**Step Response**

The step response measurement is very useful in our analysis. From it, we can derive a curve of impedance versus frequency.

Test setup:

![Diagram](image)

Pulse generator repeatedly applies step input to DUT.

Rules of thumb in characterizing the DUT using the step response:

- Resistors display a flat step response, i.e., at time $t = 0$, the output rises and remains at a fixed value.
Step Response

- Capacitors display a *rising* step response, i.e., at time $t = 0$, the output starts rising and later reaches its full value.

- Inductors display a *sinking* step response, i.e., at time $t = 0$, the output rises instantly to its full value and then later decays back toward 0.

Capacitors and inductors subdivide into *ordinary* and *mutual* categories.

Ordinary capacitance and inductance (two-terminal devices) can be a help or hindrance.

Mutual capacitance and inductance usually creates unwanted crosstalk.
Capacitance

The capacitor is a 2-terminal element in which the branch voltage and current variables are related by integral and differential equations:

\[ v(t) = \frac{1}{C} \int_{t_0}^{t} i(\tau) d\tau + v(t_0) \]  

(Voltage on cap depends on history of i)

and

\[ i(t) = C \frac{dv}{dt} \]

Charge and voltage are related by the linear relationship:

\[ q(t) = Cv(t) \]

Power is negative or positive depending on the value of the term \( v(t) \frac{dv}{dt} \) in the following expression

\[ p(t) = Cv(t) \frac{dv}{dt} \]
**Capacitance**

But energy (the integral of power) is always positive or zero:

\[ w(t) = \frac{Cv^2(t)}{2} \]

Therefore, it’s a passive element (like the resistor) but it is **non-dissipative** (unlike the resistor).

All the energy supplied to the cap. is stored in the electric field.

Note that the voltage appearing across a capacitor must always be a continuous function (voltage steps not allowed -- require an infinite \( i \)).

Current, on the other hand, is allowed to change instantaneously.
Capacitance

The energy stored in the electric field of a capacitor is supplied by the driving circuit.

Since the driving source is a limited source of power, the voltage takes a finite time to build up.

The reluctance of voltage to build up quickly in response to injected power (or decay quickly) is called *capacitance*.

\[
\begin{align*}
X(t) & \quad I(t) \\
Y(t) & \quad Y(t)
\end{align*}
\]

\[R_s = 30\Omega\]

\[\max \text{value } V_{CC}/R\]

\[\text{voltage approaches } V_{CC}\]

\[\frac{Y(t)}{I(t)}\]

\[1 \text{ ns}\]

\[\text{short circuit} \quad \text{open circuit}\]
Capacitance

Bear in mind that a capacitor behaves like an *inductor* at high frequencies (unfortunately).

This is due to the mounting leads on capacitors.

This inductance causes the step response to have a tiny pulse (a couple hundred ps) at time 0, followed by a drop to 0 and then a capacitive ramp.

Note that you will not be able to see this unless your step source rise time is sharp.

At $T_r$, you can characterize the circuit element for frequencies up to:

$$F_A = \frac{0.5}{T_r}$$

Reactance on leading edge (to est. distortion in digital wfm by a cap.):

$$X_C = \frac{T_r}{\pi C}$$
Capacitance Test Gig

A measurement setup ideal for characterizing capacitors:

Note that the resistances are known, so by measuring the rise time of the resulting waveform, the capacitance of the DUT can be computed.

The test gig is dimensioned at 1 square inch to ensure it behaves in a lumped fashion.

The test gig should include a ground plane of 1 square inch.
Inductance

The inductor is a 2-terminal element in which the branch voltage and current variables are related by integral and differential equations:

\[ i(t) = \frac{1}{L} \int_{t_0}^{t} v(\tau)d\tau + i(t_0) \quad \text{and} \quad v(t) = L \frac{di}{dt} \]

Flux linkages (in weber-turns) and current are related by the relationship:

\[ \lambda(t) = n\phi(t) = Li(t) \]

Power is negative or positive depending on the value of the term \( i(t)di/dt \) in the following expression

\[ p(t) = i(t)v(t) = i(t)L \frac{di}{dt} \]
Inductance

But energy (the integral of power) is always positive or zero:

\[ w(t) = \frac{Li^2(t)}{2} \]

Therefore, it’s a passive element (like the resistor) but it is **non-dissipative** (unlike the resistor).

All the energy supplied to the inductor is stored in the magnetic field.

Note that the current flowing through an inductor must always be a continuous function (current steps not allowed -- require an infinite \( v \)). Voltage, on the other hand, is allowed to change instantenously.
Inductance

The energy stored in the magnetic field of a inductor is supplied by the driving circuit.

Since the driving source is a limited source of power, the current takes a finite time to build up.

The reluctance of current to build up quickly in response to injected power (or decay quickly) is called inductance.

\[ X(t) \]
\[ I(t) \]
\[ Y(t) \]
\[ R_s = 30 \, \Omega \]

1 ns

current approaches \( V_{CC}/R \)
voltage output decays to 0.
open circuit short circuit

\( \frac{Y(t)}{I(t)} \)
Inductance Test Gig

A measurement setup ideal for characterizing inductors:

Reactance on leading edge (to est. distortion in digital wfm by an inductive load):

\[ X_L = \frac{\pi L}{T_r} \]
Measuring Inductance

Inductance can be measured in a similar way to capacitance by computing the time, for example, in the response waveform to the 63% point.

A second method for inductance involves computing the area under the response waveform’s curve:

\[
\nu(t) = L \frac{di}{dt} \quad \rightarrow \quad \int_{0}^{\infty} V_{\text{ind}}(t) dt = L \int_{0}^{\infty} \frac{dI_{\text{ind}}(t)}{dt} dt
\]

\[
\int_{0}^{\infty} V_{\text{ind}}(t) dt = L[I(\infty) - I(0)]
\]

\[
\text{area} = L[I(\infty) - I(0)]
\]

\[
L = \frac{\text{area}}{\Delta I} = \frac{(\text{area})R_s}{\Delta V}
\]

where \(R_s\) and \(\Delta V\) are the open circuit response values (see Example 1.2)
Mutual Capacitance

*Mutual capacitance* coupling between two circuits is simply a parasitic capacitor connected between circuit $A$ and circuit $B$.

The coefficient of interaction is in units of farads or **amp-seconds/volt**.

Mutual capacitance $C_M$ injects a current $I_M$ into circuit $B$ proportional to the rate of change of voltage in circuit $A$:

$$I_M = C_M \frac{dV_A}{dt}$$

This simplification works if:

- The coupled current flowing in $C_M$ is much smaller than the primary signal current in circuit $A$, i.e. $C_M$ does not load circuit $A$.
- The coupled **signal** voltage in circuit $B$ is small and can be ignored. Therefore, the voltage difference between $A$ and $B$ is just $V_A$.
- The mutual capacitance represents a large impedance compared to the impedance to ground of circuit $B$. 

Mutual Capacitance
A more accurate model uses the difference in voltages between circuits $A$ and $B$ and the loading effect of $C_M$ on both circuits.

When the coupled noise voltage (*crosstalk*) is less than 10% of the signal step size (on $A$), this approximation is accurate to one decimal place.

Given:
• $C_M$ is known.
• The rise time $T_r$ and voltage step magnitude $V_A$ are known.
• The impedance in the receiving circuit, $R_B$, to ground is known.
Then *crosstalk* can be estimated as a fraction of the driving wfm $V_A$.

First derive the maximum change in voltage/time of wfm $V_A$:
\[
\frac{dV_A}{dt} = \frac{\Delta V}{T_r}
\]
where $\Delta V$ is the step height of $V_A$. 
Mutual Capacitance

Second, compute the mutual capacitive current which flows from circuit A to circuit B using:

\[ I_M = C_M \frac{\Delta V}{T_r} \quad \text{(dV}/\text{dt}) \]

Finally, multiply the interfering current \( I_M \) by \( R_B \) to find the interfering voltage (divide by \( \Delta V \) to express the result as a fractional interference level):

\[ \text{Crosstalk} = \frac{R_B I_M}{\Delta V} = \frac{R_B C_M}{T_r} \quad \text{using} \quad \frac{I_M}{\Delta V} = \frac{C_M}{T_r} \quad \text{given above.} \]

If \( C_M \) is not known, we can measure it from response wfm.

\[ R_B = 50 \, \Omega \]
**Mutual Capacitance**

We use the *area* method here as well.

\[
\text{integrated current} = \frac{\text{area}}{R_B} \\
(Q=CV)
\]

\[
C_M = \frac{\text{area}}{R_B \Delta V}
\]

We measure area of voltage wfm.

then \[
\text{Crosstalk} = \frac{R_B C_M}{T_r}
\]
**Mutual Inductance**

Whenever there are two loops of current, this is *mutual inductance*. The coefficient of interaction is in units of henries or **volt-seconds/amp**.

Mutual inductive coupling between two circuits $A$ and $B$ acts the same as a tiny transformer connecting the circuits.

Mutual inductance is usually more problematic than mutual capacitance.
**Mutual Inductance**

The mutual inductance $L_M$ injects a noise voltage $Y$ into circuit $B$ proportional to the rate of change in current in $A$:

$$Y = L_M \frac{dI_A}{dt}$$

Once again, this equation is an approximation to the actual coupled noise voltage and is valid under an analogous set of restrictions:

- Induced voltage across $L_M$ is much smaller than the signal voltage and $L_M$ does not load $A$.
- The coupled signal current in $B$ is smaller than the current in $A$ ($I_A$) and therefore, the small coupled current in $B$ can be ignored.
- Coupled impedance is small compared to impedance to ground of $B$.

*Magnetic flux in $B$ is total magnetic field strength of $A$ over loop.*
**Mutual Inductance**

Note that voltage induced in loop $B$ is proportional to the rate of change of current in loop $A$.

Also note that a magnetic field is a vector quantity, e.g. flipping loop $B$ reverses the polarity of the flux coupling and induced voltage in $B$.

Given:
- $L_M$ is known.
- The rise time $T_r$ and voltage step magnitude $V_A$ are known.
- The impedance in the driving circuit, $R_A$, to ground is known.

Then crosstalk can be estimated as a fraction of the driving wfm $V_A$.

First derive the maximum change in voltage/time of wfm $V_A$:

$$\frac{dV_A}{dt} = \frac{\Delta V}{T_r} \quad \text{where } \Delta V \text{ is the step height of } V_A.$$
Mutual Capacitance
Second, assume loop $A$ is resistively damped by $R_A$, i.e. current and voltage are proportional to each other.
Then, we can relate current to voltage using some well-defined resistance $R_A$:

$$\frac{dI_A}{dt} = \frac{\Delta V}{R_A T_r}$$ (V=IR)

Next compute the mutual inductive interference $Y$, which appears in $B$:

$$Y = L_M \frac{\Delta V}{R_A T_r}$$ from $$Y = L_M \frac{dI_A}{dt}$$

Finally, divide by $\Delta V$ to express the result as a fractional interference level:

$$\text{Crosstalk} = \frac{L_M}{R_A T_r}$$