# Impact of Mobile Node Density on Detection Performance Measures in a Hybrid Sensor Network

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Abstract—We investigate the impact of mobile node density on several detection performance measures for stationary target detection by a hybrid sensor network consisting of both static and mobile nodes. Such hybrid sensor networks are becoming attractive with the recent advances in sensor nodes equipped with mobile platforms. However, adding a large number of mobile nodes to a sensor network for continuous coverage improvement might be expensive due to mobile node's higher energy consumptions compared to that with static nodes. Motivated by these, we investigate the trade-off between the density of mobile nodes and the network performance in a hybrid sensor network with respect to several performance measures of interest, when mobile nodes perform random mobility. We derive analytical (exact and/or approximate) formulae for detection probability, detection latency and mean first contact distance, by applying the theory of coverage processes and use them to evaluate the tradeoff between the fraction of mobile nodes and these performance measures. Analytical results presented in this paper give insights on how to select optimal network parameters in designing hybrid sensor networks to achieve desired performance requirements. Validity of the derived analytical results is verified via Monte-Carlo simulations.

## I. INTRODUCTION

Target detection is one of the most common applications of wireless sensor networks. In this paper, we address the problem of detecting a stationary target using a hybrid sensor network consisting of both mobile and static nodes. Since deploying mobile nodes is not as cost effective as deploying static nodes, it is desired to investigate the optimal portion of the total nodes to be mobile to meet desired performance levels. We investigate the trade-off between the mobile node density (with respect to static nodes) and several performance measures that are important in designing hybrid sensor networks. In particular, we derive the detection probability, detection latency and the mean first contact distance at a given time of the hybrid sensor network, and investigate the impact of the mobile node density on these measures.

We assume that the static nodes and the initial locations of mobile nodes are both independently and uniformly distributed in a two dimensional plane such that node locations follow a 2-D Poisson point process (PPP). Such a deployment model for nodes is justifiable in situations where the network does not have any prior information regarding the sensing field and the target locations, or when it is more cost effective and practical to deploy nodes randomly in contrast to systematic deployment. We further assume that mobile nodes move randomly and independently in the sensing region searching for targets.

For target detection, in particular, we consider two detection models: single-sensing and k-sensing [1]. In single-sensing detection, the target is assumed to be detected if at least one sensor detects it providing the minimum guarantee on target detection [1]. In k-sensing detection, on the other hand, the target is assumed to be detected if at least k-sensors detects it where k is a design parameter. In this model, the target is detected with lower false alarm probability compared to that with single-sensing detection [1]. Under these detection and node mobility models, the detection performance of the hybrid sensor network is analyzed for detecting a stationary target. Specifically, we derive (i). detection probability which accounts for the quality of the detection, (ii). detection latency, which represents the average time that a target remains undetected after appearing in the sensing region and, (iii). the mean first contact distance at a given time, which accounts for the fact that how fast the mobile nodes approach the target at a given time. We characterize the minimum mobile node density required in order to achieve a desired performance criteria subject to given constraints.

The paper is organized as follows: In Section II, we present related work. Section III explains the sensor network, target and detection models. Section IV derives the detection performance measures (detection probability, detection latency, mean first contact distance at a given time) of stationary target detection with single-sensing and k-sensing detection models and discusses their dependence on mobile node density. The performance results are shown in Section VI. The concluding remarks are given in Section VII.

## II. RELATED WORK

Distributed detection in wireless sensor networks with stationary sensor nodes has been extensively studied by many authors in the literature. For example, in [2]–[6], decision fusion for distributed detection was considered in different contexts when the sensor network is deterministically deployed. However, in practice, random sensor deployment for sensor networks is desirable in many situations. For example, if a priori knowledge of the sensing field is not available at the deployment stage, it is more desirable to position sensors

<sup>&</sup>lt;sup>1</sup>This research was supported in part by the National Science Foundation (NSF) under the grant CCF-0830545.

randomly. Moreover, random sensor deployment is justifiable when it is more cost effective and practical to deploy nodes randomly in contrast to systematic deployment. Stationary and mobile target detection in random stationary sensor networks has been studied by [1], [7]-[10]. Since the performance of such a stationary sensor network is limited by its initial configuration, recently mobile sensor nodes are deployed in wireless sensor network applications to enhance the network performance. For example, to achieve a k-coverage in a random sensor network, with a network size of L, it needs to increase the sensor density as  $O(\log L + k \log \log L)$  at initial deployment stage [11]. On the other hand, the coverage of a static sensor network will remain the same (or reduced due to node failures) after the initial deployment stage. This leads the sensor network to have coverage holes over time. In order to cope with the unreliability, and provide dynamic on demand coverage, static nodes can be integrated with mobile nodes. Use of node mobility at deployment stage for node relocation was considered in [12], [13]. However, these studies do not provide a performance improvement on-demand after deployment stage. Liu et. al. in [14] showed that the coverage can be improved by allowing nodes to be mobile continuously in a mobile sensor network over time compared to that with a static network. In [15], detection of targets using mobile sensor networks is addressed where they analyzed the detection latency of detecting a target. Distributed tracking by mobile sensor networks is addressed in recent research, for example in [16], [17]. However, deploying mobile sensors is not as cost effective as deploying static nodes in a sensor network due to energy constraints. Thus it is desirable to allow only a fraction of total nodes to be mobile to improve the network performance depending on application requirements. Distributed detection and tracking by hybrid sensor networks is also addressed by recent work [18]-[20] when the sensor positions are deterministic.

In this paper we address the problem of detecting an arbitrary target located independently and randomly in a hybrid sensor network consisting of both static and mobile nodes. In particular, the main results presented in this paper can be listed as: (i). Derive the detection probability in stationary target detection by the hybrid sensor network for two specific random mobility models (as presented in subsection III-B) for mobile nodes. We consider two detection models; single-sensing and k-sensing detection. (ii). Derive the detection latency for both single-sensing and k-sensing schemes. (iii). Analyze the tradeoff between the mobile node density and desired the detection performance with given constraints. (iv). Derive the mean first contact distance between the target and the closest (to the target) point covered by the sensor network with at least one sensor at a given time.

## **III. SENSOR NETWORK MODEL**

We consider a hybrid sensor network made of a large number of sensor nodes, N, deployed in a large region  $\mathcal{R}$ . When a large region is to be monitored by a sensor network, it is desirable to deploy a large number of inexpensive, low power sensor nodes to achieve the expected performance. It

was shown in [21]-[23], that large scale sensor networks consisting of a large number of mobile and static nodes are considered to be candidates for many applications including environmental monitoring and event detection in the near future. We assume that there are  $N_s$  number of static nodes and  $N_m$  number of mobile nodes. Denote  $(x_{sk}, y_{sk})$  to be the location of the k-th static node where  $x_{sk}$  and  $y_{sk}$  are assumed to be independently and uniformly distributed in [-b/2, b/2] where  $b \times b$  is the assumed dimension of the sensor network. Denote  $\lambda = \frac{N}{b^2}$  to be spatial density of the nodes and  $\lambda_m = \frac{N_m}{N}$  and  $\lambda_s = \frac{N_s}{N}$  to be the fractions of mobile and static nodes, respectively. Note that we assume that the total number of sensor nodes, N and network dimension,  $b \times b$ are large enough so that assumptions made in the rest of the paper are valid. Let  $\mathcal{V}$  be the set containing all node indices in the network and  $\mathcal{V}_m$  and  $\mathcal{V}_s$  be the sets containing mobile and static node indices, respectively.

## A. Target model

We consider stationary target detection by the hybrid sensor network, where the target location is assumed to be an independently and uniformly distributed arbitrary point  $R_0$  in the region  $\mathcal{R}$ .

## B. Node mobility models

In this paper, we consider two random mobility models: In the first model (model 1), a mobile node moves independently in a direction  $\theta$  selected randomly and uniformly where  $\theta \sim \mathcal{U}[0, 2\pi)$ , with an average speed of  $\bar{v}$  which is assumed to be the same for all mobile nodes. Note that we use  $X \sim \mathcal{U}[a_1, a_2]$  to denote that X is uniformly distributed in the interval  $[a_1, a_2]$ . Then at any time  $t = nT_s$ , a mobile node has moved a distance of  $n\bar{v}T_s$  on a straight line where  $T_s$ is the length of each time step [14]. Second, in model 2, we consider that the k-th mobile node follows a 2-dimensional random walk [24] of n steps at time  $nT_s$  where each step has a length  $\mu = \bar{v}T_s$ . Random and independent mobility models, together with random initial node deployments in large scale, are justifiable in scenarios where nodes do not have any prior knowledge of sensing field or target existence, for example in remote environment monitoring and remote target detection applications. Also random node mobility models are desirable when minimum node coordination is required. Model 1 assumed in the paper is the simplest mobility model which requires minimum control and coordination. Random walk mobility model can be justifiable when mobile nodes are characterized by uncontrolled dynamics, such as random ON-OFF transitions at each time step [25]. These two random models for a mobile node are illustrated in Fig. 1.

## C. Detection model

We assume that each (mobile or static) node has identical effective sensing range r with the sensing area of  $\pi r^2$ . Although we assume homogeneous sensor nodes for simplicity, the results can easily be extended for heterogeneous sensor nodes where mobile and static nodes have different sensing ranges.



Fig. 1. Random mobility models of a mobile node

We assume a binary detection model in which the point  $R_0$ is considered to be detected with probability 1 by the sensor  $s_k$ at time  $t = nT_s$  if it lies in sensor-coverage area  $C_k(nT_s)$  [12], where  $C_k(nT_s)$  is the coverage area of node  $s_k$  at time  $nT_s$ for  $n = 0, 1, 2, \cdots$ . Formally, we can express the probability that the node  $s_k$  detects the target at time interval  $[0, nT_s)$  as:

$$P_{d_k}(nT_s) = \begin{cases} 1 & \text{if } R_0 \in C_k(nT_s) \\ 0 & \text{if } \text{otherwise} \end{cases}$$

Note that for a static node, the coverage area  $C_k(nT_s)$  is constant over time. That is, if the target is not detected by a static node initially, it will never be detected. However, with a mobile node, since the coverage is varied over time, there is a possibility to detect the target as the time goes.

#### D. Preliminaries

1) Boolean model: Let  $\Omega \equiv \{\alpha_i, i \geq 1\}$  in  $\mathbb{R}^k$  is a point process and  $\{\beta_i, i \ge 1\}$  be a sequence of independently and identically distributed random sets, independent of  $\Omega$ . The collection of sets  $C = \{\alpha_i + \beta_i, i \ge 1\}$  is called a coverage process [26]. When C is driven by a stationary Poisson point process (i.e.  $\Omega$  is a stationary Poisson point process), the coverage process C is called a Boolean model [26]. Since we assume that static node locations and initial mobile node locations are independently and identically distributed in a vast two dimensional area, the sensor locations can be modeled as a two-dimensional Poisson point process with intensity  $\lambda$ , when the total number of nodes and the sensing region are large. With the considered random mobility models, since mobile nodes make independent and identical random movements, at any time instance  $t = nT_s$ , sensor locations still form a 2-D Poisson point process with the same intensity [27] when the area  $b^2 \lim \infty$ .

2) Notation: We use  $\mathcal{A}(S)$  and  $\mathcal{P}(S)$  to denote the area and perimeter of the set S. Denote by R + S the set centered at R with a shape of S.



Fig. 2. (a). Hybrid network at times t=0 and  $t=nT_s$  (b). Realization of random shapes at time  $nT_s$ 

# IV. STATIONARY TARGET DETECTION PERFORMANCE WITH MOBILITY MODEL 1

## A. Detection probability

In the following, we consider two modes of detection: Single-sensing detection and k-sensing detection [1]. In Single-sensing detection, the target is considered as detected if it is captured by at least one sensor. In this case, target's presence is obtained with the minimum guarantee. On the other hand, detection by multiple sensors ensure lower false alarms. In k-sensing detection model, the target is considered as detected if it is detected by at least k sensors where k is a design parameter [1].

In this section, we analyze the detection performance with the random node mobility model 1 as shown in Fig. 1, where each mobile node moves in a straight line after selecting the direction independently and uniformly from  $[0, 2\pi)$ . Figure 2 (a) illustrates the coverage area of the sensor network at time t = 0 and time  $t = nT_s$  with mobility model 1. With the assumption that the initial node locations are independent and uniform, after mobile nodes have moved a distance  $\bar{v}t = \bar{v}nT_s$ , we can model the coverage of the sensor network as a Boolean model in which the driving point process is the initial Poisson point process with intensity  $\lambda$  and the shape distribution is varied with the time. Further, denote  $T_0$  to be the average time a mobile node takes to leave the sensing region  $\mathcal{R}$ . Since we assume that the sensing region is large enough and the speed of a mobile node is small (e.g. for example, Robomote [28] mobile nodes have speed of  $0.5 \sim 2m/s$ ),  $T_0$  is assumed to be large. Thus the main focus in this paper is to analyze the detection performance in the region where  $nT_s \leq T_0$ . The corresponding coverage area  $S(nT_s)$  at time  $t = nT_s \leq T_0$  is distributed as

$$S(nT_s) = \begin{cases} S_1(nT_s) & \text{with prob } \lambda_m \\ S_2 & \text{with prob } 1 - \lambda_m \end{cases}, \qquad (1)$$

where  $S_1(nT_s)$  and  $S_2$  are as shown in Fig. 2 (b). The coverage area of k-th static sensor at time  $nT_s \leq T_0$  is given by,

 $C_k^s(nT_s) = C_k^s = \mathcal{A}(S_2) = \pi r^2$ , and the coverage area of the k-th mobile node at time  $t = nT_s$  is given by (corresponding to shape  $S_1(nT_s)$ )  $C_k^m(nT_s) = \mathcal{A}(S_1(nT_s)) = \pi r^2 + 2rnT_s \bar{v}$ . Note that for  $nT_s \ge T_0$ , we have  $C_k^m(nT_s) = \mathcal{A}(S_1(nT_s)) = \pi r^2 + 2rT_0 \bar{v}$  while  $C_k^s(nT_s) = C_k^s = \pi r^2$ .

The probability that the target is detected at time  $t = nT_s$  is given by the following theorem.

Theorem 1: (Detection probability) The probabilities of detection with single-sensing and the k-sensing models ( $k \ge 1$ ) at time  $t = nT_s$  are given by,

$$P_D^1(nT_s) = \begin{cases} 1 - e^{-\lambda(\pi r^2 + 2\lambda_m r\bar{v}nT_s)} & if \quad nT_s \le T_0 \\ 1 - e^{-\lambda(\pi r^2 + 2\lambda_m r\bar{v}T_0)} & if \quad nT_s > T_0 \end{cases}$$

and

$$\begin{split} P_D^k(nT_s) &= \\ \begin{cases} 1 - \sum_{j=0}^{k-1} \frac{(\lambda(\pi r^2 + 2\lambda_m r\bar{v}nT_s))^j e^{-\lambda(\pi r^2 + 2\lambda_m r\bar{v}nT_s)}}{j!}, & \text{if } nT_s \leq T_0 \\ 1 - \sum_{j=0}^{k-1} \frac{(\lambda(\pi r^2 + 2\lambda_m r\bar{v}T_0))^j e^{-\lambda(\pi r^2 + 2\lambda_m r\bar{v}T_0)}}{j!}, & \text{if } nT_s > T_0 \end{cases} \end{split}$$

respectively.

*Proof:* See Appendix A.

Since allowing more nodes to be mobile is not desirable in many applications due to energy constraints, it is required to determine the minimum fraction of mobile nodes to be deployed in order to achieve the desired performance during a given time interval. The following theorem states the minimum fraction of mobile nodes required to achieve a desired probability level within a desired time interval for single sensing detection.

Theorem 2: (Minimum mobile node density required with single sensing detection) Let  $\eta_D$  be the desired detection probability to be achieved by the hybrid sensor network at time  $t_D \leq T_0$ . The minimum fraction of mobile nodes to be used to achieve  $\eta_D$  at time  $t_D (\leq T_0)$  with single-sensing detection model is given by,

$$\lambda_m^{min} = \begin{cases} \frac{-\log(1-\eta_D) - \lambda \pi r^2}{2\lfloor \frac{t_D}{T_s} \rfloor \lambda r \bar{v} T_s}, & \text{if } \eta_s \le \eta_D \le \eta_t \\ \text{infeasible}, & \text{otherwise}, \end{cases}$$
(3)

where  $\eta_s = 1 - e^{-\lambda \pi r^2}$  and  $\eta_t = 1 - e^{-\lambda [\pi r^2 + 2\lfloor \frac{t_D}{T_s} \rfloor r \bar{v} T_s]}$ . *Proof:* See Appendix B.

In the case of k-sensing detection, the minimum fraction of mobile nodes can be found by finding the minimum  $\lambda_m$ which satisfies the following inequality:

$$1 - \sum_{j=0}^{k-1} \frac{(\lambda(\pi r^2 + 2\lambda_m r \lfloor \frac{t_D}{T_s} \rfloor \bar{v}T_s))^j e^{-\lambda(\pi r^2 + 2\lambda_m r \lfloor \frac{t_D}{T_s} \rfloor \bar{v}T_s)}}{j!} \ge \eta_D.$$

However, if the desired delay constraint is such that  $\lfloor \frac{t_D}{T_s} \rfloor \leq \frac{\pi r}{2\overline{v}T_s}$ , the minimum fraction of mobile nodes can be found by finding the minimum  $\lambda_m$  which satisfies the following inequality:

$$\lambda_m - \frac{\log(f_1(k-1) + \lambda_m f_2(k-1))}{2\lambda r \lfloor \frac{t_D}{T_s} \rfloor \bar{v}T_s} \ge \frac{-\log(1-\eta_D) - \lambda \pi r^2}{2 \lfloor \frac{t_D}{T_s} \rfloor \lambda r \bar{v}T_s}$$

where  $f_1(k-1) = \sum_{j=0}^{k-1} \frac{(\lambda \pi r^2)^j}{j!}$  and  $f_2(k-1) = \frac{2r\lfloor \frac{t_D}{T_s} \rfloor \overline{v} T_s}{\pi r^2} \sum_{j=1}^{k-1} \frac{(\lambda \pi r^2)^j}{(j-1)!}$ .

#### B. First contact length for single-sensing detection

An important measure to evaluate the quality of the target detection is to analyze the mean distance between the target and the closest point (to the target) covered by at least one sensor by the sensor network at any time instant. This is called the first contact distance of the target with single-sensing. When there are mobile nodes in the network, this measure essentially reflects how fast each point in the sensor network is covered over time.

The following theorem states the mean length of the first contact distance for single-sensing detection.

(2) Theorem 3: (Mean first contact distance) Denote  $X^1(nT_s)$  to be the distance between the target, located at any arbitrary point in region  $\mathcal{R}$ , and the closest point covered by the sensor network by at least one sensor at time  $t = nT_s$ . Denote  $\bar{X}^1(nT_s) = \mathbb{E}\{X^1(nT_s)\}$  to be the corresponding mean distance. Then  $\bar{X}^1(nT_s)$  is given by,

$$X^{1}(nT_{s}) = \begin{cases} \frac{1}{\sqrt{\lambda}}e^{\frac{1}{\pi}\lambda\lambda_{m}^{2}\bar{v}^{2}n^{2}T_{s}^{2}}Q\left(\sqrt{\frac{\lambda}{2\pi}}(2\pi r + 2\lambda_{m}\bar{v}nT_{s})\right), \text{ if } nT_{s} \leq T_{0}\\ \frac{1}{\sqrt{\lambda}}e^{\frac{1}{\pi}\lambda\lambda_{m}^{2}\bar{v}^{2}T_{0}^{2}}Q\left(\sqrt{\frac{\lambda}{2\pi}}(2\pi r + 2\lambda_{m}\bar{v}T_{0})\right), \text{ if } nT_{s} > T_{0} \end{cases}$$

and is upper bounded by,

$$\bar{X}^{1}(nT_{s}) \leq \begin{cases} \frac{1}{2\sqrt{\lambda}}e^{-\lambda(\pi r^{2}+2\lambda_{m}r\bar{v}nT_{s})}, & if \quad nT_{s} \leq T_{0} \\ \frac{1}{2\sqrt{\lambda}}e^{-\lambda(\pi r^{2}+2\lambda_{m}r\bar{v}T_{0})}, & if \quad nT_{s} > T_{0} \end{cases}$$
(4)

where Q-function is defined as  $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-\frac{t^2}{2}} dt$ . *Proof:* See Appendix C.

For  $nT_s \leq T_0$ , from (4) it can be seen that when  $\lambda_m$  or n is increased, the mean length of the first contact distance is decreased for fixed  $\lambda$  and r. On the other hand, if there is only a stationary sensor network,  $\bar{X}^1(nT_s)$  can be decreased by only increasing either  $\lambda$  or r. Note that, (4) shows the proper trade-off between  $\bar{X}^1(nT_s)$ ,  $\lambda_m$  and n when the total node density  $\lambda$  and r are fixed.

#### C. Detection latency

In a hybrid sensor network embedded with mobile nodes, it is important to analyze the time delay till the target is first detected after appearing in the sensor network, which is called the *detection latency* [15]. This measure essentially reflects the monitoring capability and how fast the target can be detected by allowing nodes to be mobile. First, we explore the dependence of the detection latency on mobile node density with single-sensing detection. In this discussion we assume that the target needs to be detected before mobile nodes leave the sensing region and the average time that a mobile node requires to leave the region under mobility model 1,  $T_0$ , is sufficiently large.

Theorem 4: (Average detection latency for single-sensing detection) Define the random variable  $\tau_1$  to be the time until the target is first detected by the hybrid sensor network with single-sensing. Then the average detection latency  $\bar{\tau}_1$  of the hybrid sensor network in single-sensing detection (when  $T_0 \lim \infty$ ) is given by,

$$\bar{\tau}_1 = \frac{e^{-\lambda \pi r^2}}{2\lambda \lambda_m \bar{v} r}.$$
(5)

It can be seen from (5) that for a given total node density  $\lambda$  and sensing range r, the average detection latency can be reduced by increasing the fraction of mobile nodes  $\lambda_m$  or speed of mobile nodes,  $\bar{v}$ .

*Proof:* Let  $\tau_1$  be the random variable which represents the time until the target is first detected by the hybrid sensor network with single-sensing. Then we have,

$$Pr(\tau_1 > t) = Pr(target \ is \ not \ detected \ until time \ t \ (\leq T_0)) = e^{-\lambda(\pi r^2 + 2r\lambda_m \bar{v}t)}$$
(6)

Then the mean value of  $\tau_1$  is given by,

$$\bar{\tau}_{1} = \int_{0}^{T_{0}} Pr(\tau_{1} > t) dt$$

$$= \frac{e^{-\lambda \pi r^{2}}}{2\lambda \lambda_{m} \bar{v}r} \left(1 - e^{-2r\lambda \lambda_{m} \bar{v}T_{0}}\right),$$

$$\lim_{T_{0} \to \infty} \bar{\tau}_{1} = \frac{e^{-\lambda \pi r^{2}}}{2\lambda \lambda_{m} \bar{v}r}.$$
(7)

Detection latency with k-sensing detection for  $T_0 \lim_{\to} \infty$  is given by the following theorem.

Theorem 5: (Average detection latency for k-sensing detection) The average detection latency with k-sensing detection is given by

$$\bar{\tau}_k = \frac{e^{-\lambda\pi r^2}}{2\lambda\lambda_m\bar{v}r} \sum_{j=0}^{k-1} (\lambda\pi r^2)^j \tilde{f}(j), \tag{8}$$

where  $\tilde{f}(j) = \sum_{i=0}^{j} \frac{1}{(j-i)!} \left(\frac{1}{\pi r^2}\right)^i$ . *Proof:* See Appendix D.

## V. DETECTION PERFORMANCE WITH RANDOM NODE MOBILITY MODEL 2 (RANDOM WALK)

In this Section, we consider that the mobile nodes follow 2-D random walk mobility model at each time step  $nT_s$  as shown in Fig. 1. With random walk mobility model, since it is difficult to obtain a closed form solution for detection probability, in the following we find an approximation for single-sensing and k-sensing detection models. Let us assume that the sensing region can be viewed as a virtual square lattice having a total of  $\approx \frac{b^2}{\mu^2}$  square sites where  $\mu = \bar{v}T_s$  is the lattice side length. The k-th mobile node is assumed to be at the center of a site. If the mobile node starts to move at time t = 0, the expected number of distinct sites visited by time  $nT_s$ ,  $\mathbb{E}{G(nT_s)}$  can be approximated by [24], [29],

$$\mathbb{E}\{G(nT_s)\} \approx \frac{b^2}{\mu^2} \left(1 - \left(\frac{cb^2}{\mu^2}\right)^{-\frac{\pi nT_s}{\frac{b^2}{\mu^2}\log^2\left(\frac{cb^2}{\mu^2}\right)}}\right),$$

where c = 1.8456....

In this paper we consider only the case  $r \leq \mu$ , since if the step size  $\mu$  is selected such that  $\mu \ll r$ , there are large overlaps in the sensing areas at consecutive steps [24]. Thus it is more desirable to select step size of the random walk such that  $\mu \geq r$ , which results in a larger coverage area at each step of the random walk. Since each mobile node performs independent

and identical random walks at each time step, and the sensing range of each mobile node is identical, it can be seen that,  $\{C_k^m(nT_s)\}_{k\in\mathcal{V}_m}$  are a set of independently and identically distributed random sets where  $C_k^m(nT_s)$  is the area covered by the k-th mobile node at time  $nT_s$ . Denote  $\bar{C}_k^m(nT_s) = \bar{C}^m(nT_s)$  to be the average coverage area of the k-th mobile node at time  $nT_s$ . A lower bound for the average area covered by a mobile node at time  $nT_s$ ,  $\bar{C}^m(nT_s)$  is then given by the following theorem.

Theorem 6: (Minimum average coverage area of a mobile node) Assuming that  $\mu \ge r$ , the minimum average area covered by any single mobile node at time  $nT_s$  is given by,

$$\bar{C}_{min}^{m}(nT_s) = \pi r^2 + (\mathbb{E}\{G(nT_s)\} - 1)^+ 2r\mu - (\mathbb{E}\{G(nT_s)\} - 2)^+ (1 - \frac{\pi}{4})r^2.$$
(9)

**Proof:** Assuming  $\mu \geq r$ , when there is  $\mathbb{E}\{G(nT_s)\}$ number of distinct sites visited at time  $nT_s$ , there should be at least  $\mathbb{E}\{G(nT_s)\}-1$  number of steps to ensure that each point is connected to at least one lattice point. Then the minimum coverage area results if these lattice points are located such that each transition is orthogonal to the previous transition (That is, then the maximum amount of overlapping will occur with the minimum number of transitions). Based on geometric simplification, it can be shown that the minimum coverage area is given by,

$$\bar{C}_{min}^{m}(nT_s) = \pi r^2 + (\mathbb{E}\{G(nT_s)\} - 1)2r\bar{v}T_s 
- (\mathbb{E}\{G(nT_s)\} - 2)(1 - \frac{\pi}{4})r^2.$$

Note that this result can be shown to be valid for  $r \leq \frac{\mu}{2}$ , where there is no overlapping of the sensing range, as well as for  $\frac{\mu}{2} \leq r < \mu$  where there is overlapping of sensing range, between two consecutive steps.

Then lower bounds for the detection probability in singlesensor and k-sensor detections can be shown as,

$$P_D^1(nT_s) \ge 1 - e^{-\lambda \bar{C}_{min}(nT_s)},$$
 (10)

and  $P_D^k(nT_s) \geq 1 - \sum_{j=0}^{k-1} \frac{(\lambda \bar{C}_{min}(nT_s))^j e^{-\lambda \bar{C}_{min}(nT_s)}}{j!}$ , respectively, with  $\bar{C}_{min}(nT_s) = \lambda_m \bar{C}_{min}^m(nT_s) + (1-\lambda_m)\pi r^2$  where  $\bar{C}_{min}^m(nT_s)$  is given by (9).

Let  $\eta_D$  be the desired detection probability lower bound to be achieved by the hybrid sensor network at time  $t_D$ . The minimum fraction of mobile nodes  $\lambda_m^{min}$  that should be used in order to achieve this probability bound, within the desired time is stated in the following theorem:

Theorem 7: (Minimum fraction of mobile nodes required to achieve a desired detection probability lower bound at a given time) With single-sensing detection, if the desired detection probability lower bound,  $\eta_D$ , is to be achieved within a time interval  $t_D$ , the minimum fraction of mobile nodes that should be deployed in the hybrid network with single-sensing detection is given by

$$\lambda_m^{min} = \frac{-\log(1-\eta_D) - \lambda \pi r^2}{\lambda \left( \bar{G}_1(\lfloor \frac{t_D}{T_s} \rfloor T_s) 2r \bar{v} T_s - \bar{G}_2(\lfloor \frac{t_D}{T_s} \rfloor T_s)(1-\frac{\pi}{4}) r^2 \right)}.$$



Fig. 3. Detection probability with single-sensing detection Vs desired delay constraint with mobility model 1: r = 20m, b = 1000m, N = 500,  $\lambda = 0.0005$ 

for  $\mu \geq r$  where  $\bar{G}_1(\lfloor \frac{t_D}{T_s} \rfloor T_s) = (\mathbb{E}\{G(\lfloor \frac{t_D}{T_s} \rfloor T_s)\} - 1)$  and  $\bar{G}_2(\lfloor \frac{t_D}{T_s} \rfloor T_s) = (\mathbb{E}\{G(\lfloor \frac{t_D}{T_s} \rfloor T_s)\} - 2).$ 

*Proof:* The proof follows directly from (10) and (9).

# VI. SIMULATION RESULTS

## A. With node mobility model 1

We verify the analytical results obtained in this paper via extensive Monte-Carlo simulations. The dimension of the sensing area is assumed to be b = 1000m, such that the area is  $1000 \times 1000m^2$ . This figure for the network size is consistent with network sizes used in existing literature for large scale sensor networks, for example in [23]. Unless specified, for each figure in the following, 10<sup>5</sup> Monte-Carlo runs were performed. Mobile node speed is set to  $\bar{v} = 1m/s$ , which is consistent with the speeds of some of the currently existing mobile nodes, for example in [28]. Initially a total of N = 500sensor nodes are deployed independently and uniformly in the sensing field, such that the node density  $\lambda = 0.0005$ . A fraction  $\lambda_m$  of 500 total nodes, is directed to move according to the random mobility model 1 as described in subsection III-B. With these parameters, it can be shown by simulations that the average time a mobile node takes to leave the sensing region with the mobility model 1 is,  $T_0 = 473.31655s$ .

In the first experiment, the time varying detection probability is investigated when the fraction of mobile nodes is varying for a given sensing range for mobile and static nodes. Figure 3 shows the analytical and simulated results which reflect the time varying detection probability of the hybrid sensor network for single-sensing detection when the fraction of mobile nodes deployed is varied. In Fig. 3, we assume that the sensing range of a sensor, r = 20m, which is a valid figure for sensing range of certain currently existing sensors [23], [30]. From Fig. 3, we can see the derived analytical results almost exactly match with the simulation results for  $nT_s \leq T_0$ and  $nT_s > T_0$ . It can be seen from Fig. 3, that after a certain time period, the detection probability reaches a steady state, which essentially means that the area is maximally covered



Fig. 4. Detection probability Vs fraction of mobile nodes in the network for single-sensing and 2-sensing detection models for mobility model 1; Desired detection delay is  $t_D = 60s$ .

by the mobile nodes (with static nodes) before they leave the sensing region. Interestingly, we see that when the fraction of mobile nodes is increasing, this steady state probability becomes 1 and it is reached well before the nodes leave the sensing region. This means that when  $\lambda_m$  increases, the network can be completely covered by the hybrid network within a shorter time (compared to  $T_0$ ) with the mobility model 1. This phenomenon essentially reflects the trade-off between the fraction of mobile nodes and the probability of detecting the target before it disappears in the field. For example, if the target appearing time is shorter, it is desired that the total area is covered as quickly as possible to detect it before disappearing, which needs a relatively larger fraction of mobile nodes. On the other hand, if the target appearing time is longer, then with a relatively small number of nodes is enough to cover the area with the desired quality. Also it is noted from Fig. 3 that, at earlier time intervals before the probability reaches steady state, the detection probability has rapid increment compared to the stationary configuration, and increases slowly as it approaches the steady state probability. Moreover, it is seen for Fig. 3 that by adding a small fraction of mobile nodes will boost the detection performance significantly compared to the stationary configuration, and the rate of performance improvement eventually decreases as  $\lambda_m$  increases.

In the next experiment, the detection performance is evaluated with varying sensing ranges for single-sensing and 2sensing detection models. Figure 4 shows the detection probabilities for single-sensing (top plot) and 2-sensing (bottom plot) detection models of the hybrid sensor network Vs the fraction of mobile nodes for a given desired delay constraint, when the sensing range is varied. In Fig. 4 we let the delay constraint  $t_D = 60s < T_0$  in which the network has not reached the steady state performance. Note that, with mobility model 1, our interest is more on the dynamic performance results in the hybrid network before it reaches the steady state (i.e. before the mobile nodes leave the sensing region). Different plots in Fig. 4 are corresponding to varying sensing ranges (for r = 20m, r = 30m and r = 40m). From Fig



Fig. 5. Minimum fraction of mobile nodes required to achieve a desired performance level within a desired delay constraint for mobility model 1

4, it can be seen that the derived analytical results perfectly match with the simulation results. It can also be seen that the detection probability is nearly-linearly increasing, when the fraction of mobile nodes is increasing, for a given sensing range around the considered delay constraint (i.e. around relatively lower delay constraints). Also, when the sensing range is increasing the increment in the detection probabilities over  $\lambda_m$  occurs at a lower rate for both single and 2-sensing detection models.

In Fig. 5, the minimum fraction of mobile nodes required to achieve a desired performance level within a desired delay constraint ( $< T_0$ ) is shown for r = 20m and r = 30m with single sensing detection. It is seen that when the desired delay constraint is small, the minimum fraction of mobile nodes is increasing to achieve a desired performance level. Moreover, the effect of the mobile node density on the detection performance is more significant when the sensing range of the nodes is low, which is the most practical scenario in many sensor networks. It can be seen from Fig. 5 that when the sensing range is increasing, the variation of the required fractions of mobile nodes to achieve different detection thresholds, is less compared to that with lower sensing ranges.

The next experiment is performed to evaluate the performance of hybrid sensor network in terms of the mean first contact distance at a given time. Figure 6(a) shows the performance of the mean first contact distance derived in subsection IV-B, with the mobile node density. In Figure 6(a), we let r = 20m and plots are corresponding to different delay constraints. From Fig. 6 (a), it can be seen that the derived results for the mean first contact distance fairly match with the simulated results. Note that the mean first contact distance at a given time essentially means that how much, in average, that an given arbitrary point is closer to any point in the network covered by at least one sensor at a given time. It can be seen from Fig. 6(a), as the time elapsed, any arbitrary point is getting closer to an point covered by the sensor network by at least one sensor much faster until a certain fraction of mobile nodes, and after that the mean distance reaches slowly



Fig. 6. (a). Mean value of the first contact distance for single-sensing detection with mobility model 1; r = 20m,  $\bar{v} = 1m/s$  (b). Average detection latency for single-sensing and 2-sensing detection models with mobility model 1; r = 40m,  $\bar{v} = 1m/s$ 

to zero. This reflects the proper trade-off between the fraction of mobile nodes required and the delay constraint in order to cover any arbitrary point in the network as time goes.

Figure 6(b) depicts the average detection latency for singlesensing and 2-sensing detection models with the fraction of mobile nodes. It can be seen that, for a given sensing range, with a smaller fraction of mobile nodes, the average delay of detection with 2-sensing model is significantly increased compared to that with single-sensing model. However, as  $\lambda_m$  is increasing, the difference of average detection delays of two sensing models becomes smaller. This essentially implies that to obtain the system performance with a higher confidence level (increasing k) with a smaller fraction of mobile nodes, it is required to wait a longer time compared to that with single-sensing model (lower or minimum possible performance level). Moreover, as  $\lambda_m$  increases, the average detection latency required to achieve a performance level with a higher confidence, is not significantly long compared to single-sensing detection model.

From the results in the Figures 3, 4, 5 and 6 it can be seen that the Boolean model is a good approximation for the hybrid sensor network considered in this paper when the number of nodes and the sensing area are relatively large. To further illustrate the suitability of Boolean/Poisson model with reduced number of nodes and network sizes, in Fig. 7 we plot the time varying detection probability for b = 500mand N = 125 such that the node density is still  $\lambda = 0.0005$ . With these parameter values, it can be shown that the average time that a mobile node needs to leave the sensing region,  $T_0 = 236.4925s$ . From Fig. 7, it can be seen that the Boolean approximation does not give very accurate results when N and  $b^2$  are relatively small.

## B. With node mobility model 2

With random walk mobility model, we perform Monte-Carlo simulations to obtain the exact detection probability to compare the performance of the derived detection probability



Fig. 7. Detection probability with single-sensing detection Vs desired delay constraint with mobility model 1: r = 20m, b = 500m, N = 125,  $\lambda = 0.0005$ 



Fig. 8. Detection probability lower bound with single-sensing detection Vs fraction of mobile nodes in the network with random walk mobility model after completing n = 20 steps: for  $\mu = \sqrt{2}r$  and  $\mu = 2r$ : r = 20m

lower bound. Figure 8 shows the analytical detection probability lower bound and the exact detection probability vs the fraction of mobile nodes, with random walk mobility model after completing n = 20 steps. In Fig. 8, we let the step sizes of the random walk to be  $\mu = \sqrt{2}r$  and  $\mu = 2r$  where r is set to r = 20m. From Fig. 8, it can be seen that the derived lower bound is a good match for the exact detection probability. Moreover, when the step size of the random walk is selected relatively larger compared to the sensing radius of the node, it can be seen that the derived lower bound becomes much tighter for the exact detection probability. For a given sensing range, selecting a larger step size compared to the sensing range is more desirable in performing 2-D random walk, since then the overlapping of sensing coverage at consecutive steps is reduced.

## VII. CONCLUSION

In this paper, the impact of mobile node density on the detection performance in different perspectives of a hybrid sensor network consisting of both static and mobile nodes was addressed. We considered two random mobility models for mobile nodes where in the first one, mobile nodes move on a straight line after selecting a random direction initially and in the second one, mobile node follow a 2-D random walk. With the mobility model 1, we derived the detection performance, in terms of detection probability, detection latency and mean first contact distance for single-sensing and k-sensing detection models of the hybrid sensor network. With mobility model 2, we derived reasonable approximations for the average coverage area and the detection probability for single and k sensing detection models. We investigated the trade-off between the mobile node density and the desired (exact or approximated) performance gain with given constraints. The analytical results derived in this paper help to select design parameters in hybrid sensor networks for on-demand application requirements.

## APPENDIX A

## **PROOF OF THEOREM 1**

In single sensing detection, the target is considered as detected, if at least one sensor captures it. If  $C \equiv \{\alpha_i + S_i, i \geq 1\}$ is a Boolean model with shapes  $S_i$  are distributed as S, the number of sets (shapes) that intersects an arbitrary point (or the number of sets that covers an arbitrary point) in the Boolean model has a Poisson distribution with mean  $\lambda \mathbb{E} \{ \mathcal{A}(S) \}$  [26]. Note that with the mobility model 1, the average area covered by a mobile node within the time interval  $[0, nT_s)$  is given by  $\bar{C}^m(nT_s) = \mathcal{A}(S_1(nT_s)) = \pi r^2 + 2rnT_s \bar{v} \text{ if } nT_s \leq T_0 \text{ and}$  $C_k^m(nT_s) = \mathcal{A}(S_1(T_0)) = \pi r^2 + 2rT_0\bar{v}$  if  $nT_s > T_0$ . Now, as can be seen from the right plot of Fig. 2 (a), at time  $t = nT_s$ , the hybrid sensor network can be considered as a Boolean model in which the diving point process is the initial Poisson point process and the shape distribution is given by (1), in which the average coverage areas are determined depending on whether  $nT_s \leq T_0$  or  $nT_s > T_0$ . Denote  $P_{R_0}(m, nT_s)$  to be the probability that m number of sensors cover the point  $R_0$  at time  $t = nT_s$ , which is given by [26]

$$P_{R_0}(m, nT_s) = \frac{\left(\lambda \bar{C}(nT_s)\right)^m e^{-\lambda \bar{C}(nT_s)}}{m!}$$

where  $\bar{C}(nT_s) = (\lambda_m \bar{C}^m (nT_s) + (1 - \lambda_m)C^s)$  is the average coverage area of the network at time  $nT_s$ . Then the probability that no sensor covers the point  $R_0$ ,  $P_{R_0}(0, nT_s)$ , at time  $nT_s$  is given by  $P_{R_0}(0, nT_s) = e^{-\lambda \bar{C}(nT_s)}$ . The probability of the single-sensing detection at time  $t = nT_s \leq T_0$  is thus given by,

$$P_D^1(nT_s) = 1 - P_{R_0}(0, nT_s) = 1 - e^{-\lambda C(nT_s)}$$
  
=  $1 - e^{-\lambda (\pi r^2 + 2\lambda_m r n \bar{v} T_s)}$ .

For  $t = nT_s > T_0$ , we will get,  $P_D^1(nT_s) = 1 - e^{-\lambda(\pi r^2 + 2\lambda_m r \bar{v} T_0)}$ . In k-sensing detection, the target is considered to be detected if at least k sensors detect it. Probability that the point  $R_0$  is covered by at least k sensors at time  $nT_s$  is given by,

$$P_D^k(nT_s) = 1 - Pr(R_0 \text{ is covered by } k - 1 \text{ or less sensors})$$
$$= 1 - \sum_{j=0}^{k-1} P_{R_0}(j, nT_s)$$

$$= \begin{cases} 1 - \sum_{j=0}^{k-1} \frac{(\lambda(\pi r^2 + 2\lambda_m r\bar{v}nT_s))^j e^{-\lambda(\pi r^2 + 2\lambda_m r\bar{v}nT_s)}}{j!}, & \text{if } nT_s \leq T_0 \\ 1 - \sum_{j=0}^{k-1} \frac{(\lambda(\pi r^2 + 2\lambda_m r\bar{v}T_0))^j e^{-\lambda(\pi r^2 + 2\lambda_m r\bar{v}T_0)}}{j!}, & \text{if } nT_s > T_0 \end{cases}$$

## APPENDIX B

#### **PROOF OF THEOREM 2**

If the tolerable detection delay is  $t_D (\leq T_0)$ , and the desired detection probability is  $\eta_D$ , the minimum  $\lambda_m$  is characterized by,

s.t. 
$$P_D^1\left(\left\lfloor \frac{t_D}{T_s} \right\rfloor T_s\right) \ge \eta_D$$

where  $P_D^1\left(\lfloor \frac{t_D}{T_s} \rfloor T_s\right)$  is given by (2). This leads to

$$\lambda_m \ge \frac{-\log(1-\eta_D) - \lambda \pi r^2}{2\lfloor \frac{t_D}{T_s} \rfloor \lambda r \bar{v} T_s}.$$
(11)

Note that (11) holds for a desired delay constraint, only if the desired detection probability  $\eta_D$  satisfies the condition  $\eta_s \leq \eta_D \leq \eta_t$  where  $\eta_s = 1 - e^{-\lambda \pi r^2}$  and  $\eta_t = 1 - e^{-\lambda [\pi r^2 + 2\lfloor \frac{t_D}{T_s} \rfloor r \bar{v} T_s]}$  are the detection probabilities achieved by the network if all nodes are stationary ( $\lambda_m = 0$ ), and if all nodes are allowed to move ( $\lambda_m = 1$ ), respectively.

## APPENDIX C

## **PROOF OF THEOREM 3**

To prove theorem 3 we use the following theorem regarding set intersection whose proof can be found in [26]. An isotropic random set is a set in which the distribution is invariant under independent and uniform rotations.

Theorem 8: Consider the Boolean model as defined in section III-D1 with the shapes S distributed as isotropic convex sets. Let  $S_0$  be a fixed convex subset in  $\mathbb{R}^2$ . Then the number of sets in the Boolean model that intersects  $S_0$  is poisson distributed with mean  $\lambda(\mathcal{A}(S_0) + \mathbb{E}\{\mathcal{A}(S)\} + \frac{1}{2\pi}\mathcal{P}(S_0)\mathbb{E}\{\mathcal{P}(S)\})$ .

## Proof of Theorem 3

2

Let the stationary target be located at any arbitrary point  $R_0 \in \mathcal{R}$ . Let  $R_0 + S_0(x)$  represents the disk centered at  $R_0$  with a shape defined by  $S_0(x)$  with a radius of x. Let the distance between  $R_0$  and the closest (to  $R_0$ ) point covered by the sensor network at time  $nT_s$  be  $X^1(nT_s)$ . Then the probability of  $X^1(nT_s) > x$  is equivalent to,

$$Pr(X^{1}(nT_{s}) > x) = Pr(no \ set \ intersects \ the \ disk$$
$$R_{0} + S_{0}(x)at \ time \ nT_{s})$$
$$- e^{-\lambda(\mathcal{A}(S_{0}(x)) + \mathbb{E}\{\mathcal{A}(S(nT_{s}))\} + \frac{1}{2\pi}\mathcal{P}(S_{0}(x))\mathbb{E}\{\mathcal{P}(S(nT_{s}))\})}$$
(12)

where the last step is obtained by applying theorem 8. In our case,  $\mathcal{A}(S_0(x)) = \pi x^2$ ,  $\mathbb{E}\{\mathcal{A}(S(nT_s))\}$  equals to  $\pi r^2 + 2\lambda_m r \bar{v} n T_s$  if  $nT_s \leq T_0$  and  $\pi r^2 + 2\lambda_m r \bar{v} T_0$  if  $nT_s > T_0$ ,  $\mathcal{P}(S_0(x)) = 2\pi x$  and  $\mathbb{E}\{\mathcal{P}(S(nT_s))\}$  equals to  $2\pi r + 2\lambda_m \bar{v} n T_s$  if  $nT_s \leq T_0$  and  $2\pi r + 2\lambda_m \bar{v} T_0$  if  $nT_s > T_0$ . Hence the mean distance  $\bar{X}^1(nT_s)$  equals to

$$\bar{X}^1(nT_s) = \mathbb{E}\{X^1(nT_s)\} = \int_0^\infty Pr(X^1(nT_s) > x)dx$$

$$= \begin{cases} \frac{1}{\sqrt{\lambda}} e^{\frac{1}{\pi}\lambda\lambda_m^2 \bar{v}^2 n^2 T_s^2} Q\left(\sqrt{\frac{\lambda}{2\pi}} (2\pi r + 2\lambda_m \bar{v}nT_s)\right), & \text{if } nT_s \leq T_0 \\ \frac{1}{\sqrt{\lambda}} e^{\frac{1}{\pi}\lambda\lambda_m^2 \bar{v}^2 T_0^2} Q\left(\sqrt{\frac{\lambda}{2\pi}} (2\pi r + 2\lambda_m \bar{v}T_0)\right), & \text{if } nT_s > T_0 \end{cases}$$
(13)

where the last step results by using (12). The upper bounds in (4) for  $\bar{X}^1(nT_s)$  (for  $nT_s \leq T_0$  and  $nT_s > T_0$ ) are obtained by applying the upper bound for the *Q*-function,  $Q(x) \leq \frac{1}{2}e^{-\frac{x^2}{2}}$  in (13).

#### APPENDIX D

#### **PROOF OF THEOREM 5**

Let  $\tau_k$  be the random variable which represents the time until the target is first detected by the hybrid sensor network with k-sensing. Then  $Pr(\tau_k > t)$  is given by,

$$\begin{aligned} Pr(\tau_k > t) &= Pr(the \ target \ is \ not \ detected \ by \\ k - sensing \ until \ time \ t) \\ &= \sum_{i=0}^{k-1} \frac{[\lambda(\pi r^2 + 2\lambda_m \bar{v}rt)]^j}{j!} e^{-\lambda(\pi r^2 + 2\lambda_m \bar{v}rt)}, \end{aligned}$$

Then we have,

$$\begin{split} \bar{\tau}_k &= \int_0^\infty \sum_{j=0}^{k-1} \frac{[\lambda(\pi r^2 + 2\lambda_m \bar{v}rt)]^j}{j!} e^{-\lambda(\pi r^2 + 2\lambda_m \bar{v}rt)} dt \\ &= \sum_{j=0}^{k-1} \frac{\lambda^j}{j!} e^{-\lambda \pi r^2} \int_0^\infty [\pi r^2 + 2\lambda_m \bar{v}rt]^j e^{-2\lambda \lambda_m \bar{v}rt} dt \\ &= \frac{e^{-\lambda \pi r^2}}{2\lambda \lambda_m \bar{v}r} \sum_{j=0}^{k-1} (\lambda \pi r^2)^j \sum_{i=0}^j \frac{1}{(j-i)!} \left(\frac{1}{\pi r^2}\right)^i, \end{split}$$

where we have used the integral identity  $\int_0^\infty x^{i-1}e^{-x}dx = \Gamma(i) = (i-1)!$  for an integer *i* where  $\Gamma(.)$  is the Gamma function.

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