\frac{\sqrt{2}|\rho_w||x_1|}{(1 - \rho_w^2)\tilde{\sigma}^2}, \frac{\sqrt{2}|x_1|}{(1 - \rho_w^2)\tilde{\sigma}^2} \right) \right\}$ where $Q(a, b)$ is the Marcum's Q-function. Since $0 \leq |\rho_w| \leq 1$ we may substitute in (III) the alternative form $Q(a, b) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{(1 + \beta \sin \phi) e^{-\frac{b^2}{2}(1 + 2\beta \sin \phi + \beta^2)}}{1 + 2\beta \sin \phi + \beta^2} d\phi$ of the Marcum's Q-function (which is valid when $0 \leq \beta = \frac{a}{b} \leq 1$) [5]. Since X_1 is Rayleigh with the parameter $\tilde{\sigma}^2 + E_b$, for non-coherent PPM in a narrowband system

$$P_e^{NB} = \frac{1}{2} \left[1 - \tilde{\lambda} \left(1 + 2(1 + \tilde{\lambda}) \frac{1 + \rho_w^2}{1 - \rho_w^2} + (1 + \tilde{\lambda})^2 \right)^{-\frac{1}{2}} \right], \quad (9)$$

where we have defined $\tilde{\lambda} = \frac{E_b}{\tilde{\sigma}^2}$. It can be verified from (9) that the best signal design for PPM narrowband systems with non-coherent reception also corresponds to a δ value that results in $\rho_w = 0$. However, it is well known that in the case of coherent detection the best signal design correspond to the value of δ that results in minimum ρ_w .

IV. NUMERICAL RESULTS

A commonly used received UWB mono-pulse is the second derivative of a scaled Gaussian pulse given by $w(t) =$

¹Note that although, perhaps surprisingly, the derivation given here and the resulting error probability expression (9) for non-coherent PPM are not available in the existing literature (to the best of our knowledge), the conclusions are known.

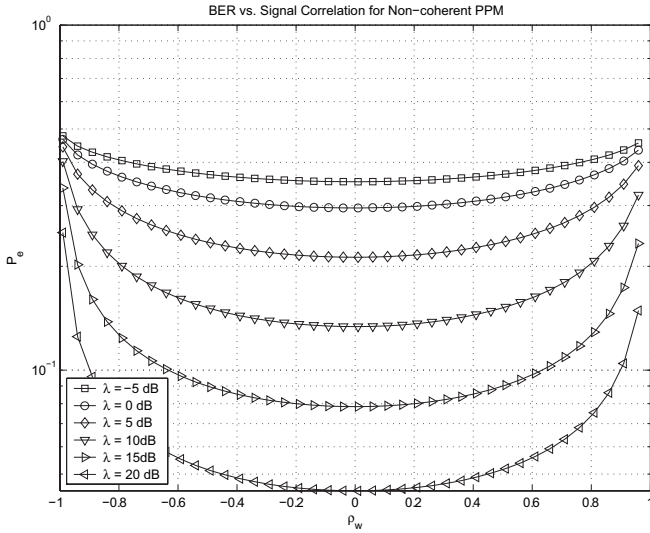


Fig. 1. Probability of Error as a Function of Signal Correlation ρ_w for PPM modulated Non-coherent UWB in the Presence of Fading.

$\sqrt{\frac{8}{3\tau}} \left[1 - 4\pi \left(\frac{t}{\tau} \right)^2 \right] \exp \left(-2\pi \left(\frac{t}{\tau} \right)^2 \right)$. To be concrete, we may use the pulse parameter $\tau = 0.7531$ ns that was used in [1] so that $T_w \approx 2$ ns. The signal correlation ρ_w is then given by $\rho_w = \left[1 - 4\pi \left(\frac{\delta}{\tau} \right)^2 + \frac{4\pi^2}{3} \left(\frac{\delta}{\tau} \right)^4 \right] e^{-\pi \left(\frac{\delta}{\tau} \right)^2}$. Hence, the best time shift value for the coherent reception can be seen to be $\delta^c = 0.410$ which results in the minimum possible correlation $\rho_w^{(\min)} = -0.6181$. However, as we observed above the optimal value δ^{nc} for non-coherent reception is any time shift that results in $\rho_w = 0$. Thus, for the above mono-pulse there are three possible choices for optimal δ :

$$\delta^{nc} = \begin{cases} \tau \sqrt{\frac{\frac{3}{2} - \sqrt{\frac{3}{2}}}{\pi}} \approx 0.2229 \\ \tau \sqrt{\frac{\frac{3}{2} + \sqrt{\frac{3}{2}}}{\pi}} \approx 0.7014 \\ \infty \end{cases} \quad (10)$$

It is easy to see that the most robust performance in the presence of possible timing mismatches is attained by the last solution. In practice, $\delta^{nc} = T_w \approx 2$ ns can provide a very good approximation to the last solution. If we were to plot the probability of error of non-coherent PPM modulated UWB with optimal $\delta^{nc} = 0$ choice and $\delta^{nc} = \delta^c$, it can be seen that there is an SNR penalty. For error probability values less than 10^{-1} this SNR penalty is more than 2dB. This shows that it is important to base signal design for non-coherent PPM on the optimization of (8).

Finally, in Fig. 2 we have shown the sensitivity of the error probability to the time shift parameter choice. Figure 2 shows the SNR penalty that will result in if the value of the time shift parameter δ were to depart from the each of the three optimal solutions given in (10) (we have approximated the third solution by $\delta = T_w \approx 2$ ns). Clearly, the first solution $\delta = 0.2229$ is not a good choice since even a slight timing mismatch could result in a large SNR penalty as is evident from Fig. 2. The best choice of course is to employ $\delta = T_w \approx 2$ ns. However, if it is desirable to maximize the processing gain (in a time hopping multiuser UWB system), it may be desirable to employ the second solution $\delta = 0.7014$ ns. As

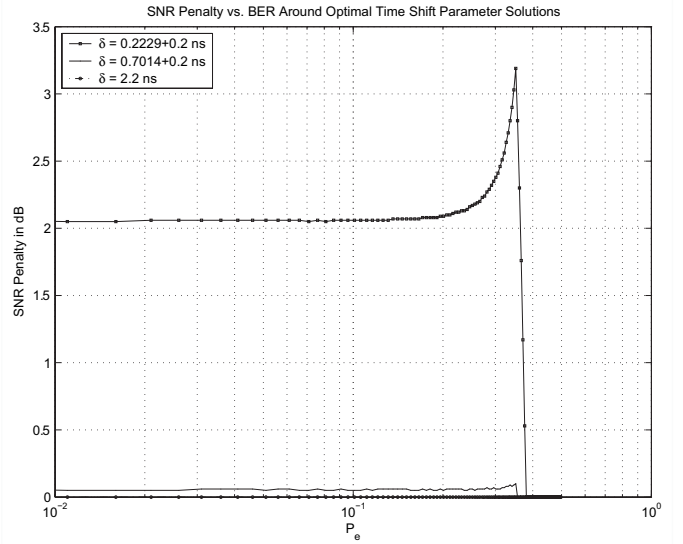


Fig. 2. SNR Penalty Vs. BER for Variations Around the Optimal Time Shift Parameter Solutions for the Gaussian Mono-pulse.

can be seen from Fig. 2, the error performance is relatively robust around this value of δ and the SNR penalty for even about 0.2 ns of timing mismatch is very small.

V. CONCLUSIONS

By deriving a closed form expression for the probability of error of a binary PPM system with non-coherent detection we established that in order to minimize the probability of error for a given SNR the time shift parameter should be chosen so that the two signalling waveforms $w(t)$ and $w(t - \delta)$ are uncorrelated. By taking the second derivative of the Gaussian pulse as the received mono-pulse, we also showed that in a non-coherent UWB system, while it is desirable to employ $\delta \approx T_w$ in order to avoid large SNR penalties that could result in due to timing mismatches, there are other optimal time shift values such that $\delta < T_w$ that can provide good performance trade-offs in terms of maximizing the processing gain vs. minimizing the SNR penalty.

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