University of New Mexico

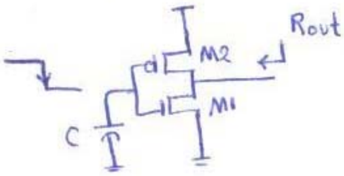
Department of Electrical and Computer Engineering

ECE321-Electronics I (Fall 2015) **Homework Solution** #11

Problem 1:

Based on hint:

1. Connecting a load of 100pf to the gate



$$I_{av} = \frac{1}{2}\left[I_{ds}(V_{out}=0) + I_{ds}\left(V_{out} = \frac{V_{dd}}{2}\right)\right]$$

2. Calculating the LH propagation delay

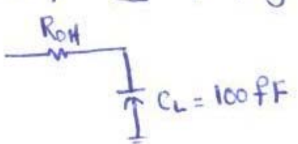
$V_{out} = 0$ $\quad I_{ds} = \frac{k_p'}{2}\left(\frac{W}{L}\right)_p \left[V_{gs} - |V_{tp}|^2\right](1+\lambda V_{ds}) = 802\mu A$

$V_{out} = \frac{V_{dd}}{2}$ $\quad I_{ds} = \frac{k_p'}{2}\left(\frac{W}{L}\right)_p \left[2(V_{gs} - |V_{tp}|)\frac{V_{dd}}{2} - \frac{V_{dd}^2}{4}\right] = 555\mu A$

$$I_{av} = \frac{1}{2}(802\mu A + 555\mu A) = 678.5\mu A$$

$$t_{pd} = \frac{C_L\, V_{dd}/2}{I_{av}} = \frac{(100fF)((1.5/2)V)}{678.5\mu A} = 11.05\ \text{ns}$$

3. Equating propagation delay to a simple RC network



$$t_{pLH} = 0.7\, R_{OH} \cdot C_L \longrightarrow R_{OH} = \frac{t_{pLH}}{0.7 C_L} = \frac{11.05\ \text{ns}}{0.7\,(100fF)} \qquad R_{OH} = 1579\,\Omega$$

Problem 2:

Based on Elmore technique:

$$\begin{aligned}\tau = {} & (1.579k\Omega + 120\Omega)\,75fF \\ & + (1.579k\Omega + 120\Omega)(15fF + 75fF + 50fF) \\ & + (1.579k\Omega + 120\Omega + 165\Omega)\,100fF \\ & + (1.579k\Omega + 120\Omega + 165\Omega)\,95fF \\ & + (1.579k\Omega + 120\Omega + 165\Omega + 105\Omega)\,65fF \\ & + (1.579k\Omega + 120\Omega + 165\Omega + 105\Omega + 220\Omega)\,200fF\end{aligned}$$

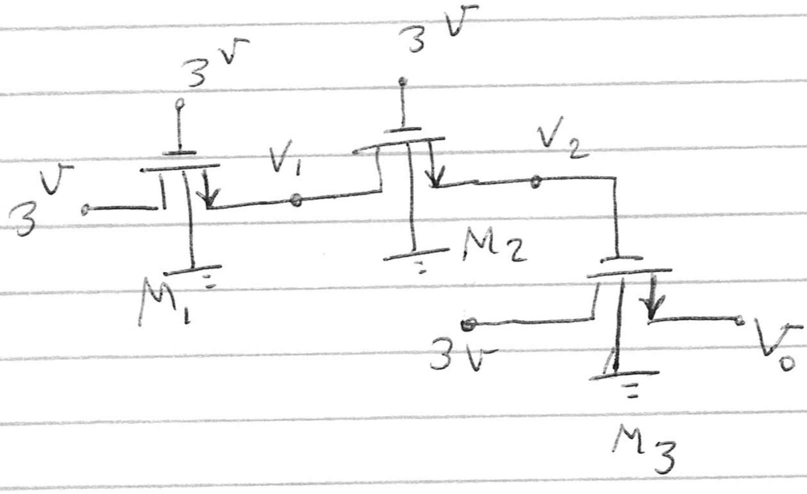
$\longrightarrow$ $\tau = 1.24\,ns$

$t_{pLH} = 0.7\tau = 0.7 \times 1.24ns = 871ps$

3) $V_{T_0} = 0.5^V$

$\gamma = 0.3\ V^{1/2}$

$|\varphi_F| = 0.35$



For M1: $\begin{cases} V_1 = 3 - V_{T_1} \\ V_{T_1} = V_{T_0} + \gamma\left(\sqrt{2\varphi_F + V_{SB}} - \sqrt{2\varphi_F}\right) \\ V_{SB} = V_1 \end{cases}$

$\Rightarrow \quad V_1 = 3 - \left\{0.5 + 0.3\left(\sqrt{0.7 + V_1}\right) - \sqrt{0.7}\right)\right\}$

Solve for V_1 $\Rightarrow$ $V_1 = 2.23687^V$

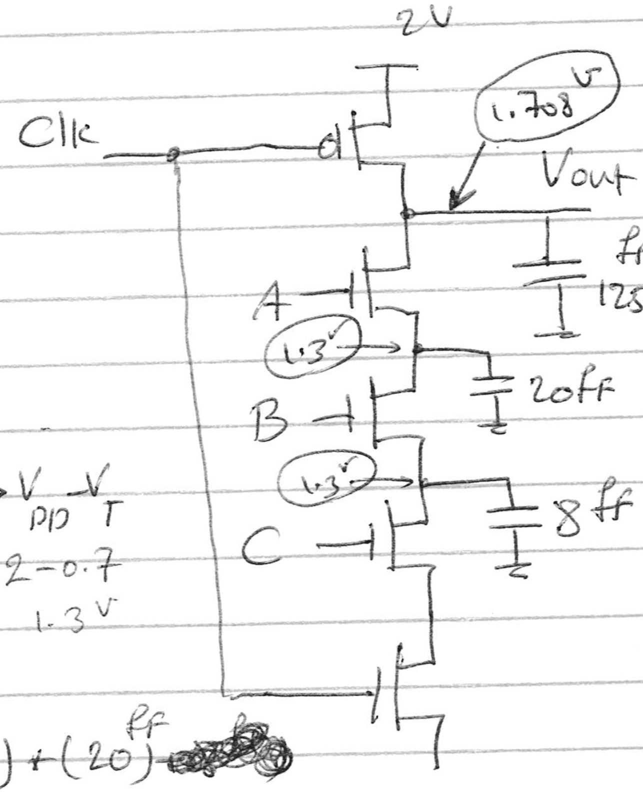
For M2: (Same as M1) $V_2 = V_1 = 2.23687^V$

For M3: $\begin{cases} V_o = V_2 - V_{T_3} \\ V_{T_3} = V_{T_0} + \gamma\left(\sqrt{2\varphi_F + V_{SB}} - \sqrt{2\varphi_F}\right) \\ V_{SB} = V_o \end{cases}$

$\Rightarrow \quad V_o = 2.23687 - \left\{0.5 + 0.3\left(\sqrt{0.7 + V_o} - \sqrt{0.7}\right)\right\}$

Solve for V_o $\Rightarrow$ $V_o = 1.53898^V$

4) $V_A = V_B = 2^V$, $V_C = 0$



$Q_{initial} = 125^{ff} * 2^V = 250^{fC}$

Case 1: $Q_{final} = (125^{ff} + 20^{ff} + 8^{ff}) * V_x$

$Q_{initial} = Q_{final} \Rightarrow V_x = \cancel{1.634^V} > V_{DD} - V_T$

impossible

$2 - 0.7$

1.3^V

Case 2: $Q_{final} = 125^{ff} * V_x + (V_{DD} - V_T) * (20^{ff})$
$+ (V_{DD} - V_T) * (8^{ff})$

$= 125^{ff} * V_x + 1.3^V * 20^{ff} + 1.3^V * 8^{ff} = 125^{ff} V_x + 36.4^{fC}$

$Q_{initial} = Q_{final} \Rightarrow 125^{ff} V_x + 36.4^{ff} = 250^{ff}$

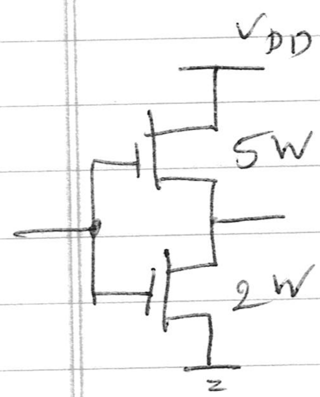
$\Rightarrow V_x = 1.7088^V$

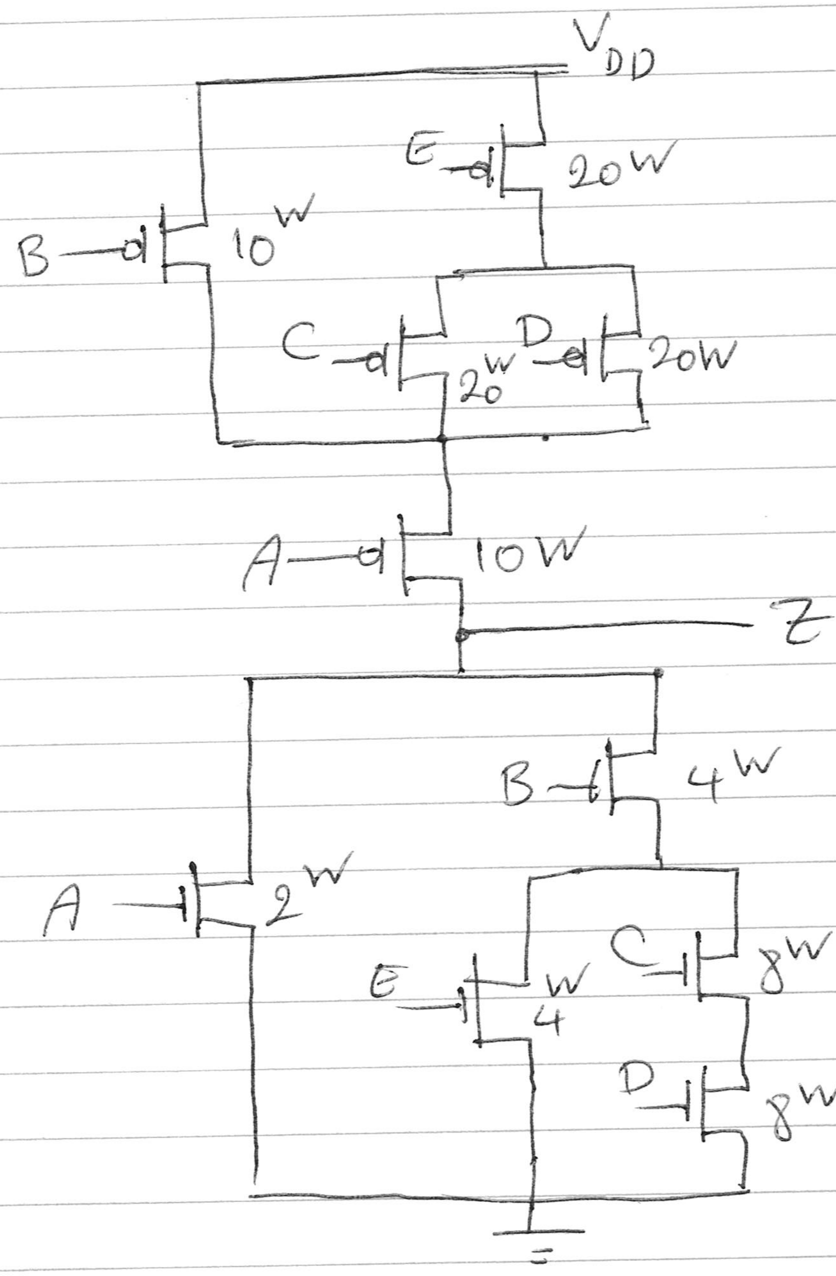
$V_{125ff} = 1.7088^V$

$V_{20ff} = 1.3^V$

$V_{8ff} = 1.3^V$

5)





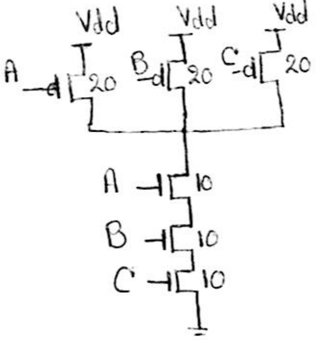
6) $P_{leakage} = \frac{1}{2} V_{DD} [I_{off(nmos)} + I_{off(pmos)}]$

$W_n = 58.5^{\mu m} \rightsquigarrow I_{off(NMOS)} = 702^{nA}$

$W_p = 85.5^{\mu m} \rightsquigarrow I_{off(PMOS)} = 2223^{nA}$

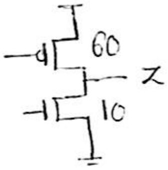
$\Rightarrow P_{leakage} = \frac{1}{2} * 0.5^{V} * (702^{nA} + 2223^{nA})$

$\Rightarrow P_{leakage} = 1.755^{\mu W}$



Problem 7:

$\simeq$

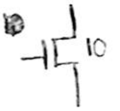


For a nMos: $W_{eq} = \dfrac{1}{1/30 + 1/30 + 1/30} = 10$

For pMos: $W_{eq} = 20 + 20 + 20 = 60$

1) 000 ⟶ 111

The 3 Nands n-chanel transistors in series are on and the p-channel transistors are off hence, the current across the nMos transistor is:

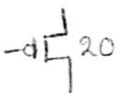


$I = \dfrac{K_n'}{2} \left(\dfrac{W}{L}\right) (V_{gs} - V_{tn})^2$ where $V_{gs} = V_{dd}$

$I = \dfrac{100 \mu A/V^2}{2} (10)(1.5V - 0.4V)^2 = 605 \mu A$

$t_{pLH} = \dfrac{C_L \, V_{dd}/2}{I} = \dfrac{(100 fF)(1.5V/2)}{605 \mu A} = 123.97 \text{ ps}$

2) 111 ⟶ 110



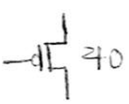
The pMos transistor with C input will be on and the other pMos transistors will be off, then the current for the pMos transistor is:

$I = \dfrac{K_p'}{2} (W/L)(V_{gs} - |V_{tp}|)^2$

$I = \dfrac{60 \mu A/V^2}{2} (20)(1.5 - 0.4)^2 = 726 \mu A$

$t_{pLH} = \dfrac{C_L V_{dd}/2}{I_p} = \dfrac{(100fF)(1.5V/2)}{726 \mu A} = 103.31 \text{ ps}$

3) 111 ⟶ 100



The pMos transistor with B input and C input will be on and the nMos transistor with A input will be on; then, the current for the pMos transistor is:

$I_p = \dfrac{K_p'}{2} (W/L)(V_{gs} - |V_{tp}|)^2$

$I_p = \dfrac{60 \mu A/V^2}{2} (40)(1.5V - 0.4V)^2 = 1452 \mu A$

$t_{pLH} = \dfrac{C \, V_{dd}/2}{I_p} = \dfrac{(100fF)(1.5V/2)}{1452 \mu A} = 51.65 \text{ ps}$

4) 111 ⟶ 000 All the pMos transistors are on and all the nMos transistors are off; then, the current for the pMos transistor is:

$$I_p = \frac{k'_p}{2} (W/L) (V_{gs} - |V_{tp}|)^2$$

$$I_p = \frac{60 \mu A/V^2}{2} (60)(1.5V - 0.4V)^2 = 2178 \mu A$$

$$t_{pLH} = \frac{C V_{dd}/2}{I_p} = \frac{(100fF)(1.5V/2)}{2178 \mu A} = 34.43 ps$$

b) $t_{pd} = 0.7RC \longrightarrow R = \frac{t_{pd}}{0.7C}$

1) 000 ⟶ 111 $R_{eff} = \frac{123.97 ps}{0.7(100fF)} = 1771 \Omega$

2) 111 ⟶ 110 $R_{eff} = \frac{103.31 ps}{0.7(100fF)} = 1475.86 \Omega$

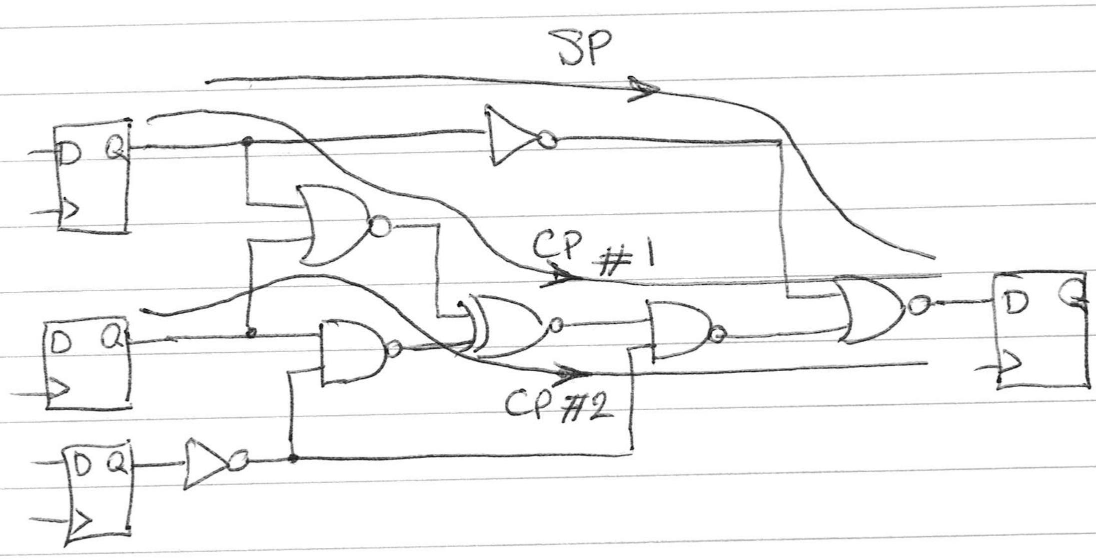
3) 111 ⟶ 100 $R_{eff} = \frac{51.65 ps}{0.7(100fF)} = 737.86 \Omega$

4) 111 ⟶ 000 $R_{eff} = \frac{34.43 ps}{0.7(100fF)} = 491.86 \Omega$

Transition (ABC)	Propagation Delay	Effective Output Resistance
000 ⟶ 111	123.97 ps	1771 Ω
111 ⟶ 110	103.31 ps	1475.86 Ω
111 ⟶ 100	51.65 ps	737.86 Ω
111 ⟶ 000	34.43 ps	491.86 Ω

8)

a)



b) There are 2 critical paths (CP #1, CP #2) and one short path (SP)

$T > t_{c2q} + t_{logic} + t_{setup}$

$T > 1^{ns} + (550^{ps} + 750^{ps} + 350^{ps} + 450^{ps}) + 3^{ns}$

$T > 6.1^{ns} \Rightarrow f_{max} = \frac{1}{T_{min}} = \frac{1}{6.1^{ns}} = 163.93$ MHz

c) SP is the short path.

d) $t_{hold} \overset{?}{<} t_{c2q} + t_{logic, CD}$

$2^{ns} \overset{?}{<} 1^{ns} + (150^{ps} + 450^{ps})$

$2^{ns} \not< 1.6^{ns}$ ⇝ This is a hold time violation

9) a) $Z = \overline{A(BC+DE)}$

b) ABCDE = 11010

c) if ABCDE = 11000 , $V_{DD} = 1.2^V$

$$\begin{cases} Q_{initial} = 20^{fF} * 1.2^V \\ Q_{final} = (20^{fF} + 3^{fF} + 2^{fF}) * V_X \end{cases} \Rightarrow V_X = 0.96^V > 0.8^V$$

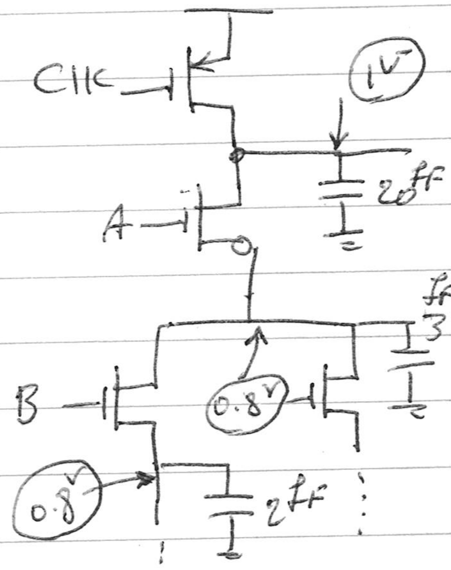
(impossible)

$V_{DD} - V_T = 1.2 - 0.4^V = 0.8^V$

rework ⇒

$$\begin{cases} Q_{initial} = 20^{fF} * 1.2^V \\ Q_{final} = 20^{fF} * V_X + 3^{fF} * (1.2^V - 0.4^V) + 2^{fF} * (1.2^V - 0.4^V) \end{cases}$$

$\Rightarrow V_X = 1.0^V$ ✓



d)
$$\begin{cases} t_{0 \to 90\%} = 250^{ps} \\ \Delta V = 0.9 * 1.2^V = 1.08^V \\ C_{total} = 20^{fF} + (3^{fF} + 2^{fF} + 2^{fF} + 3^{fF}) = 30^{fF} \\ I = \frac{k'_p}{2} \left(\frac{W}{L}\right)_p (|V_{GS}| - |V_{Tp}|)^2 \end{cases}$$

$= \frac{50^{\mu A/V^2}}{2} \left(\frac{W}{L}\right)_p (1.2 - 0.5)^2$; $t_{0-90\%} = \frac{C_L \Delta V}{I}$

$$\Rightarrow 250^{ps} = \frac{30^{fF} * (1.08^V)}{\frac{50^{\mu A/V^2}}{2} \left(\frac{W}{L}\right)_p (1.2 - 0.5)^2} \Rightarrow \left(\frac{W}{L}\right)_p = 10.58$$

$W_p = 1.058^{\mu m}$ or $L = 100^{nm}$

10)