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Combining RF Sources Using C_N Symmetry

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Abstract

This paper considers the use of point symmetry groups (rotation and reflection) for designing arrays of RF sources. Symmetry is helpful for operating all the sources under the same conditions of voltage, current, etc. Besides having the sources all operate at the same frequency one needs to control the relative phases so that the signals can be appropriately added.

I. Introduction

In designing RF (radio frequency) sources for larger powers one can encounter various limitations associated with peak electric fields, electron densities, etc. This can, of course, be a function of operating frequency and pulse width. Here we are considering some kind of source which produces some sort of pulsed waveform such as a damped sinusoid [2]. However, the details of the envelope of the sinusoid at some frequency f_s are not critical, but the pulse width can be significant [6]. This canonical damped sinusoid is characterized by a complex frequency

$$\begin{aligned} s_s &= \Omega_s + j\omega_s \\ \omega_s &= 2\pi f_s \\ \Omega_s &< 0 \text{ (damping)} \end{aligned} \tag{1.1}$$

The pulse width is then proportional to $-\Omega_s^{-1}$. Such a source is also characterized by some equivalent peak voltage V_o' such that the peak power is $V_o'^2/Z_o$ and the average power (standard convention of averaging over one cycle) has the peak value of

$$\begin{aligned} P_{o_{avg}} &= \frac{1}{2} \frac{V_o'^2}{Z_o} \\ Z_o &= \sqrt{\frac{\mu_o}{\varepsilon_o}} \equiv \text{wave impedance of free space} \end{aligned} \tag{1.2}$$

The subscript "O" refers to a single source. Assuming N sources we have a total average power of

$$P_{avg} = N \, P_{o_{avg}} \tag{1.3}$$

available. The total energy (RF) available is just

$$\begin{aligned} U &= N \, U_o \\ U_o &= P_{o_{avg}} \left[-2 \, \Omega_s\right]^{-1} \end{aligned} \tag{1.4}$$

For energy purposes, then the effective pulse width is just $[-2\,\Omega_s]^{-1}$.

Now there are many kinds of RF sources that one may wish to consider [7,8,12]. Let us refer to one of these as an Xatron (pronounced exatron) where X indicates some general type of source. The list can include klystron, magnetron, vircatron, milotron, ubitron, gyrotron, etc. (Note the rationalization of a few of the terms.)

For effective use of the power the N Xatrons need to operate essentially as identical phase-locked sources. This allows one to combine the signals as a single mode on a waveguide [3] which goes to an antenna feed, or as signals on N waveguides (or some other number of waveguides) going to separate horns comprising one or more feeds to one or more reflector antennas [1,3]. The phases of the waves coming from every horn need to be controlled relative to each other if one is to effectively radiate the power in a given direction [1]. There are various ways to achieve phase locking of the sources and such has been achieved by coupling two sources together [9,10]. For N sources one may wish to link them all together. Alternatively, if these sources are operated with output phase locked to some lower level input signal (perhaps as an amplifier, but not necessarily so), then all N may be controlled by a common source.

To assure identical conditions at each source symmetry can be employed. These conditions include voltage and current from some pulse power sources(s), identical signals for phase control, and (where applicable) identical magnetic fields to control electron flow.

II. C_N Symmetry for Source Array

For present purposes an appropriate symmetry group is C_N, the finite rotation group in two dimensions [11]. As illustrated in Figure 2.1 we have coordinates

$$\begin{aligned} \vec{r} &= (x,y,z) = x\,\vec{1}_x + y\,\vec{1}_y + z\,\vec{1}_z \quad &\text{(cartesian)} \\ &= \Psi\,\vec{1}_\Psi + z\,\vec{1}_z \quad &\text{(cylindrical)} \\ x &= \Psi\cos(\phi)\ ,\ y = \Psi\sin(\phi) \end{aligned} \tag{2.1}$$

With the z axis as the axis of rotation let us define

$$\begin{aligned} \phi_n &\equiv 2\pi\frac{n}{N} \\ n &= 1,2,\ldots,N \end{aligned} \tag{2.2}$$

Let us position the N Xatrons on some common radius (say $\Psi = a$) and at $\phi = \phi_n$ for the nth Xatron. Note that in going from ϕ_n to ϕ_{n+1} the Xatron is also rotated an angle ϕ_1 if the Xatron itself does not already have appropriate rotation symmetry about an axis of its own parallel to the z axis.

Other parts of the source array should also follow the C_N symmetry. The pulse power source(s) can be centered on the z axis with a common connection to the Xatron cathodes. This connection might take the form of a conducting circular cylindrical conductor of radius of the order of a, and electrical connections at each ϕ_n. Similarly, the return (anode) connections might take the form of a similar structure surrounding the first. Together they could form a coaxial geometry for the pulse power. A master source for phase control could be placed on the z axis with identical waveguides on each ϕ_n leading to the Xatrons. The outputs from the N Xatrons can also be fed back toward the z axis or led to some common output structure with C_N symmetry such as a conducting circular disk and/or cylinder. By such techniques one can assure that there are "identical" (within accuracies) voltages and currents on each Xatron.

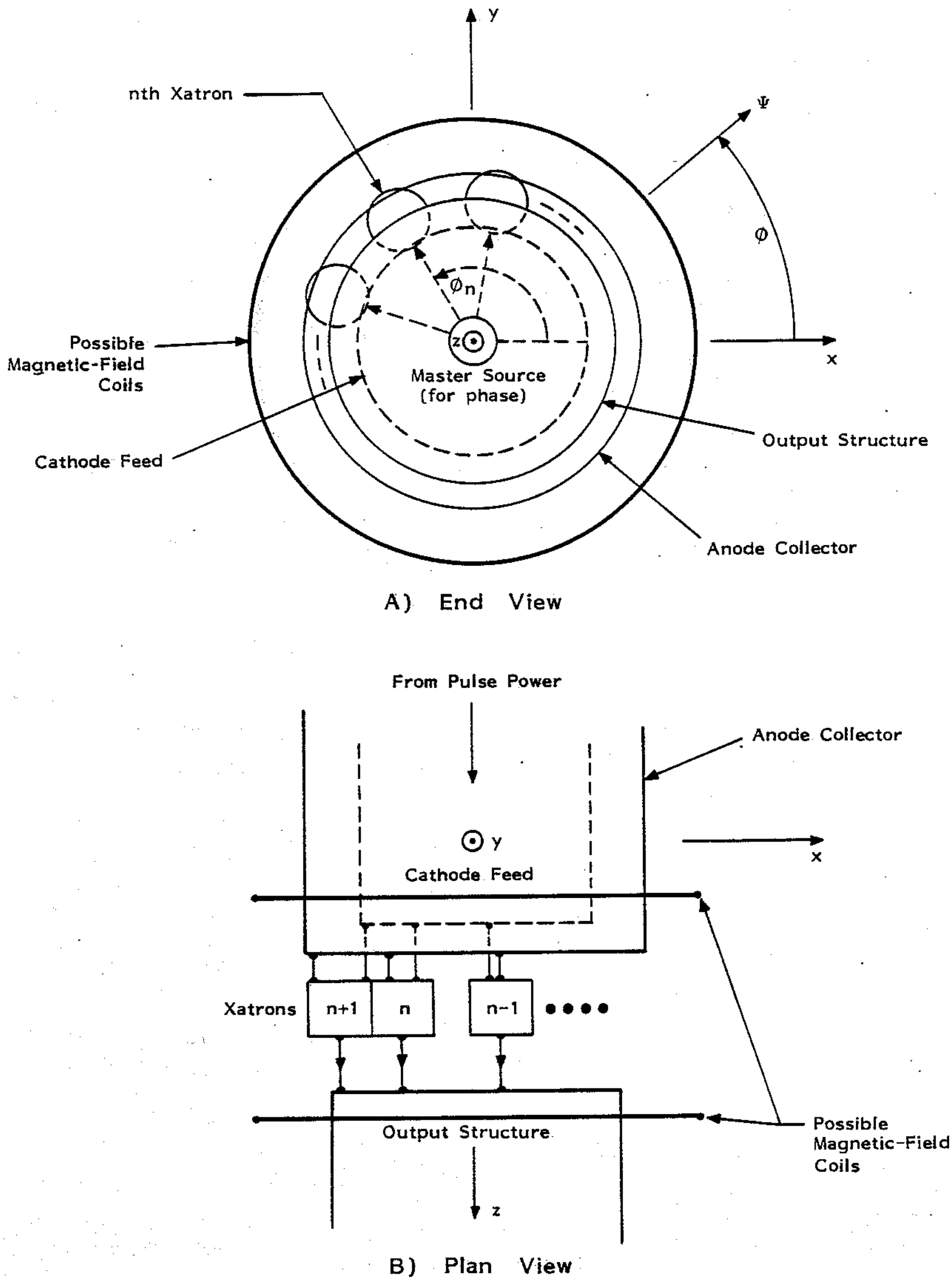

Figure 2.1. Source Array with C_N Symmetry

As a symmetry group C_N is a cyclic group containing N elements (order N) as

$$C_N = \left\{ \left(C_N\right)_1 , \left(C_N\right)_2 , \ldots , \left(C_N\right)_{N-1} , (1) \right\}$$

$$\left(C_N\right)_n \equiv \text{rotation by } \phi_n \text{ about rotation axis (z axis)}$$

$$\equiv \text{group element} \tag{2.3}$$

$$(1) \equiv \text{identity}$$

$$= \left(C_N\right)_1^N = \left(C_N\right)_N$$

As indicated in Figure 2.1 the structure is invariant to rotation not only by ϕ_1, but also by ϕ_n, or equivalently to operation by

$$\left(C_N\right)_n = \left(C_N\right)_1^n \tag{2.4}$$

There are various subgroups of C_N based on the factors of N. If N is divisible by ℓ we have the subgroup (cyclic of order N/ℓ) as

$$C_{\frac{N}{\ell}} = \left\{ \left(C_{\frac{N}{\ell}}\right)_1 , \left(C_{\frac{N}{\ell}}\right)_2 , \ldots , \left(C_{\frac{N}{\ell}}\right)_{\frac{N}{\ell}-1} , (1) \right\}$$

$$\left(C_{\frac{N}{\ell}}\right)_1 \equiv \text{rotation by } \phi_\ell = \text{rotation by } \ell\,\phi_1 = \left(C_N\right)_1^\ell$$

$$(1) \equiv \text{identity} = \left(C_{\frac{N}{\ell}}\right)_1^{\frac{N}{\ell}} = \left(C_{\frac{N}{\ell}}\right)_{\frac{N}{\ell}} = \left(C_N\right)_1^N \tag{2.5}$$

$$\frac{N}{\ell} = \text{positive integer} = \text{order of subgroup}$$

Stated another way an N-fold axis (the z axis) is also an (N/ℓ)-fold axis for all ℓ that give positive integer N/ℓ. For example, for N=12 we can have ℓ=2,3,4,6 while for prime numbers N there are no (proper) subgroups.

Note that as a practical matter the symmetry is not perfect (due to mechanical and electrical tolerances, possible instabilities, etc.). The subgroups can be used to help organize the modes (eigenvalues and eigenfunctions) of the response.

Some types of Xatron have magnets or current-carrying coils to generate magnetic fields to control the electron flow in various desirable ways. These biasing magnetic fields should also follow the C_N symmetry. This can be achieved by separate structures at each Xatron with electrical connections carrying identical currents consistent with C_N symmetry. Assuming that the desired biasing magnetic fields are to be parallel to the z axis, they can be replaced by a common set of field coils which produce a biasing magnetic field for the entire Xatron array. Assuming circular field coils centered on the z axis, the biasing magnetic field then has only B_z and B_Ψ components, both independent of ϕ. This has C_∞ symmetry of which C_N is a subgroup. Of course, for C_∞ symmetry the biasing magnetic field should not be perturbed by the other conducting structures which have only C_N symmetry.

III. Adjunction of Symmetry Planes

There are possible additional symmetries in the configuration of Figure 2.1. As discussed in [11] the point symmetry groups consist of only rotations and reflections. Rotations allow one to bring an object into congruence with itself by a continuous nondeforming motion (rotation). Reflections, however, do not correspond to a rigid displacement of the object; points must be pushed through to opposite sides of the symmetry plane to replicate the object. One can visualize this as squeezing the object until it is coincident with the symmetry plane, and then re-expanding the object with original points on opposite sides of the plane.

As discussed in [4,5] a symmetry plane in an object decomposes the electromagnetic response into two noninteracting parts designated symmetric and antisymmetric. This also applies to eigenmodes (of integral equations) and natural modes.

Denoting the operation of reflection in a plane by (R) we have the group R with elements

$$(R)^2 = (1) \tag{3.1}$$

Without introducing new rotations [11], we have two kinds of reflections (groups R_h and R_v)

$$\left(R_h\right) \equiv \text{reflection in plane perpendicular to rotation axis (z axis)}$$

$$\left(R_v\right) \equiv \text{reflection in plane containing rotation axis (z axis)}$$

These operations can be adjoined to the operations in C_N to form the groups C_{Nh} and C_{Nv} respectively, both of order 2N.

Considering R_h symmetry consult Figure 2.1. In this configuration there is no such symmetry plane since pulse power is on one side of the Xatron array with the output structure on the other. A reconfiguration could have the Xatron outputs radially outward to an output structure with pulse power inside, or conversely. Such a radial configuration is quite different from that in Figure 2.1, but gives in general two alternative configurations with possible C_{Nh} symmetry. There is still the question of the biasing magnetic field, but this is also illustrated in the following discussion.

Next, in the context of the "axial" configuration in Figure 2.1, consider R_v symmetry. This generates a set of N planes [11] containing the z axis. For N odd we can define a plane P_n by the angle ϕ_n. Note that P_n also is described by the angle $\phi_n + \pi$ so that besides passing through one Xatron at ϕ_n it passes (symmetrically) between two Xatrons at $\phi_n + \pi$. For N even there are N/2 planes defined by ϕ_n ($\phi_n + \pi$ giving the same plane), and another N/2 planes defined by an angle $\phi_1/2$ as an initial angle plus successive rotations by ϕ_1. The ones at ϕ_n pass through two Xatrons; the ones at $\phi_1/2 + \phi_n$ pass between two adjacent Xatrons on opposite sides of the Xatron ring.

Now for convenience, as in Figure 3.1, consider some plane P_n at some angle ϕ_n passing through some nth Xatron. Now this may be a symmetry plane of this Xatron mechanical structure, but there are still other electromagnetic properties to consider. In particular consider as in Figure 3.1 some biasing magnetic field parallel to the z axis to control an electron beam propagating in the z direction. Note the sense of rotation of the current (or electrons) about the biasing magnetic field. On reflection through P_n an electron is mirrored to the other side, but its velocity vector is mirrored (antisymmetric reflection), and the corresponding z component of the magnetic field is reversed. Particularly if the electron distribution at any time is not symmetric with respect to P_n then this is a non symmetrical situation with respect to P_n (e.g., electron bunches may be rotating). So C_{Nv} symmetry may not be applicable in such cases. Of course, if there is not such biasing magnetic field, then C_{Nv} symmetry is applicable if the Xatrons are symmetric with respect to respective P_ns. However, the order of C_{Nv} is 2N, more than needed to impose symmetry on N voltages and N currents.

Suppose, however, that N is even and alternate Xatrons have opposite biasing magnetic fields and the electrons in adjacent Xatrons rotate in opposite directions. This is a case of $C_{\frac{N}{2}v}$ symmetry with symmetry planes between the Xatrons. In this case the Xatrons need not be spaced at angles ϕ_n but at angles $2\phi_n \pm \phi'$ with symmetric orientation of the Xatron pairs with respect to the symmetry planes between them. The input and output connections to the Xatrons need only have the same $C_{\frac{N}{2}v}$ symmetry. Note that this group is of order N, adequate to impose symmetry on the N voltages and N currents.

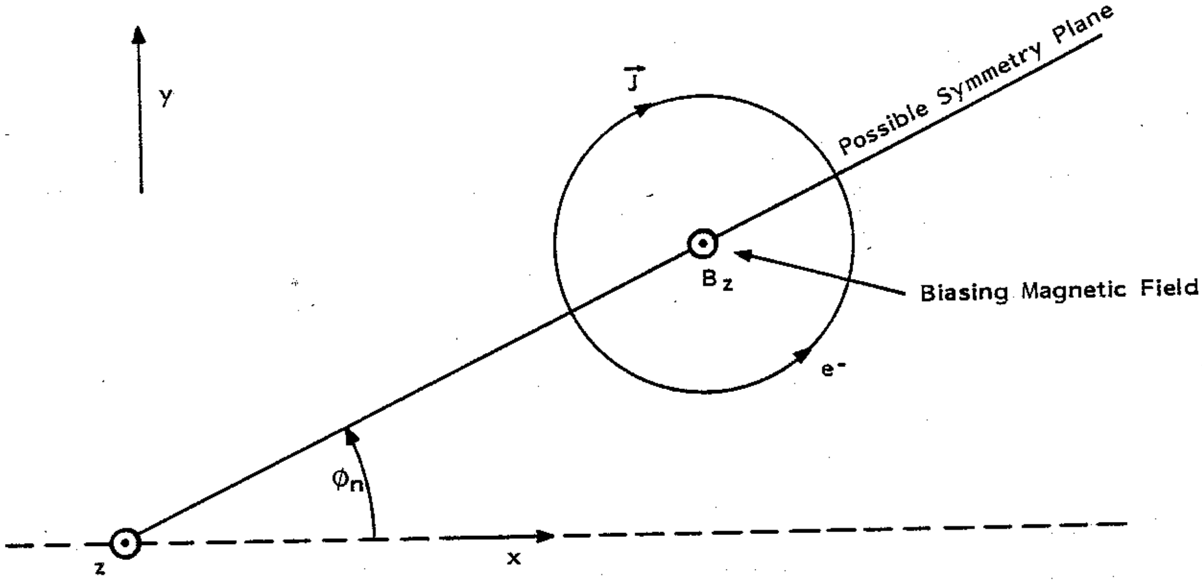

Figure 3.1. Possible Symmetry Plane Through nth Xatron

IV. Concluding Remarks

Symmetry is then conceptually useful for designing RF source arrays, helping to obtain "identical" outputs from each source, especially with controlled phase. For this purpose the point symmetry groups (rotation and reflection) are applicable.

Here C_N symmetry with adjunction of symmetry planes has been explored. Perhaps for other applications other point symmetry groups will be useful.

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