High Voltage Notes
Note 1

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Electrostatic Grading Structures

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This note briefly summarises some data obtained in a couple of days' work with electrolytic tanks on the optimum profiles for field grading electrodes. Symmetrical cases only were considered and Figure 1 shows the nomenclature used for both the 1-D and 2-D cases. In each case the thickness of the slabs forming the two electrodes is chosen and the question is asked for a given separation of these, what the optimum profile is so that a minimum field enhancement exists around the edges. For the 2-D case a second question is what is the field enhancement in the centre of the electrodes. The electrolytic tank is a particularly powerful tool for this work, since even in the limited work noted here, some 20 configurations had to be optimised, each configuration requiring four or five attempts at the profile before a satisfactory one was obtained. Allowing for calibration, rechecks, etc., the actual time to determine these was about four hours.

The accuracy aimed at was only 10 per cent or so, since the finished electrodes were not intended to be of great accuracy. As such, no extrapolation techniques were used to obtain the field on the electrodes. However, in general the radius of curvature of the electrodes was considerably greater than ten times the half separation of the voltage probes used to measure the field. A second and probably more serious error is in determining the field at the centre of the disc electrodes with the optimum profile. These measurements had to be made in a wedge electrolytic tank and the central field corresponds with the region where surface tension has important effects. However, the scale of the electrodes was such that this field varied only slowly in this region and it is felt that the curves that give this data are probably within 20 per cent of reality, and these are included to give some measure of this parameter. The more important parameter, the field enhancement factor (FEF), is defined as maximum field around the electrode edge divided by the mean field between the electrodes, i.e.

$$\text{FEF} = \frac{E_{\text{max}}}{(V/h)}.$$ 

This factor of course does not involve the field at the centre of the disc and also is the one which is by far the most useful practically.

Apart from the approximate nature of the profile to be expected in large, cheap electrodes, the electrostatic solution to the problem is only the first step in obtaining this profile when pulse voltage breakdown is considered. Some of the additional factors which have to be included when approaching the true optimum profile and field enhancement factor are

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(a) the area term;

(b) any streamer transit time, which means the shorter parts will break first;

(c) any radius of curvature effects - this effect in air can mean that at the sharp upper edge, up to 30 per cent more or so field can be tolerated in reasonably small electrodes;

(d) roughness and the statistical probability of distribution of sites of initiating electrons for gases.

However, most of these effects tend to mean that the electrostatic first order profile is on the safe side and usually it is not seriously affected by any of them. However, to an accuracy of better than 10 per cent they should be considered, hence there is little point in refining the first order solution beyond this point for pulse work.

I am aware that this work has almost certainly been done much better and published but the search to find it would have taken more time than the experimental approach, also the sensitivity of the FEF to incorrect profiles was not felt likely to be covered.

The curves enable the FEF and profile to be obtained for the symmetrical case and of course for a single electrode over a ground plane. In the case, which is of some interest, of a generator sitting some way above an earth plane with its bottom earthed, the top and bottom electrodes should have different thicknesses, the top one of course being the thicker. If the equipotential which is reasonably flat in the central region can be found, with an electrolytic tank or by field sketching, the problem can then be split into two parts and an adequate solution then obtained.

Figure 2 gives the FEF for the optimum profiles. The 1-D case is covered by the line $Y/X = \alpha$. It is clear that even for this case a uniform field gap cannot be made with $H/X > 3$.

Figure 3 gives the factor $\text{Edge/Edcne}$ for different values of $H/X$ and $Y/X$.

Figure 4 plots the FEF factor at the centre of the disc for different $H/X$ and $Y/X$. These are the curves which should be treated with rather more caution than the previous ones. As an example of the gain to be had from using these profiles rather than, say, large spheres, a case is chosen where the FEF factor of 1.5 is taken as a required input. For a ratio $Y/X$ of 2.5, this is achieved with $H/X$ of 9. Hence
for unit spacing \((H = 1)\) \(X = 0.11\) and the diameter of the
disc = \(2Y = 0.55\). The same PEF for equal spheres with unit
spacing requires a diameter of 1.4. Thus compared with spheri-
cal electrodes, the optimum electrodes in this case are only
0.4 the diameter and 0.08 the thickness.

When the various profiles were compared, it was found that
over the range of parameters investigated these changed little
and to a first approximation depended mainly on the factor
\(H/Y\). Figure 5 shows a couple of profiles for \(H/Y < 0.5\) and
\(H/Y > 1.5\). In a couple of typical cases the sensitivity of the
PEF to the profile was approximated to by substituting the
wrong one. The average increase of the PEF given in Figure 2
was 7 per cent and hence over most of the range covered either
profile could be used within the stated accuracy. However,
certain sorts of departure from the profile are quite important,
in particular bumps with fairly large derivatives of the slope:
thus it is moderately important to keep the profile smooth. In
making quite large electrodes with a file, the correct profile
can be obtained without checking after only a little experience
and a bumpiness which is acceptable is easily obtained. This
judgment of bumpiness is difficult to quantify but was of
course directly experienced in optimising the profiles in the
electrolytic tank.

With regard to making the electrodes cheaply and quickly,
the following approaches have been found to be useful by the
members of the SSWA pulse group at AWRE.

For oil systems, plywood worked over with a file and cov-
ered in copper foil stuck down with an impact adhesive was
found to be useful. Corners (the 2-D case) were formed by cut-
ting the few thou. (mil.) copper and soldering the joints down.

For water, balsa wood can be quickly worked and covered
with thin lead sheet which can be gently hammered and worked
onto the shaped wood backing. No serious deleterious effect on
the water resistivity was observed.

For electrodes for use in air and other gases, polyure-
thane foam of density between 2 and 6 lbs per cubic foot can be
quickly worked and is nice and light. This is then covered
with about 2 thou. aluminium foil. Double-sided sticky tape is
then stuck to the flat foil before it is formed over the con-
tour. The creases and folds are then pushed into the foam by
smoothing with a wooden stick (the handle of a hammer of modest
size works fine). The finished metal surface is quite smooth
and such defects as there are in it are pushed inwards and
small smooth dents are not important. A large electrode of di-
mensions 3 feet by 2 feet by 4 inches can be made in about two
and a half hours.
All the approaches mentioned above are largely self-healing, in the sense that a discharge between the electrodes pushes the metal into the underlying material. In general this leaves a smooth contoured dent which does not lead to subsequent discharges at the same site.

In concluding it should be re-emphasised that the data produced is not of great accuracy, but is adequate for the design of cheap, large field shapers. In particular it is warned that the data in Figure 4 is less accurate than the rest. While the data it gives can be derived from Figures 2 and 3, the smoothing was done independently for the three graphs and is reflected in differences of two or three per cent between the internally related graphs. As such, this is a technological user note rather than one in the purest traditions of Science, something for which I make absolutely no apologies.
F. E. F. = \frac{E_{\text{EDGE}}}{V_{H}}

1D CASE

2D CASE

FIGURE 1
RATIO OF EDGE FIELD TO CENTRAL FIELD

\[ \frac{E_{\text{edge}}}{E_{\text{centre}}} \]

\( \frac{H}{X} = 2.5 \)
\( \frac{H}{X} = 5 \)
\( \frac{H}{X} = 7.5 \)
\( \frac{H}{X} = \infty \)

FIGURE 3
CENTRAL FIELD ENHANCEMENT FACTOR

$Y_X = 2.5$
$Y_X = 5$
$Y_X = 7.5$

$E_{centre}$
$V/H$

$H_{/X}$

FIGURE 4

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