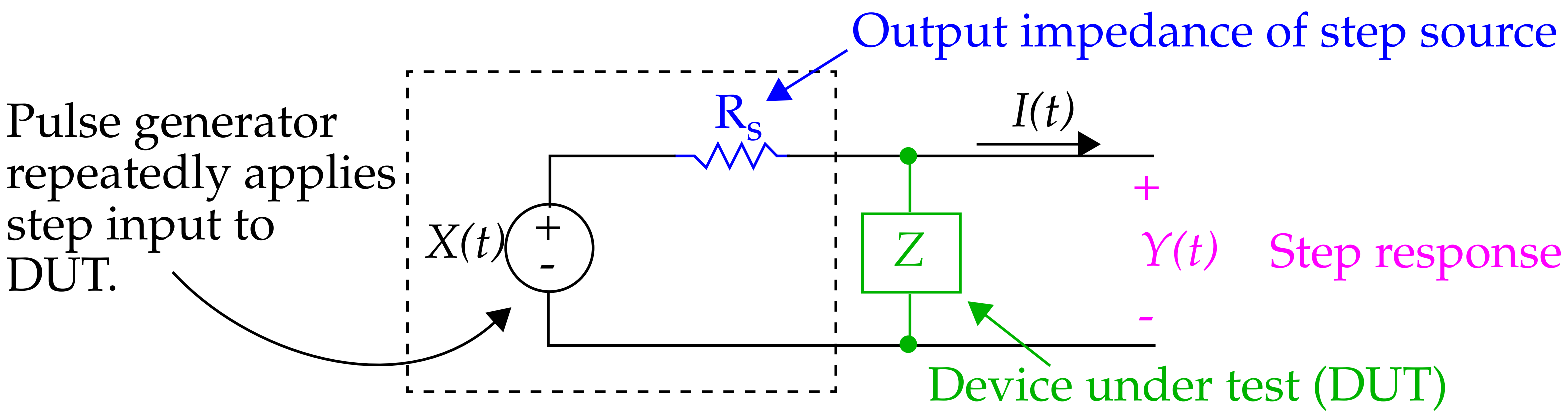
Step Response

The step response measurement is very useful in our analysis.

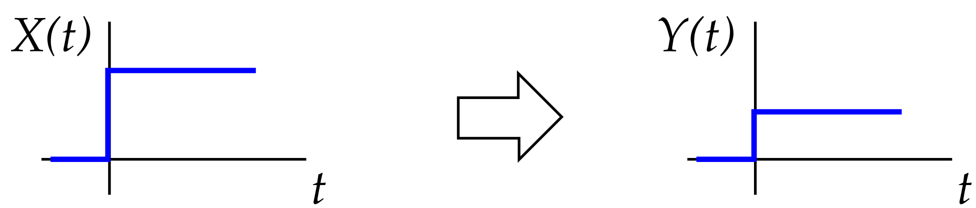
From it, we can derive a curve of impedance versus frequency.

Test setup:



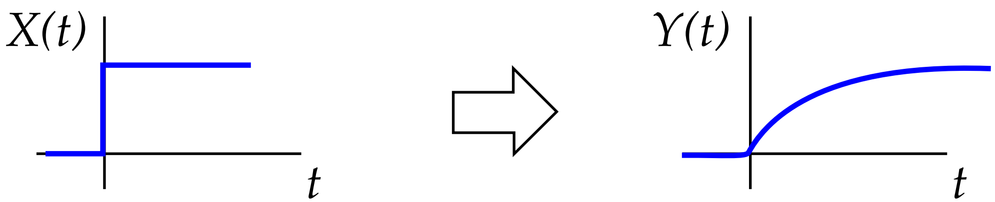
Rules of thumb in characterizing the DUT using the step response:

- Resistors display a flat step response, i.e., at time $t = 0$, the output rises and remains at a fixed value.

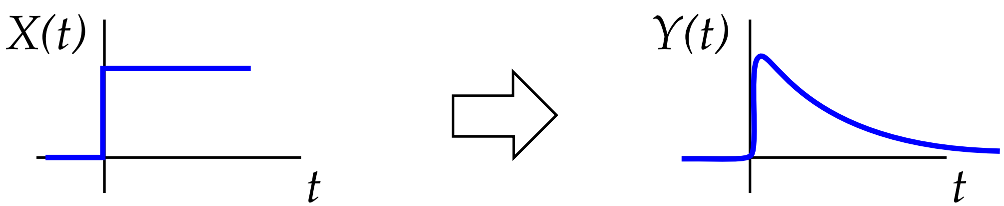


Step Response

- Capacitors display a *rising* step response, i.e., at time $t = 0$, the output starts rising and later reaches its full value.



- Inductors display a *sinking* step response, i.e., at time $t = 0$, the output rises instantly to its full value and then later decays back toward 0.



Capacitors and inductors subdivide into *ordinary* and *mutual* categories.

Ordinary capacitance and inductance (two-terminal devices) can be a help or hindrance.

Mutual capacitance and inductance usually creates unwanted crosstalk.

Capacitance

The capacitor is a 2-terminal element in which the branch voltage and current variables are related by integral and differential equations:

$$v(t) = \frac{1}{C}\int_{t_0}^{t} i(\tau)d\tau + v(t_0)$$

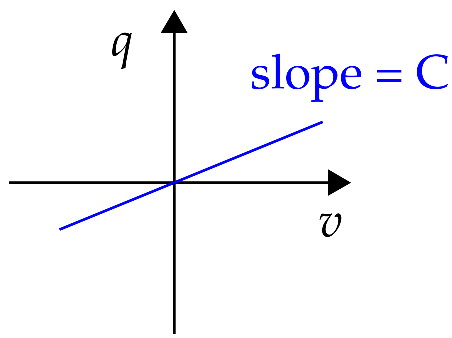
(Voltage on cap depends on history of *i*)

and

$$i(t) = C\frac{dv}{dt}$$

Charge and voltage are related by the linear relationship:

$$q(t) = Cv(t)$$



Power is negative or positive depending on the value of the term *v(t)dv/dt* in the following expression

$$p(t) = Cv(t)\frac{dv}{dt}$$

Capacitance

But energy (the integral of power) is always positive or zero:

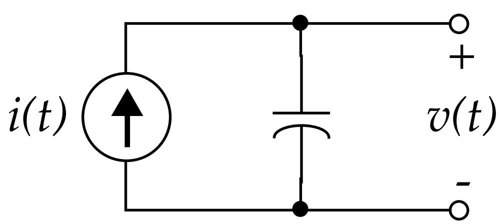
$$w(t) = \frac{Cv^2(t)}{2}$$

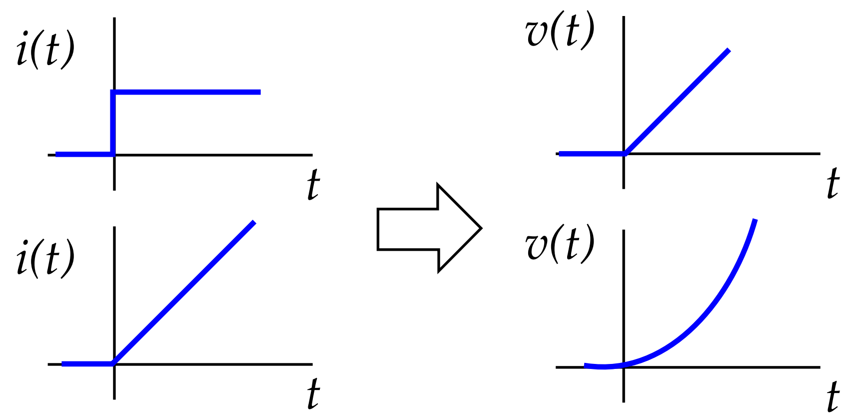
Therefore, it's a passive element (like the resistor) but it is **non-dissipative** (unlike the resistor).

All the energy supplied to the cap. is stored in the electric field.

Note that the voltage appearing across a capacitor must always be a continuous function (voltage steps not allowed -- require an infinite *i*).

Current, on the other hand, is allowed to change instantenously.



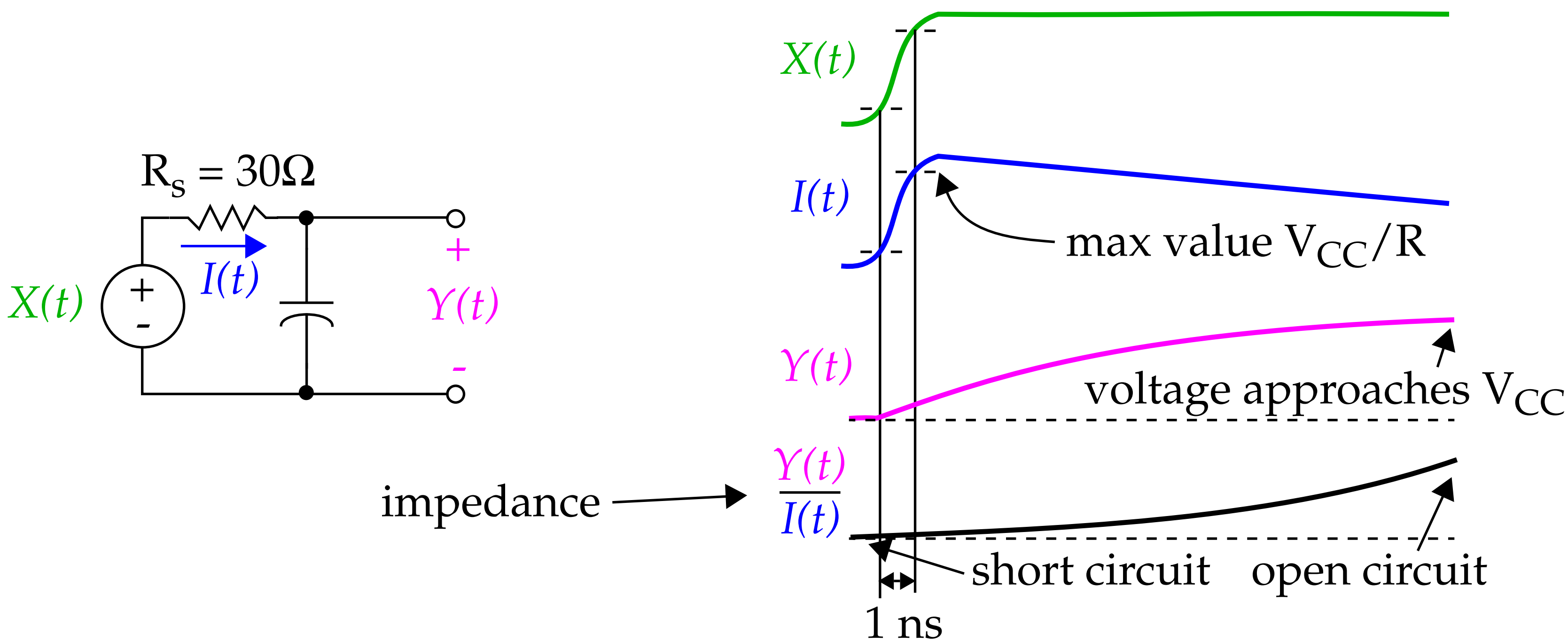


Capacitance

The energy stored in the electric field of a capacitor is supplied by the driving circuit.

Since the driving source is a limited source of power, the voltage takes a finite time to build up.

The reluctance of voltage to build up quickly in response to injected power (or decay quickly) is called *capacitance*.



Capacitance

Bear in mind that a capacitor behaves like an ***inductor*** at high frequencies (unfortunately).

This is due to the mounting leads on capacitors.

This inductance causes the step response to have a tiny pulse (a couple hundred ps) at time 0, followed by a drop to 0 and then a capacitive ramp.

Note that you will not be able to see this unless your step source rise time is sharp.

At T_r, you can characterize the circuit element for frequencies up to:

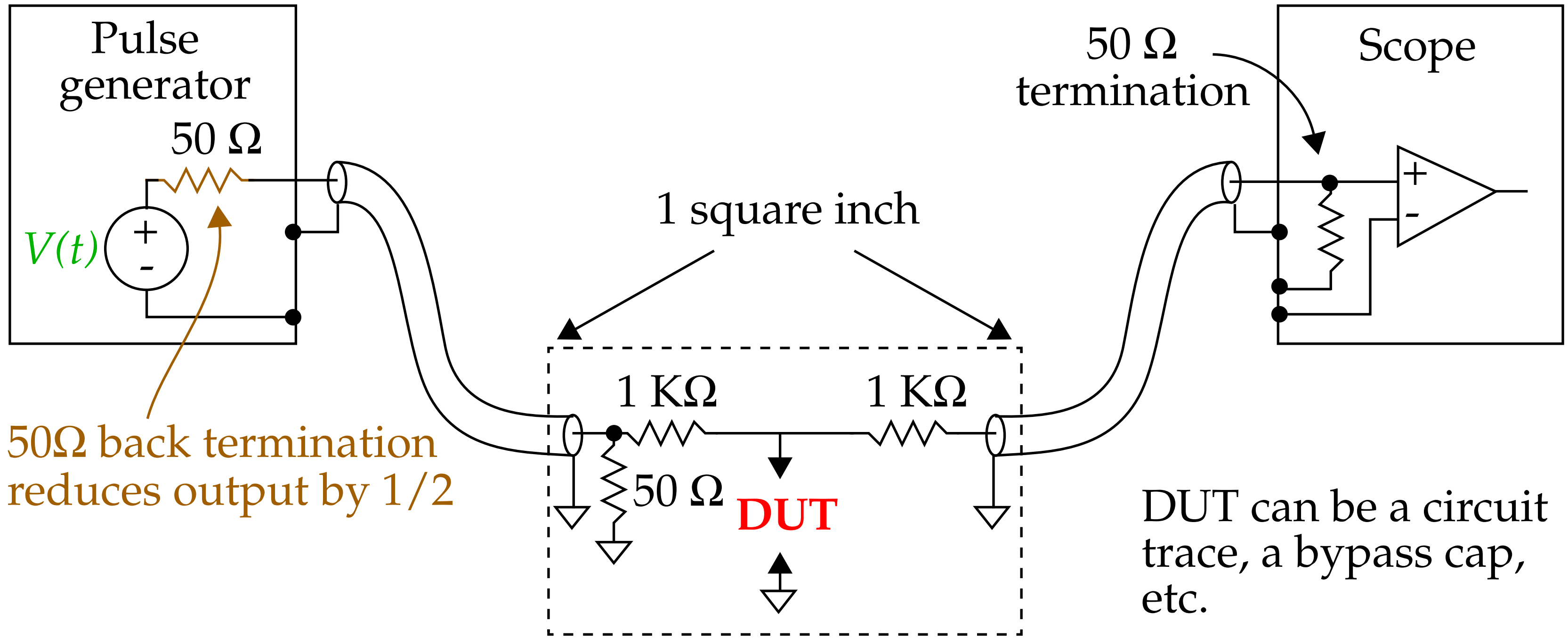
$$F_A = \frac{0.5}{T_r}$$

Reactance on leading edge (to est. distortion in digital wfm by a cap.):

$$X_C = \frac{T_r}{\pi C}$$

Capacitance Test Gig

A measurement setup ideal for characterizing capacitors:



DUT can be a circuit trace, a bypass cap, etc.

Note that the resistances are known, so by measuring the rise time of the resulting waveform, the capacitance of the DUT can be computed.

The test gig is dimensioned at 1 square inch to ensure it behaves in a lumped fashion.

The test gig should include a ground plane of 1 square inch.

Inductance

The inductor is a 2-terminal element in which the branch voltage and current variables are related by integral and differential equations:

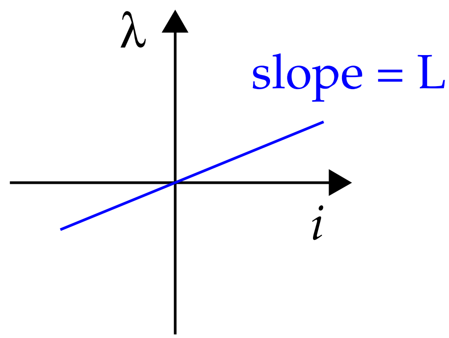
$$i(t) = \frac{1}{L}\int_{t_0}^{t} v(\tau)d\tau + i(t_0) \qquad \text{and} \qquad v(t) = L\frac{di}{dt}$$

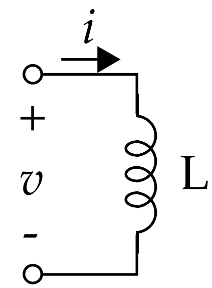
Flux linkages (in weber-turns) and current are related by the relationship:

n turns of wire

$$\lambda(t) = n\phi(t) = Li(t)$$

produces flux





$$v(t) = \frac{d\lambda}{dt}$$

Power is negative or positive depending on the value of the term *i(t)di/dt* in the following expression

$$p(t) = i(t)v(t) = i(t)L\frac{di}{dt}$$

Inductance

But energy (the integral of power) is always positive or zero:

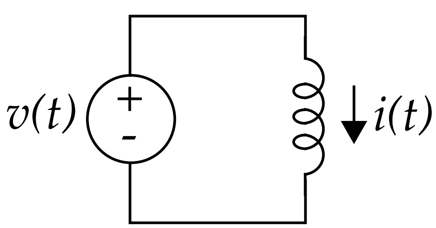
$$w(t) = \frac{Li^2(t)}{2}$$

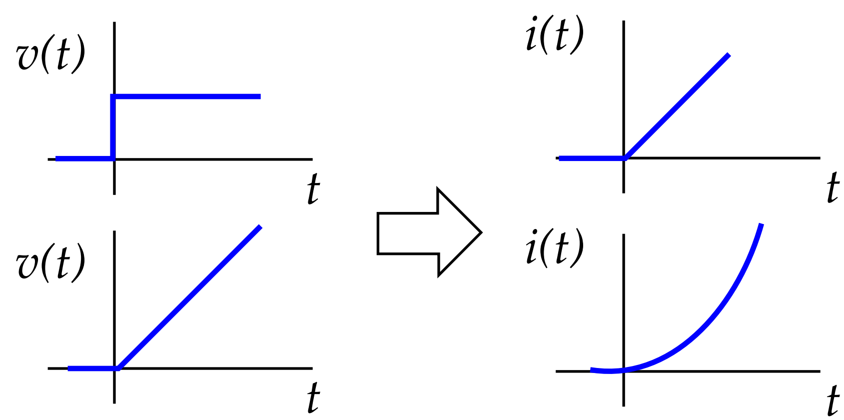
Therefore, it's a passive element (like the resistor) but it is **non-dissipative** (unlike the resistor).

All the energy supplied to the ind. is stored in the magnetic field.

Note that the current flowing through an inductor must always be a continuous function (current steps not allowed -- require an infinite v).

Voltage, on the other hand, is allowed to change instantenously.



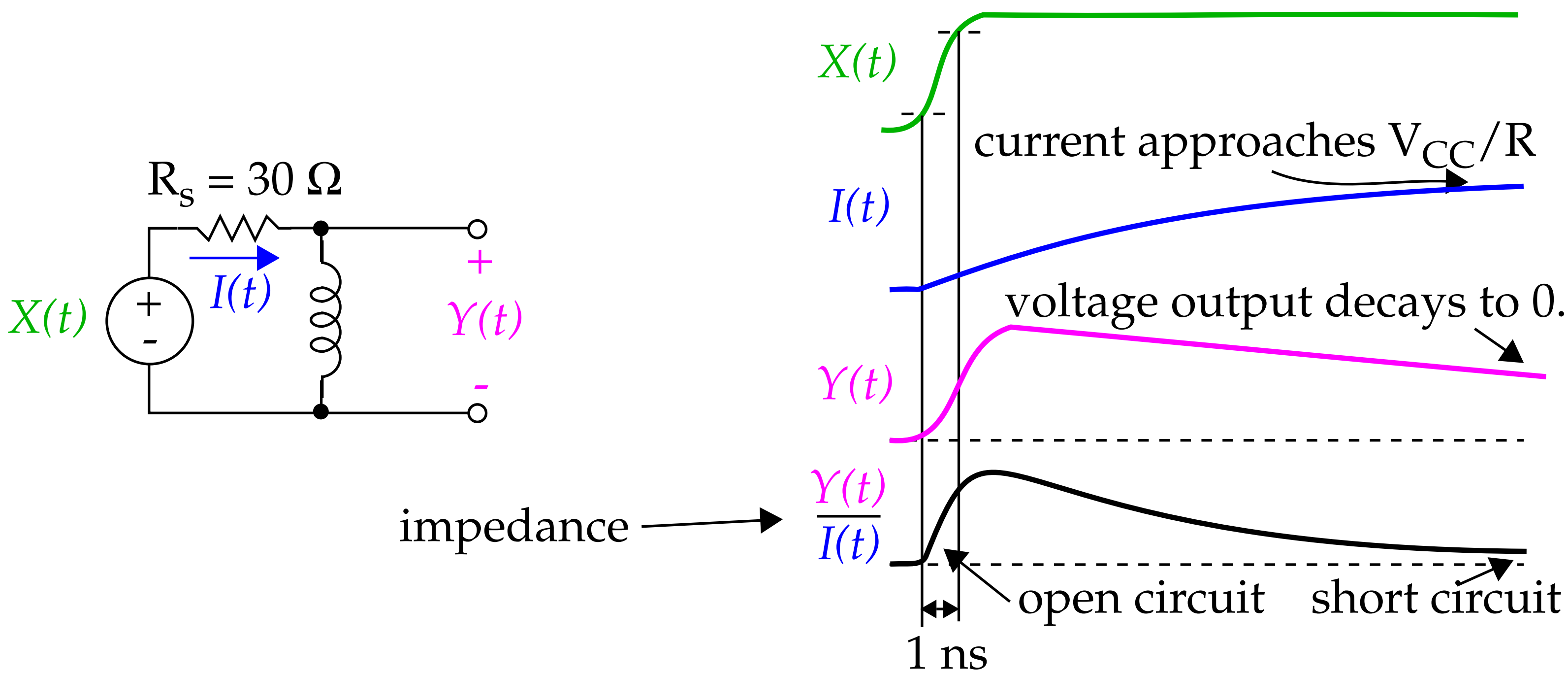


Inductance

The energy stored in the magnetic field of a inductor is supplied by the driving circuit.

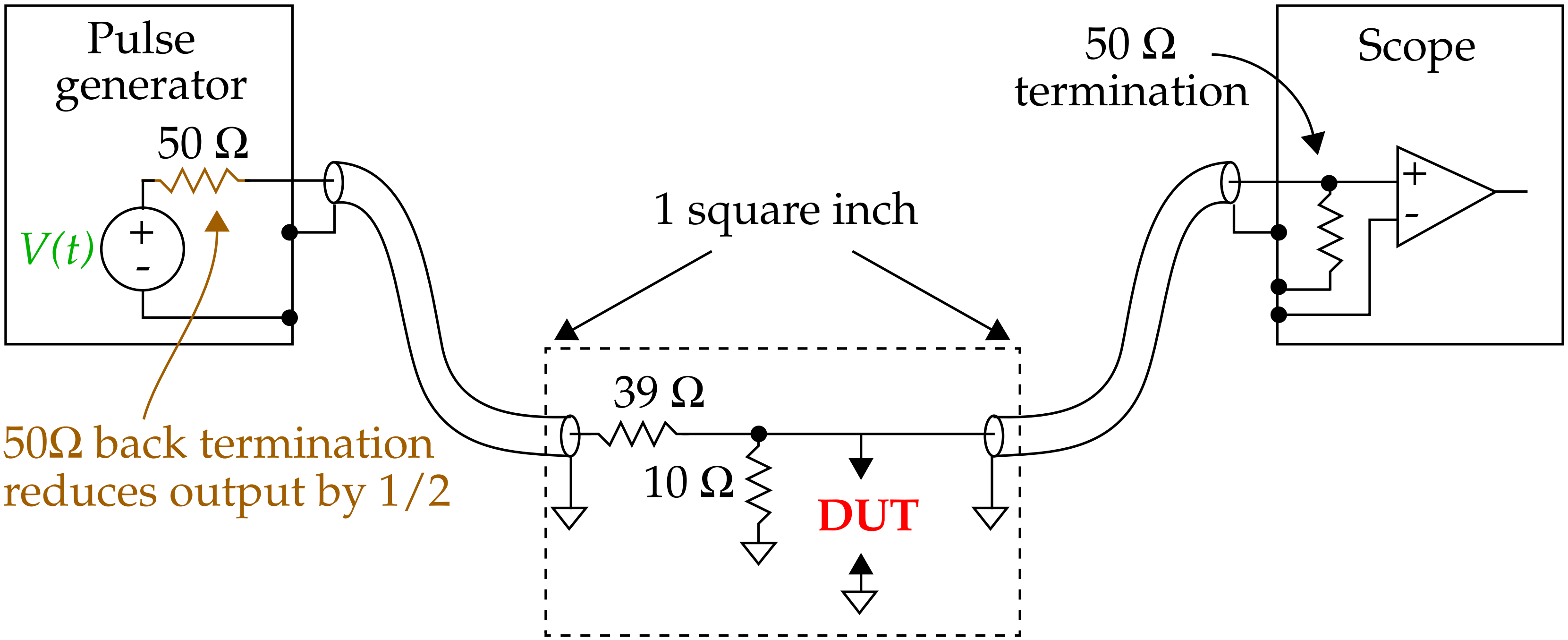
Since the driving source is a limited source of power, the current takes a finite time to build up.

The reluctance of current to build up quickly in response to injected power (or decay quickly) is called *inductance*.



Inductance Test Gig

A measurement setup ideal for characterizing inductors:



Reactance on leading edge (to est. distortion in digital wfm by an inductive load:

$$X_L = \frac{\pi L}{T_r}$$

Measuring Inductance

Inductance can be measured in a similar way to capacitance by computing the time, for example, in the response waveform to the 63% point.

A second method for inductance involves computing the area under the response waveform's curve:

$$v(t) = L\frac{di}{dt} \longrightarrow \int_0^\infty V_{ind}(t)dt = L\int_0^\infty \frac{dI_{ind}(t)}{\mathrm{dt}}dt$$

$$\int_0^\infty V_{ind}(t)dt = L[I(\infty) - I(0)]$$

Inductor acts as a short circuit at time (infinity) therefore delta V/R_S gives the current.

$$area = L[I(\infty) - I(0)]$$

$$L = \left[\frac{area}{\Delta I}\right] = \frac{(area)R_S}{\Delta V}$$

where R_S and ΔV are the open circuit response values (see Example 1.2)

$$\Delta I = \frac{\Delta V}{R_S}$$

Mutual Capacitance

Mutual capacitance coupling between two circuits is simply a parasitic capacitor connected between circuit *A* and circuit *B*.

The coefficient of interaction is in units of farads or **amp-seconds/volt**.

Mutual capacitance C_M injects a current I_M into circuit *B* proportional to the rate of change of voltage in circuit *A*:

$$I_M = C_M \frac{dV_A}{dt}$$

This simplification works if:

- The coupled current flowing in C_M is much smaller than the primary signal current in circuit *A*, i.e. C_M does not load circuit *A*.
- The coupled **signal** voltage in circuit *B* is small and can be ignored. Therefore, the voltage difference between *A* and *B* is just V_A.
- The mutual capacitance represents a large impedance compared to the impedance to ground of circuit *B*.

Mutual Capacitance

A more accurate model uses the **difference** in voltages between circuits *A* and *B* and the loading effect of C_M on both circuits.

When the coupled noise voltage (*crosstalk*) is less than 10% of the signal step size (on *A*), this approximation is accurate to one decimal place.

Given:

- C_M is known.
- The rise time T_r and voltage step magnitude V_A are known.
- The impedance in the receiving circuit, R_B, to ground is known.

Then *crosstalk* can be estimated as a fraction of the driving wfm V_A.

First derive the maximum change in voltage/time of wfm V_A:

$$\frac{dV_A}{dt} = \frac{\Delta V}{T_r}$$ where ΔV is the step height of V_A.

Mutual Capacitance

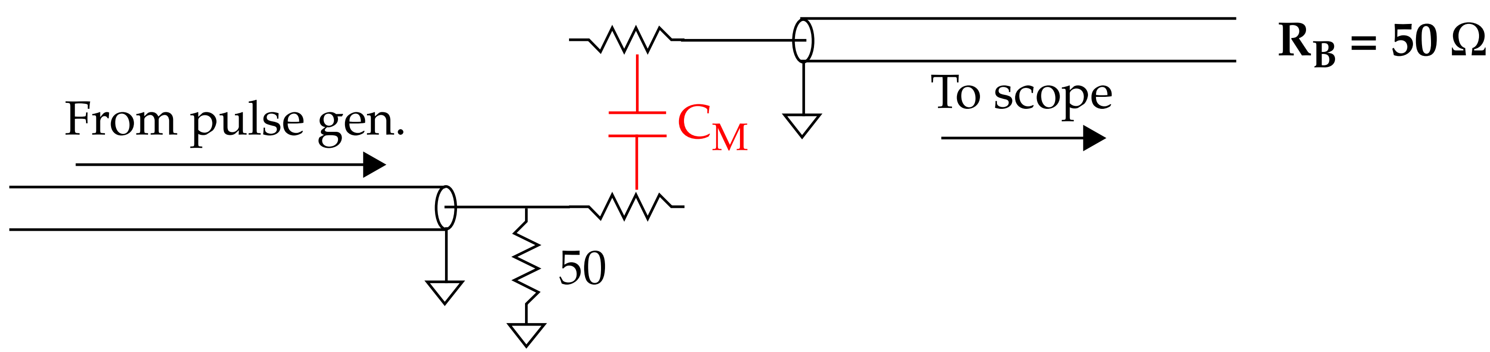
Second, compute the mutual capacitive current which flows from circuit *A* to circuit *B* using:

$$I_M = C_M \frac{\Delta V}{T_r} \longleftarrow (dV/dt)$$

Finally, multiply the interfering current I_M by R_B to find the interfering voltage (divide by ΔV to express the result as a fractional interference level):

$$\text{Crosstalk} = \frac{R_B I_M}{\Delta V} = \frac{R_B C_M}{T_r} \quad \text{using} \quad \frac{I_M}{\Delta V} = \frac{C_M}{T_r} \text{ given above.}$$

If C_M is not known, we can measure it from response wfm.



Mutual Capacitance

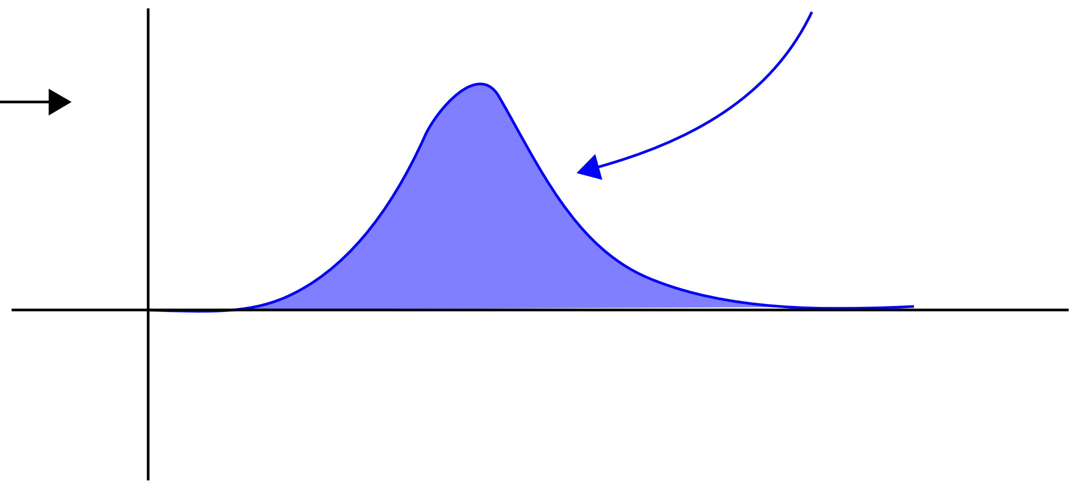
We use the *area* method here as well.

$$\text{integrated current} = \frac{area}{R_B} \longrightarrow C_M = \frac{area}{R_B \Delta V}$$

(Q=CV)

We measure area of voltage wfm.

Scope display →



t (ps/div)

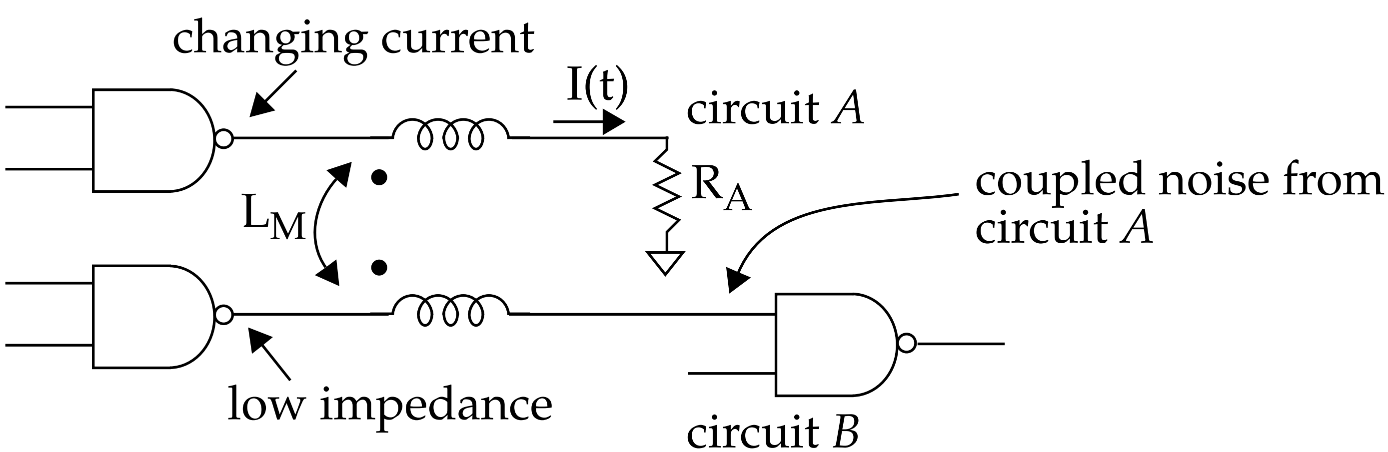
then $\text{Crosstalk} = \dfrac{R_B C_M}{T_r}$

Mutual Inductance

Whenever there are two loops of current, this is *mutual inductance*.

The coefficient of interaction is in units of heneries or **volt-seconds/amp**.

Mutual inductive coupling between two circuits *A* and *B* acts the same as a tiny transformer connecting the circuits.



Mutual inductance is usually more problematic than mutual capacitance.

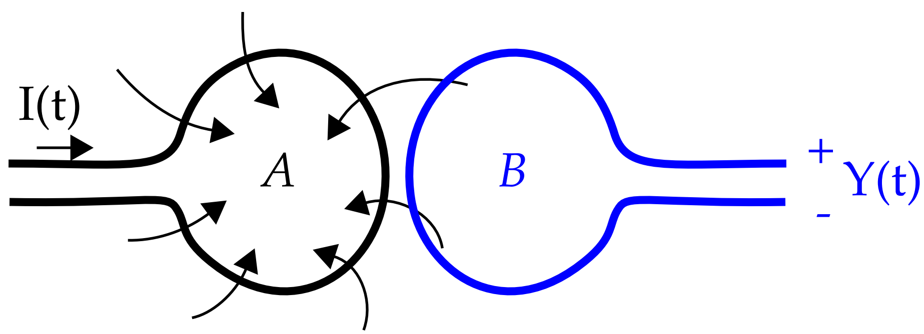
Mutual Inductance

The mutual inductance L_M injects a noise voltage *Y* into circuit *B* proportional to the rate of change in current in *A*:

$$Y = L_M \frac{dI_A}{dt}$$

Once again, this equation is an approximation to the actual coupled noise voltage and is valid under an analogous set of restrictions:

- Induced voltage across L_M is much smaller than the signal voltage and L_M does not load *A*.
- The coupled signal current in *B* is smaller than the current in *A* (I_A) and therefore, the small coupled current in *B* can be ignored.
- Coupled impedance is small compared to impedance to ground of *B*.



Magnetic flux in *B* is total magnetic field strength of *A* over loop.

Mutual Inductance

Note that voltage induced in loop *B* is proportional to the **rate of change** of current in loop *A*.

Also note that a magnetic field is a vector quantity, e.g. flipping loop *B* reverses the polarity of the flux coupling and induced voltage in *B*.

Given:

- L_M is known.
- The rise time T_r and voltage step magnitude V_A are known.
- The impedance in the *driving* circuit, R_A, to ground is known.

Then *crosstalk* can be estimated as a fraction of the driving wfm V_A.

First derive the maximum change in voltage/time of wfm V_A:

$$\frac{dV_A}{dt} = \frac{\Delta V}{T_r}$$ where ΔV is the step height of V_A.

Mutual Capacitance

Second, assume loop A is resistively damped by R_A, i.e. current and voltage are proportional to each other.

Then, we can relate current to voltage using some well-defined resistance R_A:

$$\frac{dI_A}{dt} = \frac{\Delta V}{R_A T_r} \qquad \text{(V=IR)}$$

Next compute the mutual inductive interference Y, which appears in B:

$$Y = L_M \frac{\Delta V}{R_A T_r} \qquad \text{from} \qquad Y = L_M \frac{dI_A}{dt}$$

Finally, divide by ΔV to express the result as a fractional interference level:

$$\text{Crosstalk} = \frac{L_M}{R_A T_r}$$