Transmission Lines

At high frequencies, transmission lines are superior to point-to-point wiring for several reasons:

- Less distortion
- Less radiation (EMI)
- Less crosstalk

However, this comes at a cost: they consume more power.

The improved signal integrity and performance makes this worthwhile.

Let’s analyze an example point-to-point wiring using wire-wrap.

The parameters of the system are:

- The average length of a signal net is 4 in.
- The average height above GND of the nets is 0.2 in.
- The wire size is 0.01 in. diameter (AWG 30)
- The signal rise time is 2.0 ns
- $F_{knee}$ is 250 MHz (0.5/2.0 ns).
Signal Distortion in Point-to-Point Wiring

A 2 ns rise time has an electrical length of:

\[ l = \frac{\text{rise time (ps)}}{\text{propagation delay (ps/in)}} = \frac{2000 \text{ ps}}{85 \text{ ps/in}} = 23.5 \text{ in.} \]

Is this system **lumped** or **distributed**?

Critical dimension is \( l/6 = 3.9 \text{ in.} \)

Is it true that this system will **not** experience ringing (since the average net length is \( \sim 4 \text{ in.} \))?

Note that **distributed** systems ALWAYS ring, i.e., have **overshoot** and **undershoot**, unless terminated.

The \( Q \) determines if **lumped** systems ring: it measures how quickly signals die out.

Low-\( Q \) circuits damp quickly while high-\( Q \) circuits cause signals to bounce around.
Signal Distortion in Point-to-Point Wiring

Remember $Q$ definition from chapter 3: ratio of energy stored to energy lost per radian of oscillation.

$$Q \approx \frac{\sqrt{L/C}}{R_S}$$

The following can be used to approximate the maximum overshoot voltage (under assumption that $Q > 0.5$):

$$\frac{V_{\text{overshoot}}}{V_{\text{step}}} = e^{-\left[\frac{\pi}{\sqrt{4Q^2 - 1}}\right]}$$

$V_{\text{overshoot}}$ = voltage rise above $V_{\text{step}}$

$V_{\text{step}}$ = nominal steady-state level

The ideal 2nd order circuit decays with time constant $2L/R$
Signal Distortion in Point-to-Point Wiring

After $t_0$, driver is 0 V.

Cap still fully charged right after $t_0$

$R \quad L$  $V_o$

Voltage across capacitor

$\exp\left(-t \frac{R}{2L}\right)$

Exponential decay envelope

$-V_o \exp\left[\frac{-\pi}{\sqrt{4Q^2 - 1}}\right]$  

Max overshoot

Ringing period:  $2\pi \sqrt{LC}$
Signal Distortion in Point-to-Point Wiring

Rule of thumb:
A digital circuit with a $Q$ of $1$, in response to a perfect step input, produces a $16\%$ overshoot.
A $Q$ value of $2$ displays a $44\%$ overshoot.
A $Q$ less than $0.5$ has no overshoot or ringing.

Also note that ringing is related to the natural ringing frequency of the circuit and the rise time of the driver.

The basic problem with the example circuit is the high inductance associated with the point-to-point wiring.
Large wiring inductance working into a heavy cap load yields a high-$Q$ circuit.

For the example system, the wire inductance is approximated by:

$$L = X(5.08 \times 10^{-9}) \ln \left( \frac{4H}{D} \right) = 4(5.08 \times 10^{-9}) \ln \left( \frac{4 \times 0.2}{0.01} \right) = 89nH$$
Signal Distortion in Point-to-Point Wiring

Then $Q$ can be computed assuming a TTL driver $R$ of 30 $\Omega$:

$$Q \approx \sqrt{\frac{L}{C R_S}} = \sqrt{\frac{89 \times 10^{-9}}{15 \times 10^{-12}} \frac{30}{30}} = 2.6$$

The expected worst case overshoot voltage with $V_{\text{step}}$ 3.7 V (TTL step output) is:

$$V_{\text{overshoot}} = V_{\text{step}} \exp\left(\frac{-\pi}{\sqrt{4Q^2 - 1}}\right) = 3.7e^{\frac{-3.14159}{\sqrt{4(2.6)^2 - 1}}} = 3.7e^{-0.616} = 2.0V$$

Note that this worst case overshoot only occurs if the logic drivers can transmit significant energy at frequencies above the ringing frequency:

$$F_{\text{ring}} = \frac{1}{2\pi \sqrt{LC}} = \frac{1}{2\pi \sqrt{[89 \times 10^{-9} 15 \times 10^{-12}]}} = 138 MHz$$

The system spec. was 250MHz so full amplitude ringing occurs.

Expect amplitude of ringing to be about 1/2 that predicted if $F_{\text{knee}}$ was 138 MHz (i.e., if rise time is about 1/2 the ringing period).
EMI in Point-to-Point Wiring

Large *EMI* (electromagnetic interference) fields are generated from large, fast current loops.

The magnetic fields from these loops radiate into space.

Transmission lines dramatically reduce EMI by keeping the return currents **close** to the signal path (magnetic fields cancel).

Wire-wrap current loops return currents at some distance from the signal line, increasing the total loop area.

The magnetic fields are directly proportional to the loop area.

EMI is proportional to wire height above GND.

Radiates 32dB less EM energy/wire.
Crosstalk in Point-to-Point Wiring
Crosstalk arises from changing magnetic fields, e.g., loop A fields penetrate loop B:

\( \frac{dI}{dt} \) in loop A change magnetic flux encircled by loop B, introducing a noise voltage in loop B called crosstalk.

For our example system, assume we have two adjacent loops, 4 in. x 0.2 in. high \((h)\), running parallel at a separation, \(s\), of 0.1 in.

\[
L_M = L \left[ \frac{1}{1 + (s/h)^2} \right] = 71 \text{nH}
\]

The self-inductance of each net, \(L\), was computed earlier as 89 nH.

The closeness of these numbers indicates a highly coupled condition.
Crosstalk in Point-to-Point Wiring

The crosstalk is computed by finding the maximum $dI/dt$ in loop $A$ and multiplying by the mutual inductance, $L_M$, to obtain a crosstalk voltage.

We analyzed the ringing in loop $A$ after the driving gate launches the transition.

Here, we indicated that the maximum overshoot at the load cap $C$ occurs at one-half the ringing period ($7.2 \text{ ns} / 2 = 3.6 \text{ ns}$ in our example) after $t_0$.

Plugging this value for rise time (our best guess), into:

$$
\frac{dI}{dt} = \frac{1.52 \times \Delta V}{T_r^2} C = \frac{(1.52)(3.7)}{(3.6 \times 10^{-9})^2} \times 15 \times 10^{-12} = 6.5 \times 10^6 \text{ A/s}
$$

Yields a crosstalk of about 12%:

$$
\text{Crosstalk} = \frac{dI}{dt}(max) L_M = (6.5 \times 10^6)(71 \times 10^{-9}) = 0.46 \text{ V}
$$

Since crosstalk adds linearly, bundling together 10 or 20 wires to form a bus is a really bad idea.

10 wires easily fits in 1/10 in. -- yields a 50% value for crosstalk!
Infinite Uniform Transmission Lines
The transmission line forms that we will study include:

Coaxial cable
- outer jacket
- outer shield
- inner dielectric
- inner conductor

Microstrip
- conductor
- dielectric
- GND plane

Twisted pair
- jacket (dielectric)
- first conductor
- second conductor

Stripline
- GND plane
- dielectric
- conductor
- GND plane

Twisted pair is called balanced, while the others are called single-ended or unbalanced.
For unbalanced, signal current flows out the signal wire and back along the GND connection.
The GND connection is larger and can be shared with other signal wires.
Infinite Uniform Transmission Lines

An ideal transmission line consists of two perfect conductors (zero resistance, uniform in cross-section and extend forever).

Ideal transmission lines have three properties:
- The line is infinite in extent.
- Signals propagate without distortion.
- Signals propagate without attenuation.

The voltage at any point along an ideal transmission line is a perfect delayed copy of the input wfm.

**Propagation delay**: The delay per unit length along a transmission line in ps/in.

**Propagation velocity**: Inverse of propagation delay -> in./ps. Sometimes expressed as a percentage of the velocity of light in a vacuum, given as 0.0118 in./ps or as delay, 84.7 ps/in.
Infinite Uniform Transmission Lines

The propagation delay of any transmission line is related to its *series inductance* and *parallel capacitance* per unit length.

The fine balance between these is responsible for distortionless signal propagation.

Measuring transmission line cap. and ind. (need an impedance meter).

- Measure cap. here
- Leave open

\[
C = 2.6 \text{ pF/in.}
\]

- Measure ind. and series resistance here
- Short this out

\[
L = 6.4 \text{ nH/in.}
\]

The series resistance is very small (but not zero), 0.9 m\(\Omega\)/in.
Characteristic Impedance of Infinite Uniform Transmission Lines

The input impedance is computed from the C/in. and propagation delay.

**EM wave theory** gives propagation delay as:

\[
\text{Delay} = \sqrt{L \text{/ in.})(C \text{/ in.})} \times 10^{12} \text{ ps/in.}
\]

Given C/in. and prop. delay, we can compute input impedance.

First compute average current needed to sustain the signal propagation:

A step of \( V \) volts propagates

Delay, \( t_1 - t_0 = (Y-X)(LC)^{1/2} \)

\( \text{ps} = (\text{in.}) \times (\text{ps/in.}) \)
Characteristic Impedance of Infinite Uniform Transmission Lines

The capacitance between points X and Y charges to voltage V over the time interval T.

To compute the current, first compute the capacitance over this distance:

\[ C_{XY} = (C/\text{in.})(Y - X) \]

And then the total charge supplied by the driver is computed from \( Q = CV \):

\[ Q = C_{XY}V = (C/\text{in.})(Y - X)V \]

The time interval during which \( C_{XY} \) must be charged:

\[ T = (Y - X)\sqrt{(L/\text{in.})(C/\text{in.})} \]

Then average current is:

\[ I = \frac{\text{charge}}{T} = \frac{(C/\text{in.})(Y - X)V}{(Y - X)\sqrt{(L/\text{in.})(C/\text{in.})}} \]
Characteristic Impedance of Infinite Uniform Transmission Lines

This gives us the current flow required to sustain a propagating step edge of $V$ volts.

Solving for $R$ (or $Z$) in $V = IR$ gives the characteristic impedance:

$$Z_0 = \frac{V}{I} = \sqrt{\frac{L}{\text{in.}} \frac{\text{in.}}{C}}$$

Note that the input impedance is a constant, with no imaginary part and independent of frequency.

The constant ratio is a function of the line’s physical geometry.

Triax cable is 10 $\Omega$ while TV antenna connections are 300 $\Omega$.

The characteristic impedance of the RG-58 cable is:

$$Z_0 = \sqrt{\frac{6.4nH}{2.6pF}} = 50\Omega$$
Characteristic Impedance of Infinite Uniform Transmission Lines

PCB characteristic impedances vary from 50 to 75Ω.

The following shows the dimensions needed to build transmission lines on a PCB with characteristic impedances of 50 and 75Ωs.

**Microstrip, 75Ω**

![Microstrip, 75Ω Diagram]

**Microstrip, 50Ω**

![Microstrip, 50Ω Diagram]

**Stripline, 75Ω**

![Stripline, 75Ω Diagram]

**Stripline, 50Ω**

![Stripline, 50Ω Diagram]

With FR-4 substrates

Note that the tolerance here is +/- 30%!
Characteristic Impedance of Infinite Uniform Transmission Lines
An ideal transmission line looks totally resistive:

\[
\begin{align*}
V_0 & \quad \text{(A)} \\
V_0 & \quad \text{RL} \\
& \quad 75 \Omega \\
\end{align*}
\]

\[
\begin{align*}
V_0 & \quad \text{(B)} \\
V_0 & \quad \text{RL} \\
& \quad 75 \Omega \\
\end{align*}
\]

Voltage that propagates down the transmission line is given by volt. div. eq.

\[
V_0(1 - e^{-t/\tau})
\]

Input acceptance equation

\[
V_0 \frac{Z_0}{Z_0 + R_S + R_L}
\]

\[
Z_0 = 75 \Omega
\]