# ECE321 – Electronics I

## Lecture 12: CMOS Inverter: Noise Margin & Delay Model

**Payman Zarkesh-Ha**

*Office: ECE Bldg. 230B*
*Office hours: Tuesday 2:00-3:00PM or by appointment*
*E-mail: payman@ece.unm.edu*



## *Review of Last Lecture*

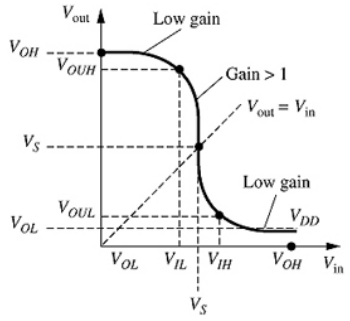
- **CMOS Inverter**
- **Voltage Transfer Characteristics (VTC)**
  - **Switching threshold voltage**
  - **Output high voltage**
  - **Output low voltage**
  - **Input high voltage**
  - **Input low voltage**
- **Current Transfer Characteristics (ITC)**
  - **Peak current**

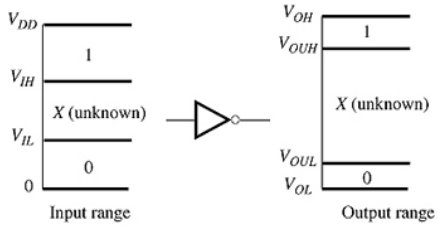
# Today's Lecture

- Noise Margin Definition
- Approximation of Noise Margin for CMOS Inverter
- Propagation Delay
- Rise and Fall Times
- Input and Self Loading (Load) Capacitances
- Delay Approximation



# Review: Voltage Transfer Characteristics

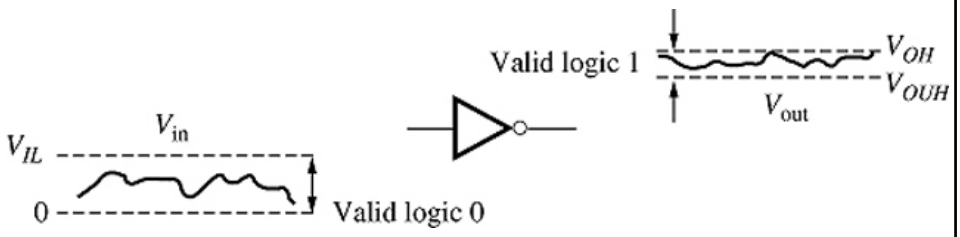



***Important Parameters on VTC:***

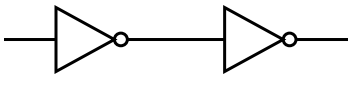
- Switching Threshold Voltage $V_S$ or $V_M$
- Gain at $V_S$ or $V_M$
- Output High Voltage $V_{OH}$
- Output Low Voltage $V_{OL}$
- Input High Voltage $V_{IH}$
- Input Low Voltage $V_{IL}$

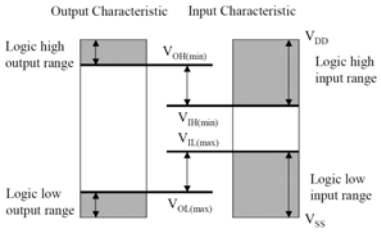
# Effect of Noise on a CMOS Inverter






# Noise Margin





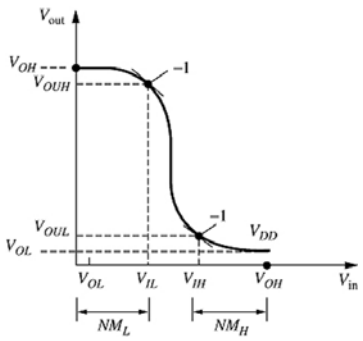

- **Noise Margin**

$$NM_H = V_{OH} - V_{IH}$$

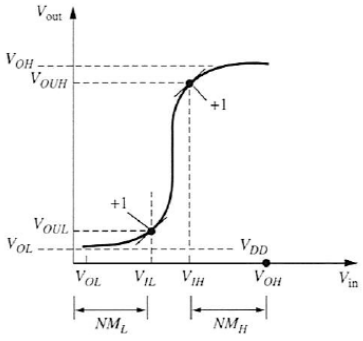
$$NM_L = V_{IL} - V_{OL}$$

- **It is better to have:**
  - $V_{OH} = V_{DD}$
  - $V_{OL} = V_{SS}$
  - Large $NM_H$
  - Large $NM_L$

# Noise Margin in Inverter and Buffer




*VTC of an inverter*



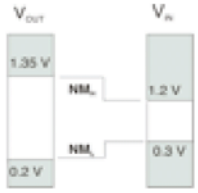

*VTC of a buffer*



# Example: Noise Margin Calculation

An IC with $V_{DD} = 1.5$ V shows $V_{OH} = 1.35$ V, $V_{OL} = 0.2$ V, $V_{IH} = 1.2$ V, and $V_{IL} = 0.3$ V. Calculate the $NM_L$ and $NM_H$ for this IC.

Sketch the NM rectangles




$NM_H = 1.35 - 1.2 = 150$ mV

$NM_L = 0.3 - 0.2 = 100$ mV

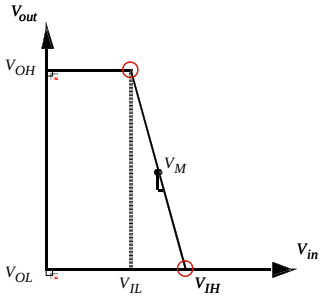
# Noise Margin Approximation

- How to compute Noise Margin
  - Usually it is harder to compute the exact value of NM
  - Use approximation (gain factor)
  - Determine gain at $V_M$
  - Extrapolate $V_{IL}$ and $V_{IH}$
  - $V_{OL}$ and $V_{OH}$ are easy to compute
- Example:
  - $NM_L$ and $NM_H$ in CMOS inverter

$$V_{IH} - V_{IL} = -\frac{(V_{OH} - V_{OL})}{g} = \frac{-V_{DD}}{g}$$

$$V_{IH} = V_M - \frac{V_M}{g} \qquad V_{IL} = V_M + \frac{V_{DD} - V_M}{g}$$

$$NM_H = V_{DD} - V_{IH} \qquad NM_L = V_{IL}$$




  - where

$$g = \frac{-2}{\lambda_n + |\lambda_p|}\left(\frac{1}{V_M - V_{Tn}} + \frac{1}{V_{DD} - V_M - |V_{Tp}|}\right)$$



# Example: Noise Margin Approximation

- A CMOS inverter has $V_{DD}$=5V is designed to have $V_M$=2.9V. If $V_{Tn}$=0.7, $V_{Tp}$=-0.5, $\lambda_n$=0.05 $V^{-1}$ and $\lambda_p$=-0.08 $V^{-1}$. Find the noise margins NMH and NML.

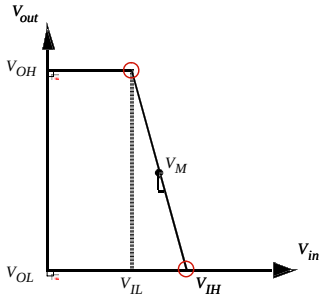
$$g = \frac{-2}{0.05 + 0.08}\left(\frac{1}{2.9 - 0.7} + \frac{1}{5 - 2.9 - 0.5}\right) = -16.6$$

$$V_{IL} = 2.9 + \frac{5 - 2.9}{-16.6} = 2.77V$$

$$V_{IH} = 2.9 - \frac{2.9}{-16.6} = 3.1V$$

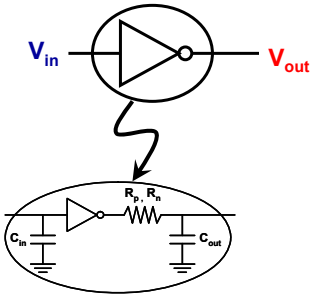
$$NMH = 5 - 3.1 = 1.9V$$

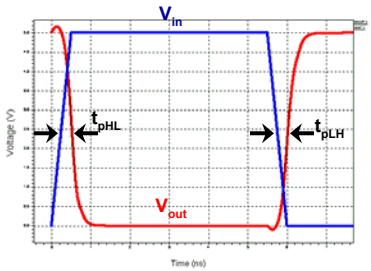
$$NML = 2.77 - 0 = 2.77V$$



# Dynamic Behavior of CMOS Inverter

- Changing of the input doesn't instantaneously change the out pf an inverter
- This is mostly due to the time it takes to chrgae or dischage the output/load capacitor
- It is important to know how long it takes to get the signal out of the inverter or any CMOS logic gate

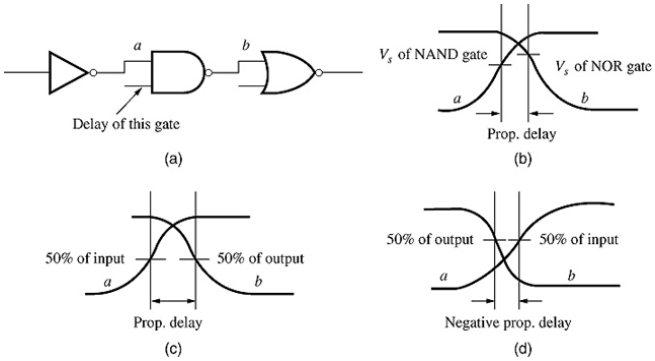




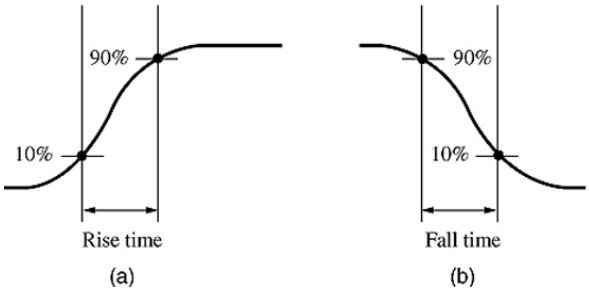

# Definition: Propagation Delay

- Definition of propagation delay
  - is delay from where input crosses 50%Vdd to where output crosses 50%Vdd
  - Remember: the value of 50%Vdd is from switching threshold voltage ($V_M$)
  - $t_{pHL}$ is propagation delay when ***output*** switches from "High to Low"
  - $t_{pLH}$ is propagation delay when ***output*** switches from "Low to High"
- To compute delay, the inverter must be simplified



# Definition: 10%-90% Rise/Fall Times

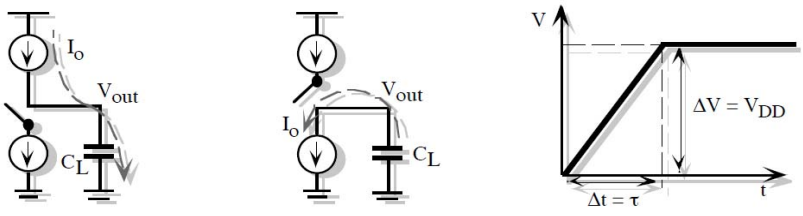
- **Definition of 10%-90% rise time**
  - **is delay from 10%Vdd to 90%Vdd in the output**
  - **$t_r$ is the rise time when *output* switches from "Low to High"**
- **Definition of 90%-10% fall time**
  - **is delay from 90%Vdd to 10%Vdd in the output**
  - **$t_f$ is the fall time when *output* switches from "High to Low"**






# Definition: Linear Rise/Fall Times

- **Definition of rise time (Hawkin's book)**
  - **is delay from 0 to Vdd in the output assuming a constant current source model**
  - **$t_r$ is the rise time when *output* switches from "Low to High"**
- **Definition of fall time (Hawkin's book)**
  - **is delay from Vdd to 0 in the output assuming a constant current source model**
  - **$t_f$ is the fall time when *output* switches from "High to Low"**

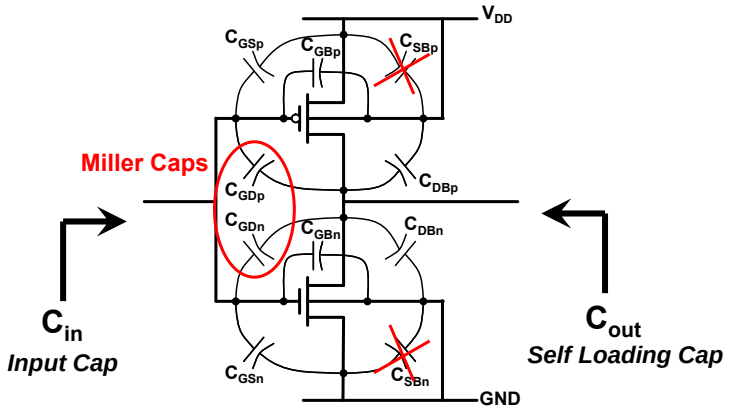


# Delay Calculation in CMOS Inveretr

- It is not easy to accurately calculate delay in CMOS inverter, because
  - CMOS inverter is a non-linear circuit, therefore exact delay calculation requires solving a non-linear differential equation
  - Most of the elements in the circuit is voltage dependent (transistor drive current, parasitic capacitances, channel length modulation, etc.)
- A simplified model is required for basic calculations and design process
- Note that SPICE actually does solve the non-linear circuit accurately, but is only good for final verification, not the design



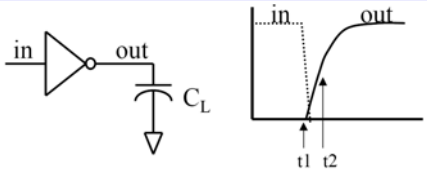
# Gate Input/Output Capacitances Components




$$C_{in} = (C_{GDp} + C_{GSp} + C_{GBp}) + (C_{GDn} + C_{GSn} + C_{GBn})$$

$$C_{out} = (2C_{GDp} + C_{DBp}) + (2C_{GDn} + C_{DBn})$$

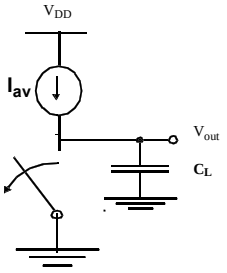
# Propagation Delay Model




$$Q = I \cdot \Delta t = C \cdot \Delta V$$

$$\Delta t = \frac{C \cdot \Delta V}{I}$$

- Propagation delay is defined as the time between the input reaching $V_{DD}/2$ and the output reaching $V_{DD}/2$
- To simplify the model, let's assume I is a constant $I_{av}$

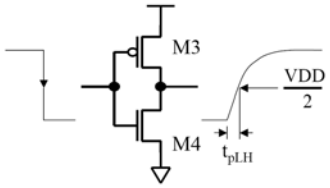



$$t_{pLH} = t_2 - t_1 = \frac{C_L \cdot (V_{DD}/2)}{I_{av}}$$



# Propagation Delay Model

- How to compute $I_{av}$ ?
  - Assume step input
  - NMOS goes into cutoff and stays there
  - PMOS goes into saturation at first because $|V_{DS}| > |V_{GS}| - |V_T|$
  - PMOS will transition to linear, however, before $V_{out}$ reaches $V_{DD}/2$




$$I_{av} = \frac{I_{DS}(V_{out} = 0) + I_{DS}(V_{out} = V_{DD}/2)}{2}$$

$$I_{av} = \left(\frac{K'_p}{2}\right)\left(\frac{W_p}{L_p}\right)\left(\frac{(V_{DD} - |V_{tp}|)^2}{2} + \frac{V_{DD}(V_{DD} - |V_{tp}|)}{2} - \frac{V_{DD}^2}{8}\right)$$

## Propagation Delay Model

- A simpler model for $I_{av}$ can be obtained by assuming that the PMOS stays in saturation the whole time, therefore acts as an ideal current source

$$I_{av} = \left(\frac{K'_p}{2}\right)\left(\frac{W_p}{L_p}\right)\left(V_{DD} - |V_{tp}|\right)^2$$

$$t_{pLH} = t_2 - t_1 = \frac{C_L \cdot (V_{DD}/2)}{I_{av}} \quad \Rightarrow \quad t_{pLH} = \frac{C_L \cdot V_{DD}}{K'_p\left(\frac{W_p}{L_p}\right)\left(V_{DD} - |V_{Tp}|\right)^2}$$

- Assuming $V_{DD} >> V_{Tp}$

$$t_{pLH} = \frac{C_L}{K'_p\left(\frac{W_p}{L_p}\right)V_{DD}}$$

- Same arguments hold for $t_{pHL}$



## Minimum Delay Design Techniques

- Reduce $C_{in}$ and $C_{out}$ – Careful layout, keep drain diffusion area as small as possible
- Reduce wiring capacitance – Careful layout, keep devices as close as possible
- Increase (W/L) of devices – Need to be careful not to get into self-loading effect
- Increase $V_{DD}$ – Need to be careful not to get into $V_{DSAT}$ or velocity saturation

# *Example: CMOS Inverter Delay*

- **A CMOS inverter has $V_{DD}$=5V is designed such that $(W/L)_n$=10 and $(W/L)_p$=20. Assume that $V_{Tn}$=0.7, $V_{Tp}$=-0.6, $K'_n$=100 uA/V$^2$, $K'_p$=-60 uA/V$^2$, and the load capacitance is 100fF.**

  **1) Use $I_{av}$ model to find $t_{pHL}$, $t_{pLH}$, $t_{r(10\%-90\%)}$, and $t_{f(90\%-10\%)}$.**

  **2) Use constant current source model to find $t_{pHL}$, $t_{pLH}$, $t_{r(10\%-90\%)}$, $t_{f(90\%-10\%)}$, $t_r$, and $t_f$.**

- *Answers:*

  *1) $t_{pHL}$=29.64 ps, $t_{pLH}$=23.73 ps, $t_{r(10\%-90\%)}$=56.71 ps, and $t_{f(90\%-10\%)}$=70.98 ps*

  *2) $t_{pHL}$=27.04 ps, $t_{pLH}$=21.52 ps, $t_{r(10\%-90\%)}$=34.43 ps, and $t_{f(90\%-10\%)}$=43.27 ps, $t_r$=43.04 ps and $t_f$=54.98 ps.*