DC Resistance in Transmission Lines

The resistance is non-zero in real transmission lines, causing them to dissipate a portion of the signal power, introducing attenuation and distortion.

Attenuation indicates that signals shrink as they propagate.

Distortion indicates that signals at different frequencies are attenuated and phase-shifted by different amounts as they propagate.

You must consider the resistance of both the signal and ground wires.

DC resistance per meter given by

$$R_{DC} = \frac{k_a \rho}{a}$$

a is cross-sectional area of conductor
k_a is correction factor that accounts for additional DC resistance of return path
ρ is resistivity of the conductor (Ω-m)

For annealed copper at room temperature, ρ is 1.724×10^{-8} Ω-m.

DC Resistance in Transmission Lines

$k_a = 1$ for a PCB trace with wide flat return path (e.g., ground plane)

$k_a = 2$ for twisted-pairs (signal and return wires same size).

For coax, it's more difficult to compute -- use manufacturing spec.

Resistance in long cables is measured in Ohms/1000 ft.

For computing resistance of round copper wires:

- Every 3 AWG points halves the cross-sectional area and doubles the wire resistance.

  AWG 24 (0.02 in.) has a resistance of 26 Ω/1000 ft. (52 Ω/1000 ft. for twisted pair) (room temperature)

  RG-58/U coax uses AWG 20 with 10.3 Ω/1000 ft.

- Resistance of copper increases 0.39 % for every 1 degree C increase.

DC Resistance in Transmission Lines

AWG formula:

$$\text{diameter in inches} = 10^{-(AWG + 10)/20} \qquad \text{(AWG 20 is about 32 mils)}$$

$$\text{R per 1000 ft} = \frac{0.0104\Omega}{(\text{diameter})^2} \qquad \text{(at 25 degrees C)}$$

PCB trace resistance is a function of both the copper thickness and trace width.

1-oz. (1.37 mil or 34.8 μm) or 2-oz. (2.74 mil)

$$R = \frac{0.65866\times10^{-6}}{W \times T}\Omega/\text{in.}$$

W is width of line (in.)
T is thickness of line (in.)

What is the resistance of a 5 in. trace in 1-oz. that’s 20 mil wide?

Skin Effect

High frequency current does not flow uniformly throughout the cross-sectional area of the conductor.

Magnetic fields within the conductor adjust the distribution of current, forcing it to flow only in a *shallow band* just underneath the surface.

This increases the apparent resistance of the conductor (**skin effect**).

It is related to the rate of change of magnetic fields and therefore increases in intensity at higher frequencies.

The thickness of the conduction band δ is called the *skin depth*.

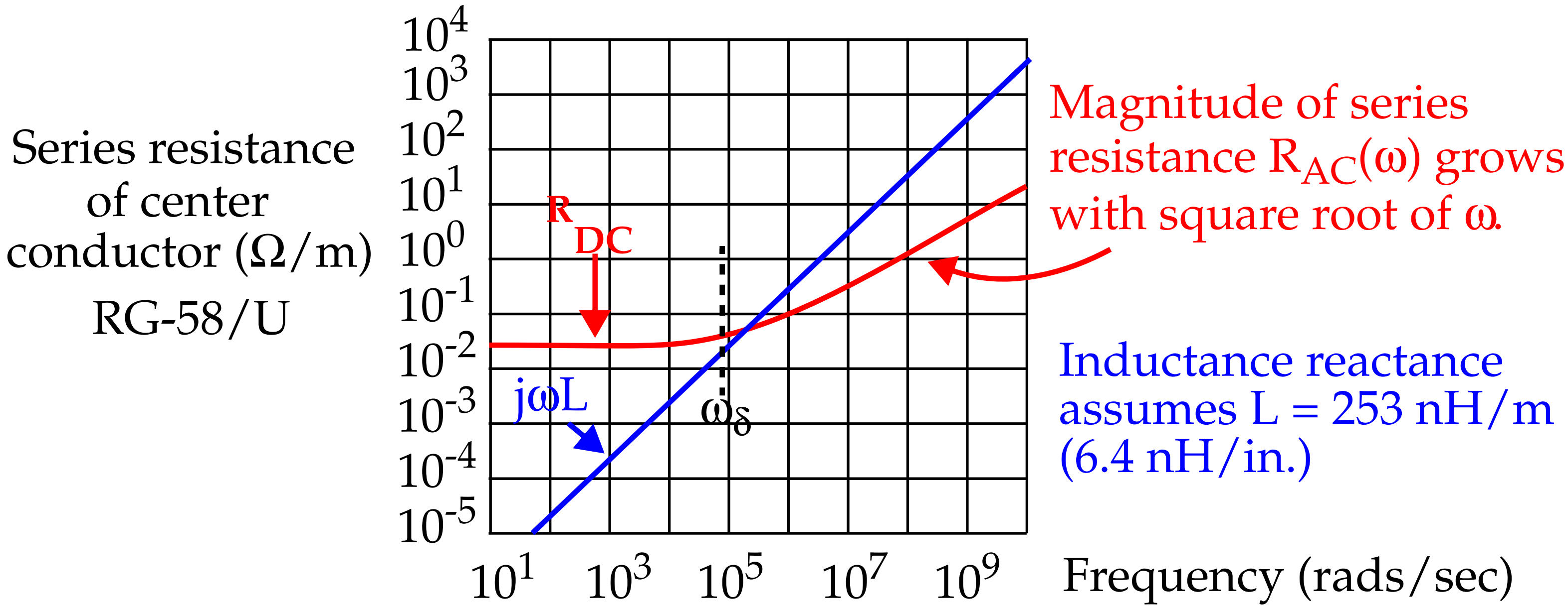
Below a cut-off frequency ω_δ, skin effect is not noticable.

Above ω_δ, the AC resistance of the conductor increases, proportional to the *square root of frequency*.

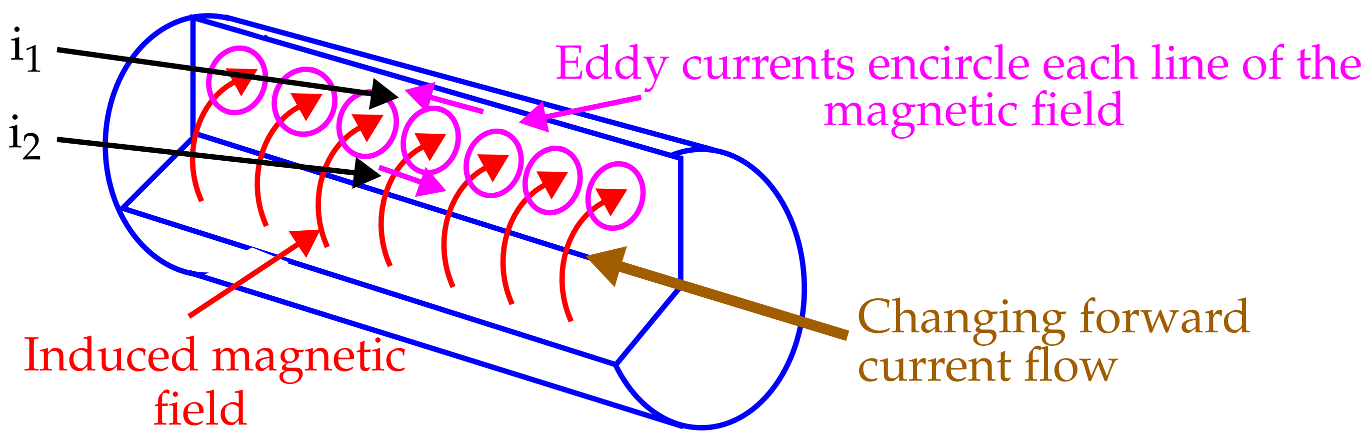
Skin effect governs the behavior of all conductors.

Skin Effect

For example, consider a coax cable, RG-58.



Changing magnetic fields induce eddy currents within the conductor.



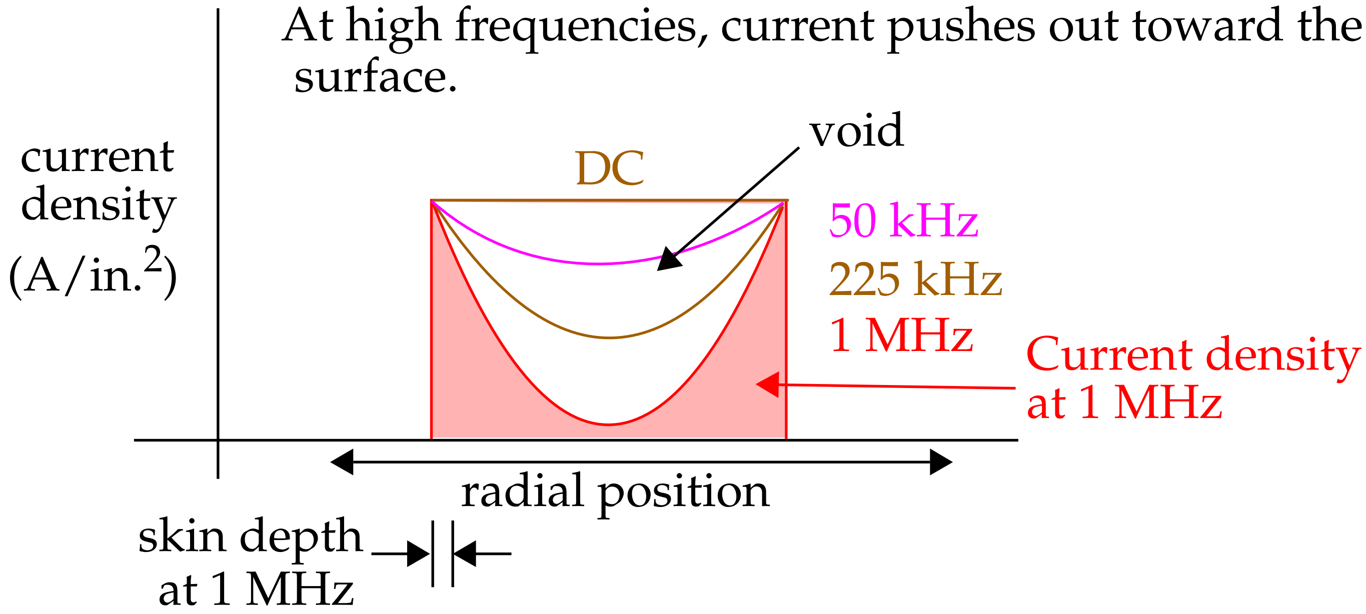
Skin Effect

Note that the magnetic fields cut through the body of the conductor, and induce eddy currents, whose magnitudes are proportional to ω.

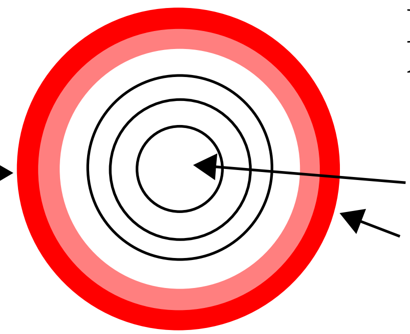
Current i_1 flows near the surface of the conductor *in the direction* of current flow, while i_2 flows in the **opposite direction**.

Therefore, the eddy currents increase the effective current density on the surface and decrease it in the middle.

At high frequencies, current pushes out toward the surface.



Skin Effect



Also, the eddy currents induce their own magnetic fields, in an opposing direction of that shown on slide 5.

These opposing fields partially *cancel* the magnetic field shown.

For frequencies significantly above ω_δ, the fields cancel completely in the center of the conductor and no current flows.

The magnetic fields decay **exponentially** as you move from the surface towards the center, effectively *shielding* the core area of the conductor.

Skin Effect

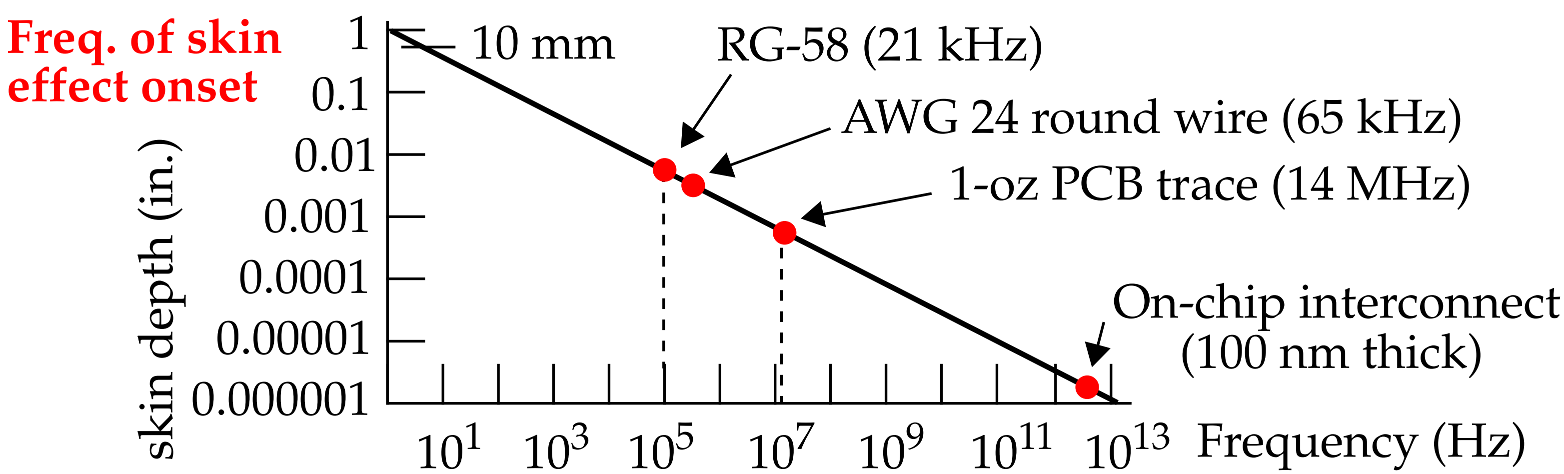
Skin depth is defined as the depth where the internal magnetic field intensity within the conductor is reduced by a factor of $1/e$.

The effective depth of the conduction band is given by

$$\delta = \sqrt{\frac{2}{\omega\mu\sigma}}$$

δ is the skin depth needed to attenuate by $1/e$
$\omega = 2\pi f$ is the frequency
μ is the magnetic permeability of the conductor (H/m)
σ is the conductivity of the conductor (S/m)
(annealed copper $\sigma = 5.80 \times 10^{7}$ S/m)
(non-magnetic materials $\mu = 4\pi \times 10^{-7}$ H/m)

Skin depth of a 1/2-oz copper layer (17.4 µm) at 100 MHz is 6.6 µm.



Skin Effect Resistance

At high frequencies, i.e., $\omega >> \omega_\delta$, current flow is restricted to an annular ring of thickness δ and perimeter p, yielding a cross-sectional area of $p\delta$.

The effective resistance, assuming wire radius greatly exceeds the skin depth, is given by

$$Re[R_{AC}] = \frac{k_p k_r}{p\delta\sigma}$$

Re(R_{AC}) is real part of skin-effect impedance (Ω/m)
p is perimeter (m)
δ is skin depth at some particular frequency
σ is conductivity of conductor (S/m)
k_p is a correction factor for proximity effect
k_r is a correction factor surface roughness effect

Folding in the δ expression given earlier yields the freq. dependancy

$$Re[R_{AC}] = \frac{k_p k_r}{p\sqrt{\frac{2}{\omega\mu\sigma}}\sigma} = \frac{k_p k_r \sqrt{\omega\mu}}{p\sqrt{2\sigma}}$$

Here, we see skin-effect resistance grows proportional to the square root of ω.

Skin Effect Impedance

When you alter the path of current, you also alter the inductance.

Therefore, to fully characterize skin effect, you need to consider both the changes in resistance and inductance with frequency.

However, a simplier model can be derived that approximates the *impedance*.

The high-frequency series *impedance* of a wire is given by

$$R_{AC} = (1 + j)R_0\sqrt{\frac{\omega}{\omega_0}} \ \ \Omega/m$$

ω is the frequency of operation

ω_0 is a frequency beyond the onset of skin effect

R_0 is $\mathrm{Re}[R_{AC}]$ given above evaluated at ω_0

The term (1 + j) signifies the real (resistive) and imaginary (inductive) parts are equal (phase angle is 45 degrees).

The impedance at any frequency is approximated by

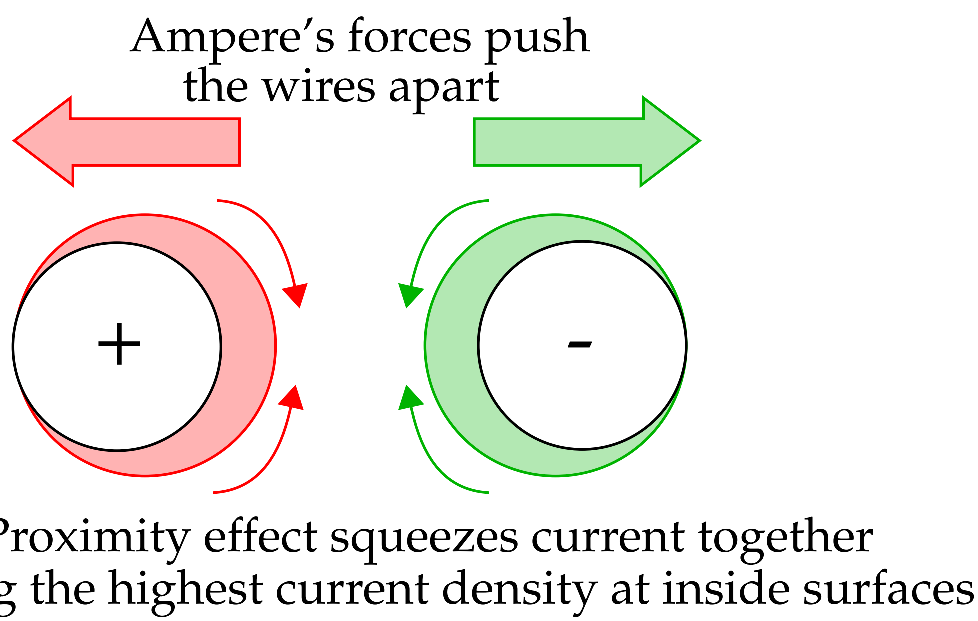
$$z(\omega) = \sqrt{(R_{DC})^2 + (R_{AC})^2}$$

Proximity Effect

Skin effect forces current to the surface of the wire, but it does **not** have a *uniform distribution* around the perimeter.

The magnetic field from the wire and its return path cause non-uniformity, which in turn, increases resistance beyond that of skin effect alone.

In parallel conductors, the additional increase in resistance is called *proximity effect*.

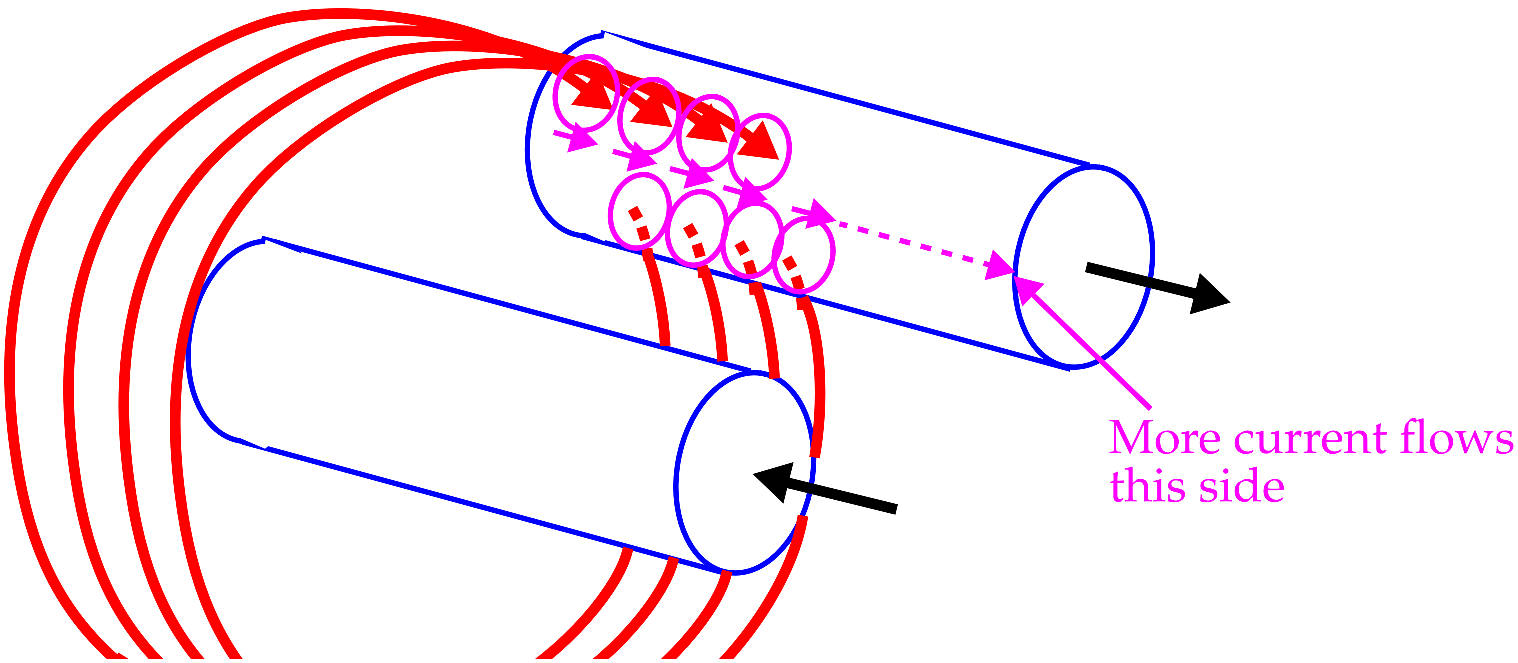


Proximity Effect

Unlike Ampere's discovery that adjacent wires carrying opposing DC currents repel (with mechanical force), proximity effects mearly re-distributes current density.

Proximity effect is an *inductive mechanism* where high frequency current density migrates toward a **minimum-inductance** configuration.

Once achieved for some onset frequency, it does not change for higher frequencies (upto non-TEM modes).



Proximity Effect

Therefore skin effect and proximity effect are two manifestations of the same principle, and both take effect at ω_δ.

Skin effect is a reaction to magnetic fields generated by current flowing **within** the affected conductor.

Proximity effect is a reaction to magnetic fields generated by current flowing in **other conductors**.

We incorporated proximity effect in an equation given earlier as k_p

$$Re[R_{AC}] = \frac{k_p k_r \sqrt{\omega\mu}}{p\sqrt{2\sigma}}$$

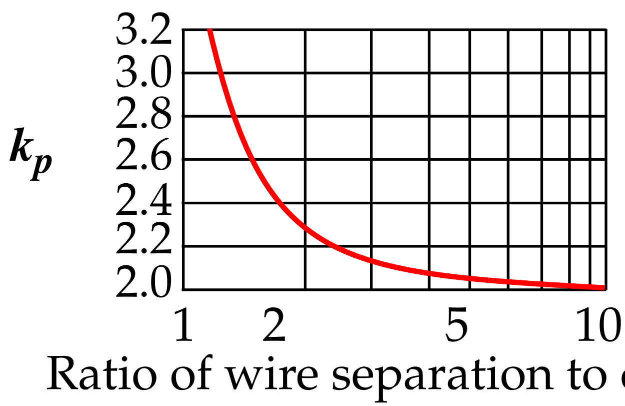
At frequencies below ω_δ, k_p has no effect.

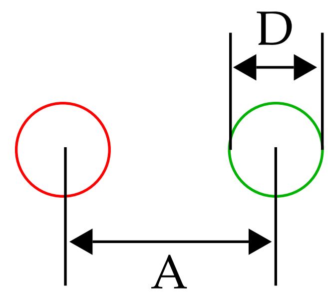
Any conductor well separated from a low-resistance return path has $k_p = 1$.

Differential (twisted pair) configuration have $k_p = 2$.

Proximity Effect

k_p grows as the ratio of separation/diameter gets smaller, i.e., as the wires are moved closer together.





The same forces responsible for the proximity effect also cause ground plane return current to follow closely underneath a signal conductor.

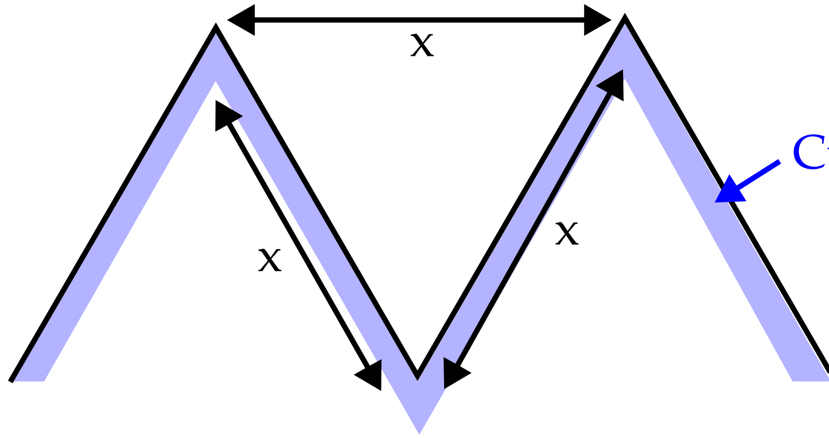
Current flows in a formation that minimizes the *total loop inductance*.

Nature picks a current density distribution that **minimizes** the total energy stored in the magnetic fields surrounding the system.

Surface Roughness

High frequency currents cling to the surfaces of the conductors.

If the conductor has a rough surface, current must flow over hills and valleys



Bumpy surface profile causes current to flow 2x the distance.

The onset frequency is related to the *root-mean-square* (RMS) height, h_{RMS}, of the surface bumps

$$\omega_{rough} = \frac{2}{\mu\sigma h_{RMS}^2}$$

μ is magnetic permeability (H/m)
σ is conductivity (S/m)
ω_{rough} is freq. at which roughness effect has assumed 60% of its ultimate effect (rad/s)

Dielectric Effects

The amount of incident EM power converted by a dielectric material into heat is called **dielectric loss**.

(Put a piece of FR4 board in the microwave for 1 minute -- it heats up)

For transmission lines, this translates into signal attenuation.

Dielectric losses scale in proportion to both frequency and length.

Loss in FR-4 (glass epoxy) is negligible for digital apps below 1 GHz, but grows above this frequency.

Ceramic substrates, *alumina*, are used for apps with $f > 1$ GHz.

The loss is often lumped with the skin effect loss.

Impedance in Series with the Return Path

The return wire adds an impedance, z_g, in series with the signal wire, which modifies the input impedance and transmission coefficient.

$$Z_C = \sqrt{\frac{z + z_g}{y}} \qquad \gamma = \sqrt{(z + z_g)y}$$