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RESISTANCE EFFECTS ON THE PERFORMANCE OF MAGNETICALLY-DRIVEN FLYER PLATES

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ABSTRACT

The kinetic energy and impulse of a magnetically-propelled flyer plate is shown to be very dependent on system resistance. A system representative of those presently in use is chosen as an example.

CONTENTS

RESISTANCE EFFECTS ON THE PERFORMANCE OF MAGNETICALLY-DRIVEN FLYER PLATES

Introduction

The differential equation governing the discharge of a capacitor bank through a flyer plate load implies that the efficiency of energy conversion and the subsequent impulse which may be delivered to a target are very dependent upon system resistance for particular combinations of other system parameters. This dependency is examined and verified using a digital computer code developed at Sandia Laboratories, Livermore (SLL). Several curves are presented showing the magnitude of this resistance effect for several capacitor banks of general interest.

Theory

The circuit for the capacitor bank and the flyer may be represented as a series R-L-C combination. The differential equation for this system is

$$\frac{d^2}{dt^2}(Li) + R\frac{di}{dt} + \frac{i}{C} = 0 \qquad (1)$$

where i is the instantaneous current and R, L, and C are the lumped values of resistance, inductance, and capacitance, respectively.

The circuit inductance can be expressed as the parallel-plate inductance of the flyer plus a constant inductance representing the fixed-bank value:

$$L = L_B + \frac{\mu \ell}{W}(x + x_s) \qquad (2)$$

where

L_B = fixed-bank inductance

μ = permeability of free space

ℓ = flyer length

W = flyer width

x = instantaneous distance of the flyer from the return path

x_s = a distance to account for skin effect in the flyer and return path.

The quantity x_s is the average skin depth of the return path plus the thickness or skin depth (whichever is less) of the flyer. This term, which accounts for the internal inductance of the flyer-return path portion of the circuit, is significant before flyer movement because the initial spacing of the flyer from the return path is of the same order as the skin depth. A more accurate determination of this quantity requires knowledge of the transient diffusion phenomenon governing the current density in the flyer and return path. Since this phenomenon is relatively independent of flyer motion, any error in the determination of the quantity x_s will be reflected only as an error in the fixed value of assumed bank inductance and will not appreciably affect the analysis.

The system resistance, similarly, can be expressed as the combination of a fixed-bank value and a value representing the flyer-return-path load.

$$R = R_B + \frac{\rho_1 \ell}{W\delta_1} \quad \frac{\rho_2 \ell}{W\delta_2} \tag{3}$$

where

R_B = fixed-bank resistance

ρ_1 = resistivity of the flyer material

ρ_2 = resistivity of the return-path material

δ_1 = thickness or skin depth (whichever is less) of the flyer

δ_2 = skin depth of the return path.

Resistance is also dependent upon the transient diffusion phenomenon; however, in this presentation, it is assumed to be constant.

Equation (1) may be rewritten as

$$L\frac{d^2i}{dt^2} + \left(R + 2\,\frac{dL}{dt}\right)\frac{di}{dt} + \left(\frac{1}{C} + \frac{d^2L}{dt^2}\right)i = 0 \qquad (4)$$

Now

$$\frac{dL}{dt} = \frac{dL}{dx}\,\frac{dx}{dt}$$

or

$$\frac{dL}{dt} = \frac{\mu\ell}{W}\,v \qquad (5)$$

and

$$\frac{d^2L}{dt^2} = \frac{dL}{dx}\,\frac{d^2x}{dt^2}$$

or

$$\frac{d^2L}{dt^2} = \frac{\mu\ell}{W}\,a \qquad (6)$$

where

v = instantaneous flyer velocity

a = instantaneous flyer acceleration

Substituting Equations (5) and (6) into Equation (4) yields

$$L\frac{d^2i}{dt^2} + \left(R + 2\,\frac{\mu\ell}{W}v\right)\frac{di}{dt} + \left(\frac{1}{C} + \frac{\mu\ell}{W}\,a\right)i = 0 \qquad (7)$$

The instantaneous acceleration can be related to the instantaneous current by considering the magnetic pressure exerted on the flyer. The pressure is

$$P = \frac{1}{2}\frac{B^2}{\mu} \qquad (8)$$

where B = magnetic flux density. For the parallel-plate case, the flux density may be written in terms of the instantaneous current and flyer width as

$$B = \frac{\mu i}{W} \qquad (9)$$

yielding

$$P = \frac{1}{2}\mu\frac{i^2}{W^2} \qquad (10)$$

as an alternative expression for pressure. Dividing Equation (10) by the mass per unit area of the flyer yields the instantaneous acceleration

$$a = \frac{\mu i^2}{2\rho\tau W^2} \qquad (11)$$

where

ρ = flyer density

τ = flyer thickness

If realistic values are assumed for bank parameters and flyer dimensions, the terms containing velocity and acceleration in Equation (11) may be compared to the resistance and capacitance, respectively.

Assume an aluminum flyer such that

ρ = 2.71 gms/cm^3

μ = $4\pi \times 10^{-7}$ henries/meter

ℓ = 0.6 meters

W = 0.3 meters

$$\tau = 0.33 \text{ millimeters}$$

$$I_{max} \doteq 5 \text{ megamps}$$

$$v \doteq 0.02 \text{ cm/microsecond}$$

$$R \doteq 3 \text{ milliohms}$$

$$C = 100 \text{ microfarads}$$

Using these values and Equation (11), it is observed that

$$\frac{\mu \ell}{W} a = \frac{\mu^2 \ell i^2}{2\rho \tau W^3} \doteq 5 \times 10^2$$

is only five percent of the quantity.

$$\frac{1}{C} = 10^4$$

even at the time of peak current.

However, the term which contains the flyer velocity is

$$\frac{2\mu \ell}{W} v = 10^{-2}$$

which is somewhat greater than the resistance term

$$R = 3 \times 10^{-3} .$$

Hence, for purposes of gaining insight into system efficiency, the coefficient of the first derivative in Equation (7), i. e.,

$$D = R + \frac{2\mu \ell}{W} v \tag{12}$$

may be most useful.

Multiplying Equation (12) by the square of the instantaneous current yields

$$i^2 D = i^2 R + \frac{2\mu \ell}{W} v\, i^2 . \tag{13}$$

Recalling that the total force on the flyer is

$$F = P\ell W = \frac{1}{2}\frac{\mu\ell}{W} i^2 ,$$

Equation (13) may be rewritten as

$$i^2 D = i^2 R + 4Fv \tag{14}$$

The first term on the right of Equation (14) is the instantaneous power dissipated in the system resistance, and the quantity Fv in the second term is the instantaneous power delivered to the flyer.

The velocity may be written in terms of the instantaneous acceleration and, hence, the instantaneous current.

$$v = \int_0^t a \, dt = \frac{\mu}{2\rho\tau W^2} \int_0^t i^2 \, dt \tag{15}$$

Substituting this expression into Equation (12) gives

$$D = R + \frac{\mu^2 \ell}{\rho \tau W^3} \int_0^t i^2 \, dt . \tag{16}$$

System Efficiency

Kinetic Energy

The following conclusions regarding system efficiency, i. e., the percent of the initial stored energy delivered to the flyer, may be drawn with the aid of Equation (16).

1. The system resistance should be made as low as possible for the most efficient energy conversion. The increase in efficiency gained from a lower resistance will depend upon the relative values assigned to the remaining parameters.

2. The system efficiency will be inversely dependent upon some power of the mass per unit area of the flyer. The dependency is not a direct

inverse because the instantaneous current will be higher for the case of a more massive flyer. (The more massive flyer will not accelerate as fast; hence, the inductance of the circuit will be lower for a longer time.)

3. The system efficiency will be very dependent upon flyer width, particularly if the bank inductance is high compared to that of the flyer. (If the bank inductance is high, the current will be relatively independent of flyer width. The second term in Equation (16) will then vary approximately as the inverse cube of flyer width.)

4. The system efficiency will be dependent on the initial stored energy. The square of the instantaneous current is approximately proportional to the bank energy; thus, the second term on the right of Equation (16) will be energy dependent. Because of the scope of this paper, this dependence is not determined.

Flyer Momentum

Often flyer momentum density is the desired performance criterion. The expression for the momentum density of the flyer is

$$M = \rho \tau v$$

which, using Equation (15) for velocity, may be written as

$$M = \frac{1}{2} \frac{\mu}{W^2} \int_0^t i^2 \, dt \qquad (17)$$

The conclusions regarding flyer efficiency must be modified somewhat to apply to momentum density.

1. It is still desirable to maintain system resistance as low as possible for a given flyer width. (It should be noted, however, that increasing flyer width to obtain lower resistance is not desirable.)

2. The mass per unit area of the flyer will affect the impulse in a manner opposite to that in which it affects efficiency; the more massive flyer will accelerate at a slower rate, resulting in a higher current in Equation (17). Thus, while a more massive flyer results in lower efficiency, it will result in a higher impulse if it represents a significant part of the total system inductance.

3. The flyer width affects impulse in approximately the same manner as it affects efficiency.

4. The impulse is almost directly proportional to the initial stored energy.

Examples

I. The circuit equation (Equation 1) was solved using a modified Runge-Kutta technique in a code developed by the Sandia Laboratories, Livermore Environmental Test Division.[1] This code allows any or all of the parameters to be varied.

To determine the effects of resistance on flyer performance, the following parameters were used as inputs to the code:

Bank capacitance	96 microfarads
Initial charge voltage	55 kilovolts
Fixed-bank inductance	6, 9, 12 nanohenries
Fixed-bank resistance	0.5, 1.0, 1.5, 2.0, 2.5, 3 milliohms
Flyer length	24 inches
Flyer width	21 and 10.5 inches
Flyer thickness	0.012 inches
Initial flyer-return path spacing	0.010 inches
Flyer material	Al, Cu

Typical curves showing flyer velocity and displacement for a fixed-bank inductance of six nanohenries are shown in Figures A1 through A6 in Appendix A. Similar curves may be plotted for the remaining values of fixed-bank inductance and the results combined to obtain plots of flyer impulse and kinetic energy as a function of resistance for various flyer displacements. Since it is not practical to allow a flyer of this thickness to travel more than three to five millimeters before impact, the plots shown in Figures A7 through A12 show the values of impulse and kinetic energy at a displacement of three millimeters and those shown in Figures A13 through A18 show the values at five millimeters.

These curves corroborate the general conclusions drawn from Equation (16) and also indicate the relative importance of the factors involved in determining flyer performance. System resistance is

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[1] To be published under separate cover by R. W. Reynolds, 8125.

observed to be a quite important factor for the range of chosen inductance values and appears to be a good trade-off parameter in bank design, e. g., for the case of the six-nanohenry bank driving the 10. 5 x 24-inch aluminum flyer, a reduction in system resistance of from three to two milliohms results in a twenty-five percent increase in flyer impulse at a displacement of three millimeters. It is also interesting to note from Figures A13 through A16 that for a given resistance, the discharge efficiency is, indeed, less for the more massive copper flyer than for the aluminum flyer of the same dimensions, while the flyer impulse is higher.

The kinetic energy and impulse appear to be insensitive to the system inductance if the system is allowed to complete the energy conversion process. However, lower system inductance is generally advantageous to assure complete conversion before flyer impact.

II. The curves presented in the above example characterize a typical state-of-the-art system. The following curves A19 through A24 show the result of decreasing resistance and inductance to very low values. It is observed that very significant gains can be achieved if resistance in the submilliohm range can be realized, e. g., Figure A22 shows the impulse at a displacement of three millimeters may be doubled by reducing the resistance from 2 to 0. 5 milliohms for the 3-nh bank.

Conclusions

System resistance has been shown to be an important consideration in the design of capacitor banks intended to drive flyers for impulse testing. The system sensitivity to resistance exhibits an approximate inverse relationship, i. e., the rate of impulse gain increases as resistance is decreased.

This relationship should be considered when choosing the materials for the current-carrying elements of a system. System resistance will vary as the ratio of the square root of material resistivities. (This variation is not a direct ratio because of the skin effect.)

For example, if copper rather than aluminum is used in a system, this ratio becomes

$$\frac{\sqrt{\rho_{Cu}}}{\sqrt{\rho_{Al}}} = \frac{\sqrt{1.72}}{\sqrt{2.828}} = 0.78$$

This means that a 3-milliohm bank would become a 2.34-milliohm bank. From Figure A12, it is observed that this reduction would result in a twenty-five per cent increase in the resultant impulse. To obtain an equivalent increase by increasing the bank size would require approximately twenty-five per cent more capacitance.

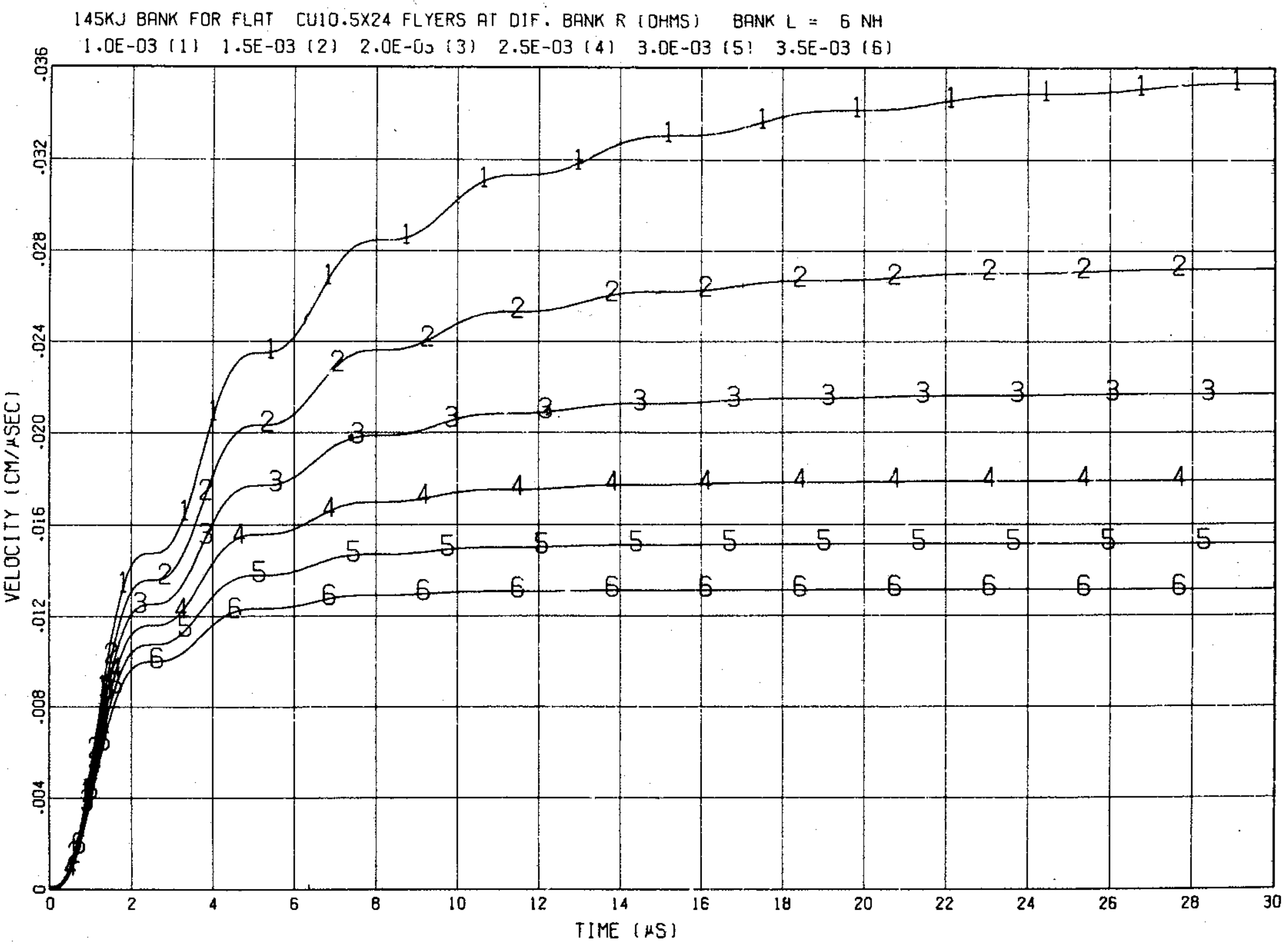

Figure A1. Velocity of Copper Flyer (10.5 x 24 inch) - R = 1.02 mΩ, (2) R = 1.52, (3) R = 2.02, (4) R = 2.52, (5) R = 3.02, and (6) R = 3.52. L_B = 6 nh

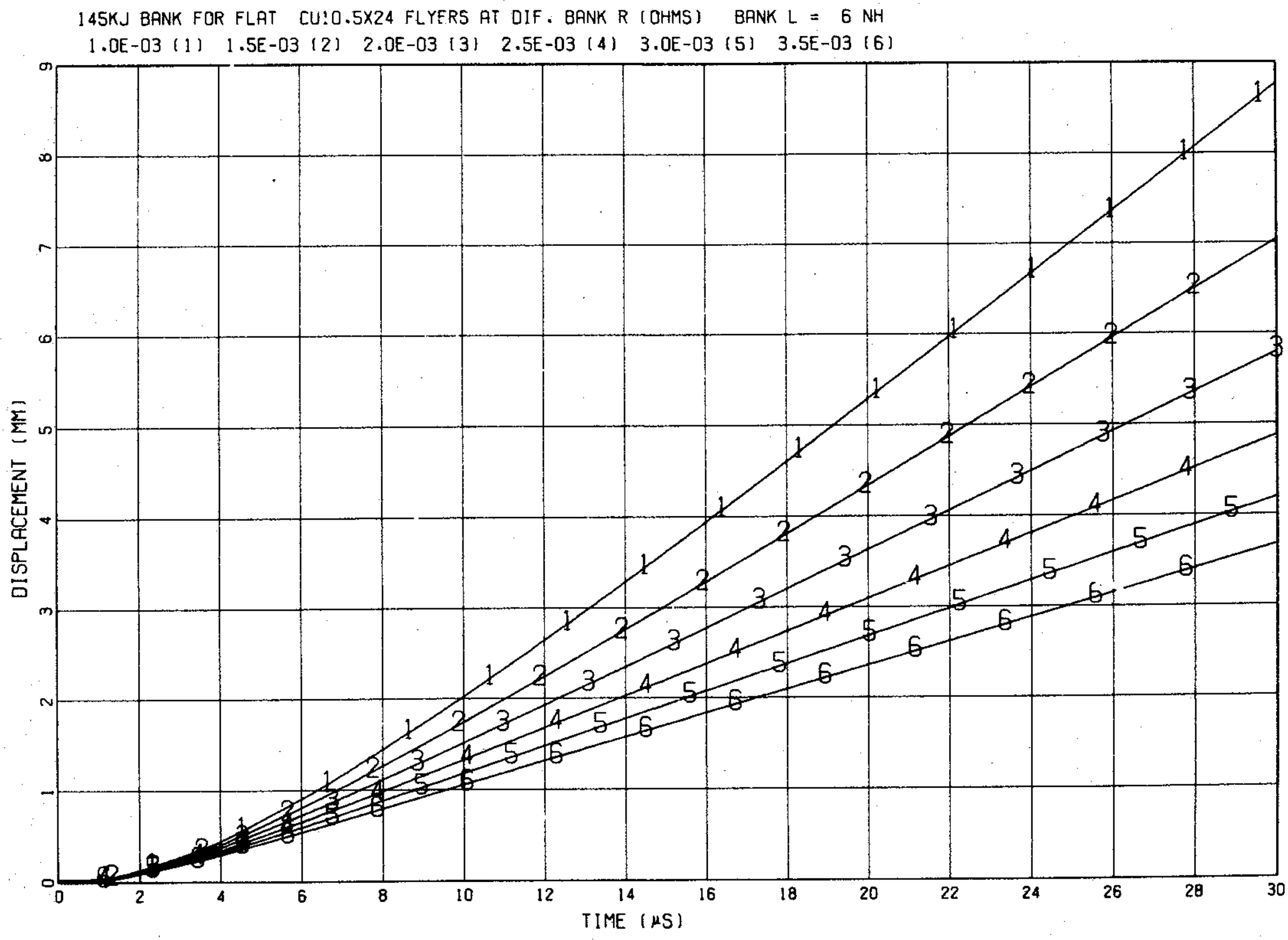

Figure A2. Displacement of Copper Flyer (10.5 x 24 inch) - R = 1.02 mΩ, (2) R = 1.52, (3) R = 2.02, (4) R = 2.52, (5) R = 3.02, and (6) R = 3.52. L_B = 6 nh

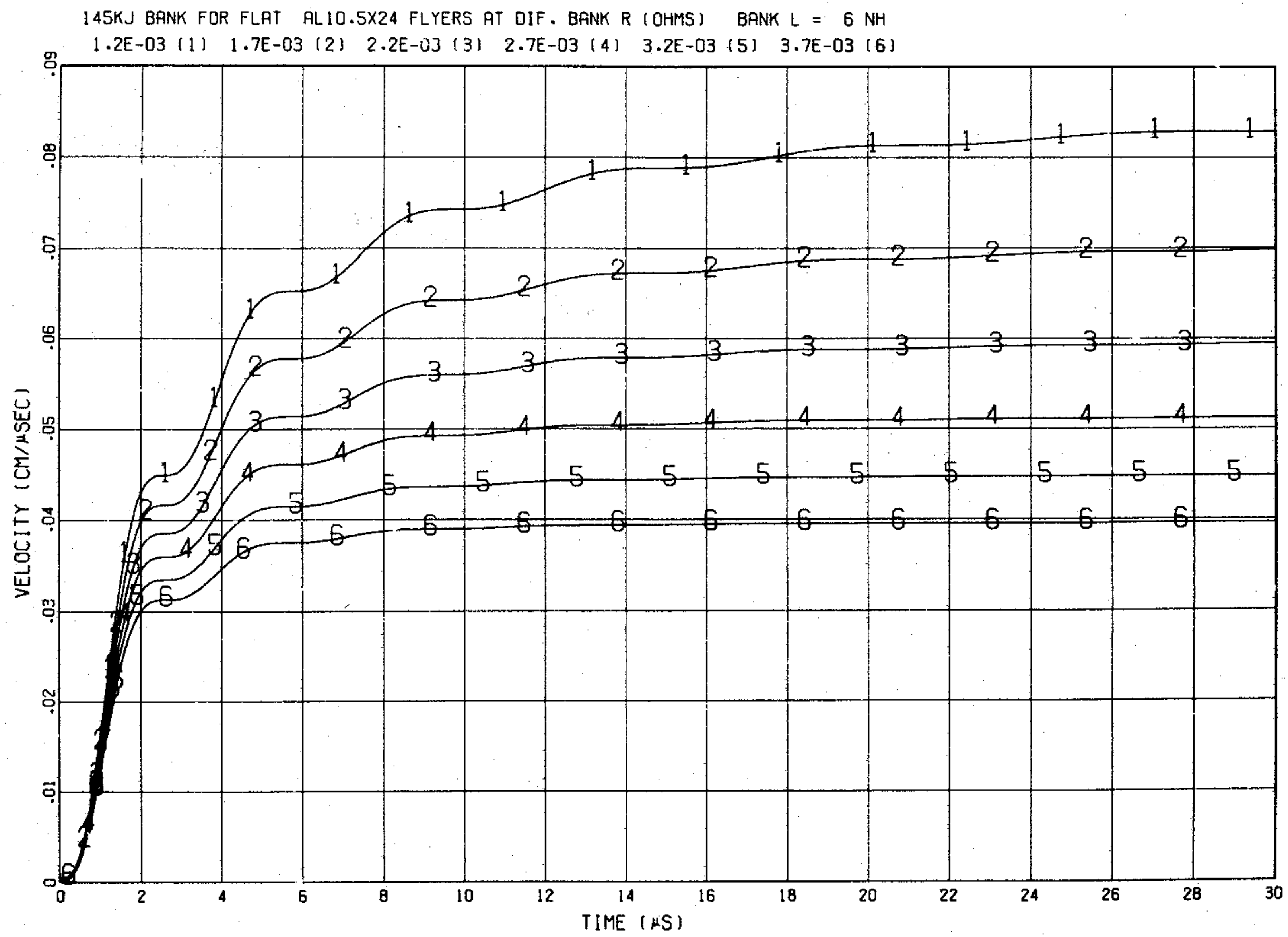

Figure A3. Velocity of Aluminum Flyer (10.5 x 24 inch) - (1) R = 1.17 mΩ, (2) R = 1.67, (3) R = 2.17, (4) R = 2.67, (5) R = 3.17, and (6) R = 3.67. L_B = 6 nh

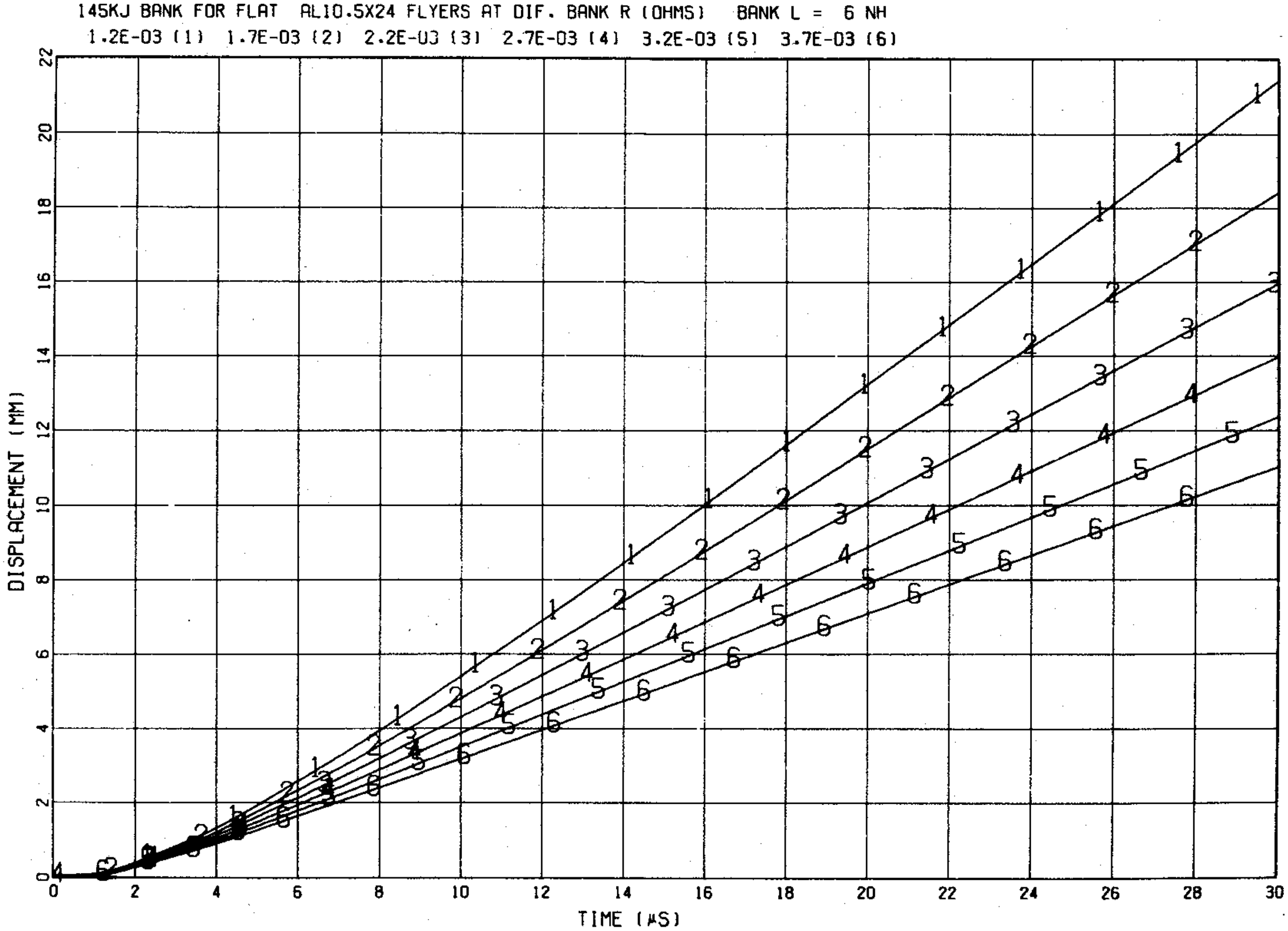

Figure A4. Displacement of Aluminum Flyer (10.5 x 24 inch) - (1) R = 1.17 mΩ, (2) R = 1.67, (3) R = 2.17, (4) R = 2.67, (5) R = 3.17, and (6) R = 3.67. L_B = 6 nh

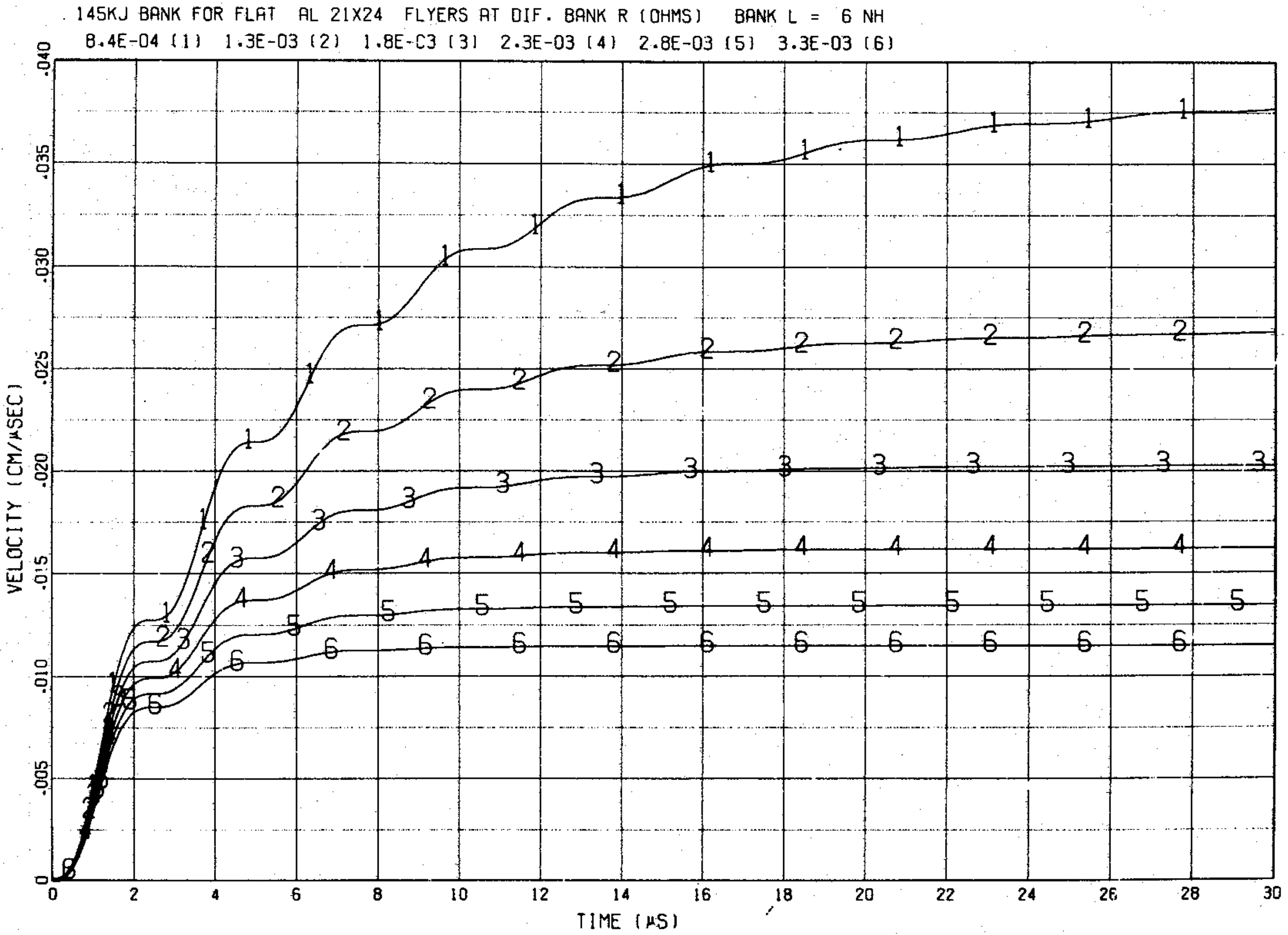

Figure A5. Velocity of Aluminum Flyer (21 x 24 inch) - (1) R = 0.84 mΩ, (2) R = 1.34, (3) R = 1.84, (4) R = 2.34, (5) R = 2.84, and (6) R = 3.34. L_B = 6 nh

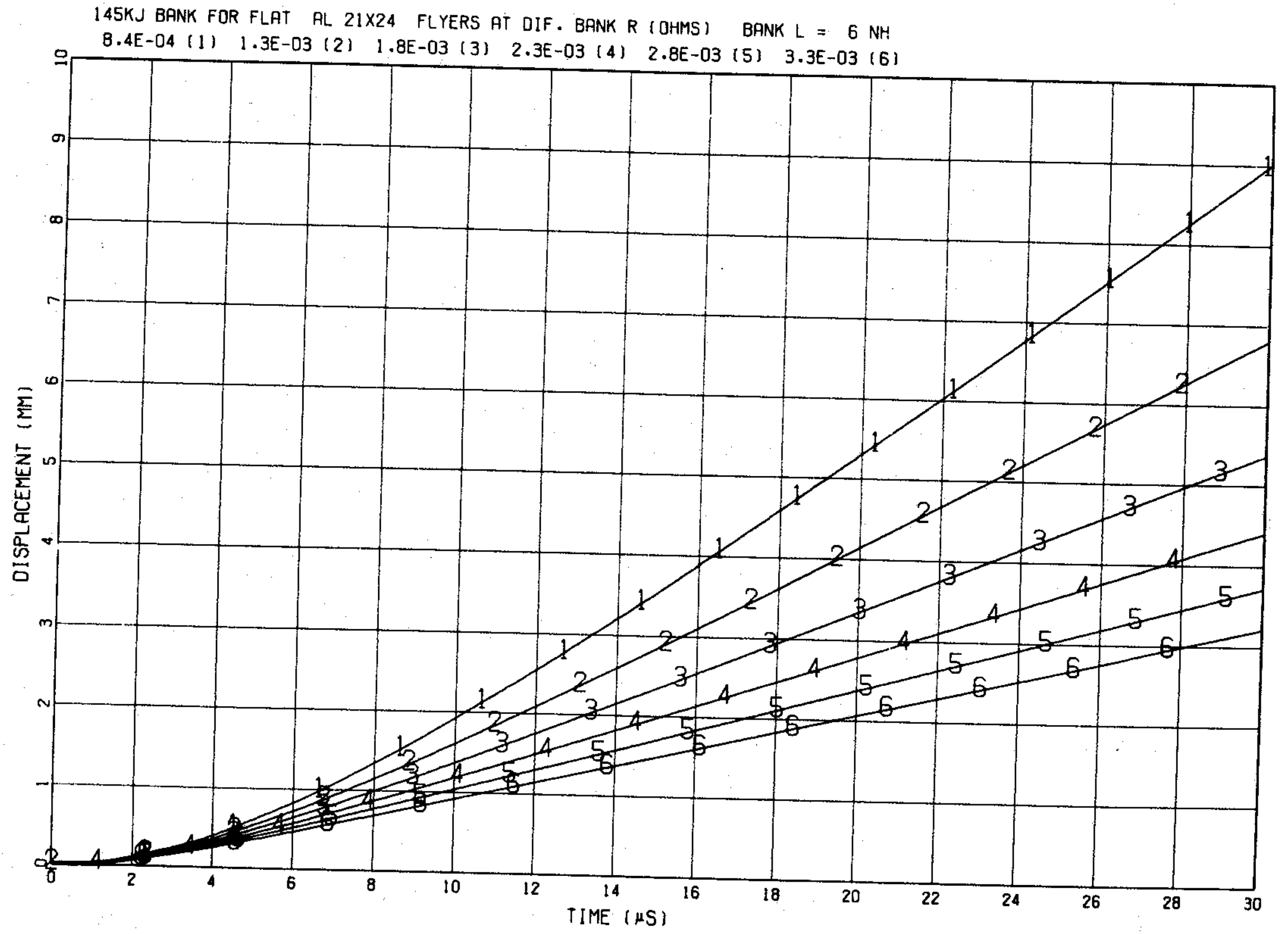

Figure A6. Displacement of Aluminum Flyer (21 x 24 inch) - (1) R = 0. 84 mΩ, (2) R = 1. 34, (3) R = 1. 84, (4) R = 2. 34, (5) R = 2. 84, and (6) R = 3. 34. L_B = 6 nh

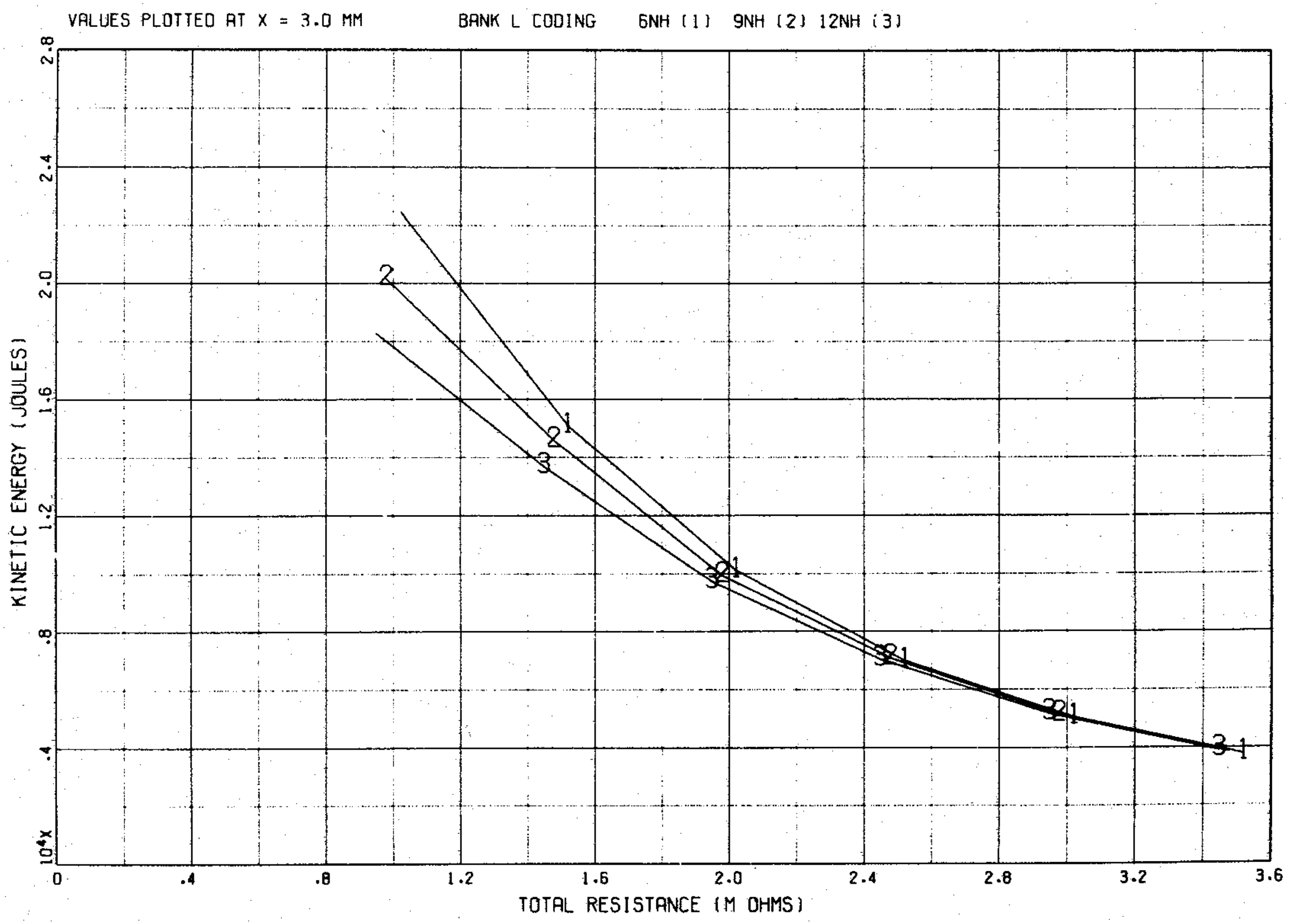

Figure A7. Kinetic Energy of Copper Flyer (10.5 x 24 inch) at a Displacement of 3 mm. (1) L_B = 6 nh, (2) L_B = 9 nh, and (3) L_B = 12 nh

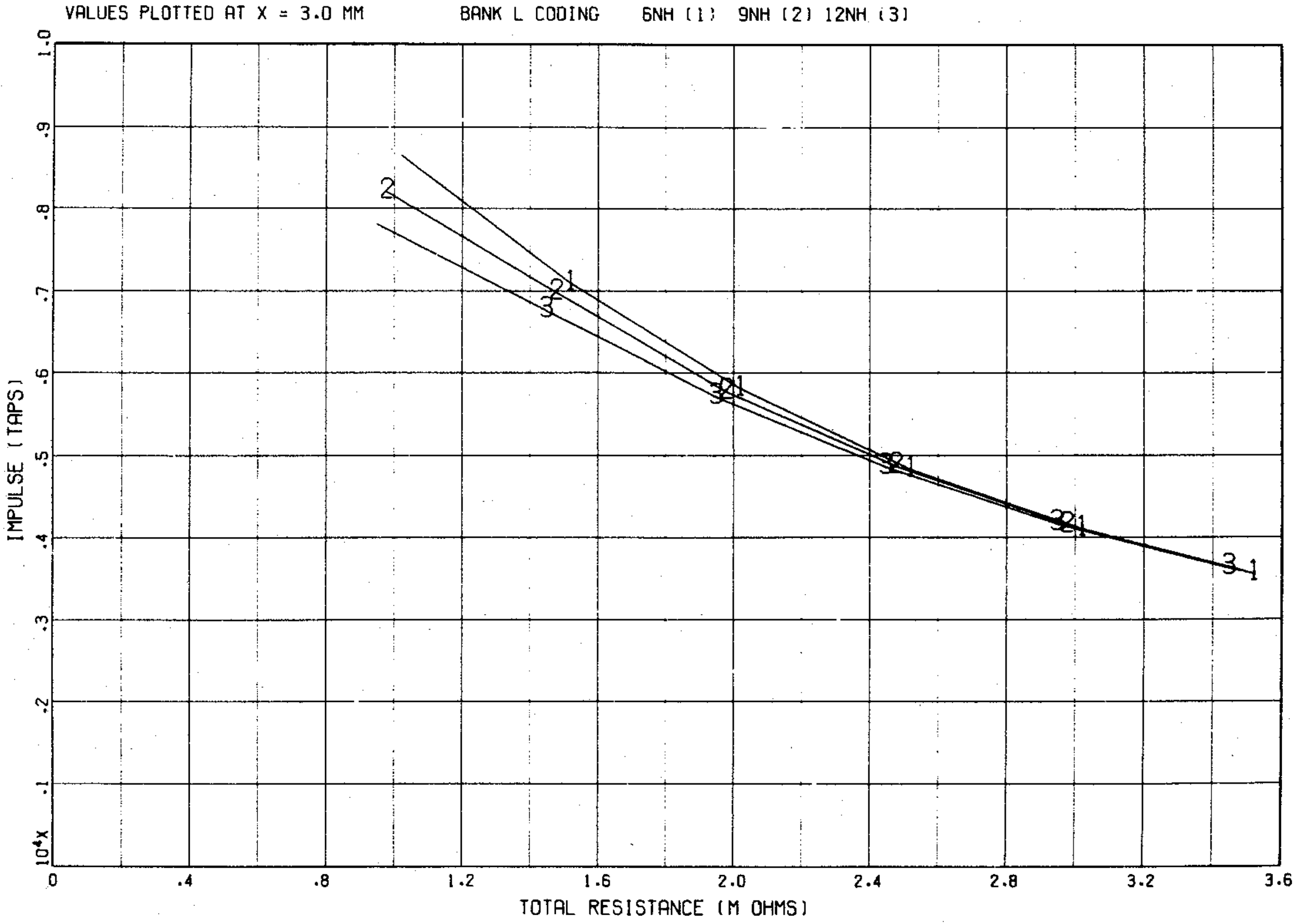

Figure A8. Impulse of Copper Flyer (10.5 x 24 inch) at a Displacement of 3 mm. (1) L_B = 6 nh, (2) L_B = 9 nh, and (3) L_B = 12 nh

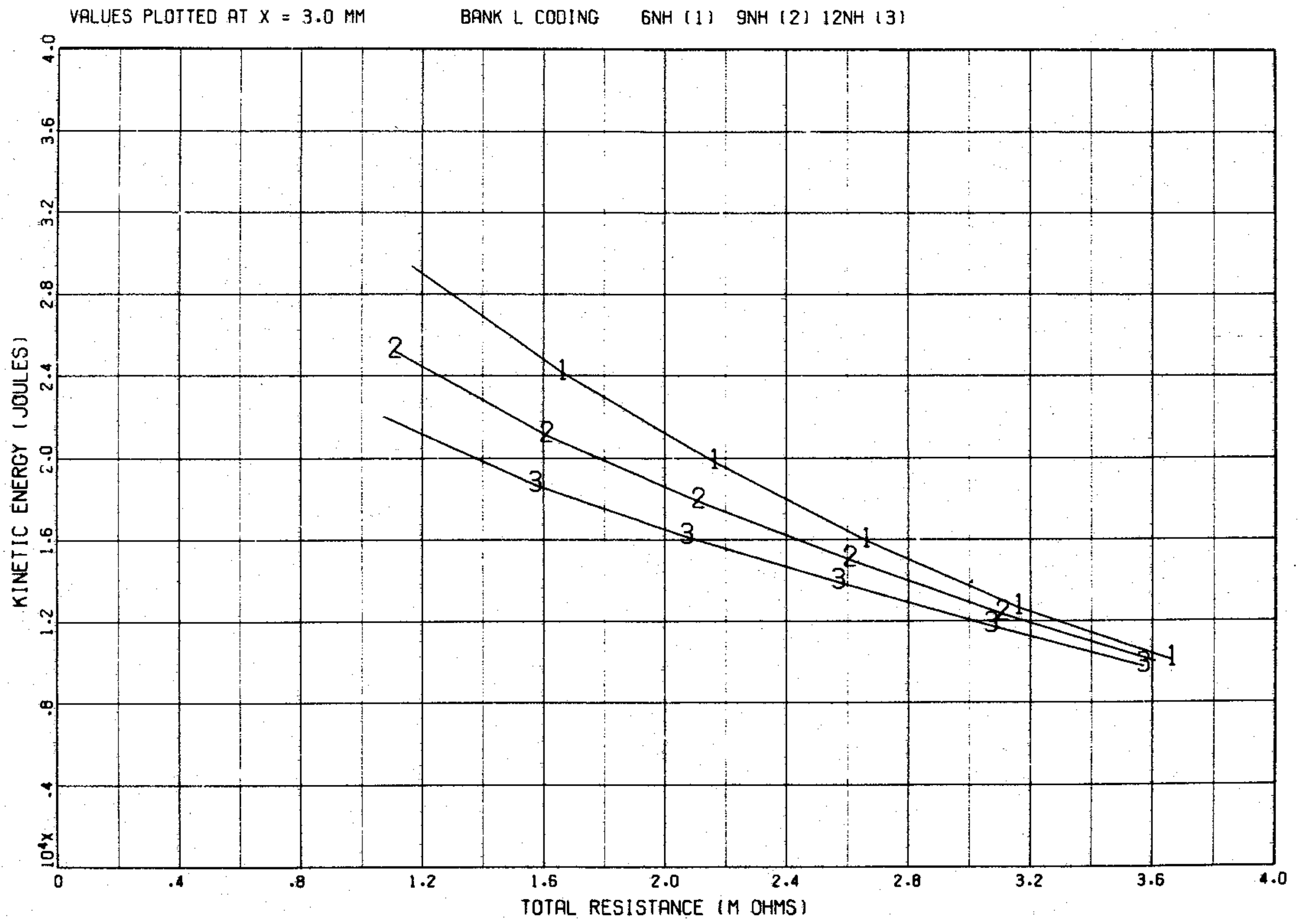

Figure A9. Kinetic Energy of Aluminum Flyer (10.5 x 24 inch) at a Displacement of 3 mm. (1) L_B = 6 nh, (2) L_B = 9 nh, and (3) L_B = 12 nh

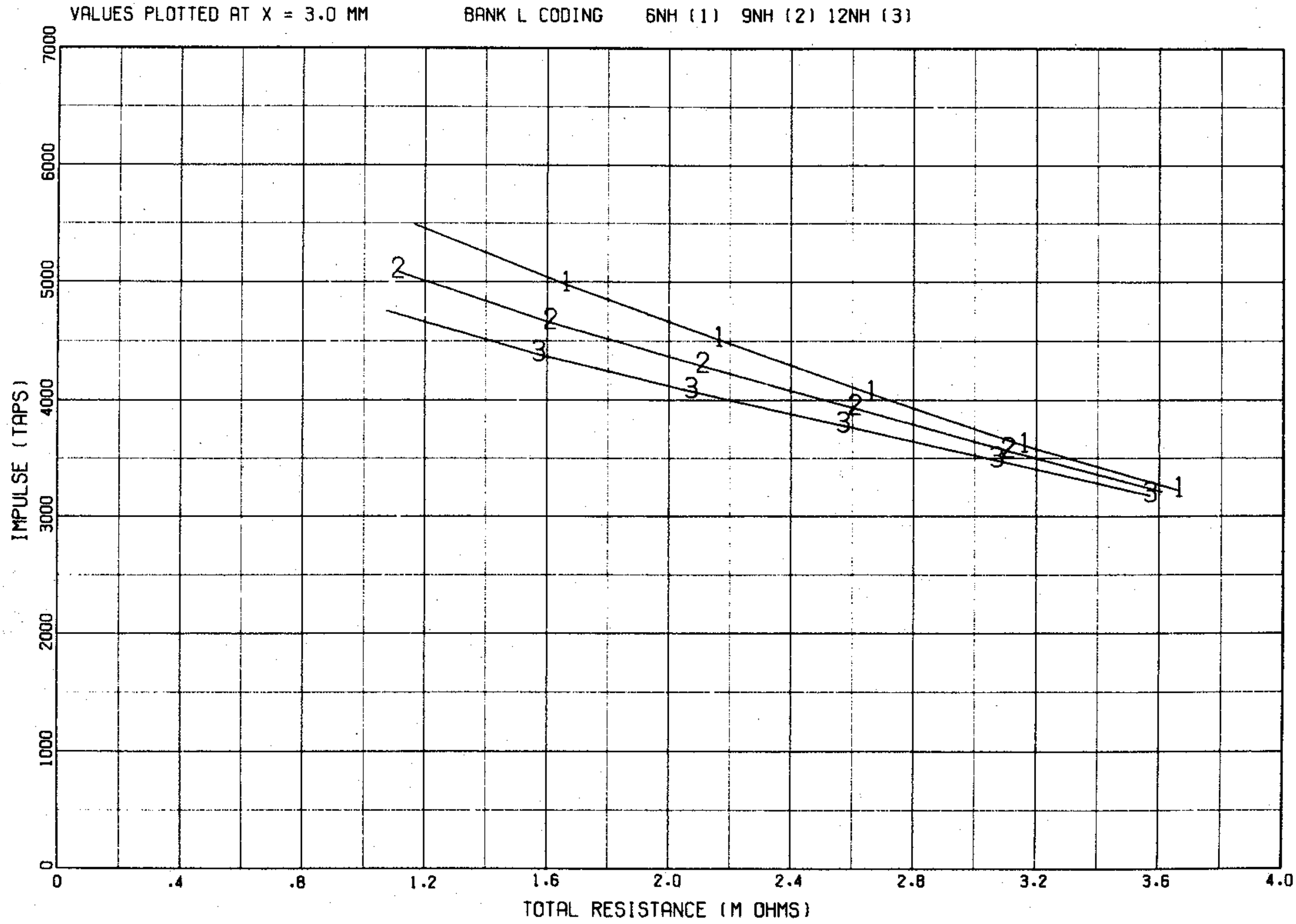

Figure A10. Impulse of Aluminum Flyer (10.5 x 24 inch) at a Displacement of 3 mm. (1) L_B = 6 nh, (2) L_B = 9 nh, and (3) L_B = 12 nh

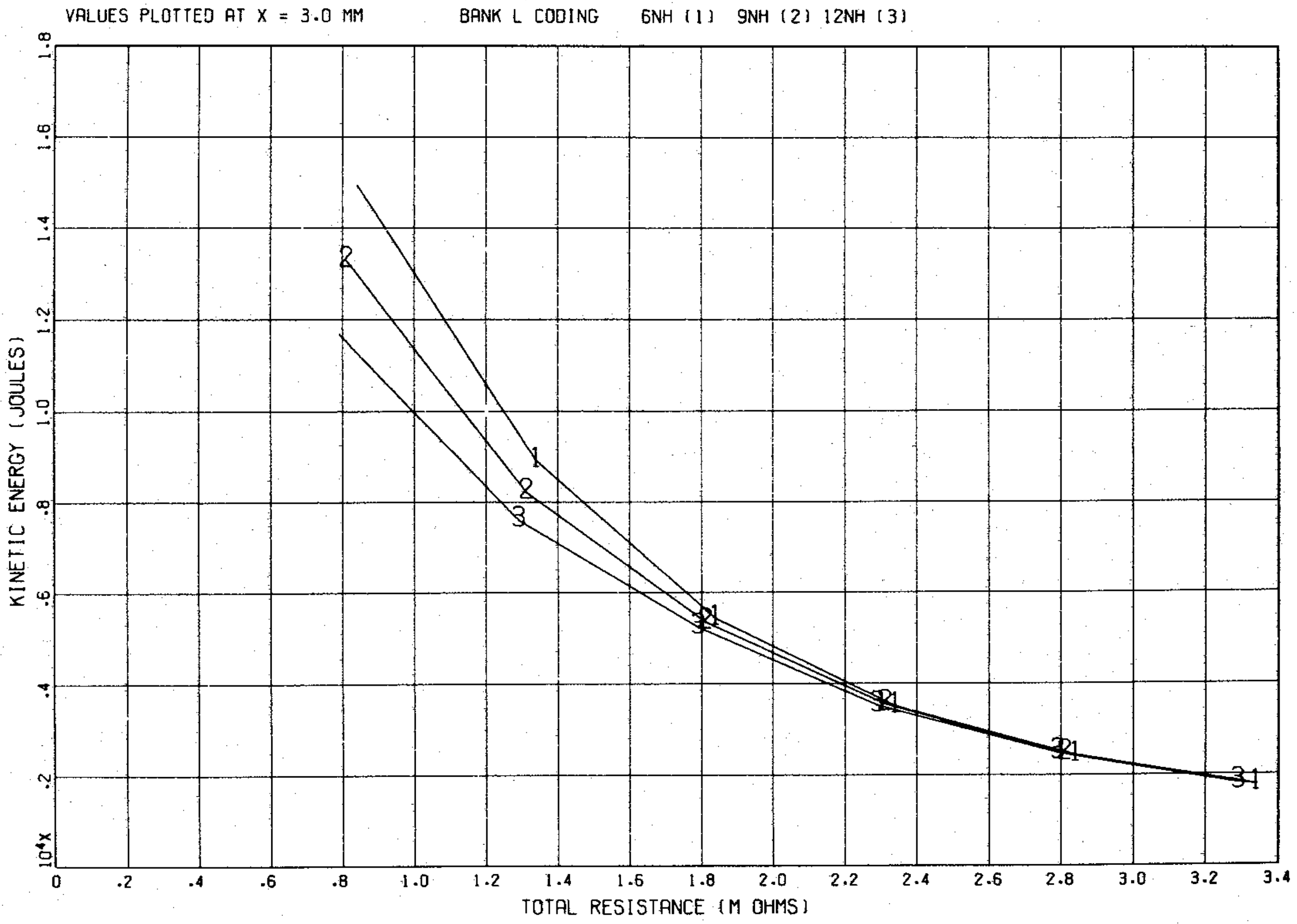

Figure A11. Kinetic Energy of Aluminum Flyer (21 x 24 inch) at a Displacement of 3 mm. (1) L_B = 6 nh, (2) L_B = 9 nh, and (3) L_B = 12 nh

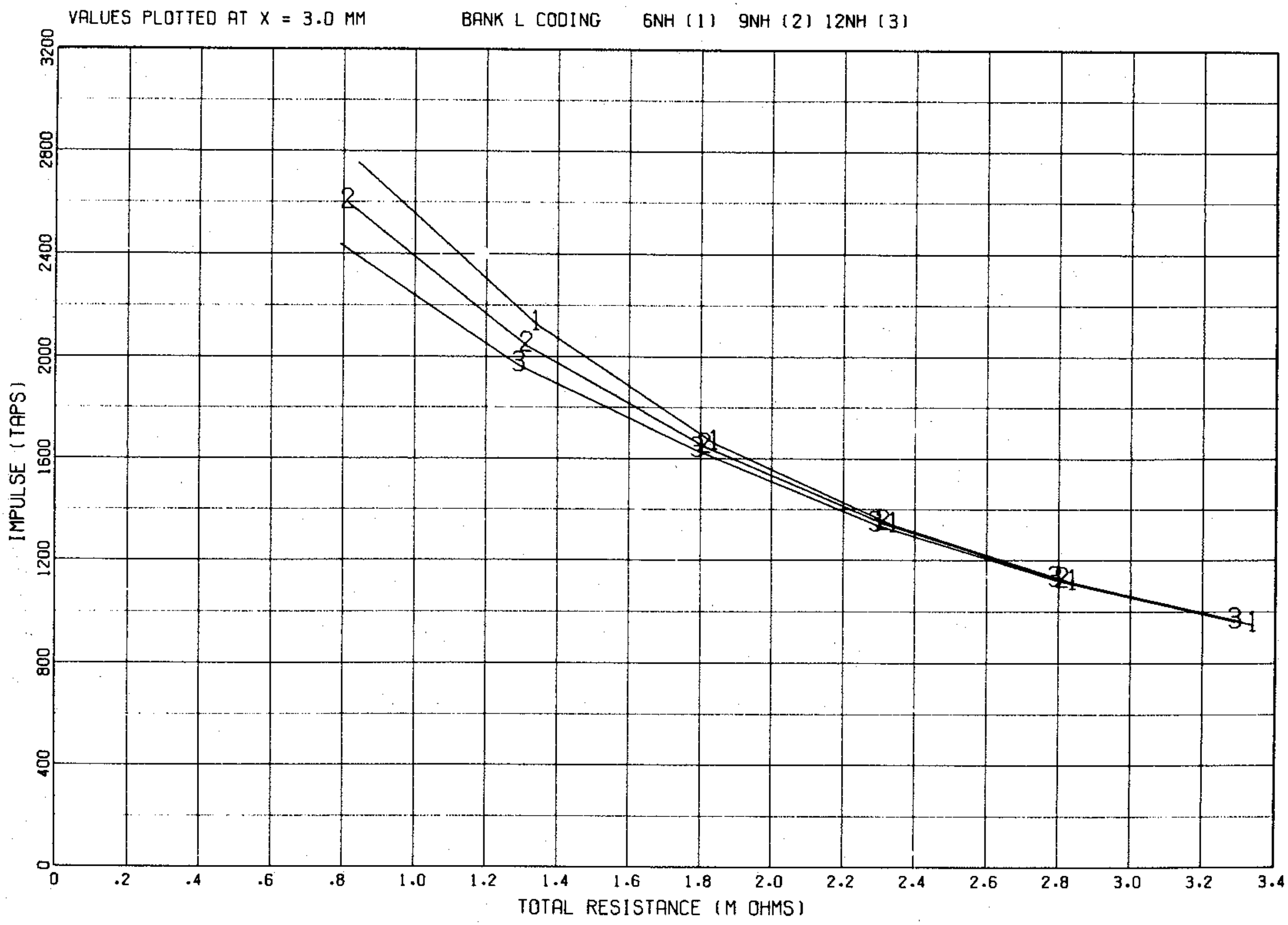

Figure A12. Impulse of Aluminum Flyer (21 x 24 inch) at a Displacement of 3 mm. (1) L_B = 6 nh, (2) L_B = 9 nh, and (3) L_B = 12 nh

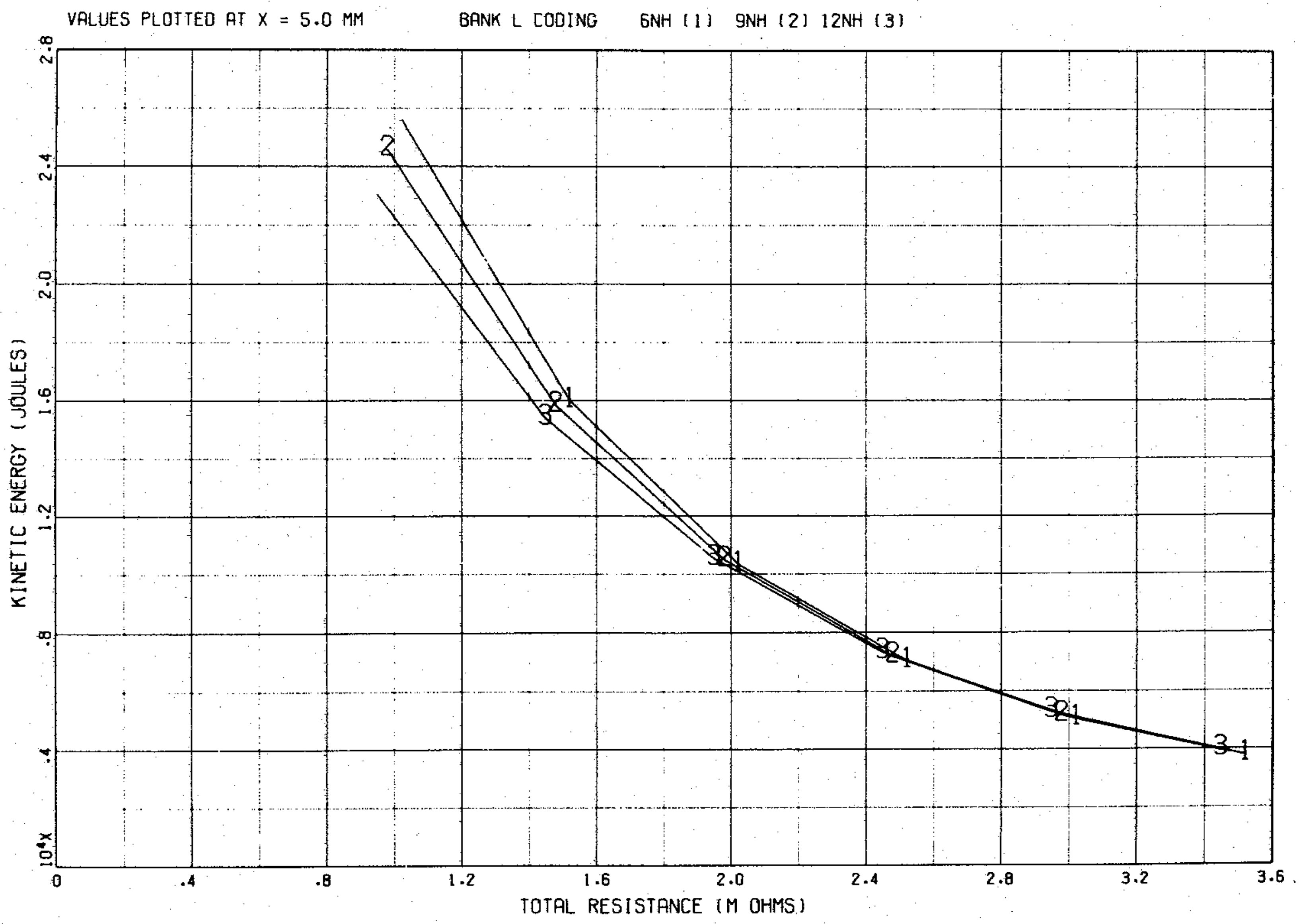

Figure A13. Kinetic Energy of Copper Flyer (10. 5 x 24 inch) at a Displacement of 5 mm. (1) L_B = 6 nh, (2) L_B = 9 nh, and (3) L_B = 12 nh

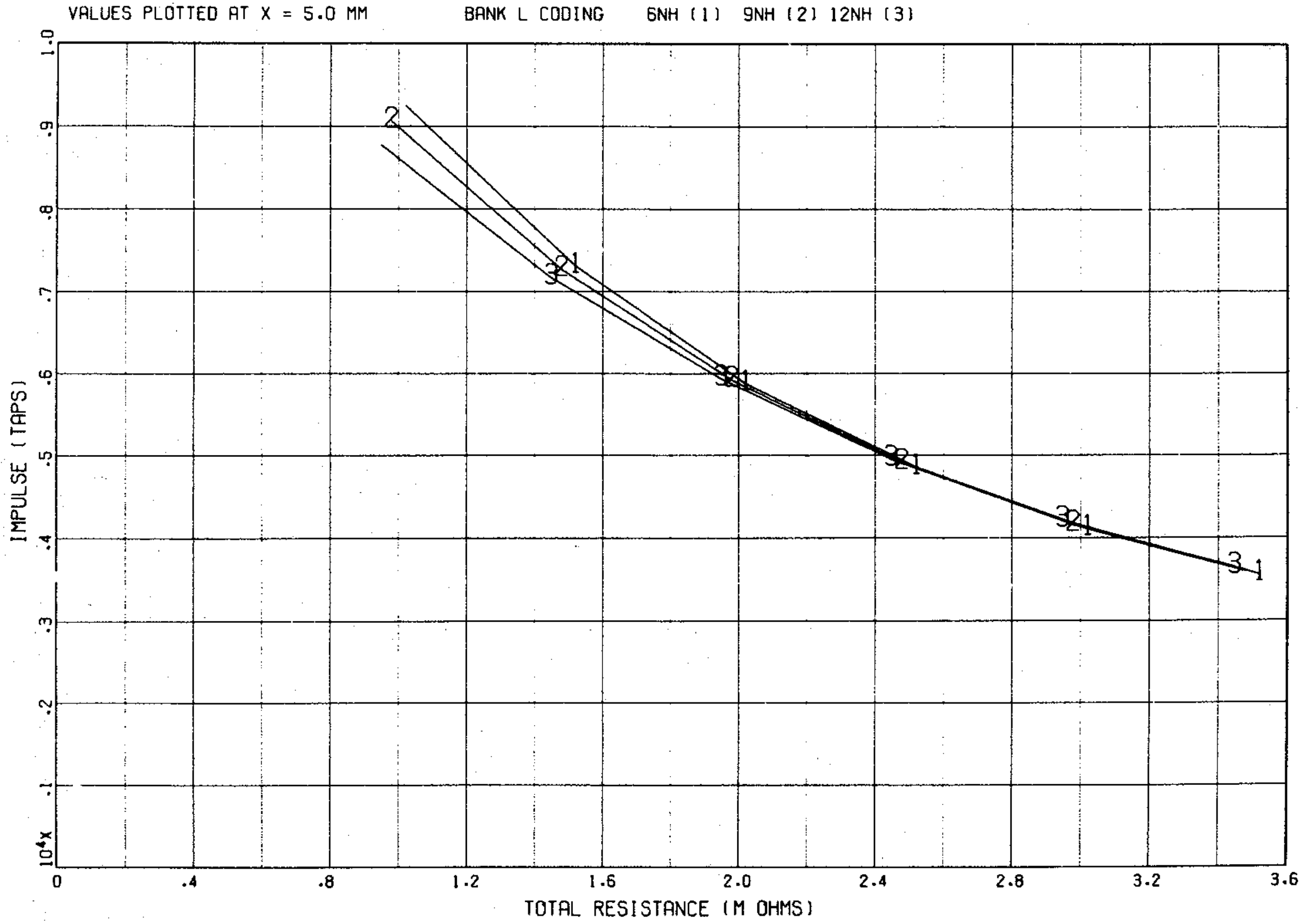

Figure A14. Impulse of Copper Flyer (10. 5 x 24 inch) at a Displacement of 5 mm. (1) L_B = 6 nh, (2) L_B = 9 nh, and (3) L_B - 12 nh

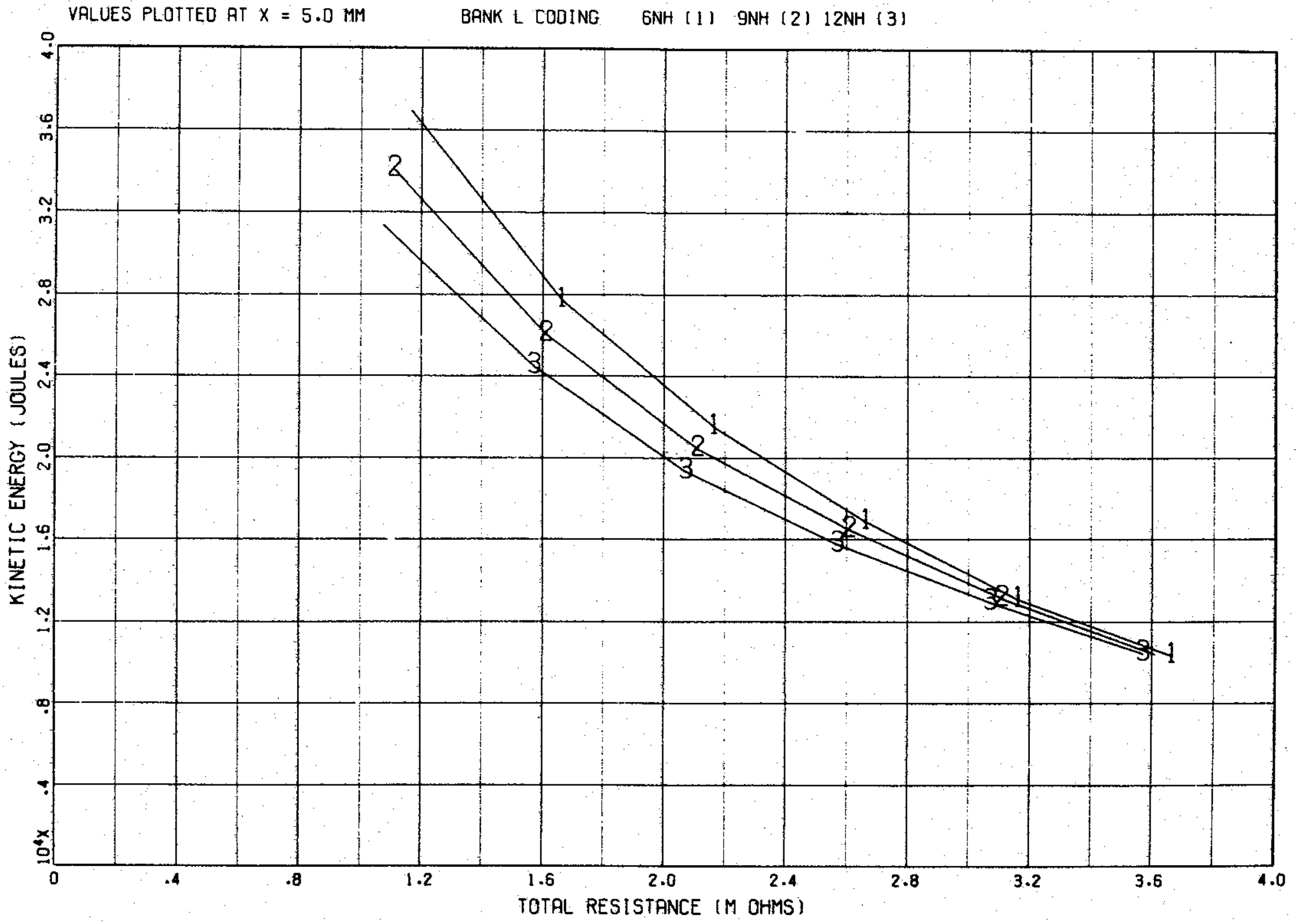

Figure A15. Kinetic Energy of Aluminum Flyer (10.5 x 24 inch) at a Displacement of 5 mm. (1) L_B = 6 nh, (2) L_B = 9 nh, and (3) L_B = 12 nh

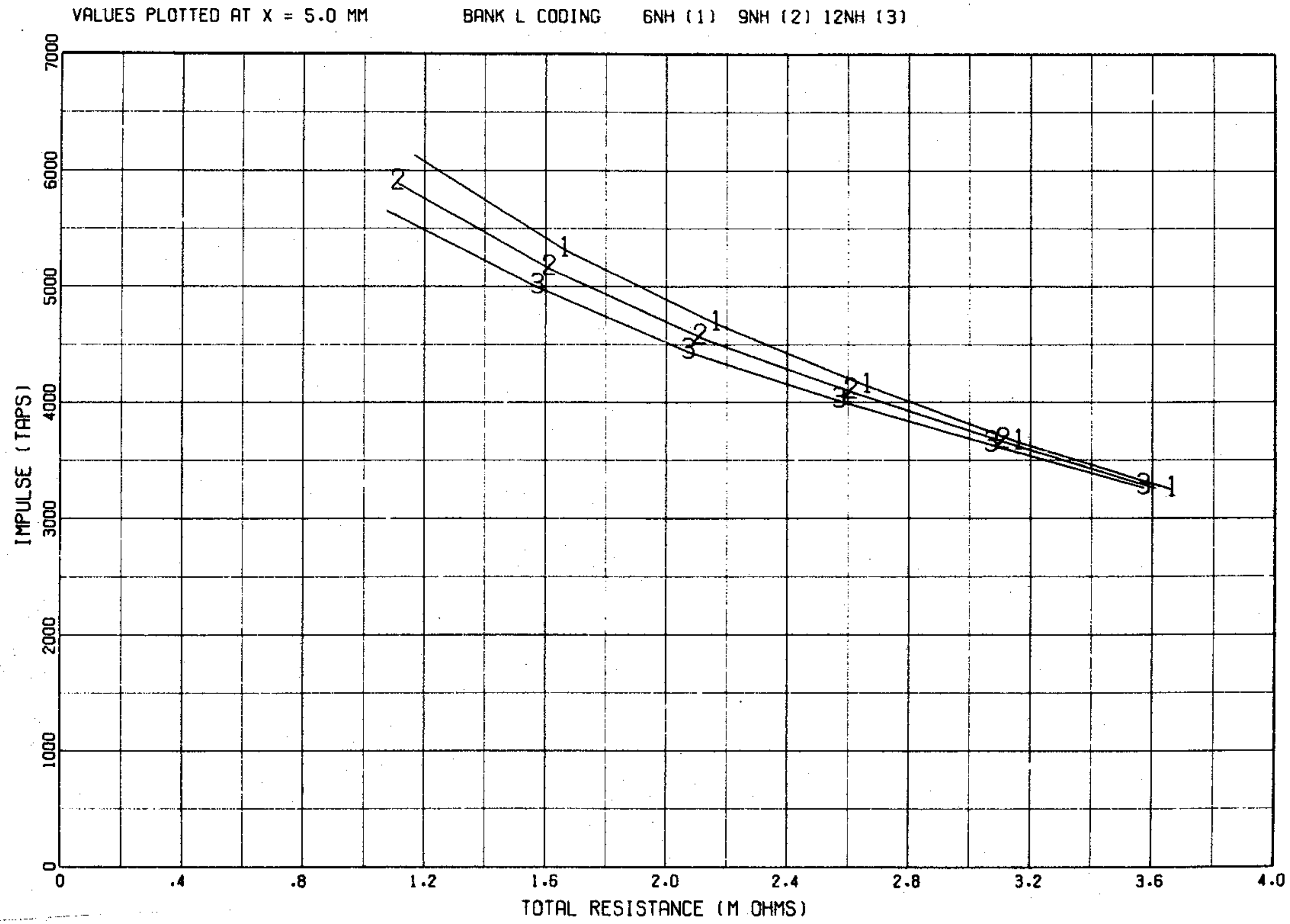

Figure A16. Impulse of Aluminum Flyer (10.5 x 24 inch) at a Displacement of 5 mm. (1) L_B = 6 nh, (2) L_B = 9 nh, and (3) L_B = 12 nh

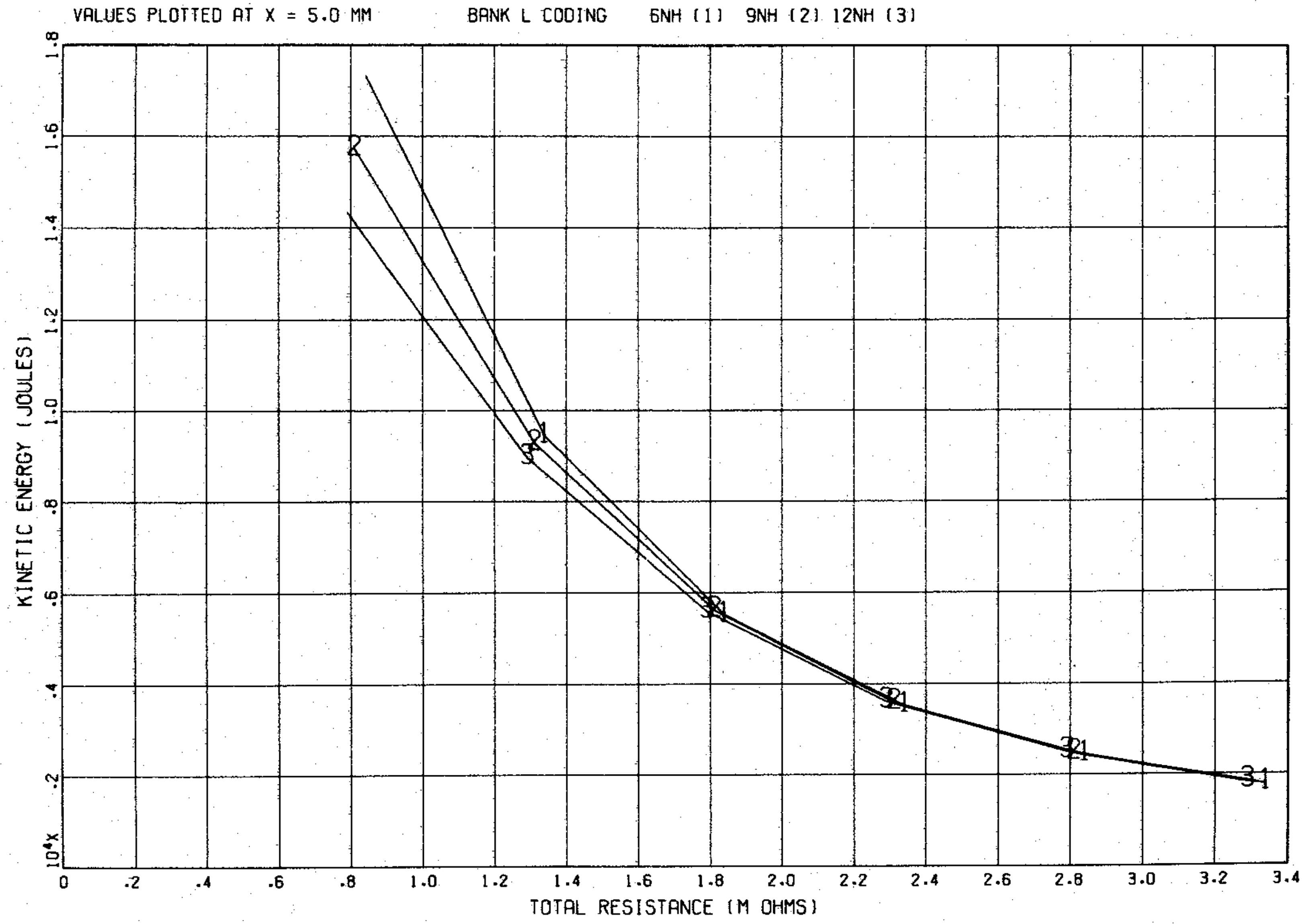

Figure A17. Kinetic Energy of Aluminum Flyer (21 x 24 inch) at a Displacement of 5 mm. (1) L_B = 6 nh, (2) L_B = 9 nh, and (3) L_B = 12 nh

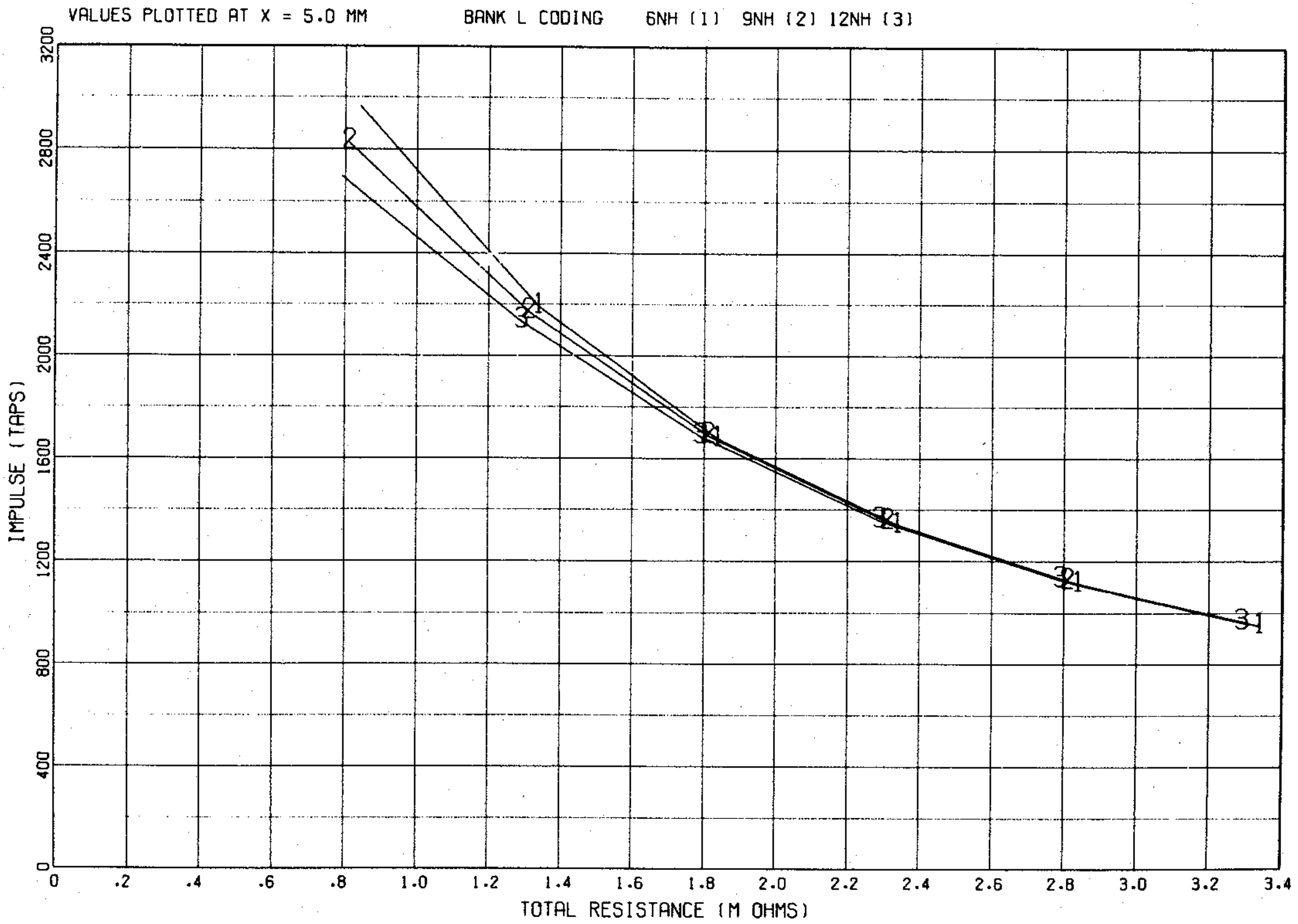

Figure A18. Impulse of Aluminum Flyer (21 x 24 inch) at a Displacement of 5 mm. (1) L_B = 6 nh, (2) L_B = 9 nh, and (3) L_B = 12 nh

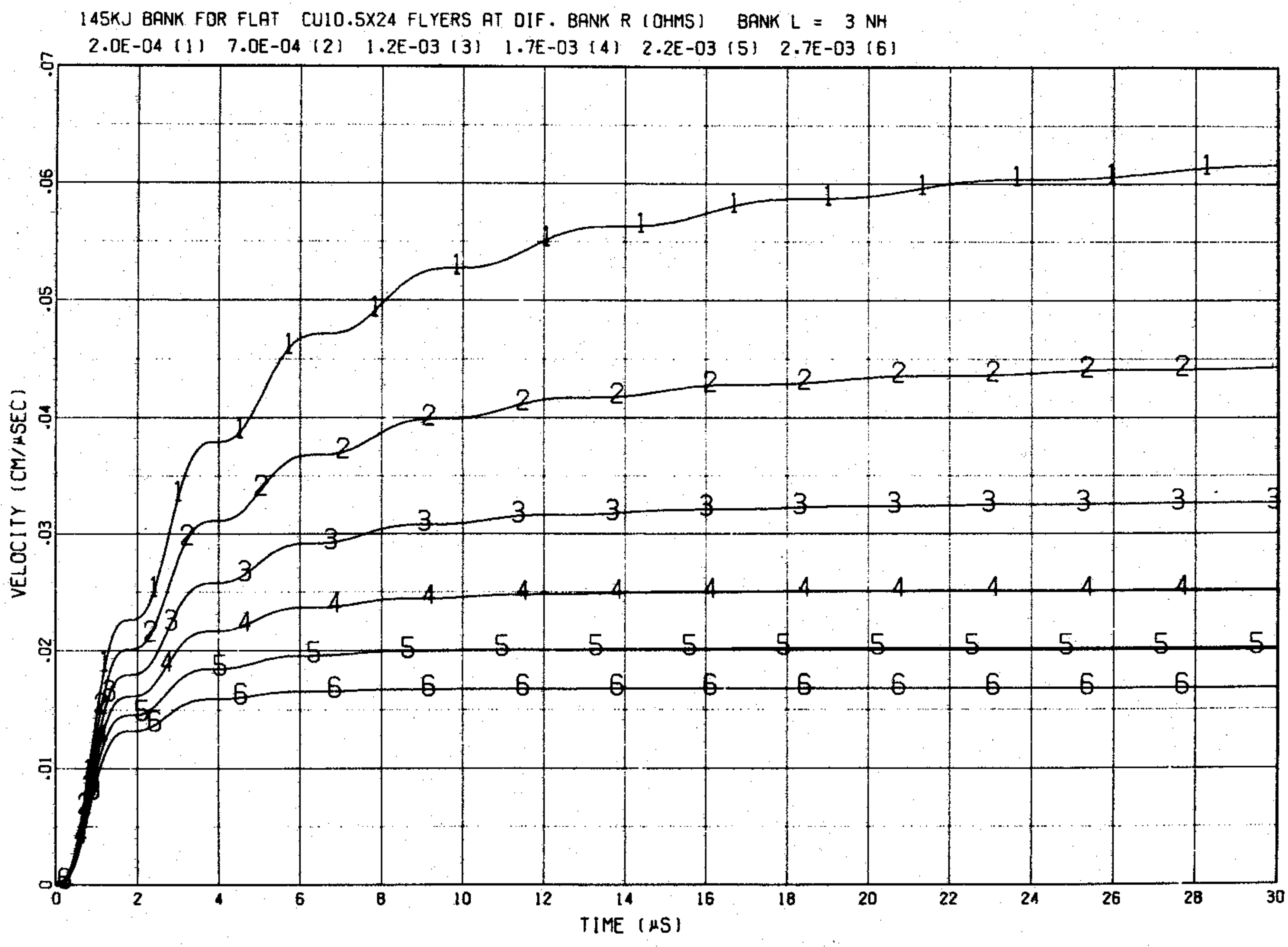

Figure A19. Velocity of Copper Flyer (10.5 x 24 inch). (1) R = 0.2 mΩ, (2) R = 0.7, (3) R = 1.2, (4) R = 1.7, (5) R = 2.2, and (6) R = 2.7. L_B = 3 nh

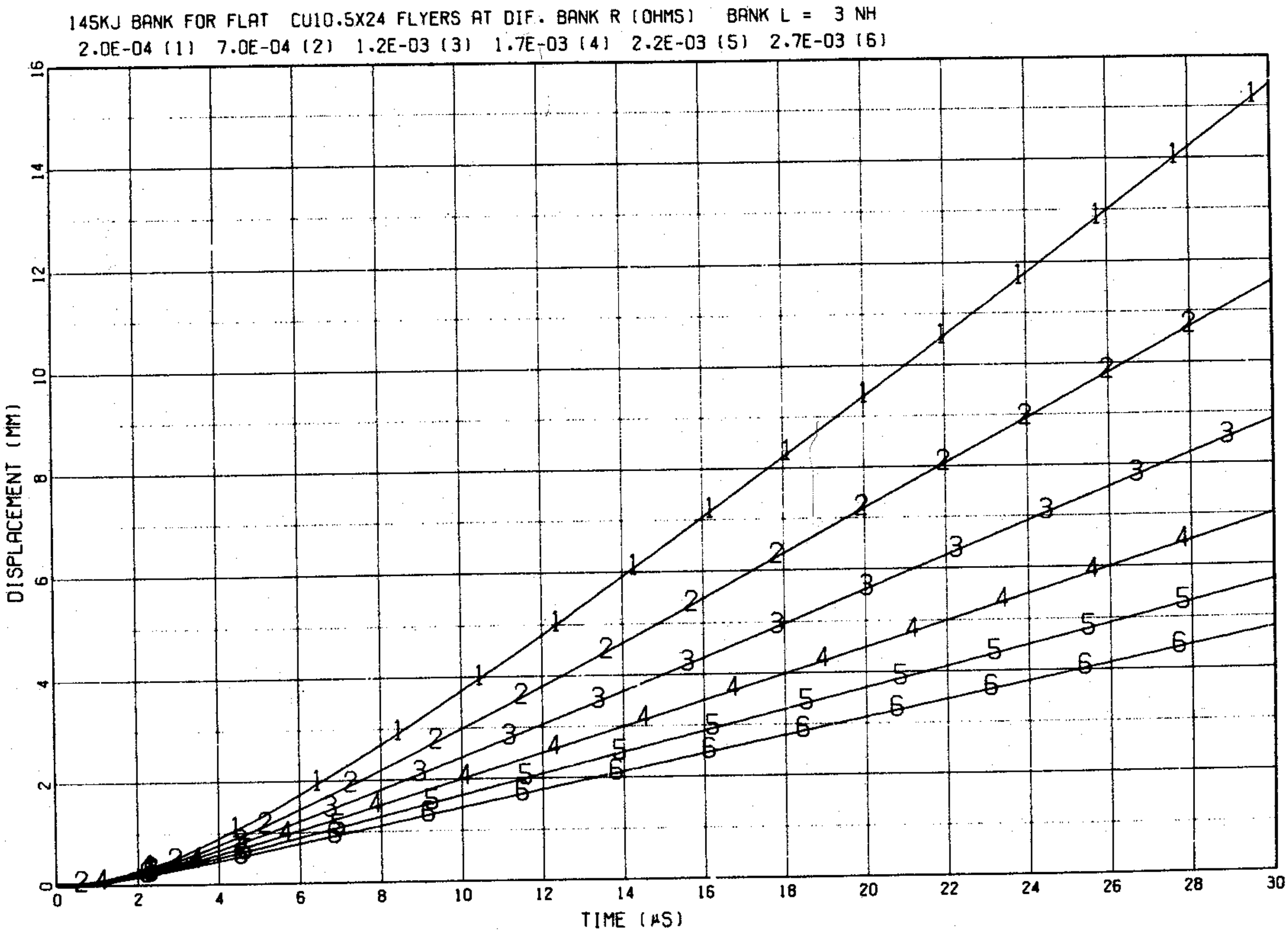

Figure A20. Displace of Copper Flyer (10.5 x 24 inch) (1) R = 0.2 mΩ, (2) R = 0.7, (3) R = 1.2, (4) R = 1.7, (5) R = 2.2, and (6) R = 2.7. L_B = 3 nh

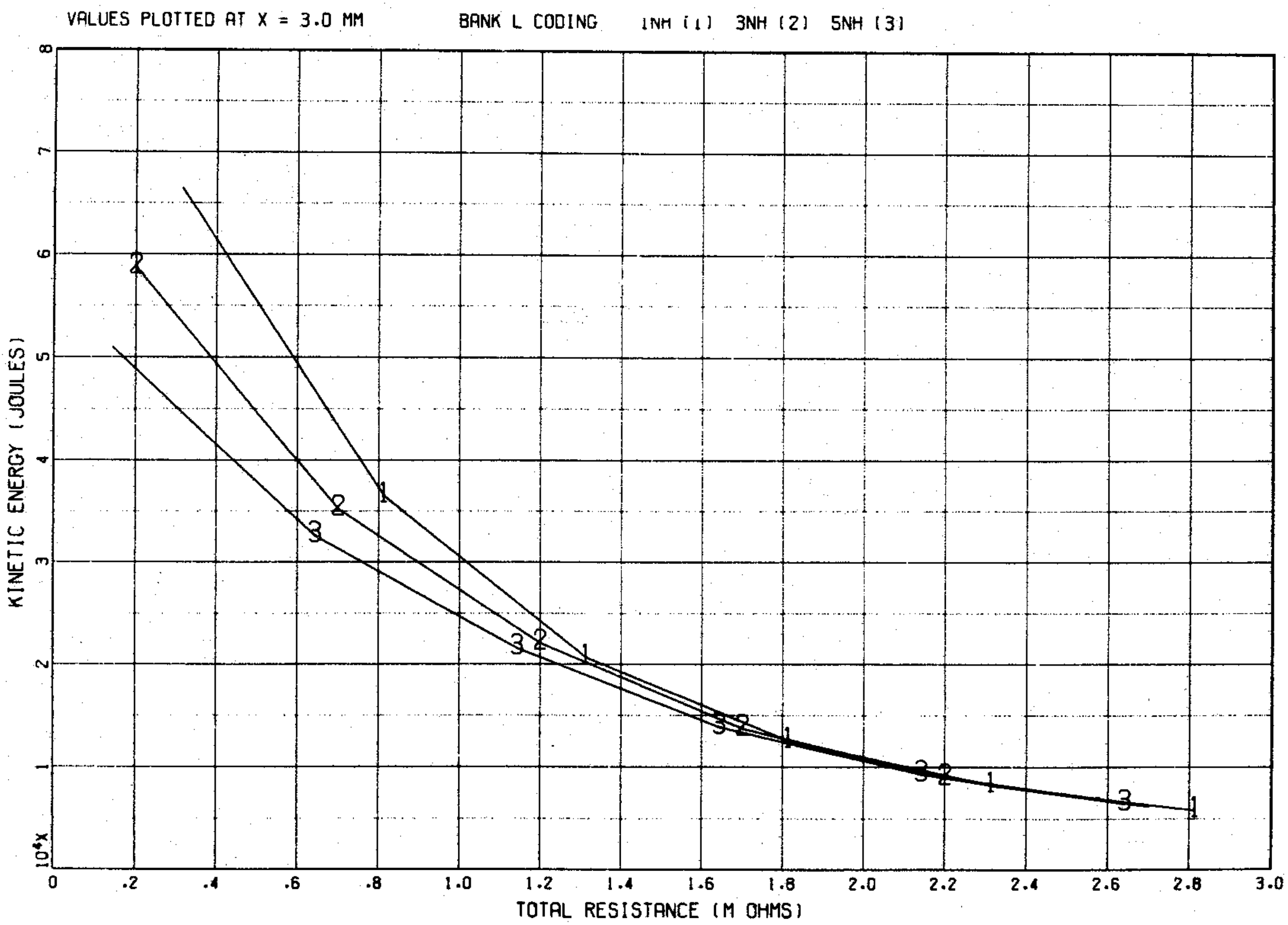

Figure A21. Kinetic Energy of Copper Flyer (10.5 x 24 inch) at a Displacement of 3 mm. (1) L_B = 1 nh, (2) L_B = 3 nh, and (3) L_B = 5 nh

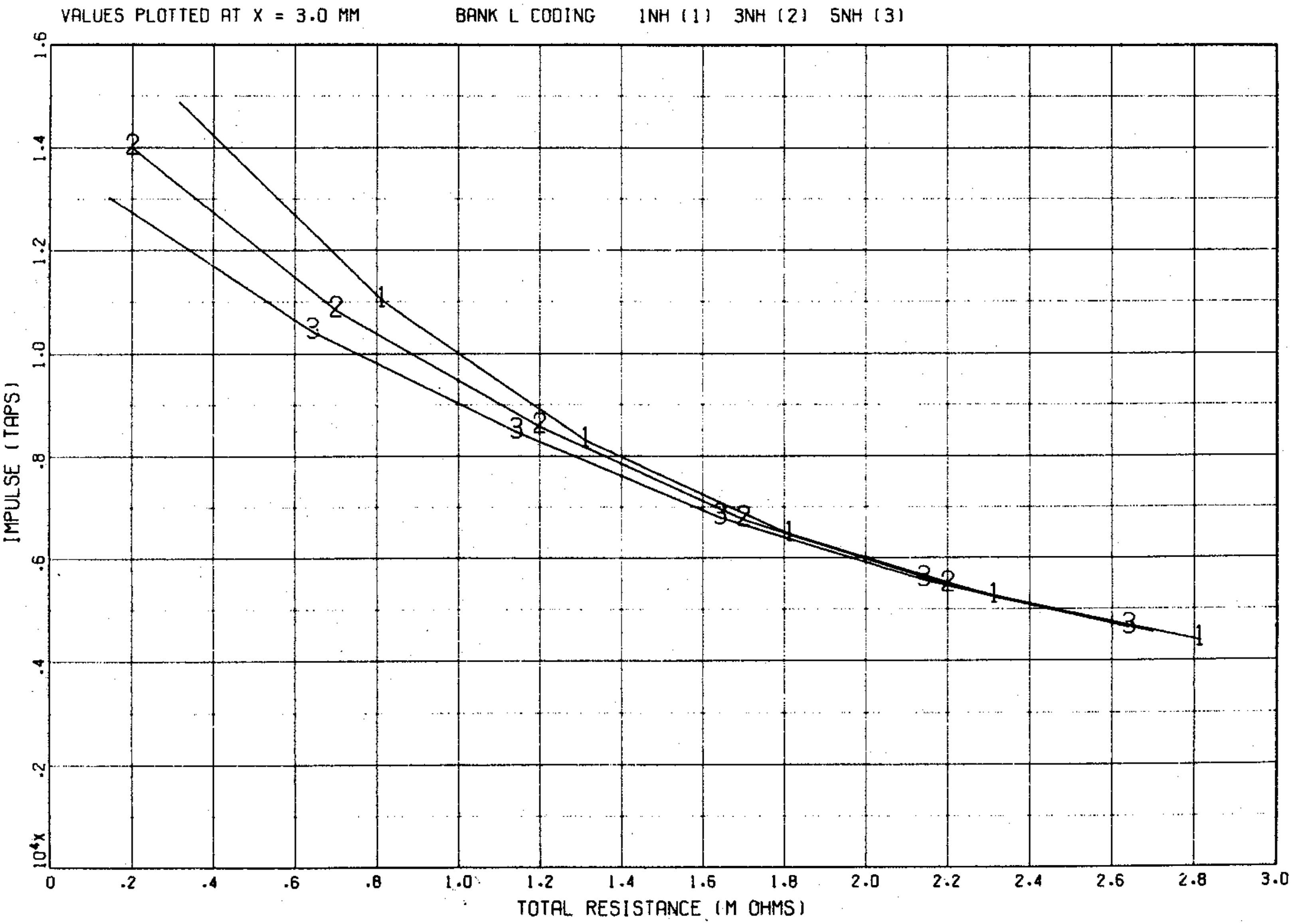

Figure A22. Impulse of Copper Flyer (10. 5 x 24 inch) at a Displacement of 3 mm. (1) L_B = 1 nh, (2) L_B = 3 nh, and (3) L_B = 5 nh

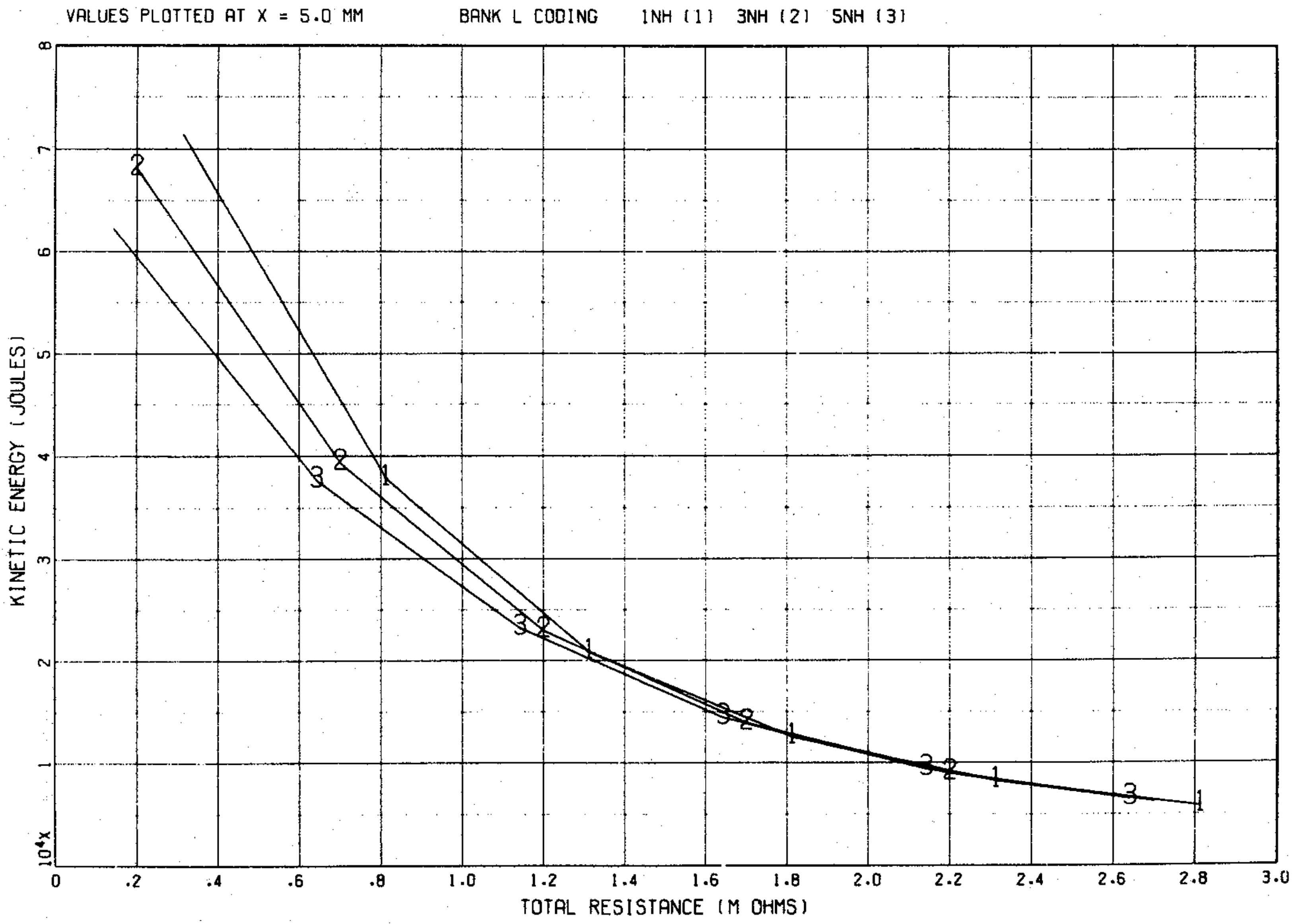

Figure A23. Kinetic Energy of Copper Flyer (10.5 x 24 inch) at a Displacement of 5 mm. (1) L_B = 1 nh, (2) L_B = 3 nh, and (3) L_B = 5 nh

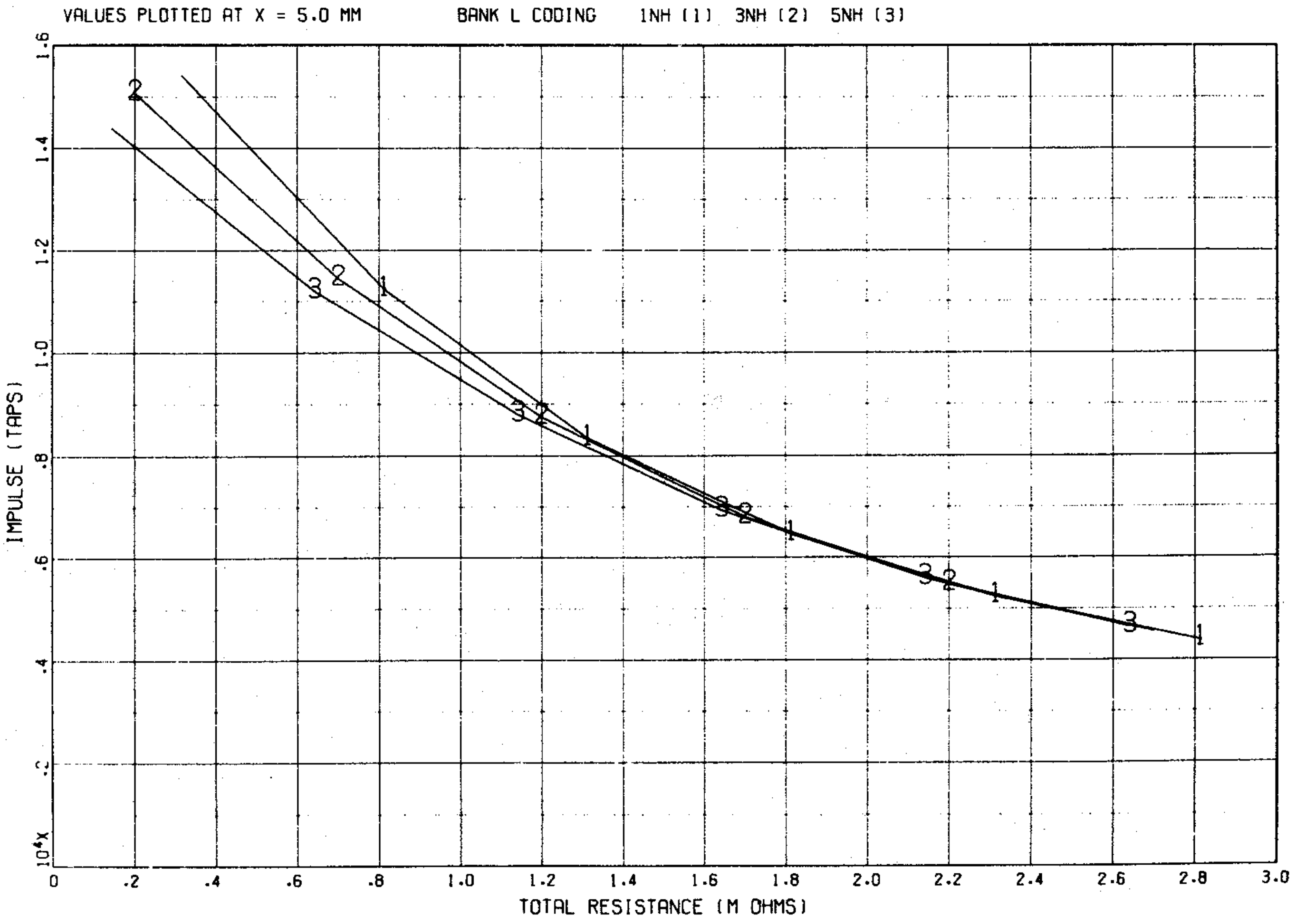

Figure A24. Impulse of Copper Flyer (10.5 x 24 inch) at a Displacement of 5 mm. (1) L_B = 1 nh, (2) L_B = 3 nh, and (3) L_B = 5 nh

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