Throughput Optimization in Multi-hop Wireless Networks with Multi-packet Reception and Directional Antennas

J. Crichigno*,†, M. Y. Wu‡, S. K. Jayaweera†, W. Shu†
*Engineering Department, Northern New Mexico College, Espanola, NM - USA
†Electrical & Computer Engineering Dept., University of New Mexico, Albuquerque - NM, USA
‡Dept. of Computer Science & Engineering, Shanghai JiaoTong University, Shanghai, China
†{jcrichigno, jayaweera, shu}@ece.unm.edu, ‡wu-my@cs.sjtu.edu.cn

Abstract—Recent advances in the physical layer have enabled the simultaneous reception of multiple packets by a node in wireless networks. We address the throughput optimization problem in wireless networks that support multi-packet reception (MPR) capability. The problem is modeled as a joint routing and scheduling problem, which is known to be NP-hard. The scheduling subproblem deals with finding the optimal *schedulable sets*, which are defined as subsets of links that can be scheduled or activated simultaneously. We demonstrate that any solution of the scheduling subproblem can be built with $|E|+1$ or fewer schedulable sets, where $|E|$ is the number of links of the network. This result is in contrast with previous works that stated that a solution of the scheduling subproblem is composed of an exponential number of schedulable sets. Due to the hardness of the problem, we propose a polynomial time scheme based on a combination of linear programming and approximation algorithm paradigms. We illustrate the use of the scheme to study the impact of design parameters on the performance of MPR-capable networks, including the number of transmit interfaces, the beamwidth and the receiver range of the antennas.

Index Terms—Wireless networks, multi-hop, multi-packet reception, directional antennas, NP-hard.

1 Introduction

Recent advances in multiuser detection techniques open up new opportunities for resolving collisions at the physical layer. These techniques permit the simultaneous reception of multiple packets by a node, which in turn increases the capacity of wireless networks [1]. However, to fully exploit multi-packet reception (MPR) capability, new architectures and protocols should be devised. These new schemes need to reformulate a historically underlying assumption in ad hoc wireless networks which states that any concurrent transmission of two or more packets results in a collision and failure of all packet receptions [2].

Recently, researchers started focusing on theoretical upper and lower bounds on the throughput of MPR-capable wireless networks [1], [3], [4]. Garcia-Luna-Aceves et al. [1] demonstrated that architectures exploiting MPR capability increase the capacity of random wireless networks by a logarithmic factor with respect to the Protocol model of Gupta et al. [5]. Subsequent work considered alternative schemes to compute asymptotic bounds on the throughput capacity under some homogeneous assumptions, such as nodes transmit to a single base station or access point [2], or nodes are equipped with a single omni-directional antenna[1] [6].

In this paper, we present a generalized model for the throughput optimization in MPR-capable wireless networks, where nodes have one or more transmit antennas. The problem is known to be NP-hard, since finding the cardinality of the maximum independent set of a graph is equivalent to finding the optimal throughput in a wireless network [7]. The model divides the problem into two subproblems: routing and scheduling. For the scheduling subproblem, we demonstrate that any solution can be built with $|E|+1$ or fewer schedulable sets, where E is the set of links of the network, $|\cdot|$ denotes cardinality, and links in the same schedulable set can be simultaneously activated. This result is in contrast with a conjecture that states that a solution to the scheduling subproblem is composed of an exponential number of schedulable sets [7]. We finally propose a polynomial time scheme for the joint routing and scheduling problem.

The paper is organized as follows. Section 2 discusses related work. Section 3 presents the antenna and channel models used in the paper. Section 4 formulates the throughput optimization problem in MPR networks as a joint routing and link scheduling problem, and Section 5 presents a polynomial time scheme to approximate to the optimal solution. Section 6 shows performance studies, and Section 7 concludes our work.

2 Related Work

Scheduling problem. The throughput optimization problem in wireless networks can be seen as an extension of the maximum flow (max-flow) problem, where at

1. Antenna and interface are used interchangeably hereafter.

any time only a subset of links may be simultaneously scheduled or activated. Brar et al. [8] presented a greedy algorithm for the scheduling problem under the physical model [5]. Moscibroda et al. [9] proposed a centralized scheduling algorithm for scenarios where the traffic demands are the same on every network link.

Joint routing and scheduling. Jain et al. [7] presented a max-flow linear program for computing upper and lower bounds on the optimal throughput under the protocol model. The scheme requires to find all independent sets in the conflict graph, which is intractable. Kodialam et al. [10] proposed a polynomial time approximation algorithm for the routing problem, and a *graph-coloring* approach for the scheduling problem. Zhang et al. [11] presented a *column generation* approach to iteratively solve the joint routing and scheduling problem.

Scheduling with directional antennas. Spyropoulus et al. [12] formulated the scheduling problem as a series of maximal-weight matching in a graph. Cain et al. [13] described a distributed TDMA scheduling protocol, while Capone et al. [14] presented a max-flow formulation which results in an integer linear program.

Scheduling, routing and MPR. Garcia-Luna-Aceves et al. [1] demonstrated that MPR increases the order of capacity of random wireless networks by a logarithmic factor with respect to the protocol model [5]. The same authors [3] demonstrated that throughput is also improved with respect to the physical model [5]. Wang et al. [6] proposed a max-flow ILP formulation that considers MPR capability, and a centralized heuristic algorithm that jointly performs routing and scheduling.

3 Antenna and Channel Models

In this section we present the models used for the antenna and the channel. For an extended overview of these models, please refer to Section 1, supplemental material. We represent a wireless network as a directed graph $G = (V, E)$, where V is the set of nodes and E the set of links. The existence of a directed link $e = (i, j) \in E$ from node $i \in V$ to node $j \in V$ is determined by the antenna model. From here on, we will use the notations (i, j) and e in an interchangeable manner. However, the first notation will be used when the endpoints of a link must be specified.

3.1 Antenna Model

The antenna model considered in this paper is the one used in previous work including [15]. Sidelobes and backlobes are ignored. The interference region of an antenna is principally determined by its main lobe and a simplified radiation pattern does not substantially change the result of the throughput analysis [15]. We assume that: i) all nodes in the network lie in a two-dimension plane, so that the gain of the antenna is a function of the azimuth angle only; ii) the gain of the main lobe is constant (greater than zero), and zero outside it. The main lobe is characterized by the beamwidth β of the antenna; iii) the axis of the main lobe, namely the boresight, can be directed to only one direction at a time. Fig. 1(a) shows the radiation pattern model, where α represents the angle between the boresight of the transmit antenna and the direction of a potential receiver node. R represents the receiver range of the receiver node. The conditions for successfully receiving a packet can be stated as follows. A node j can successfully decode a packet sent by a node i if and only if (iff):

$$r_{ij} \leq R, \tag{1}$$

$$\frac{-\beta}{2} \leq \alpha_{ij} \leq \frac{\beta}{2}, \tag{2}$$

where r_{ij} is the distance between nodes i and j, and α_{ij} represents the angle between the boresight of the transmit antenna at sender node i and the direction to receiver node j. Eq. (1) requires the sender node being inside the receiver range. We will say that there exists a link $(i, j) \in E$ iff this condition is satisfied. Eq. (2) states that the receiver node must lie inside the main lobe of the radiation pattern of the transmit antenna.

3.2 Channel Model

For MPR capable nodes, the MPR protocol model [1], [3], [6] states that the reception of up to K packets sent inside a disk of radius R from the receiver is achievable if the the number of simultaneous transmitters is K or less. Transmitters outside of radio R do not affect the reception of the receiver node. We assume a single-channel Additive White Gaussian Noise (AWGN) channel with bandwidth W, where the capacity of a link (i, j) is modeled by the following equations:

$$SINR_{ij} = \begin{cases} \zeta \cdot r_{ij}^{-\gamma} & \text{if } \frac{-\beta}{2} \leq \alpha_{ij} \leq \frac{\beta}{2} \\ 0 & \text{otherwise,} \end{cases} \tag{3}$$

$$c_{ij} = W \log_2(1 + SINR_{ij}). \tag{4}$$

Eq. (3) states that the signal-to-interference-plus-noise ratio ($SINR_{ij}$) decays exponentially according to the distance r_{ij} between nodes i and j, and it is zero if receiver node j is outside of the main lobe of the radiation pattern of the transmit antenna. γ is the path loss exponent, and ζ is a term that depends on multiple factors such as path loss model and decoding technology. We simply consider ζ as an opaque value, which can be computed as desired. Eq. (4) is the Shannon capacity.

We will assume that nodes can simultaneously decode up to K packets, and that they are equipped with $M \geq 1$ transmit antennas, unless otherwise is explicitly specified. The notation (M, K, β)-network will refer to a network with M transmit antennas with beamwidth β per node, where nodes have a decoding capability of K.

3.3 Scheduling in an (M, K, β)-network

A *schedulable set* $S \subseteq E$ is a set of links which can be scheduled simultaneously. The set S can be characterized by a *schedulable vector* $\vec{S}$ of a vector space $\{0, 1\}^n$. The j^{th} element of this vector is set to one if the link $e_j \in E$ is a member of $\vec{S}$, and to zero otherwise.

In polyhedral combinatorics, a schedulable vector as defined above is also known as characteristic vector [16]

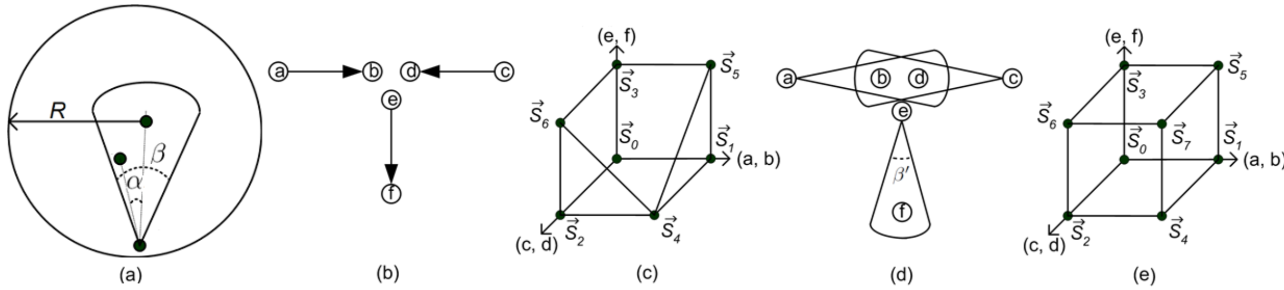

Fig. 1. (a) Radiation pattern model. (b) A wireless network where links (a, b), (c, d) and (e, f) interfere with each other. (c) Schedulable vectors and corresponding convex hull for the network in (b), assuming a (1, 2, 2π)-network. The schedulable vectors are: $\vec{S_0} = (0, 0, 0)$, $\vec{S_1} = (1, 0, 0)$, $\vec{S_2} = (0, 1, 0)$, $\vec{S_3} = (0, 0, 1)$, $\vec{S_4} = (1, 1, 0)$, $\vec{S_5} = (1, 0, 1)$ and $\vec{S_6} = (0, 1, 1)$. (d) A schedulable set for the network in (b), assuming a (1, 2, β')-network, and (e) corresponding schedulable vectors and convex hull, where vertex $\vec{S_7} = (1, 1, 1)$ is added.

or incidence vector [17]. A characteristic vector uniquely identifies a subset of a given ground set. In this paper, the ground set is the set of links E, and a subset is a schedulable set S. Since a schedulable vector cannot be formed as a convex combination of other schedulable vectors, it can be regarded as a vertex of the convex hull of the set of schedulable vectors [17]. The following example illustrates these concepts.

Example 1. Consider the network topology of Fig. 1(b), where $E = \{(a, b), (c, d), (e, f)\}$ and the links conflict with each other. For a (1, 2, 2π)-network, links can be scheduled individually, or they can be combined in groups of two (because $K = 2$). Fig. 1(c) shows the corresponding schedulable vectors and convex hull. Now, if nodes are endowed with directional antennas so that the network becomes a (1, 2, β')-network, a feasible schedulable set is shown in Fig. 1(d), which *enlarges* the convex hull as depicted in Fig. 1(e).

To define the feasibility conditions for scheduling, we need the following terminology. Let $\delta^+(i) \subseteq S$ denote the set of links in S which have node i as the transmitter node. Similarly, let $\delta^-(j) \subseteq S$ be a set of links such that, $\forall e \in \delta^-(j)$, the corresponding transmitter node of link e, say node i, is within the receiver range of j and schedules its antenna in a direction for which $\frac{-\beta}{2} \leq \alpha_{ij} \leq \frac{\beta}{2}$ (i.e., Eqs. (1) and (2) are satisfied). $\delta^-(j)$ represents the set of links for which node j is within a distance R to the corresponding transmit antennas and lies in the main lobe of them. For an (M, K, β)-network, a set $S \subseteq E$ is a feasible schedulable set if $\forall\, e = (i, j) \in S$:

$$|\delta^+(i)| \leq M, \quad (5)$$
$$|\delta^-(j)| \leq K. \quad (6)$$

The term on the left hand side of Eq. (5) is the number of links having node i as transmitter, which must be less than or equal to M. Eq. (6) states that the receiver node j can decode at most K packets.

4 Problem Formulation

4.1 Routing

We first present the routing and scheduling subproblems separately, and then the joint formulation. Let N be the set of end-to-end flows. Each flow is characterized by a 3-tuple (s_n, d_n, f_n), which denotes the source node, the destination node, and the amount of flow[2] in bits per second (bps) transmitted from s_n to d_n, respectively. f_n can be thought as a constant bit rate flow. Let x^n_{ij} be a variable representing the amount of the n^{th} flow routed on link (i, j). The routing problem is defined in Fig. 2. Eq. (7) is the aggregated throughput or amount of flow (or simply throughput), to be optimized. Eq. (8) represents the flow conservation constraint; for each flow $n \in N$, the amount of flow at each node other than its own source or destination must be zero. Eq. (9) is the capacity constraint. Eqs. (10) and (11) restrict the per-source throughput and the amount of flow on each link to be non-negative. We will refer to Eqs. (7)-(11) as routing linear program (RT-LP), which is sometimes called *sum multicommodity flow* model [18].

$$\max F_{RT\text{-}LP} = \sum_{n \in N} f_n \quad (7)$$

$$\sum_{j:(i,j)\in E} x^n_{ij} - \sum_{j:(j,i)\in E} x^n_{ji} = \begin{cases} f_n; i = s_n \\ -f_n; i = d_n \\ 0; \text{otherwise} \end{cases} \quad (8)$$
$$n \in N$$

$$\sum_{n \in N} x^n_{ij} \leq c_{ij}; (i, j) \in E \quad (9)$$

$$f_n \geq 0; n \in N \quad (10)$$

$$x^n_{ij} \geq 0; n \in N, (i, j) \in E \quad (11)$$

Fig. 2. Routing linear program (RT-LP).

4.2 Scheduling

A schedule specifies the schedulable sets and the fraction of time allocated to each of them. Let $\Gamma = \{S_1, S_2, ..., S_{|\Gamma|}\}$ be a set of *all possible schedulable sets* which satisfy Eqs. (5) and (6) in an (M, K, β)-network. Let $\lambda_k, 0 \leq \lambda_k \leq 1$, be a fraction of time allocated

2. Although a flow is characterized by (s_n, d_n, f_n), we will also use the term flow to informally refer to f_n.

to the set S_k. We may write the time interval $[0,1]$ as $\cup_k[t_k, t_{k+1}]$, where links in S_k are active for the activity period $t_{k+1} - t_k = \lambda_k$, $k \in \{1,2,...,|\Gamma|\}$, $t_1 = 0$ and $t_{|\Gamma|+1} = 1$. The variable λ_k represents the *activity period variable* corresponding to set S_k. The schedule restriction can be written as:

$$\sum_{k=1}^{|\Gamma|} \lambda_k = 1. \tag{12}$$

Eq. (12) states that the sum of the fraction of time allocated to all schedulable sets must be one. The scheduling subproblem requires reconsidering Eq. (4), which models the capacity of a link that is active at all time. In the scheduling subproblem, the amount of flow through a link must not exceed its capacity given by Eq. (4) multiplied by the fraction of time the link is active:

$$\sum_{n \in N} x_{ij}^n \leq \sum_{\forall k \in \{1,2,...,|\Gamma|\} | (i,j) \in S_k} \lambda_k c_{ij}. \tag{13}$$

4.3 Joint Routing and Scheduling Problem

In general, only a small subset $\Gamma' \subseteq \Gamma$ is needed to evaluate Eqs. (12) and (13), as we shall see in *Proposition 2*. By incorporating Eqs. (12) and (13) into RT-LP and optimizing not only over the variables given by Eqs. (10) and (11) but also over all possible set $\Gamma' \subseteq \Gamma$ of schedulable sets, the formulation of the joint routing and scheduling problem is given in Fig. 3.

$$\max_{\Gamma' \subseteq \Gamma} F_{RTSCH\text{-}LP} = \sum_{n \in N} f_n \tag{14}$$

$$\sum_{j:(i,j)\in E} x_{ij}^n - \sum_{j:(j,i)\in E} x_{ji}^n = \begin{cases} f_n; \text{if } i = s_n \\ -f_n; \text{if } i = d_n \\ 0; \text{otherwise} \end{cases} \quad n \in N \tag{15}$$

$$\sum_{n \in N} x_{ij}^n \leq \sum_{\forall k \in \{1,2,...,|\Gamma'|\} | (i,j) \in S_k} \lambda_k c_{ij}; \quad (i,j) \in E \tag{16}$$

$$\sum_{k=1}^{|\Gamma'|} \lambda_k = 1 \tag{17}$$

$$f_n \geq 0; n \in N \tag{18}$$

$$x_{ij}^n \geq 0; n \in N, (i,j) \in E \tag{19}$$

$$\lambda_k \geq 0; k \in \{1,2,...,|\Gamma'|\} \tag{20}$$

Fig. 3. Routing & scheduling linear program (RTSCH-LP).

The complexity of the routing and scheduling linear program (RTSCH-LP) is mainly determined by the scheduling subproblem. To find an optimal solution, RTSCH-LP requires searching for a set of optimal schedulable vectors in $\{0,1\}^{|E|}$, which is exponentially large in $|E|$. The hardness of the problem, alternatively, can be proved by noting that the throughput optimization problem under the Protocol model, for omni-directional antenna networks without MPR-capability [7], is a particular case of the of the joint routing and scheduling problem in an (M, K, β)-network (i.e., it can be reduced to RTSCH-LP). Since the former is an NP-hard problem, RTSCH-LP is also NP-hard.

The goal of exploring space $\{0,1\}^{|E|}$ in the context of RTSCH-LP is to obtain the best set of schedulable vectors, so that Eq. (14) is maximized. An optimal solution would require allocating a fraction of time to each corresponding schedulable set. Jain et al. [7] formalized the above observation for the throughput optimization problem under the Protocol model, for omni-directional antenna networks without MPR capability. We can extend this observation for an (M, K, β)-network. As showed in Section 3.3, a schedulable set S can be characterized by its schedulable vector $\vec{S}$, which becomes an extreme point of the convex hull of the set of schedulable vectors. Let $\vec{u} = (u_1, u_2, ..., u_{|E|})$ be an $|E|$-dimensional *utilization vector*, where u_k indicates the fraction of time allocated to link $e_k \in E$. When using the notation (i,j) instead of e_k for a link $e_k = (i,j) \in E$, we will refer to the corresponding link utilization as u_{ij}. By regarding $\vec{u}$ as a point in $\{0,1\}^{|E|}$, we have the following proposition:

Proposition 1: A solution to the scheduling subproblem given by a set $\Gamma' = \{S_1, S_2, ..., S_{|\Gamma'|}\} \subseteq \Gamma$ with corresponding activity periods $\lambda_1, \lambda_2, ..., \lambda_{|\Gamma'|}$ is feasible iff the resulting utilization vector $\vec{u}$ lies within the convex hull of the schedulable vectors.

Proof: $\Rightarrow$ Assume a feasible solution, with a set $\Gamma' = \{S_1, S_2, ..., S_{|\Gamma'|}\}$ and corresponding activity periods $\lambda_1, \lambda_2, ..., \lambda_{|\Gamma'|}$. Then $\vec{u}$ must be of the form $\vec{u} = \sum_{i=1}^{|\Gamma'|} \lambda_i \vec{S_i}$. By definition, the convex hull of all schedulable vectors, denoted by $Co(\Gamma)$, is the set of all convex combinations of all possible schedulable vectors:

$$Co(\Gamma) = \{\theta_1 \vec{S_1} + ... + \theta_{|\Gamma|} \vec{S_{|\Gamma|}}, \text{ for all } S_i \in \Gamma, \theta_i \geq 0, \theta_1 + ... + \theta_{|\Gamma|} = 1\}. \tag{21}$$

Note that the utilization vector $\vec{u}$ is a convex combination of the schedulable vectors corresponding to the sets in Γ', where the *weights* are given by the activity periods $\lambda_1, \lambda_2, ..., \lambda_{|\Gamma'|}$. Since $\Gamma' \subseteq \Gamma$, $\vec{u}$ is a particular point that satisfies Eq. (21) and therefore lies inside $Co(\Gamma)$.

$\Leftarrow$ Assume that $\vec{u}$ lies within $Co(\Gamma)$. Then, $\vec{u}$ can be expressed as a convex combination of a set of schedulable vectors, which have a corresponding set $\Gamma' = \{S_1, S_2, ..., S_{|\Gamma'|}\}$. By allocating λ_i seconds to $S_i \in \Gamma'$ (the schedulable set that has a corresponding schedulable vector $\vec{S}$), we can build a feasible schedulable, which implies that $\vec{u}$ is feasible.

Proposition 1 implies that any convex combination of schedulable vectors is schedulable. Since there is an exponential number of schedulable sets, a solution to the scheduling subproblem might be composed of $\Theta\left(2^{|E|}\right)$ schedulable sets, as claimed by Jain et al. [7] for the scheduling problem without multi-packet reception and with omni-directional antennas only. *Proposition 2*, however, shows that it is not necessary to use an exponential number of schedulable sets.

Proposition 2: Any utilization vector $\vec{u}$ can be represented as a convex combination of $|E| + 1$ or fewer schedulable vectors from $Co(\Gamma)$.

Proof: we can apply Caratheodorys's Theorem on convex sets [19]. Let $\vec{u} = \sum_{k=1}^{|\Gamma'|} \lambda_k \vec{S_k}$ be an utilization vector corresponding to a set of schedulable sets $\Gamma' \subseteq \Gamma$, where $|\Gamma'| > |E| + 1$. Denote the i^{th} scalar component of the schedulable vector $\vec{S_k}$ as S_{ki}. Then, for any $e_i \in E$, the component u_i of $\vec{u}$ can be computed as $u_i = \sum_{k=1}^{|\Gamma'|} \lambda_k S_{ki}$, which can be formulated as:

$$\sum_{k=1}^{|\Gamma'|} \lambda_k S_{ki} = u_i; e_i \in E \quad (22)$$

$$\sum_{k=1}^{|\Gamma'|} \lambda_k = 1; \quad (23)$$

$$\lambda_k \geq 0; k \in \{1, 2, ..., |\Gamma'|\} \quad (24)$$

Fig. 4. Feasibility linear program.

Every feasible linear program has a basic feasible solution [20]. In a basic feasible solution, only the basic variables are nonzero. The linear program of Fig. 4 has $|E| + 1$ basic variables, one for each equality constraint. Hence, the utilization vector obtained from this basic feasible solution corresponds to a convex combination of just $|E| + 1$ schedulable vectors of the original $|\Gamma'|$ vectors, which demonstrates *Proposition 2*.

Example 2. Consider the network of Fig. 1(a) and the corresponding convex hull of Fig. 1(d). Let $\Gamma'_1 = \{S_1, S_2, S_3, S_4, S_5, S_6\}$ be a schedule with corresponding allocation times $\lambda_i = \frac{1}{6}$ for all $i \in \{1, 2, ..., 6\}$. The schedule produces an utilization vector $\vec{u} = \sum_{i=1}^{6} \lambda_i \vec{S_i} = (\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$, activating each link for $\frac{1}{2}$ seconds. According to *Proposition 2*, we can build a schedule with no more than $|E| + 1 = 4$ schedulable sets that produces the same utilization vector. A possible solution is the set $\Gamma'_2 = \{S_0, S_7\}$ with corresponding allocation times of $\lambda_0 = \lambda_7 = \frac{1}{2}$. The allocation of $\frac{1}{2}$ seconds to S_0 means that there are more resources (i.e., time) than needed by the links, and the network is idle for half of the time. Note also that $|\Gamma'_2| = 2 < |E|+1$. In general, however, the number of schedulable sets may be $|E|+1$. For example, referring back to 1(a) but assuming that only one link can be active at any time and that an utilization vector $\vec{u} = (\frac{1}{4}, \frac{1}{4}, \frac{1}{4})$ is to be achieved, the only set that produces the desired $\vec{u}$ is $\Gamma'_3 = \{S_0, S_1, S_2, S_3\}$ with corresponding $\lambda_i = \frac{1}{4}$ for $i \in \{0, 1, 2, 3\}$ (i.e., $|\Gamma'_3| = |E| + 1$).

5 JRS Scheme

We present a polynomial time Joint Routing and Scheduling (JRS) scheme for the problem presented in Section 4. The scheme consists of three steps:

1) Solve RT-LP.
2) Create a set $\Gamma' \in \Gamma$ by using an approximation algorithm. To obtain a polynomial running time, the approximation algorithm guarantees that the number of schedulable sets found during the searching process is upper-bounded by $|E|$.
3) Solve RTSCH-LP.

Step 1

This step is intended to identify *good* paths for each flow, such that the $F_{RT\text{-}LP}$ is maximized. The output of the step 1 is the set of links which are assigned a positive amount of flow value by RT-LP, namely $E_{RT\text{-}LP} = \{(i, j) \in E | \sum_{n \in N} x_{ij}^n > 0\}$.

Step 2

Given that step 1 may produce an unfeasible solution, since RT-LP ignores the feasibility conditions for scheduling imposed by Eqs. (5) and (6), step 2 finds feasible schedulable sets for $E_{RT\text{-}LP}$. The algorithm schedules all the links in $E_{RT\text{-}LP}$ by finding a *small* number of maximal schedulable sets, so that they can be found in polynomial time. A maximal set S is defined as a schedulable set, such that, when all links in S are activated, no more links can be activated without violating the scheduling constraints.

The utilization of a link (i, j) defined in Section 4.3 can be expressed as $u_{ij} = \frac{\sum_{n \in N} x_{ij}^n}{c_{ij}}$, where $\sum_{n \in N} x_{ij}^n$ is the amount of flow through link (i, j) resulting from step 1. The scheduling algorithm schedules links so that every link (i, j) can send the amount of flow $\sum_{n \in N} x_{ij}^n$ during a schedule period. The algorithm schedules the links one by one, in an arbitrary order, until a maximal set S_1 is obtained. The fraction of time allocated to S_1 is equal to the minimum link utilization among all links in S_1, i.e.,

$$\tau_1 = \min\{u_{ij} | (i, j) \in S_1\}. \quad (25)$$

Having been scheduled for this fraction of time, the link $(i, j) = \text{argmin}\{u_{ij} | (i, j) \in S_1\}$ can send the amount of flow $u_{ij} \cdot c_{ij} = \sum_{n \in N} x_{ij}^n$. Thus, it is then removed from S_1 and is not considered for any subsequent set. The remaining links form a new set S_2, which becomes maximal by adding new links not scheduled yet. Successive sets are found iteratively in a similar way. Consider a general iteration when S_k is being created, and sets $S_1, ..., S_{k-1}$ have already been built. Define the residual link utilization of a link (i, j) as:

$$u'_{ij} = u_{ij} - \sum_{\forall k' \in \{1,2,...,k-1\} | (i,j) \in S_{k'}} \tau_{k'}. \quad (26)$$

For a general iteration k, Eq. (25) can be rewritten as:

$$\tau_k = \min\{u'_{ij} | (i, j) \in S_k\}. \quad (27)$$

The process is repeated until all link $(i, j) \in E_{RT\text{-}LP}$ have been scheduled for a fraction of time u_{ij}.

The pseudo-code of the scheduler is shown in Fig. 5. In lines (8)-(11), links are added one by one until a maximal set is created. In line 16, the link with minimum residual utilization is removed from S_k, so that a new maximal set is created in the next iteration. For an operation example of the scheduler algorithm, please refer to Section 2, supplemental material.

The throughput of the schedule produced by scheduler algorithm can be computed as follows. Let $\tau = \sum_k \tau_k$ be the schedule period, where τ_k is the fraction of

Scheduler

INPUT: $E_{RT\text{-}LP}$, $G(V,E)$
OUTPUT: Set Γ' of schedulable sets.

$u'_{ij} = u_{ij}, \forall (i,j) \in E_{RT\text{-}LP}$;
$\Gamma' = \{\}$; $S_0 = \{\}$; $t = 0$; $k = 0$;
while $(\exists\ (i,j) | u'_{ij} > 0)$ **do**
  $k = k + 1$;
  $S_k = S_{k-1}$;
  while $(\exists\ (i,j) \in E_{RT\text{-}LP} | S_k \cup \{(i,j)\}$ is a schedulable set$)$ **do**
    $S_k = S_k \cup \{(i,j)\}$;
    $E_{RT\text{-}LP} = E_{RT\text{-}LP} - \{(i,j)\}$;
  end while
  $\Gamma' = \Gamma' \cup S_k$;
  $e = \text{argmin}\{u'_e | e \in S_k\}$;
  $\tau_k = u'_e$;
  $u'_{ij} = u'_{ij} - u'_e, \forall (i,j) \in S_k$;
  $S_k = S_k - \{e\}$;
end while
return Γ';

Fig. 5. Scheduler algorithm.

time allocated to S_k (Eq. (27)). Then, the throughput is: $F = \frac{F_{RT\text{-}LP}}{\text{schedule period}} = \frac{F_{RT\text{-}LP}}{\tau}$. For particular network topologies, we have the following proposition.

Proposition 3: Let $E' = \{e_1, e_2, ..., e_{|E'|}\}$ be the set of links to be scheduled by the scheduler algorithm, and $u_1, u_2, ..., u_{|E'|}$ be the corresponding link utilizations. In a fully-connected $(M, K, 2\pi)$-network with $M \geq K$, and in a single-hop MPR-capable network where the transmissions are directed to a central node, the schedule period is guaranteed to be factor two optimal.

Proof: for the two types of network topologies referred in the proposition, K links can be simultaneously scheduled. This is illustrated in Fig. 6, where there is a *timeline* for each scheduled link. Let e_f be the link with the larger completion time (i.e., link e_f is scheduled until the completion time of the whole schedule), and $start_f$ be the time e_f is scheduled. Since the algorithm creates maximal sets, all the timelines are *busy* until $start_f$ (otherwise, the schedulable set prior to $start_f$ would not be maximal, and the algorithm would have already scheduled e_f). From this observation, it follows that $start_f$ is less than or equal to the *average timeline*:

$$start_f \leq \frac{1}{K} \sum_{e \in E'} u_e. \tag{28}$$

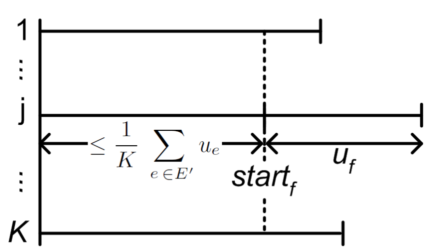

Fig. 6. Operation of the scheduling algorithm. Since it creates maximal schedulable sets, K links are always active until at $t = start_f$, the time the link with larger completion time is scheduled. This implies that $start_f \leq \frac{1}{K} \sum_{e \in E'} u_e$.

The average timeline, on the other hand, can be considered as a lower bound of any schedule period; i.e., the optimal schedule period τ_{OPT} is at least the total duration of all link utilizations divided by K. In addition, τ_{OPT} is at least equal to the largest link utilization. These two lower bounds can be expressed as:

$$\frac{1}{K} \sum_{e \in E'} u_e \quad \leq \quad \tau_{OPT}, \tag{29}$$

$$\max_{e \in E'} \{u_e\} \quad \leq \quad \tau_{OPT}. \tag{30}$$

By combining Eqs. (29) and (30) with Eq. (28), we can express the schedule period τ of the algorithm as:

$$\tau = start_f + u_f \leq 2\tau_{OPT}, \tag{31}$$

which proves Proposition 3.

Although feasible, this solution may not be optimal with respect to Γ'. The optimal solution can be found by applying step 3.

Step 3

The last step solves RTSCH-LP, which produces the optimal solution with respect to Γ' (i.e., the best convex combination of the schedulable vectors considering the schedulable sets in Γ' is obtained). The solution of RTSCH-LP gives the amount of flow x^n_{ij} routed through each link (i,j) (which may differ with x^n_{ij} found by RT-LP), and establishes the fraction of time λ_k allocated to each schedulable set S_k.

6 Performance Studies

We present numerical examples based on the scheme presented in Section 5, which was implemented as a solver in C language. For RT-LP and RTSCH-LP, the solver incorporates the package LP-solve [21]. We set $W = 1$, and a link capacity $c_{ij} = 10$ units when the distance r_{ij} between nodes i and j is equal to R (maximum distance from which a node can decode a packet). The path loss exponent γ was set to 4, which corresponds to the two-ray model. Having set these values, any link capacity can be computed according to Eqs. (3) and (4). The results were evaluated in terms of the objective function of the joint routing and scheduling problem (Eq. (14)), and normalized to the upper bound $F_{RT\text{-}LP}$ (Eq. (7)). We simulated 3 random topologies of 50 nodes over a 1000 x 1000 square-meter area, and analyzed the results of the JRS approach. Additional performance studies over a grid topology can be found in Section 3, supplemental material. The first random topology was generated by setting $R = 200$, which produced a network with an average node degree of 10.96. By varying this parameter to $R = 300$ and $R = 400$, subsequent random topologies with average node degrees of 21.92 and 35.7 were generated. Topologies can be seen in Fig. 7, supplemental material. We include results for both (M, K, β)-networks and half-duplex (HD) networks. This comparison permits us to visualize the gain obtained by the former type of networks with respect to the latter, which was proposed in previous work [6]. Ten

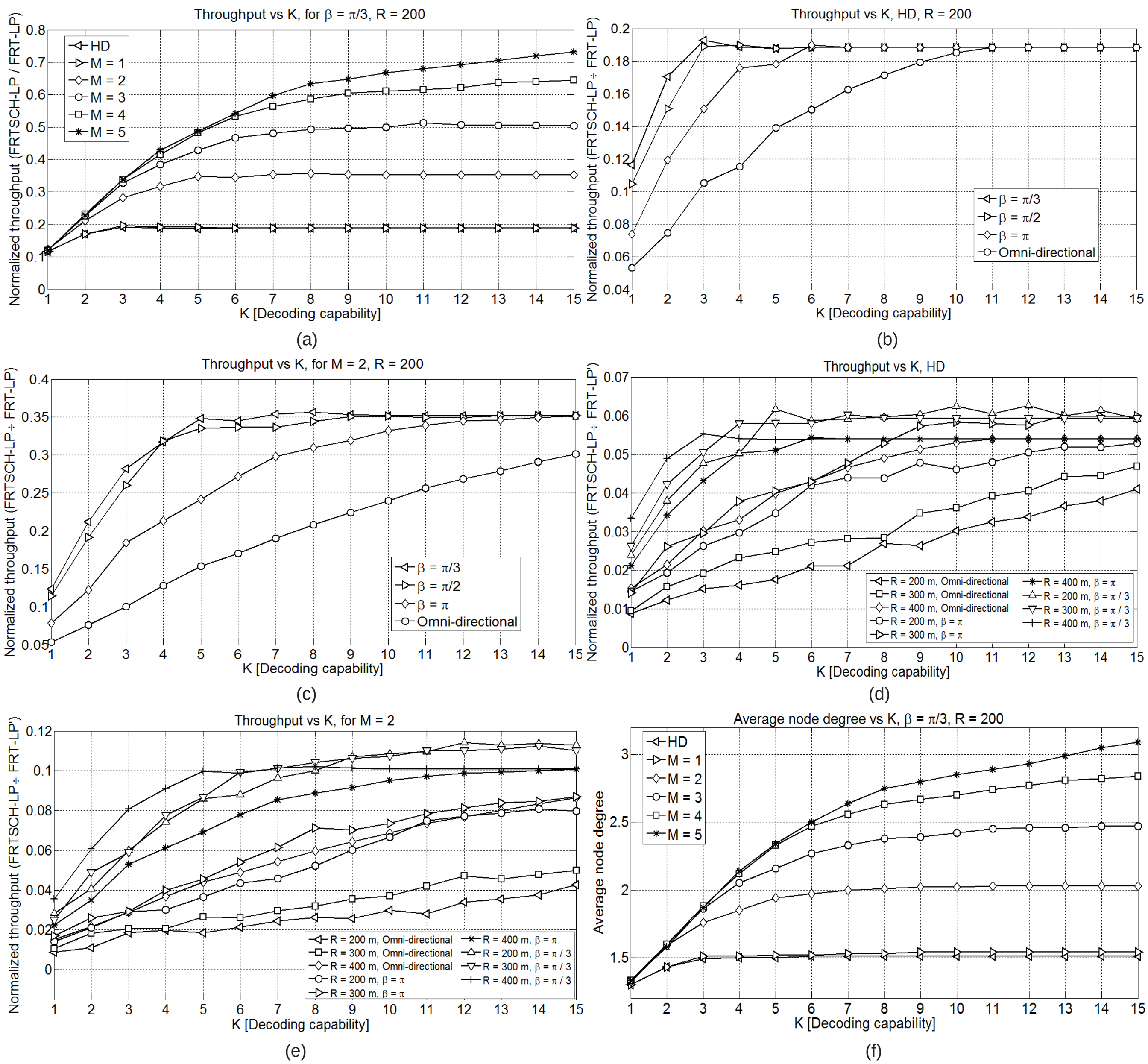

Fig. 7. Numerical results.

flows were created. The source and destination of each flow were randomly selected.

Impact of the Number of Transmit Antennas. Fig. 7(a) shows the throughput as a function of K, for different number of antennas with $\beta = \frac{\pi}{3}$. Note that the throughput for HD and $M = 1$ increases monotonically until $K = 3$, and that further increments of K have no impact. This constitutes a *transmission-oriented bottleneck*; transmitting nodes cannot generate enough beams, even when the decoding capability is increased. Consequently, the number of simultaneously scheduled links is limited by this bottleneck. The curves for HD and $M = 1$ are almost overlapped because they have the same bottleneck: only one interface can operate in transmit mode. By adding more transmit antennas, the MPR capability is better exploited. Note also that the throughput increases approximately linearly until K equals M.

Impact of the Beamwidth. Fig. 7(b) shows the curves of throughput vs K, for HD and different values of β. The best performance is obtained with the minimum beamwidth value ($\beta = \frac{\pi}{3}$), due to a better spatial reuse. However, the disadvantage of having wider beamwidth antennas can be compensated by increasing the MPR capability; for $K \geq 11$, even the use of omni-directional antennas produces an optimal performance. The effect of using two transmit antennas per node is shown in Fig. (7)(c): the maximum throughput increases from about 0.19 to 0.35. Note also that, for both, HD and $M = 2$, and for a given value of K, say 5, a beamwidth $\beta = \frac{\pi}{2}$ produces about the same result as a beamwidth

$\beta = \frac{\pi}{3}$ (a beamwidth of $\frac{\pi}{2}$ is narrow enough for optimal performance; narrower beamwidths would not produce significant improvement).

Impact of the Receiver Range R. Fig. 7(d) shows, for HD, the normalized throughput $F_{RTSCH\text{-}LP}/F_{RT\text{-}LP}'$, where $F_{RT\text{-}LP}'$ is the flow value when $R = 400$. RT-LP produces the maximum upper bound on throughput when R increases, because of a higher connectivity (the average node degrees for R = 200, 300 and 400 meters (m) were 10.26, 21.92 and 35.7 respectively). As a result, the maximum amount of flow that can *potentially* be sent from a source to a destination increases (through multiple paths). However, to achieve this goal, nodes may need to decode a very high number of transmissions. Consider the case for $\beta = \frac{\pi}{3}$ in Fig. 7(d). The throughput is an increasing function of K. For $R = 400$, however, incrementing K beyond 3 does not result on better performances because of the transmission-oriented bottleneck. Similarly, for $R = 300$ and $R = 200$, the throughput also experiences the transmission-oriented bottleneck. Note, however, that increments of K beyond 3 but less than or equal to 6 still produce improvements when $R = 300$ or $R = 200$. The reason of this is the higher number of links that can be simultaneously scheduled in the network. Since each transmission consumes a circular area of radio R and angle β, no more than K - 1 other transmissions can take place in the same sector. Similar results are obtained with $\beta = \pi$. With omni-directional antennas, the same phenomenon would be observed when $K > 15$. As shown in Fig. 7(e), incrementing the number of transmit antennas increases throughput. Let *average node degree* $= \frac{1}{|V|} \sum_{k=1}^{|\Gamma'|} \lambda_k |S_k|$; i.e., the time-averaged number of links per node activated in the network. This metric quantifies the node degree of the network, as links are scheduled. Fig. 7(f) better highlights the transmission-oriented bottleneck for the particular case when $R = 200$ and $\beta = \frac{\pi}{3}$. As M increases, nodes can transmit to multiple receivers, increasing the number of simultaneous transmission. As a consequence, the average node degree and throughput (refer to Figs. 7(d) vs 7(e)) are incremented. However, while potentially 10.26 links might be scheduled per node, the average node degree is at most approximately 3 (when $M = 5$ and $K = 15$). R, K, M, and β play key roles: throughput may increase with R, but the decoding capability should be very high (for example, for $R = 400$, in average, every transmission may affect 35.7 other nodes for omni-directional transmissions; in general the number of nodes covered by a transmission may be proportional to R^2 and to β). To exploit the multiple paths resulting from increasing R, nodes should generate a high number of transmissions.

7 Conclusion

We have presented a generalized model for the throughput optimization problem in wireless networks. We have decoupled the problem into two subproblems, routing and scheduling, and demonstrated that any solution of the scheduling subproblem can be built with $|E| + 1$ or fewer schedulable sets. Because of the hardness of the scheduling subproblem, we have also proposed a polynomial time scheme based on a combination of linear programming and approximation algorithm paradigms. Numerical results showed that to fully exploit MPR capability, nodes may need to be endowed with multiple transmit antennas. The study of increasing the MPR capability of networks with wide beamwidth antennas to achieve similar performance to networks with narrower beamwidths was discussed. Future work includes the application of the multi-access channel model to overcome current model limitations at the physical layer.

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