BAS

Sensor Notes

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Motion Measurement Techniques

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FOREWORD

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This technical note has been reviewed and is approved.

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SECTION I

INTRODUCTION

Motion measurements using seismic mass techniques take one of two general forms depending on the method used to measure the motion of the seismic mass itself. The first general form uses an electrical pick-off of the displacement of the seismic mass relative to the case of the transducer. The second general form results from an electrical pick-off of the velocity of the seismic mass relative to the velocity of the transducer case. Each form may produce an output analog of any of several motion parameters (displacement, velocity, etc.) depending on the mechanical parameter (spring constant, damping factor, and mass size) of the seismic system with the exception that the second form will not produce a displacement analog.

SECTION II

RELATIVE DISPLACEMENT PICK-OFF SYSTEMS

Consider a system of masses connected as in figure 1.

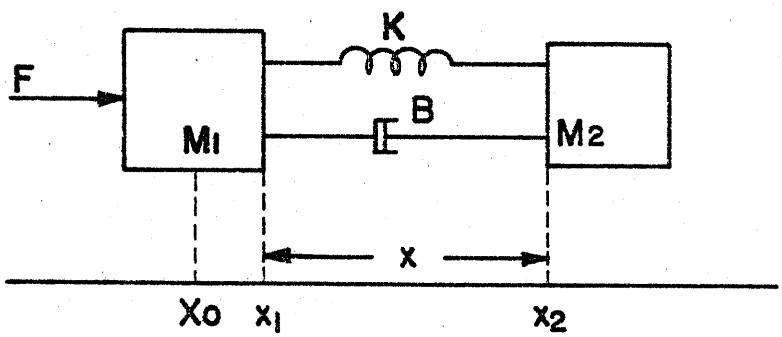

Figure 1. Seismic Mass System

Mass M_1 experiences a force F and transmits, therefore, forces to M_2 through a spring with coefficient K and a dashpot with a ~~friction~~ DAMPING coefficient B. During time period Δt, mass M_1 moves a distance x_1 and mass M_2 moves to position x_2 from some original unknown position. Using Laplace notation and presuming that the initial conditions for all time derivatives of x_1 and x_2 are zero, the expression for the motion of M_2 becomes

$$-M_2 s^2 x_2 = Bs(x_2 - x_1) + K(x_2 - x_1) \tag{1}$$

or

$$Bsx_1 = M_2 s^2 x_2 + Bsx_2 + K(x_2 - x_1)$$

$$\text{but } x_2 = x_1 + x$$

Therefore,

$$Bsx_1 = M_2 s^2 x_1 + M_2 s^2 x + Bsx_1 + Bsx + Kx \quad (2)$$

By eliminating terms and rearranging, the result is

$$-s^2 x_1 = x(s^2 + \frac{Bs}{M_2} + \frac{K}{M_2}) \quad (3)$$

Now, if M_1 is the case of an accelerometer, then $s^2 x_1$ represents the acceleration A(s) of that case. Further, let a potentiometer with resistance R be attached to M_1 and the slide arm attached to M_2. Now, let a voltage V be across the potentiometer and a voltage v_0 be measured on the slide arm as in figure 2.

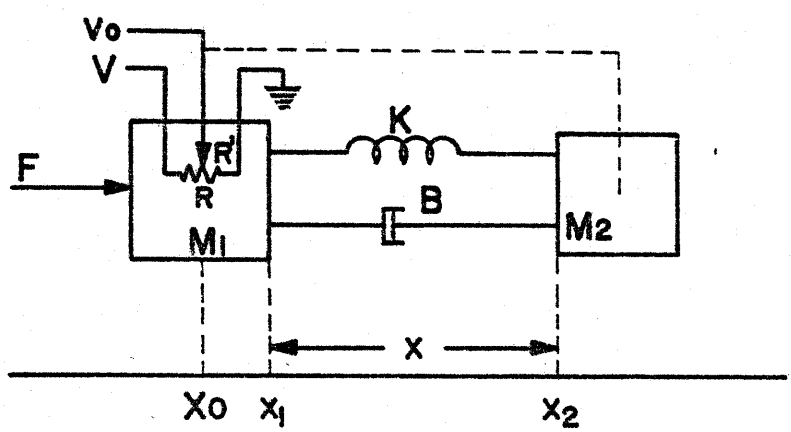

Figure 2. Seismic Mass System with Displacement Pick-off

Then

$$v_0 = V\frac{R'}{R}. \quad \text{Let } R' = -Cx$$

where C is a constant mechanical advantage and R' is the resistance from the slide arm to common.

and equation 3 becomes

$$A(s) = v_0 \frac{R}{VC} (s^2 + \frac{Bs}{M_2} + \frac{K}{M_2}) \tag{4}$$

Solving for v_0 reveals that

$$v_0 = \frac{A(s) \frac{VC}{R}}{s^2 + \frac{B}{M_2} s + \frac{K}{M_2}} \tag{5}$$

For steady state, make the $s=j\omega$ substitution. Now, obviously, for low frequencies ($\omega << \sqrt{\frac{K}{M_2}}$) and low damping (small B)

$$v_0(j\omega) \cong \frac{A(j\omega) \frac{VC}{R}}{\frac{K}{M_2}} \tag{6}$$

and the output v_0 is proportional to the case acceleration.

However, if K is small and B is large, then for

$$\sqrt{\frac{K}{M_2}} << \omega << \frac{B}{M_2}$$

$$v_0(j\omega) \cong \frac{U(j\omega) \frac{VC}{R}}{\frac{B}{M_2}}$$

and the output v_0 is approximately proportional to the case velocity if $U(j\omega)$ is the steady state case velocity.

Consider now

$$\frac{B}{M_2} << \omega$$

and a small K (low spring constant) with small B.

Then the expression for v_0 approaches the following

$$v_0(j\omega) \cong D(j\omega) \frac{VC}{R} \tag{8}$$

where $D(j\omega)$ is the steady state displacement.

SECTION III

RELATIVE VELOCITY PICK-OFF SYSTEMS

Consider the system of figure 3.

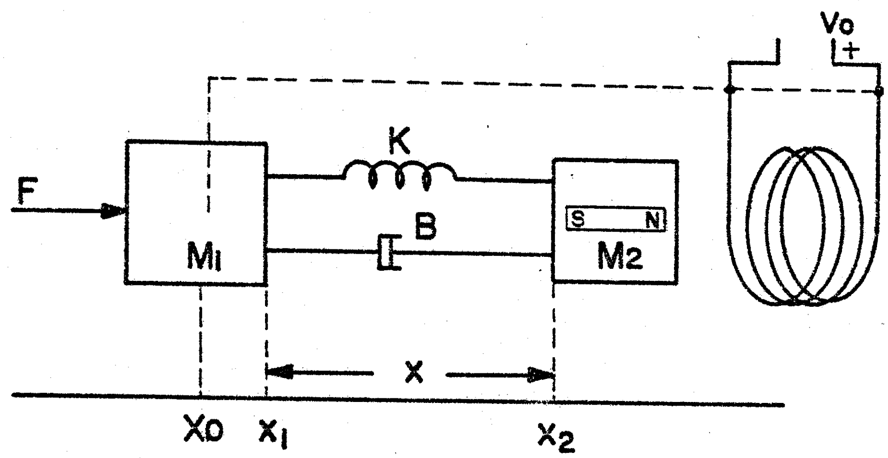

Figure 3. Seismic Mass System with Velocity Pick-off

As before, the motion description is equation 3. However, now the output voltage is from a coil which surrounds a moving magnet. Note that the coil is fixed to M_1 and the magnet to M_2. Now

$$x = \frac{-v_0}{Gs}$$

where G is a constant related to the strength of the magnetic field and the number of turns on the coil.

Substituting in equation 3 results in

$$s^2 x_1 = \frac{v_0}{Gs}\left(s^2 + \frac{B}{M_2} s + \frac{K}{M_2}\right) \tag{9}$$

or

$$v_0(s) = \frac{G\, s^3\, x_1}{s^2 + \frac{B}{M_2} s + \frac{K}{M_2}} \tag{10}$$

Let s^2x_1 be A(s) which is the case acceleration. Also, let

$$\sqrt{\frac{K}{M_2}} \ll \omega \ll \frac{B}{M_2} .$$

Then, upon substituting $s=j\omega$, equation 10 becomes

$$v_0(j\omega) \cong \frac{G\,A(j\omega)}{\frac{B}{M_2}} \qquad (11)$$

and the output is approximately proportional to acceleration.

Next, let sx_1 be U(s) which is the case velocity. In equation 10, let both M_2 and ω be very large, i.e.,

$$\frac{B}{M_2} \ll \omega .$$

Both B and K should be small; that is, there should be a weak spring and low damping.

Then, upon the substitution of $s=j\omega$, equation 10 becomes

$$v_0(j\omega) = G\,U(j\omega) \qquad (12)$$

Note that v_0 is approximately proportional to the case velocity.

SECTION IV

CONCLUSIONS

It is apparent that form 1 (displacement pick-off) may, by adjusting the seismic parameters, be used to measure acceleration, velocity, or displacement. Also, form 2 (velocity pick-off) may be used to measure acceleration or velocity, depending on the seismic parameters.

Table 1 is the summary of the seismic parameters and outputs.

Form 1

Output	Mass (M_2)	Spring Constant (K)	DAMPING ~~Friction~~ Coefficient (B)
Acceleration	Small*	Large	Small
Velocity	Small*	Small	Large*
Displacement	Large	Small	Small

Form 2

Output	Mass (M_2)	Spring Constant (K)	DAMPING ~~Friction~~ Coefficient (B)
Acceleration	Small*	Small	Large*
Velocity	Large	Small	Small

Table 1. Summary - Seismic Parameters & Outputs

*Denotes case for widest frequency response but lowest sensitivity. The converse condition gives reduced frequency response but an increased sensitivity.

It is also apparent that form 2 may be made to give the first time derivative of the acceleration; however, this condition has been omitted as the output is not normally used in seismic work.

These then are all the possible motion parameters that have voltage analogs for the two forms of pick-off. Table 1 indicates the strengths and weaknesses of the two forms. For example, it may not be desirable to use large ~~friction~~ DAMPING coefficients when making velocity measurements over wide frequency ranges for reasons of temperature sensitivity. In this instance, form 2 would have the advantage since it requires a small and presumably less temperature-sensitive ~~friction~~ DAMPING coefficient. On the other hand, it may be necessary to make a relatively light and small velocity gage. In this instance, form 1 would have the advangage as it does not require large mass for the same frequency response.

Finally, it should be noted that the pick-off schemes are models only; the important point is whether the electrical output parameter to be the input analog is proportional to relative displacement or velocity. For example, form 1 may use a transformer E core with coils wound on the arms. If the center coil is excited with a sine wave voltage, then the opposite coils may be used to sense an amplitude-modulated sine wave. Here, the E core is mounted on either mass M_1 or M_2, and either a sliding or swinging plate of ferromagnetic material is mounted on the other mass element. The result is a sine wave whose amplitude is modulated by the relative displacement of the two masses. Such a system is referred to as a linear variable-differential-transformer (LVDT) displacement pick-off system.

In any event, it is now only necessary to determine the general form of the electrical pick-off and then refer to table 1 to determine the approximate value of the mechanical parameters that will be required.