MICROWAVE ASSOCIATES, INC.

Contract No.
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Home Air Development Center
Research and Technology Division
Air Force Systems Command
United States Air Force
Griffiss Air Force Base
New York
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NANOSECOND PULSE BREAKDOWN STUDY

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MICROWAVE ASSOCIATES, INC.
Burlington, Massachusetts

Final Report
Contract No. AF30(602)-2782

Prepared for
Rome Air Development Center
Research and Technology Division
Air Force Systems Command
United States Air Force
Griffiss Air Force Base
New York
FOREWORD

The objective of this contract was to explore the possibility of obtaining meaningful experimental data on microwave breakdown at pulse widths below twenty nanoseconds. The ultimate goal is super range resolution radar through the use of short pulses. The major problems of interest included application of breakdown theory to microwave nanosecond pulses to determine experimental requirements and provide a theory to compare with experiment, the design of a suitable experimental setup, and the design of components for the experimental setup.

Microwave Associates' approach followed the general outline of major problems stated in the last paragraph. Application of theory to the nanosecond case was accomplished insofar as this was possible. The experimental scheme is based on r.f. buildup and storage in a section of waveguide which is a few wavelengths in length. The nanosecond pulse is formed when the waveguide section is quickly switched into another section of waveguide containing the test chamber. The most formidable problem is the waveguide switch, but progress on the development of a suitable switch has been encouraging to date.

Briefly, some of the more important conclusions are as follows:

1. The theory predicts substantial increases in peak power handling capability for pulse widths below twenty nanoseconds even at the poorest pressures to be encountered in practical application.

2. Further theoretical work is needed, especially as regards the effects of space charge, field distortion, differences in the breakdown mechanism at low and high pressures, gas heating, etc.
(3) The properties of high dielectric strength gases should be studied experimentally in the nanosecond region.

(4) Further improvement of the experimental scheme devised under this contract is desirable.

This contract is being extended for one year in order to accomplish the tasks outlined above.

WILLIAM C. QUINN
RADC Project Engineer
ABSTRACT

An estimate is made of the pulse breakdown characteristic for air on the basis of the theory worked out by Gould and Roberts and the available data. The theory is critically examined and the consequences of the assumptions made are investigated to insure its applicability to extremely short pulse breakdown. Several possible modifications are advanced which require experimental verification with nanosecond pulse lengths. The possible advantages of using so-called high dielectric strength gas is also discussed. Progress on development of a nanosecond pulse generation circuit is reported; several ATR switches are described and discussed.
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I. INTRODUCTION

How can one transmit very high peak power microwave signals through a waveguide system without being troubled by electrical breakdown of the gas filling the guide? There are various approaches to solving the problem, each with its own limitations of power handling ability and practicality. Specifically, one can:

a) evacuate the system
b) pressurize the system
c) fill the system with high-dielectric strength gas
d) transmit short pulses.

The work of this contract has been to examine the "short pulse" approach in detail in order to evaluate its promise.

The breakdown of a gas requires a certain length of time. For a given gas pressure this time becomes shorter as the applied electric field or transmitted power increases. Therefore, if the length of a microwave pulse is shorter than the breakdown time corresponding to its amplitude, the gas will not break down and the pulse will be transmitted undisturbed. Thus the problem becomes one of determining the breakdown field - pulse length characteristic of the gas filling the waveguide.

For significant increases in pulsed breakdown power levels over CW breakdown power at atmospheric pressure the time required is typically less than $10^{-7}$ or $10^{-8}$ sec. This fact brings up the second problem, that of producing high power pulses of such short duration.
Primary microwave sources to do this job are not available. A microwave circuit employing a resonant section of waveguide, which is charged by a long, relatively low power, magnetron pulse and discharged into the following waveguide section by means of a fast ATR-type switch, can yield the desired short pulses at higher peak power level than the magnetron can produce.

In this report an estimate is made of the pulse breakdown characteristic for air on the basis of the theory worked out by Gould and Roberts¹ and the available data. The theory is critically examined and the consequences of the assumptions made are investigated to insure its applicability to extremely short pulse breakdown. Several possible modifications are advanced which require experimental verification with nanosecond pulse lengths. The possible advantages of using so-called high dielectric strength gas is also discussed. Progress on development of a nanosecond pulse-generation circuit is reported; several ATR switches are described and discussed. Further development of the fast switch and nanosecond pulse circuit is outlined. Some experiments to measure the nanosecond pulse breakdown characteristic, as well as additional experiments to study the basic breakdown mechanisms are suggested.
II. BREAKDOWN MECHANISM AND THEORY

An electric field applied to a gas initially containing some free electrons accelerates these electrons. Some acquire enough energy to ionize gas molecules and thereby create more free electrons, which, in turn, are accelerated and produce further ionization. An electron avalanche develops. Electrons are lost from the avalanche when they diffuse to the walls confining the gas or, in electronegative gases, when they attach to neutral molecules forming negative ions. The negative ions eventually recombine with positive ions and form two neutral molecules.

Electrical breakdown of the gas is governed by the relative rates of ionization and delionization. Denote the ionization rate per electron by $v_i$ (ionizations/sec-electron), the attachment rate by $v_a$ (attachments/sec-electron), the diffusion loss rate by $v_d$ (fractional losses/sec-electron) and the electron density by $n$ (electrons/cm$^3$). The rate of increase of electron density is simply

$$\frac{dn}{dt} = v_{\text{eff}} n,$$  \hfill (1)

where $v_{\text{eff}} = v_i - v_a - v_d$ is the effective ionization rate per electron. If $v_{\text{eff}}$ is positive and constant, the density grows exponentially in time from an initial density $n_0$. 
\[ n = n_0 e^{v_{\text{eff}}t}. \]  \hspace{1cm} (2)

The rate of diffusion loss depends on the geometry of the container. For a differential volume element it is

\[ v_d n = -Dv^2 n, \]  \hspace{1cm} (3)

where \( D \) is the electron diffusion coefficient. With this term Eq. (1) becomes

\[ \frac{\partial n}{\partial t} = (v_i - v_a)n + Dv^2 n. \]  \hspace{1cm} (4)

The solution satisfying the boundary conditions, the electron density zero at the walls, is

\[ n(\vec{r}, t) = \sum_m n_m e^{\frac{(v_n - D)}{\lambda_m} t} X_m(\vec{r}) \]  \hspace{1cm} (5)

where \( v_n = v_i - v_a \) is called the net ionization rate, \( \lambda_m \) is the diffusion length of the \( m^{\text{th}} \) diffusion mode and is determined by the geometry. \( X_m(\vec{r}) \) is a spatial function describing the electron
distribution associated with the \(m\)th diffusion mode. The \(n_m\) are coefficients determining the amount of each mode present.

Infinite parallel plane electrodes separated by a distance \(d\) with a uniform gas density and uniform electric field is the simplest case. In this geometry \(\Lambda_m = d/mv\) and \(X_m = \cos x/\Lambda_m\), where \(m = 1, 3, 5, 7 \cdots\) and \(x\) is the coordinate perpendicular to the electrodes as measured from the midplane between them. The lowest mode, \(m = 1\), has the largest effective ionization rate and it is generally possible to neglect the higher modes. Then, we have

\[
n = n(x, t) = n_1 e^{\left[v_n - D(\pi/d)^2\right] t \cos \frac{\pi x}{d}}
\]

Hence, in Eq. (2) the initial density is \(n_o = n_1 \cos \pi x/d\) and \(v_{\text{eff}} = v_n - D(\pi/d)^2\). If \(v_{\text{eff}} > 0\) for a pulse of length \(\tau\), the ratio of final to initial electron density is

\[
\frac{n_f}{n_o} = e^{v_{\text{eff}} \tau}.
\]

Taking logarithms, we obtain

\[
v_{\text{eff}} = \frac{1}{\tau} \ln \frac{n_f}{n_o}.
\]
or

\[ v_n = \frac{1}{\tau} \ln \frac{n_f}{n_0} + D(n_d)^2 \]  

(7)

Eq. (7) is the pulse breakdown relation used by Gould and Roberts in which they set \( n_f/n_0 = 10^8 \) or \( \ln n_f/n_0 = 18.4 \) as giving best agreement with experiment.

The breakdown electric field is determined by Eq. (7) and the dependencies of \( v_i \), \( v_d \) and D on the amplitude of the field \( E_0 \), its frequency and the gas pressure. The experimentally measured ionization parameter is the first Townsend ionization coefficient \( \alpha \), defined as the number of ionizations one electron makes in one centimeter drift along a dc electric field. Similarly, the empirical attachment coefficient \( \eta \) is the number of attachments per cm per electron. The electrons acquire a drift velocity \( v_d \) which is roughly proportional to the field. The constant of proportionality is the electron mobility \( \mu \), so that \( v_d = \mu E \). The diffusion coefficient is \( D = 2\mu \bar{u}/3 \), where \( \bar{u} \) is the average electron energy. A physically important quantity is the amount of energy an electron acquires in a mean free path between collisions. Since the mean free path is inversely proportional to gas pressure, the energy gained per mean free path is proportional to the "reduced field" \( E/p \) volts/cm-mm Hg. The quantities \( \sigma/p \), \( \eta/p \), \( v_d \), \( \mu p \), and \( \bar{u} \) are all functions of \( E/p \) only and \( \mu p \) is essentially constant. They have been measured in dc electric
fields over limited ranges of \( E/p \), as summarized in Table I. The net ionization rate is \( \nu_n = (\alpha - \eta)v_d \), hence \( \nu_n/p = (\alpha/p - \eta/p)v_d \) is a function of \( E/p \) only in a dc field.

**Table I**

**Available Data on Breakdown Parameters for Air**

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<tr>
<th>Parameter</th>
<th>Range of ( E/p )</th>
<th>Reference</th>
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<tr>
<td>( \alpha/p )</td>
<td>20-900</td>
<td>3, p 534; 4, pp 128-132</td>
</tr>
<tr>
<td>( \eta/p )</td>
<td>25-60</td>
<td>4, p 176</td>
</tr>
<tr>
<td>( v_d/p )</td>
<td>0-22</td>
<td>4, p 58</td>
</tr>
<tr>
<td>( \bar{u} )</td>
<td>0.1-20</td>
<td>2</td>
</tr>
<tr>
<td>( \kappa )</td>
<td>0.1-20</td>
<td>2</td>
</tr>
</tbody>
</table>

All of the parameters depend on \( E/p \) by way of the electron energy \( \bar{u} \). In an rf field of amplitude \( E_0 \) and radian frequency \( \omega \), the energy of the average electron is

\[
\bar{u} = u_o \left[ (1 + a \cos(2\omega t - \phi)) \right] (1 - e^{-\kappa v_o t}) \tag{8}
\]

where
\[ \nu_0 = \frac{e^2 \varepsilon_0}{2 \kappa \mu \nu_c^2 (1 + \frac{\nu^2}{\nu_c^2})} = \frac{e^2}{\kappa \mu \nu_c^2 \left( \frac{\varepsilon_0}{p} \right)} \]

\[ a = \sqrt{1/1 + \frac{1 + \nu^2}{\kappa \mu \nu_c^2}} \quad \text{(modulation factor)} \]

and

\[ \phi = \tan^{-1} \frac{\nu}{\nu_c} \]

The quantity \( \kappa \) is the energy loss parameter, that is the fraction of its energy an electron loses in a non-ionizing collision with a gas atom. The electron collision frequency \( \nu_c \) is proportional to pressure, making \( \nu_c/p \) a function of \( \tilde{u} \) or \( E/p \). Both \( \kappa \) and \( \nu_c/p \) depend upon \( E/p \) as shown by the data of Crompton, Huxley, and Sutton\(^2\) in Fig. 1. The derivation of Eq. (8) assumes them to be constant. The measured \( \tilde{u}(E/p) \) at dc is also given in Fig. 1. An rf field is less efficient than a dc field in heating the electrons and causing ionization. The actual rf amplitude must be multiplied by the frequency factor \( 1/\sqrt{2(1 + \nu^2/\nu_c^2)} \) to obtain the equivalent dc field \( E_e \). For air, Gould & Roberts use \( \nu/\nu_c = 36/p\lambda \), where \( \lambda \) is the free space wavelength.
of the microwave signal in cm. They took \( v_e/p = 5.3 \times 10^9 \text{ sec}^{-1} \text{ mm Hg}^{-1} \) which is higher than reported in Ref. 2, however we have continued to use that value for purposes of comparison. Gould and Roberts have accounted for energy modulation by averaging \( v_n/p \) over an rf cycle and computing a correction curve from the \( \phi \lambda = 0 \) (\( \omega/v_c = \infty \)) limit. We shall therefore omit energy modulation effects and work with the infinite frequency limit as the reference breakdown curve.

The net ionization frequency is plotted in Fig. 2 as a function of \( E_e/p \) using the known \( \alpha/p \) and \( \eta/p \). The drift velocity is extrapolated to a value of \( E/p = 900 \) by the formula

\[
v_d = (3.92 + .263 E/p) \times 10^6 \text{ cm/sec},
\]

which is approximately that used by Gould and Roberts¹. The attachment coefficient becomes negligible for \( E/p > 60 \). The net ionization frequency was extrapolated to values of \( E/p \) between 900 and 1400 by eye where the upper limit is roughly equal to the collision frequency. Actually, at these extremely high values of \( E/p \), \( v_n/p \) has to decrease again, as the ionization efficiency decreases; the maximum will be broad and flat.

The data of Fig. 2 used in Eq. (7) yields the pulse breakdown curve of Fig. 3. Gould and Roberts show experimental points confirming this theory down to \( p \tau = 1.7 \times 10^{-5} \text{ mm Hg-sec} \) at which point \( E_e/p = 53 \text{ v/cm-mm Hg} \). The extreme point was at a pressure of 21 mm Hg
and pulse length of 0.8 μs, and pλ = 275 mm Hg-cm. The un-normalized rms field strength was \( E_{rms} = 1110 \) V/cm, corresponding to about 27 kW peak pulse power in S band waveguide.

Except for \( E/p > 900 \), the limitations on the data used in the extrapolation are not serious. The extrapolation of the drift velocity over such a long range is questionable. If we use \( v_d = (5 + 0.2 E/p) \times 10^6 \) cm/sec, obtained from the data in Ref. 4, a 60% smaller \( p\lambda \) is obtained at the higher values of \( E/p \), which suggests that the extrapolation error is of the order of a factor of 2.
III. APPLICABILITY OF THEORY TO VERY SHORT PULSES

A. Discussion of the Theoretical Model

The breakdown relation, Eq. (7), is based on a model in which the following assumptions are made:

1. A uniform rf electric field is impressed on a gas at uniform pressure (molecular density) between infinite, parallel plane electrodes.

2. Higher mode diffusion is neglected so that the lowest mode (cosine density distribution) exists during most of the pulse. Regardless of the actual initial distribution, the cosine distribution is established so rapidly that it is considered to be the initial distribution.

3. Breakdown has occurred when the density increases by a factor of $10^8$.

4. The net ionization frequency, diffusion coefficient, energy loss parameter, and collision frequency are all constant in space and time during the pulse, with values corresponding to the final electron energy $\bar{u}(E/p)$.

5. The electron heating transient is neglected.

6. Space charge effects are neglected.

7. The rf field is applied in a single, isolated pulse.

8. The ionization is by electron impact only ($\alpha$ process).

The question now is whether to expect this theory to remain substantially valid in the limit of very short pulses.
To neglect higher diffusion modes relative to the lowest requires first, that the decay time at the first higher mode be much shorter than the lowest. This is always true since the ratio of decay times for the first two modes between parallel planes is 1/9. Second, the pulse length must also be long compared to the higher mode decay time. These conditions will result in a cosine distribution and Eq. (7) applies. If the pulse is short enough to violate this assumption, any higher mode content, initially present, will reduce the rate of electron density growth so that Eq. (7) will predict a breakdown field which is too low. This is based upon the breakdown criterion \( n_f/n_o = 10^8 \) at the maximum in the distribution. An extreme limit of this situation, in which the electrons are initially localized in a plane parallel to the electrodes, is examined in Section B below. We shall see that this breakdown criterion, which results from fitting the theory to experiment, may lose significance when the initial electrons are highly localized and do not diffuse appreciably during the pulse.

To treat the breakdown parameters, especially the net ionization frequency as a constant in time, requires that the pulse length be long compared to the electron heating time, and that the parameters be averaged over a cycle when energy modulation is important. The validity of neglecting the electron heating transient, while taking into account the time variation of the energy loss parameter is analyzed in Section C and found to be justifiable in the short pulse limit.

The effect of space charge on the rate of growth of electron
density is not taken into account in this theory. Gould and Roberts determined breakdown by detecting the first occurrence of visible light with a phototube, it is possible that the density at this time in the development is not high enough to affect materially the transmission of a microwave signal, particularly if the ionized volume is still small. Space charge will inhibit the spatial growth of the ionized region and hence may be important in formulating a better breakdown criterion. Some theoretical work has been done on evaluating the space charge contribution to the breakdown time in dc fields in which mobility loss and secondary emission of electrons are necessarily taken into account. These processes do not apply to rf breakdown. It was noted, however, that space charge begins to become important in air at atmospheric pressure at a current density of $10^{-5}$ amp/cm$^2$. This corresponds to an electron density of the order of $10^7$ cm$^{-3}$, or an 8% impedance change at S band. More work, both theoretical and experimental are needed in this area, particularly applied to the microwave breakdown problem.

The problem of differences between low and high pressure breakdown has also received very little attention. The pressure scaling law $E/p \ (pd, \ p\lambda, \ pt)$ is generally assumed to hold at all pressures, whereas the experiments are usually performed at reduced pressure to keep breakdown power requirements down. Miller* has found both pulse length and pulse repetition rate dependence of breakdown power at atmospheric pressure with pulses 200 to 700 $\mu$s long which should be at the CW limit. Gould and Roberts augmented their theory to include

*Miller, S. J., Lincoln Laboratory, Private Communication.
repeated pulses by taking into account diffusion of electrons out of the gap between pulses. They checked this in the 25-300 mm Hg pressure range with a 0.8 µsec pulse and obtained general agreement. It is conjectured that a gas heating process occurs at sufficiently high pressure, in which the pulses may be far enough apart to be considered single and isolated as far as electron diffusion is concerned, but not for diffusion of heat in the gas. In this event, the below-breakdown electron cloud produced in one pulse will be almost all gone when the next pulse comes along. However, the energy the electrons acquire from the electric field is very rapidly transferred to the gas. This local heating of the gas will decrease its local molecular density, increasing F/p at that point. Thus the possibility exists for a series of pulses, relatively widely separated, to break the gas down at power levels less than pulse breakdown theory predicts. This heating phenomenon is also related possibly to the motion of a microwave arc in a waveguide.

There are several processes by which electrons are generated in a gap during breakdown. Free electrons may be produced from the gas atoms by electron impact or photo-ionization, or they may come from the walls by secondary, thermionic, field or photo electric emission. In general, at high pressures in rf fields any electrons emitted from the surface will not contribute significantly to the overall electron density or its growth rate. This source of electrons could be taken into account by changing the boundary condition on the solution to Eq. (4), to a non-zero value of electron density at the walls. This
would effectively increase the diffusion length of the gap slightly. Thus, when the pressure is high enough that the breakdown field is independent of $pd$, it will also be unaffected by any surface emission of electrons. Photo ionization and streamer formation are generally important on extremely non-uniform fields, and is probably negligible in other cases. It is therefore reasonable to assume that the impact ionization process is the only significant one operating in rf breakdown when diffusion is negligible, i.e., at high $pd$ or short pulse length.

B. Electron Diffusion

A breakdown avalanche may be initiated by a single electron, or perhaps a localized group of electrons such as might exist after passage of a cosmic ray. If the pulse is short compared to the diffusion time $\tau_D$, $p\tau_D \approx 10^{-6} (pd)^2$ mm Hg-sec electrons will scarcely diffuse at all and the entire avalanche may be fairly localized in space. The breakdown field predicted by Eq. (7) would appear low in that the breakdown would be incomplete and the ionized region would not fill an appreciable portion of the gap. On the other hand, the field strength is sufficient to increase the central density by 8 orders of magnitude.

Let us then assume a uniform initial electron density $n_i$ in a thin layer $\delta$ which is diffusing into adjacent regions in the presence of a uniform rf electric field. The walls may be considered infinitely remote. The density distribution $n(x,t)$ will proceed as sketched in Fig. 4a. The analytic solution, obtained by standard Fourier Integral technique*, is

\[ n(x, t) = \frac{n_1 e^{v_n t}}{2} \left[ \operatorname{erf} \left( \frac{x + \delta}{\sqrt{4Dt}} \right) - \operatorname{erf} \left( \frac{x - \delta}{\sqrt{4Dt}} \right) \right], \]

which reduces to
\[ n(x, t) = n_1 \delta \, e^{v_n t} \frac{e^{-x^2/4Dt}}{\sqrt{4\pi Dt}} \]  

(9)
as \( n_1 \to \infty \) and \( \delta \to 0 \) but \( n_1 \delta \) remains finite. The temporal growth at \( x = 0 \) given by Eq. (9) is compared with that of Eq. (6) in Fig. 4b. Note that the volume density initially drops from infinity to a minimum in a time \( t_{\text{min}} = 1/2 \, v_n \), i.e., one half the time it takes the electron density to increase by a factor of \( e \). We have taken this minimum value to be \( n_0 \), the initial central density of the lowest diffusion mode case, i.e., Eq. (7) becomes
\[ n(x, t) = \frac{n_0}{\sqrt{2\pi v_n t}} \, e^{-x^2/4Dt}. \]

Fig. 4b covers just 5 time constants, while 18.4 may be required for breakdown. Clearly the difference in the curves within the first time constant is of no real consequence on the breakdown time scale. The width of the ionized region grows with the square root of
time, which is slow compared to the exponential growth in density. The width at "half-height" is $x_{1/2} = \sqrt{4(\ln 2) \Delta t}$. At one atmosphere this is approximately $x_{1/2} \approx \sqrt{1/25} \text{cm}$, with $t$ in microseconds. Even for pulses of 1 \(\mu\text{sec}\) this is very small spreading.

If we adapt the $n_f/n_o = 10^8$ breakdown criterion to the present problem, calling the density at $t = t_{\text{min}}$ the equivalent initial density, we can write the breakdown relation as

$$\frac{v_n}{p} = \frac{21}{p t}$$

(10)

The relation of Eq. (7) for infinite gap width is

$$\frac{v_n}{p} = \frac{1}{p t} \ln 10^8 = \frac{18.4}{p t}$$

(11)

Eq. (10) yields a 14\% higher $v_n/p$ and at $pt = 10^{-8}$ a 5\% higher breakdown $E/p$ than Eq. (11). The data and the extrapolated breakdown curve of Fig. 3 are certainly not good to 5\%. This leaves little logical basis for choosing either initial condition in preference to the other using the $n/n_o = 10^8$ breakdown criterion. However, for short pulses there is no need to retain this criterion; a more useful breakdown criterion might be based on the impedance perturbation of the ionized volume in the waveguide.
This analysis has neglected the electron drift motion in the electric field, space charge and field distortion. These effects should be examined. They may help to explain the filamentary arc type of breakdown encountered at pressures greater than a few cm-Hg.

C. Electron Heating Transient

The rate of electron heating is determined by the rate of energy gained from the electric field and the rate of energy lost in collisions with atoms. It is given by

\[
\frac{du}{dt} = P - \nu_e u
\]  
(12)

where \( u \) is the energy of the average electron, \( \nu_e \) is the energy loss parameter (Fig. 5), \( \nu_e(u) \) is the collision frequency and \( P = (e^2E_e^2/m_e^2) \left[ 1 + \cos(2\omega t + \phi) \right] \) is the power input to the electron from the field. For zero initial energy and constant \( \nu_e \), the heating transient is given by

\[
u(t) = \frac{P}{\nu_e} \left(1 - e^{-\nu_t} \right)
\]  
(13)

The final energy is \( u_\infty = P/\nu_e \sim (E_e/p)^3 \) and the heating time constant is \( \tau_h = 1/\nu_e \).

The approximation of \( \nu(u) \) sketched in Fig. 5, which allows Eq. (12) to be solved in closed form, was used. The result is compared
with the constant \( \kappa = \kappa(u_\infty) \) solution in Fig. 6. Note that both time constants are essentially the same, but that the non-constant \( \kappa(u) \) case attains 90\% of \( u_\infty \) in one \( \tau_h \); whereas it is only 63\% when \( \kappa \) is constant. In both cases the heating time scales with pressure such that \( \tau_h \) is a function of \( E/p \). This is plotted for in Fig. 7 and on Fig. 3. It is readily seen that at any \( E/p \), the heating time is indeed negligible compared with the pulse length required for breakdown.
IV. HIGH DIELECTRIC STRENGTH GASES

The dielectric strength of a gas may be defined as its dc sparking potential relative to nitrogen at a pd large enough to make diffusion losses negligible. A few high-dielectric strength gases and their strengths measured at dc conditions at pd = 50 cm-mm Hg are\(^{10}\)

- N\(_2\) 1.00
- CHClF\(_2\) (Freon-22) 1.17
- SF\(_6\) 1.82
- CHCl\(_2\)F (Freon-21) 1.82
- CCl\(_2\)F\(_2\) (Freon-12) 2.02
- C\(_4\)F\(_8\) (Freon-C-318) 2.44
- CCl\(_2\)F\(_2\)-CCl\(_2\)F\(_2\) (Freon-114) 2.97
- CHCl\(_3\) 3.01
- CCl\(_4\) (low vapor pressure) 3.2

At microwave frequencies the equivalent ratios for SF\(_6\), Freon 12, Freon 114 and Freon C318 have been found to be considerably larger\(^{11}\). These gases are electronegative, and rely principally on high values of attachment coefficients and, perhaps to a lesser extent, on low values of Townsend coefficients for their high sparking potentials. The breakdown field is determined by that value of E/p for which the net ionization frequency is zero, i.e., the ionization frequency equals the attachment frequency. In general the ionization frequency increases
much more rapidly than the attachment frequency as $E/p$ increases. Thus, at the higher $E/p$ values required for pulse breakdown, attachment losses may become negligible and only the relation between ionization rate or the Townsend coefficient and $E/p$ determines the breakdown field.

The available data on the Townsend coefficient $\alpha/p$ for the high dielectric strength gases is sparse, covers small ranges in $E/p$ and is probably unreliable because of the difficulty of accurately measuring $\alpha$ in attaching gases. Data on attachment coefficients $\eta/p$ are practically non-existent. However one can make some rough comparison of gases by extrapolating what data is available.

The theory of the Townsend coefficient\(^9\) yields the semi-empirical formula

\[ \frac{\alpha}{p} = Ae^{\frac{-Bp}{E}} \]  

(14)

$B$ and $B/A$ are called the Stoletow constants of the gas. If $ln(\alpha/p)$ is plotted versus $p/E$ a straight line of slope $-B$ should result. For any simple, non-attaching molecular gases this law is obeyed over a wide range of $E/p$. It generally fails for monatomic gases. There is not enough data on the complex electronegative gases to know how well it works. Considerable work has been done on the complex hydrocarbons (non-attaching) by Heylen and Lewis\(^1\), and Devins and Grows\(^1\). These workers have measured the $A$ and $B$ coefficients for
numerous hydrocarbons over respectable ranges of E/p and find the law valid at least up to E/p \sim 200 \text{ v/cm-mm Hg}.

The log \( \alpha/p \) for several gases is plotted against \( p/E \) in Fig. 8. All the curves are transcribed from Ref. 3 except the SF\(_6\) curves labeled HG and WSB. The HG is from Birks and Hart\(^{13}\) attributed to Harrison and Geballe. The WSB is determined from breakdown data published by Wilson, Simmons, and Brice\(^{16}\). The diversity in the SF\(_6\) data points out the need for more basic data in this area. The A and B coefficients from these curves are roughly:

<table>
<thead>
<tr>
<th>Gas</th>
<th>B</th>
<th>A</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{N}_2 )</td>
<td>320</td>
<td>14</td>
</tr>
<tr>
<td>( \text{CCl}_2\text{F}_2 )</td>
<td>350</td>
<td>22</td>
</tr>
<tr>
<td>( \text{CH}_2\text{Cl} )</td>
<td>384</td>
<td>410</td>
</tr>
<tr>
<td>( \text{CF}_3\text{CF}_5 )</td>
<td>635</td>
<td>61</td>
</tr>
<tr>
<td>( \text{C}_2\text{H}_2\text{Br} )</td>
<td>933</td>
<td>7 \times 10^9</td>
</tr>
<tr>
<td>( \text{C}<em>6\text{H}</em>{12} )</td>
<td>1380</td>
<td>3 \times 10^{11}</td>
</tr>
<tr>
<td>SF(_6)(v.Engel)</td>
<td>1920</td>
<td>3 \times 10^{11}</td>
</tr>
<tr>
<td>( \text{CHCl}_3 )</td>
<td>2880</td>
<td>6 \times 10^{12}</td>
</tr>
<tr>
<td>( \text{CCl}_4 )</td>
<td>7670</td>
<td>10^{21}</td>
</tr>
</tbody>
</table>

It is virtually certain that gases with anomalously high values of A either do not follow this law, or \( \alpha/p \) was inaccurately measured.

Despite all the difficulties in estimating \( \alpha/p \) for very high E/p in the high dielectric strength gases, a most striking feature emerges. All \( \alpha/p \) tend to approach or cross that of air at high E/p. For E/p \( \geq 200 \) it would seem that the advantage over air of the high
strength electronegative gases is considerably diminished, or lost altogether. The validity of this speculation depends of course upon the behavior of the attachment coefficient at high E/p. The attachment coefficient for Freon 12 was measured by Harrison and Geballe and is given in Ref. 3. It has a maximum of about 1 attachment/cm-mm Hg at E/p = 110 v/cm-mm Hg. It has not been measured for the other gases; attempts have been made to measure attachment cross-sections in SF₆ but numerical values are still controversial. There is still much difficult work to be done in this field. Nevertheless, it is probably safe to say that v/p becomes negligible compared to α/p for E/p greater than a few hundred in all cases.
V. NANOSECOND PULSE GENERATION CIRCUIT

The basic microwave circuit for generating very short, high power pulses are shown schematically in Fig. 9. Its principle of operation is as follows. A long, low-power microwave pulse from the primary source (magnetron) is coupled into a resonant cavity through a coupling iris. The cavity consists of a straight section of waveguide, coupling iris at the input end, and an ATR switch at the output end. The ATR switch ideally presents a series open circuit in the transmission line. The standing wave in the cavity has a voltage minimum at the input iris and a voltage maximum at the output open circuit; its amplitude is determined by the dissipative and coupling losses of the cavity. At some time during the input pulse, after the cavity is charged up, the ATR switch is fired. Then the forward traveling component of the standing wave moves toward the load; the backward traveling component reflects off the input iris and joins onto the forward going wave, and a pulse of twice the electrical length of the cavity travels toward the load. The pulse rise time is determined by the speed with which the switch closes.

To get the most out of this circuit, the cavity must be designed for optimum power gain and the ATR switch for high speed and low loss. The cavity design is straightforward, however, the design of a good nanosecond microwave switch is a major problem. In Section B several switches which were tried are discussed and two lines of attack for further experimentation are considered.
A. Cavity Resonator Circuit Analysis

A transmission line diagram of the cavity at resonance is shown in Fig. 10. The unfired ATR switch is a shorted stub in the E-plane. The short is 1/4 wavelength from the cavity waveguide wall, where it appears as an open circuit. The switch conductance, g, allows for losses in the switch and the fact that it is not a perfect open circuit, permitting some power to leak through to the matched load. The coupling iris is represented by a susceptance, b across the line. The power gain of the resonator is defined as the ratio of the power carried by the forward traveling component of the standing wave, just inside the iris, to the power carried by the incident wave. The square root of the power gain at resonance is the voltage gain and is given by

\[ G_v = \frac{b}{a_1} = \frac{C}{1-L/1-G^2}, \]  

(15)

where \( C = \sqrt{1/L^2} \) is the coupling coefficient of the iris and \( L = (1/1 + 2g)e^{-2\sigma l} \) is the attenuation suffered by the wave in making one round trip in the cavity. The waveguide attenuation constant is \( \sigma \), the cavity length \( l \). For resonance the electrical length must be

\[ \theta = \frac{2\pi l}{\lambda_g} = \left[ (n-\frac{1}{2})\pi - \frac{1}{2}\tan^{-1} \frac{b}{g} \right] \]  

(16)
where \( n = 1, 2, 3 \cdots \). Maximum gain is obtained when the input susceptance is adjusted for critical coupling, that is when

\[
\sqrt{1-C^2} = \frac{b}{\sqrt{4+b^2}} = L.
\]

(17)

The power gain is plotted against the coupling for various loss factors \( L \), in percent, in Fig. 11. This, in conjunction with Fig. 12, can be used to convert standing wave measurements in the input line to power gain once the loss is known. The loss can be determined by measuring the cavity \( Q \).

The overall power gain is the ratio of the power at the output, after the switch is fired, to the input power. If the fired switch were lossless the overall gain would be just the cavity gain as plotted in Fig. 11 times the one-way waveguide attenuation \( e^{-\alpha l} \).

B. Switch Design and Results

An effective ATR switch for the nanosecond pulse generation circuit must

a) be a good open circuit in the unfired state,
b) be a good short circuit in the fired state,
c) have a switching time of the order of a nanosecond,
d) be capable of holding off about one megawatt of rf power.

It should also not seriously reduce the waveguide bandwidth when fired, nor should it fire randomly or erratically. Of the various switching media used at microwave frequencies, a gas discharge switch
appears to be best able to meet the above requirements.

Five different switch designs with various modifications were tried. They are schematically illustrated in Fig. 13. The arm of an E-plane tee, Fig. 13a, which joins the broad wall of the cavity waveguide contains the switch element and is terminated by a movable short. The short is placed so as to appear as an open circuit in the cavity wall when the switch is not fired. The gas discharge switch is placed in the junction in the plane of the cavity wall, or a half wavelength back, so as to create a short circuit at the wall when fired. There is some sacrifice in bandwidth when the switch is a half wavelength from the junction because the electrical length to the short varies with frequency.

The two simplest switches, Figs. 13b-d, consisted of a capacitive post and slot, respectively, across the junction, leaving narrow gaps. These gaps were broken down by the cavity power after triggering with ultraviolet light from an external spark gap. Another design, Fig. 13c, replaced the capacitive bar with a split post containing the trigger spark gap. This put the trigger source much closer to the main gap and offered some improvement. The switch of Fig. 13c, had a quartz tube, filled with air at approximately 200 mm Hg, situated immediately behind a resonant slot. A high voltage trigger pulse was applied across the electrodes to break the gas down and the rf power maintained the discharge. Another switch, Fig. 13f, consisted of two hemispheres mounted on opposite broad walls of the tee, with a 0.040" gap, located a half wavelength from the junction. It was triggered either by a
pointed electrode passing through the narrow wall to the gap center, or by light from an external spark gap.

All switches tried worked; but not well enough for the contemplated breakdown studies. They were satisfactory open circuits in the unfired state with typically 35 db isolation of the cavity from the load; but in the fired state the effective short circuit was inadequate. This is a consequence of the rf power in the cavity being unable to maintain a sufficiently dense discharge. Two important factors are the volume of the region being ionized and the equivalent impedance of the resonant structure through which the discharge is being initiated. Thus, the switching volume should be as small as possible and the equivalent impedance as low as is consistent with the desired power gain. The switches were evaluated with a sampling oscilloscope and the trigger was provided by a high voltage pulser which fired an auxiliary gap.

The low pressure switch had the best operating characteristics; it was a better short and fired more regularly than the others. However, because of the low pressure the switch operated slowly and could only hold off 400 kw of cavity power in standard WR90 waveguide. Pulses obtained using this switch had a 15 ns rise time and were about 80 ns long. The high pressure switches operated more rapidly; however the instrumentation was not adequate for accurate measurements.

To make a switch which produces a better short circuit, it will probably be advantageous to rely on an external source of power to maintain the discharge. To get fast switching, a high pressure discharge
using the geometry of Fig. 13e and external power should be tried. This type may produce a rounded leading edge to the pulse and require some pulse sharpening compensation, but it should be easier to minimize jitter\textsuperscript{18}. An alternative scheme which should also be tried is the low pressure, or "vacuum" switch\textsuperscript{17}. The discharge in this type is in the metallic vapor evaporated from the electrodes. A fairly long delay, of the order of 100 ns, is required before the initial weak discharge reaches the point where metallic vapor appears; from then on the dense switching plasma forms rapidly. This long delay could be desirable if it eliminates the need for a delayed trigger pulse and can be triggered simultaneously with the magnetron pulse. Again an auxiliary dc power supply is needed, but the switching time is less sensitive to voltage fluctuations than the high pressure type.

A typical pulse envelope is sketched in Fig. 14. The dotted curve represents the magnetron pulse, the solid curve the power into the load or detector. When the magnetron comes on, a small amount of power leaks through to the load before the switch fires. The trigger pulse closes the switch and the high power nanosecond pulse travels down the line, its amplitude distorted by arc loss and any other imperfections in the switch operation. Once the pulse has gone by the switch slowly recovers, reducing the output power gradually. Since there is still some power in the line, the switch may not recover fully until the magnetron pulse ends.
VI. SUGGESTED EXPERIMENTAL PROGRAM

It has been pointed out in Sections II and III that an experimental
determination of the pulse breakdown characteristic for air for \( p t < 10^{-5} \)
mm Hg-sec is necessary for reliable estimates of the power handling capac-
ity of pulsed microwave systems. Because of the possible differences
between low and high pressure breakdown mechanisms, the experiments
ought to be done as close to one atmosphere as possible. This requires
pulses of the order of \( 10^{-8} \) sec. Video pulses could be used, but the
correspondence to microwave breakdown is not assured, since electrons
drift in the electric field to the walls (mobility-controlled breakdown)
and secondary electrons may be emitted from the walls by ion impact.
These contributions to the breakdown mechanism may become significant
in the narrow gaps which are usually required to produce high fields
at reasonable voltages.

Microwave pulse breakdown measurements yield useful information
on the Townsend coefficient and can be used to study other gases at
high E/p.

If more details on the breakdown mechanism are known, it may be
possible to devise methods to suppress breakdown to a greater extent.
Therefore, it would be desirable to know a) where the electrons come
from and how the spark starts, b) how it grows in space and time,
c) how large it must be before interfering with transmission, and
d) how the pulse repetition rate influences breakdown by residual
electrons and gas heating.
A minimum experimental program would include: a) development of the nanosecond pulse circuit for reliable, repeatable pulses down to 10 ns duration, b) design and construction of the breakdown chamber, c) measurement of breakdown characteristic of air to smallest possible pt, varying p and t independently. The basic circuit arrangement is sketched in Fig. 14.

A more ambitious program would include: a) similar pulse breakdown measurements on high dielectric strength gases, in particular SF₆ and the Freons, b) experiments to measure temporal and spatial growth of the spark - probably a major undertaking, and c) experiments to determine the effect and importance of gas heating and the pulse repetition rate on lowering of breakdown fields.
VII. CONCLUSIONS

The pulse breakdown curve for air indicates that at atmospheric pressure the peak power for breakdown with a 10 ns pulse is approximately four times the CW power; with a 1 ns pulse it is about fourteen times the CW power. Modest gains in waveguide and antenna power handling capability are possible at atmospheric pressure using the nanosecond pulse technique. At reduced pressure enormous gains are possible; with a pulse with a $pt \approx 5 \times 10^{-8}$ mm Hg-sec one hundred times the CW power, at the same pressure, can be transmitted. There is also some hope of improving on this theoretical estimate by a better understanding of the breakdown mechanism and searching for ways of suppressing spark formation. Presently, in practice, the theoretical minimum breakdown powers are often not even attained, perhaps due to gas heating, surface irregularities or other effects.

In order to understand better the breakdown process, especially for very short pulses, more theoretical work is needed. The problems of space charge and field distortion, differences in the breakdown mechanism at low and high pressures; gas heating, and mode of spark propagation need particular attention. Experimental work to verify the theory is clearly required; a determination of the breakdown field pulse-length characteristic should be the essential goal of the experimental program. The properties of high dielectric strength gases are not well known and they should certainly be measured to determine the efficiency of the gases in the nanosecond pulse region.
REFERENCES

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4. Brown, Basic Data of Plasma Physics
5. Ward, Fifth Inter. Conf. on Ionization Phenomena in Gases, Munich 1961
8. Brown, Handbuch der Physik XXII, p 539
FIGURE 1a
ELECTRON ENERGY LOSS PARAMETER FOR AIR
FROM REF. 2
FIGURE 1b
ELECTRON ENERGY AND COLLISION FREQUENCY IN AN ELECTRIC FIELD IN AIR FROM REF. 2

\[ \frac{\nu}{p} \text{ collisions/sec - cm}^{-2}\text{ mmHg} \]

\[ \nu \text{ electron volts} \]

\[ \psi \text{ electron volts/cm - mmHg} \]
FIGURE 2
NET IONIZATION FREQUENCY FOR AIR

$\frac{\nu}{\nu_p}$ - Ionizations/Sec - mmHg - Electron

$E/p$ - Volts/cm - mmHg
FIGURE 3
EXTRAPOLATION OF PULSE-BREAKDOWN CURVE TO LARGE VALUES OF E/p

(1) BREAKDOWN CURVE FOR AIR
(2) ELECTRON HEATING RATE CURVE FOR AIR

COLLISION FREQUENCY ≈ IONIZATION FREQUENCY LIMITATION

E/p - volts/cm/mmHg

10^4

10^3

10^2

10^1

10^0

10^-1

10^-2

10^-3

10^-4

pτ - mmHg sec, (τ1 - PULSE LENGTH, τ2 - HEATING TIME)

EXTRAPOLATED

MEASURED

CONFIRMED BY EXPERIMENT WITH μSEC PULSES

REGION OF CURRENT INTEREST

E/p minimum

10^-9

10^-8

10^-7

10^-6

10^-5

10^-4
FIGURE 5
ELECTRON ENERGY LOSS PARAMETER
VS. ENERGY FOR AIR

\[ \kappa = \frac{10^{-4}}{u} e^{u/31} \]

- EXPERIMENTAL (REF 2)
- APPROXIMATION:

\[ 0 \leq u \leq 2.0 \]
FIGURE 7
ELECTRON HEATING TIME CONSTANT IN AIR

\( \frac{E}{p} \) – volts/cm-mmHg

\( \rho \tau_h \) – mmHg-sec

10^{-6}
10^{-7}
10^{-8}
10^{-9}

1 10 100
FIGURE 9
SCHEMATIC DIAGRAM OF MICROWAVE NANOSECOND PULSE GENERATOR
FIGURE 10
CAVITY RESONATOR CIRCUIT
FIGURE II
EQUIVALENT POWER GAIN IN RESONANT TRANSMISSION LINE VS IRIS COUPLING

POWER GAIN (db) vs COUPLING (db)

Critical Coupling
Percentage Power Lost

0% 0.2% 0.4% 0.6% 0.8% 1.0% 1.2% 1.4% 1.6% 1.8% 2.0%
(a) E-PLANE TEE

(b) CAPACITIVE POST AT JUNCTION PLANE

(c) SPLIT CAPACITIVE POST AT JUNCTION PLANE WITH TRIGGER GAP

(d) CAPACITIVE SLIT AT JUNCTION PLANE

(e) RESONANT SLOT WITH LOW PRESSURE DISCHARGE TUBE

(f) HEMISPHERICAL SPARK GAP, 1/2 WAVELENGTH FROM JUNCTION

FIGURE 13
SERIES SWITCH FOR THE NANOSECOND PULSE GENERATOR
FIGURE 14
WAVEFORMS AT THE BROAD BAND DETECTOR FOR NANOSECOND BREAKDOWN STUDY
FIGURE 15
BLOCK DIAGRAM OF NANOSECOND BREAKDOWN STUDY INSTRUMENTATION