Switched Oscillators

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Abstract

This paper considers switched oscillators as possible resonant sources for driving an antenna. By charging a transmission line to some high voltage and then shorting it by some fast-closing switch at one end, a resonant waveform is applied to an antenna at the other end. Various optimization conditions for this geometry and other oscillator geometries are discussed as well.

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1. Introduction

There are various kinds of narrowband high-power sources (microwave tubes, or Xatrons [5, 7]) which may produce a hundred cycles or so in a pulse with center frequency in the range of some hundreds of MHz to a few GHz, a range of interest [1]. There are also impulsive high-power radiators associated with impulse radiating antennas (IRAs) [8] with band ratios of the order of two decades. Let us consider a class of sources with frequency bandwidths somewhere between these two.

Instead of an Xatron as the oscillator let us consider some shock-excited resonant structure. This excitation may come in the form of the discharge of some slowly charged capacitance through a fast-closing switch [4]. The waveform delivered to an antenna will be characterized by one or more damped sinusoids. The pulse width (or number of cycles to some fraction of the peak) depends on the load (antenna) impedance and other parasitic effects (nonideal switch, etc.).
Quarter-Wave Transmission-Line Oscillator

One kind of oscillator is a charged transmission line (charged through a large impedance \( Z_{\text{charge}} \) at frequencies of interest) with a shorting switch at one end as in Fig. 2.1. With a short at one end and a high impedance (100 \( \Omega \) or so of an antenna) at the other end, this is a quarter wave (\( \lambda/4 \)) oscillator (with generally higher harmonics (3\( \lambda/4 \), etc.)). Note that we assume

\[
Z_c \ll |Z_a|
\]

\( Z_c \) = transmission-line characteristic impedance

\( Z_a \) = antenna input impedance

(2.1)

In order to simplify the subsequent discussion let us assume that \( Z_a \) can be approximated by a constant resistance. Also the high-voltage connection to the oscillator is assumed to be through a high impedance (e.g., inductor) near the oscillator in order to minimize the loading of the oscillator via this connection.

Following waves back and forth on the transmission line, beginning with the switch closure, let it close in a time \( t_s \), short compared to \( t_f \), the transit time along the transmission line. For a quarter-wave resonator we have a frequency

\[
f = \frac{\omega}{2\pi} = \frac{4}{t_f}
\]

(2.2)

for the principal resonance. A real switch will have some inductance (and resistance) which will modify the resonance somewhat. The switch will need to be physically small compared to the transmission-line length.

On closing the switch a wave (ideally a step function) of amplitude \(-V_0\) propagates to the left (in Fig. 2.1), with nearly a +1 reflection coefficient. This gives a transient voltage doubling which is characteristic of a Blumlein and is advantageous in the present application. The reflection coefficient at the antenna is more accurately

\[
\rho = \frac{1 - \frac{Z_c}{Z_a}}{1 + \frac{Z_c}{Z_a}} = 1 - 2 \frac{Z_c}{Z_a}
\]

(2.3)
A. Differential

B. Single-ended

Fig. 2.1. Low-Impedance, Quarter-Wave Transmission-Line Oscillator Feeding High-Impedance Antenna
Assuming an ideal reflection coefficient of $-1$ for the wave returning to the switch, then the second wave reaching the antenna reflects with amplitude $\rho^2$, etc. This is a geometric series with alternating signs, describing an exponential decay, the dominant frequency in this as in (2.2), but now more accurately described as a damped sinusoid. In $N$ cycles the amplitude is reduced to $\rho^{2N}$. If we set this equal to $e^{-1}$ we have

$$N = -\frac{1}{2\ln(\rho)}$$  \hspace{1cm} (2.4)

describing an effective number of cycles. With (2.3) we have for small damping

$$N = \frac{1}{4Z_c^2}$$  \hspace{1cm} (2.5)

The logarithmic decrement is defined by [6]

$$\Delta = \frac{1}{N} = -2\ln(\rho)$$  \hspace{1cm} (2.6)

There is also the commonly used quality factor

$$Q = \frac{\pi}{\Delta} = \pi N$$  \hspace{1cm} (2.7)

Here the antenna is just represented by an impedance which we approximate as a resistance for frequencies of interest. This might be a horn, perhaps feeding a reflector. Note that the high-voltage supply also charges up at least part of the antenna. This extra capacitance (transit time divided by characteristic impedance for a transmission line as might approximate a TEM horn) is ideally not too large. A large ratio $Z_a/Z_c$ also helps here. This loading may possibly be decreased by special series filters with blocking capacitors, if desired. Many variations on the antenna are possible allowing for differential as well as single ended sources [2, 3]. If a reflector is used, note that the TEM horn need not extend all the way to the reflector as done in a reflector IRA.
3. Half-Wave Transmission-Line Oscillator

3.1 Half-wave switched in center

An alternate source is the half-wave oscillator in Fig. 3.1 obtained by placing the switch in the center of the low-impedance transmission line. This doubles the length for a given oscillation frequency and increases the stored energy available to the antenna. However, in this case the switch needs to have some non-zero impedance to allow the energy to its right to propagate to the left, past the switch. This is but one example of a more sophisticated resonator design. The characteristic impedances of the left and right portions need not be the same. Variable transmission-line impedances of the left and right portions need not be the same. Variable transmission-line impedances (transmission-line transformer) are also possible.

3.2 Topological deformation into ring

Some of the aforementioned problems can be alleviated and more symmetry introduced in the configuration in Fig. 3.2. The transmission line in Fig. 3.1 has positions A (left end), B (center with switch), and C (right end). Changing from a side to a top view in Fig. 3.2, deform the straight transmission line into a circular path (ring) with A and C coming together at the antenna connection (left) with the switch symbolically indicated at the right. Now $\lambda/2$ with $\lambda$ as the resonant wavelength (in the dielectric medium of the transmission line) is the circumference of this ring oscillator. Both the AB and BC portions of the transmission feed equally into the antenna. Another way to look at this is to consider this ring as a single $\lambda/4$ transmission-line with $Z_c$ effectively cut in half. Note in Fig. 3.2 that this ring oscillator can consist of two rings separated by a dielectric (differential) or a single ring separated from a ground plane by a dielectric (single ended). For simplicity the high-voltage charging connection is not indicated.

3.3 Evolution toward circular disks

As one increases the width of the transmission-line conductors in Fig. 3.2 (thereby lowering $Z_c$ for a given spacing between conductors) this width approaches the radius of the ring and also becomes a significant portion of a wavelength. Then the structure is no longer a transmission-line structure characterized by one-dimensional wave propagation.

In the limit the ring oscillator becomes a pair of parallel, coaxial, circular disks (or a single disk parallel to a ground plane). Then, as in Fig. 3.3, the initial wave from the switch closure propagates away from the switch as a cylindrical wave which does not focus at the connection to the antenna. One could try to alter the propagation of this two-dimensional wave by varying the dielectric constant of the material between the metal plates (a two-dimensional lens) but there are also high-voltage considerations for small plate spacing (large capacitive energy storage). So one might prefer a single dielectric medium (such as transformer oil).
Fig. 3.1. Low-Impedance, Half-Wave, Transmission-Line Oscillator Feeding High-Impedance Antenna.

Fig.3.2. Half-Wave, Transmission-Line Oscillator Placed on Circular Path, Bringing Both Ends Together at Connection to Antenna.
Fig. 3.3. Circular-Disk Oscillator
4. Three-Dimensional Structures

The problem of making the wave from the switch focus at the antenna port can also be solved by moving our thinking into three dimensions for the oscillator geometry. Viewed another way let us make the transit times along geodesic paths between switch and antenna port all be the same.

4.1 Concentric spheres

A simple example of this approach is concentric spheres as shown in Fig. 4.1. Here the switch and antenna port are at opposite poles as indicated by the axis of rotation. The spacing \( h \) between the outer sphere of radius \( b \) and the inner sphere of radius \( a \) is assumed small, i.e.,

\[
h \ll a
\]

so that we need not be concerned about the differences in path lengths \( \pi b \) and \( \pi a \) along the two spherical surfaces.

Note now the necessity of having holes in the outer sphere for the antenna port and the charging port. This type of source is inherently single ended. Note that it can be recessed in a ground plane (to various depths) with the antenna port at or just above the ground plane.

As the wave propagates between the two spheres (with constant \( h \)) the effective pulse impedance at the wavefront first decreases and then increases as the equator is passed. This complicates the analysis. However, this is analyzable in spherical coordinates with the usual wave functions.

4.2 Coaxial nested bodies of revolution

A more general form such structures can take is coaxial nested (one inside the other) bodies of revolution as indicated in Fig. 4.2. In this case the switch and antenna port are at opposite poles, i.e., opposite intersections of the axis with the bodies of revolution. This configuration has all geodesic paths from the switch arriving at the antenna port with equal lengths, thereby focusing at the antenna port. Again \( h \) is assumed sufficiently small compared to radian wavelengths of interest.

In a cylindrical \((\Psi, \phi, z)\) coordinate system with the \( z \) axis as the axis of revolution, the geometry is \( \phi \) independent. Consider \( \Psi_g \) as the radius to the geodesic path. One can compute a characteristic impedance at some position \( \zeta \) along the path \((\zeta = 0 \text{ being at the switch})\) as
Fig. 4.1. Concentric Spheres
Fig. 4.2. Coaxial Bodies of Revolution
\[ Z_c(\zeta) = \frac{h(\zeta)}{2\pi \Psi_g(\zeta)} Z_w \]

\[ Z_w = \left[ \frac{\mu_0}{\varepsilon} \right]^\frac{1}{2} \equiv \text{wave impedance of dielectric medium} \]  
\[ \varepsilon \equiv \text{permittivity of dielectric medium} \]  

(4.1)

The length of the geodesic path is given by \( \zeta = \ell \) at the antenna port, and the transit time is

\[ t_r = \frac{\ell}{v} \equiv \text{transit time} \]

\[ v = \left[ \mu_0\varepsilon \right]^\frac{1}{2} \equiv \text{wave speed in dielectric medium} \]  

(4.2)

If one wishes one can choose \( Z_c(\zeta) \) to be a constant to give a uniform transmission line by setting

\[ \frac{h(\zeta)}{\Psi_g(\zeta)} = 2\pi \frac{Z_c}{Z_w} \text{ for } 0 < \zeta < \ell \]  

(4.3)

Note that near \( \zeta = 0, \ell \), one needs to allow for the switch and antenna port, appropriately centered on the rotation axis. Near these poles as \( \Psi_g(\zeta) \) becomes small \( h(\zeta) \) is proportionately smaller. In these regions the two bodies of revolution asymptotically approach circular cones, allowing one to think in these cases in terms of conical transmission lines (or antennas).

For such a constant \( Z_c \), the discussion has come full circle, the above giving a quarter-wave oscillator as in Section 2.
5. Concluding Remarks

There are various geometries that one can consider for switched oscillators. As one lowers the transmission-line characteristic impedance, $Z_c$, relative to the antenna input impedance, $Z_a$, the number of cycles $N$ in the pulse increases. Viewed another way, for a given $V_0$ the stored capacitive energy in the oscillator increases for lower $Z_c$.

In the present discussion the impedance of the closed switch has been assumed negligible. However, it will have some inductance, lowering the resonant frequency, and some resistance, increasing the damping of the resonance. These effects need to be minimized. More accurate models can be developed to more precisely compute the performance.
References


