EMP Interaction Notes

Note I

Electric Field - Induced Cable Currents

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Abstract:

The solution for the current induced in a long, straight conductor imbedded in a lossy dielectric by a parallel CW electric field is found. A suitable model for the conductor-dielectric system is described, and the use of the model in solving the problem for more complicated geometries is discussed.

I. Introduction.

The EMP problem divides naturally into two areas of concern. First, one must understand the character of the EMP in a simple and somewhat idealized environment. Second, one must understand the coupling of the EMP into various types of systems. This second problem is by far the more difficult, and often understanding of the coupling can be had only through an empirical approach. However, certain simple but significant cases of the coupling problem can be treated analytically. This paper will treat one of these cases.*

Facilities for power distribution and communications generally contain very long conductors near to or buried in the ground. It is possible that the EMP may induce large surge currents in these conductors through either a magnetic field interaction or an electric field interaction. This paper will consider only the electric field interaction, although several of the observations made are applicable to the magnetic interactions as well.

II. Impedance of the Conductor.

Initially, consider a long, circular, cylindrical conductor imbedded in an infinite homogeneous lossy dielectric. The homogeneous form of Maxwell's equations can be solved explicitly in the frequency domain for such a system.** Outside of the conductor, the fields are

$$E_z = aH_0^{(1)}(\lambda_2 r)\,F \qquad (1)$$

$$E_r = -a\frac{ih}{\lambda_2}H_1^{(1)}(\lambda_2 r)\,F \qquad (2)$$

* Extensive work in this field has been done by E. D. Sunde in his book, Earth Conduction Effects in Transmission Systems, D. Van Nostrand & Co., Ltd, New York (1949)

** See Stratton, Electromagnetic Theory, pp. 346f and pp. 524f.

$$H_\theta = -a\frac{ik_2^2}{\mu_2\omega\lambda_2} H_1^{(1)}(\lambda_2 r) F \tag{3}$$

where

$H_n^{(1)}$ is the Hankel function, $J_n + iN_n$

a is an arbitrary complex constant for the external fields

k_1, the propagation constant in the conductor, =

$$\sqrt{\mu_1\varepsilon_1\omega^2 + i\mu_1\sigma_1\omega} \tag{4}$$

k_2, the propagation constant in the dielectric, =

$$\sqrt{\mu_2\varepsilon_2\omega^2 + i\mu_2\sigma_2\omega} \tag{5}$$

h, the propagation constant for the system, $\approx k_2$

$$F = \exp(ihz - i\omega t) \tag{6}$$

$$\lambda_1^2 = k_1^2 - h^2 \tag{7}$$

$$\lambda_2^2 = k_2^2 - h^2 \text{ where } \lambda_2 \text{ is defined by the equation} \tag{8}$$

$(\lambda_2 b)^2 \ln\frac{\gamma\lambda_2 b}{2i} = i\frac{\mu_1}{\mu_2}\frac{k_2^2 b}{k_1}$, where b is the radius of the conductor and

$\gamma \approx 1.781$

The current in the wire is given by

$$I_z = a\,\frac{2\pi b\sigma_1\mu_1 k_2^2\, H_1^{(1)}(\lambda_2 b)}{\lambda_2\mu_2 k_1^2} F \tag{9}$$

or, solving for a,

$$a = \frac{\lambda_2}{2\pi b\sigma_1}\,\frac{\mu_2 k_1^2}{\mu_1 k_2^2}\,\frac{I_o}{H_1^{(1)}(\lambda_2 b)} \tag{10}$$

where I_o is the magnitude of I_z.

There are modes that are not symmetric in θ which satisfy the boundary conditions, but these modes are heavily damped and need not be considered. There are also possible transverse electric modes, but these, too, are heavily damped.

The radial current per unit length flowing out of the wire at z_o is

$$I_r(z_o,b) = - \left.\frac{dI_z}{dz}\right|_{z_o} = -ihI_z(z_o,b) \text{ amp/m} \tag{11}$$

We now define the impedances:

For the transverse impedance, Z_T,

$$Z_T = \frac{V_r}{I_r} \quad \text{ohm m} \tag{12}$$

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$$V_r = \int_b^\infty E_r dr \tag{13}$$

Solving for a in terms of I_z from (7) and using this value in (2), we obtain

$$E_r = -iI_z \frac{\mu_2 k_1^2 h}{\mu_1 k_2^2 \cdot (2\pi b\sigma_1)} \frac{H_1^{(1)}(\lambda_2 r)}{H_1^{(1)}(\lambda_2 b)} \tag{14}$$

Noting that

$$\int_b^\infty \frac{H_1^{(1)}(\lambda_2 r)}{H_1^{(1)}(\lambda_2 b)} dr = + \left[\frac{H_o^{(1)}(\lambda_2 r)}{\lambda_2 H_1^{(1)}(\lambda_2 b)}\right]_\infty^b = \frac{H_o^{(1)}(\lambda_2 b)}{\lambda_2 H_1^{(1)}(\lambda_2 b)} \simeq -b \cdot \ln\left(\frac{\gamma\lambda_2 b}{2i}\right) \tag{15}$$

(the last approximation requires that the cylinder be much less than a wave-length in the dielectric in radius), we obtain

$$Z_T = -\frac{\mu_2 k_1^2}{2\pi\mu_1\sigma_1 k_2^2} \ln \frac{\gamma\lambda_2 b}{2i} \tag{16}$$

for a conductor

$$k_1^2 = i\omega\mu_1\sigma_1 \tag{17}$$

this reduces to

$$Z_T = \frac{-1}{2\pi(\sigma_2 - i\omega\varepsilon_2)} \ln \frac{\gamma\lambda_2 b}{2i} \quad \text{ohm meters} \tag{18}$$

Next, define the impedance Z_s by

$$Z_s = \frac{E_z}{I_z}\bigg|_{r=b} = + \frac{\lambda_2}{2\pi b\sigma_1} \frac{\mu_2 k_1^2}{\mu_1 k_2^2} \frac{H_o^{(1)}(\lambda_2 b)}{H_1^{(1)}(\lambda_2 b)} \qquad (19)$$

With the approximation (15), and noting that $(\lambda_2 b)^2 \ln \frac{\gamma\lambda_2 b}{2i} = i \frac{\mu_1}{\mu_2} \frac{k_2^2 b}{k_1}$ * (20)

$$Z_s = \frac{(1-j)}{\sqrt{2}} \cdot \frac{1}{2 b} \cdot \sqrt{\frac{\omega\mu_1}{\sigma_1}} \text{ ohms/meter} \qquad (21)$$

which is just the skin impedance of the conductor when the skin depth in the wire is less than the wire radius. Finally, we shall consider the "inductance" per unit length L through the relation:

$LI_z = \phi$, the flux per unit length

$$\phi = \int_b^\infty B_\theta \cdot dr \qquad (22)$$

and, with an analysis similar to that used in finding Z_T, we find that

$$L = -\frac{\mu_2}{2\pi} \ln \frac{\gamma\lambda_2 b}{2i} \qquad (23)$$

and therefore

$$Z_L = \frac{i\omega\mu_2}{2\pi} \ln \frac{\gamma\lambda_2 b}{2i} \qquad (24)$$

III. The Effect of a Driving Electric Field.

Now consider the case of the conductor current being driven by a uniform electric field in the z direction, E_{zo}. Since the wire is a conductor, the electric field at its surface will be much smaller than the value of E_z at large distances from the conductor. Therefore, the current in the wire must generate an electric field which nearly cancels E_{zo} at the surface of the wire. The resultant field at the surface should be the field generated by I_z flowing through the skin impedance. To find the effect of E_{zo}, start with the equation

$$\oint E \cdot dl = -\int \dot{B} \cdot dA, \qquad (25)$$

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* Stratton, pp. 528

and integrate around the path and over the area shown in figure 1.

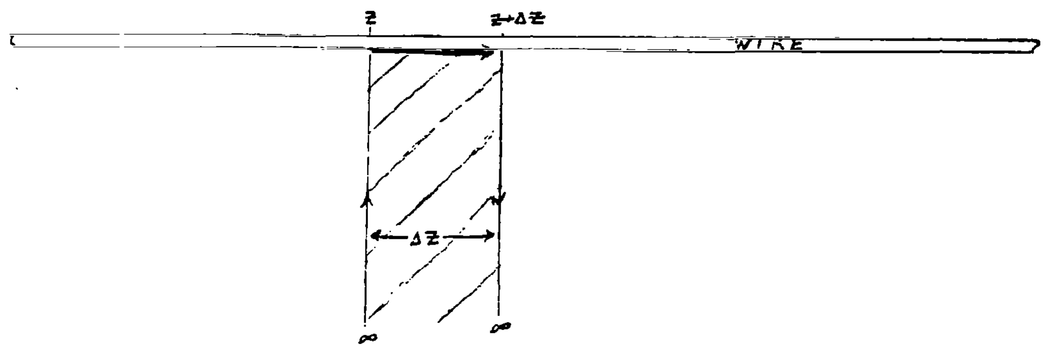


Figure 1

$$\int_{\infty}^{b} E_r(z)\cdot dr + \int_{z}^{z+\Delta z} E_z \cdot dz + \int_{b}^{\infty} E_r\,(z+\Delta z)\, dr = -\int \dot{B}\cdot dA \qquad (26)$$

If we take the limit of this equation divided by Δz as $\Delta z \to 0$, we have

$$\frac{d}{dz}\int_{b}^{\infty} E_r \cdot dr + E_z = -\frac{d}{dt}\int_{b}^{\infty} B \cdot dr \qquad (27)$$

By (11), (12), and (13)

$$\frac{d}{dz}\int_{b}^{\infty} E_r \cdot dr = -Z_T \frac{d^2 I_z}{d_z^2} \qquad (28)$$

By (22), and (24),

$$\frac{-d}{dt}\int_{b}^{\infty} B \cdot dr = -Z_L I_z \qquad (29)$$

E_z is composed of two terms, the first by $Z_s I_z$, the skin impedance field, and the second by the canceling field $-E_{zo}$.

Therefore, equation (27) becomes

$$+ Z_T \frac{d^2 I_z}{dz^2} - (Z_L + Z_s)\, I_z = -E_{zo} \qquad (30)$$

(30) is just the inhomogeneous form of the Helmholtz equation which describes the transmission line shown in figure 2.

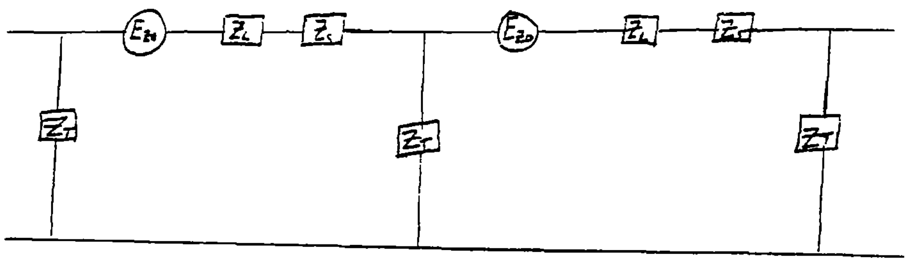


Figure 2. The Equivalent Dispersive Transmission Line

Indeed, the forms of equations (16), (21), and (24) are very suggestive of the coaxial transmission line.

IV. Transmission Line Model

Near the conductor imbedded in the loss dielectric, we would expect the electric field lines to have the shape shown in figure 3.

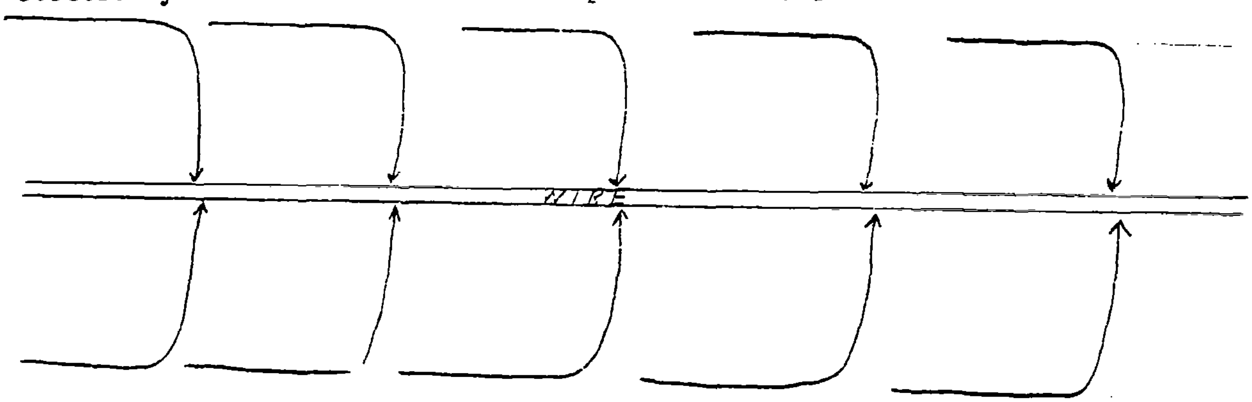


Figure 3.

The point at which the field tends toward the conductor occurs about a skin depth away from the conductor. At this distance, a cylinder drawn parallel to, and coaxial to, the conductor intersects the field lines at nearly right angles, so that such a surface is approximately an equipotential. Thus, if we were to place a conducting cylinder around the conductor at this radius, the field path between the cylinder and the conductor would not be severely distorted. We have now constructed a coaxial transmission line. The impedances of the line are easily calculable:

$$Z_L = \frac{i\omega\mu_2}{2\pi} \ln \frac{b}{b+\delta_2}$$

$$Z_T = \frac{-1}{2\pi(\sigma_2 - i\omega\varepsilon_2)} \ln \frac{b}{b+\delta_2}$$

$$Z_s \quad \frac{(1-i)}{\sqrt{2}} \; \frac{1}{2\pi b} \; \sqrt{\frac{\omega\mu_1}{\sigma_1}}$$

where δ_2 *is the skin depth in the lossy dielectric.*

As one might guess, the skin impedance from the exact analysis and the transmission line models are the same. Furthermore, Z_L *and* Z_T *differ only in the logarithmic factor. A comparison of the value of the appropriate and exact logarithmic terms is given in Tables I and II.*

Table I. Number 10 copper wire,

Wire Radius = 1.28×10^{-3} *m*

Wire Conductivity = 5.88×10^{7} *mho/m*

Wire Magnetic Permeability = $4\pi \times 10^{-7}$ *henry/meter*

Ground Conductivity = 2.9×10^{-2} *mho/m*

Ground Magnetic Permeability = $4\pi \times 10^{-7}$ *henry/meter*

Even at 1 mc, the displacement current is < 1% of the conduction current, $\therefore$ ε_2 *was set to zero.*

Frequency	$\ln \frac{b}{b+\delta_2}$	$\ln \frac{\gamma\lambda_2 b}{2i}$ Real	Imaginary
10^2 cps	-12.34	-12.75	-1.25
10^3 cps	-11.19	-12.15	-1.23
10^4 cps	-10.04	-11.55	-1.23
10^5 cps	- 8.89	-10.95	-1.23
10^6 cps	- 7.74	-10.34	-1.24

Table II. Lead Sheath Cable

Wire Radius = 2.07×10^{-2} m

Wire Conductivity = 4.45×10^{6} mho/m

Wire Magnetic Permeability = $4\pi \times 10^{-7}$ henry/m

Ground Conductivity = 2.9×10^{-2} mho/m

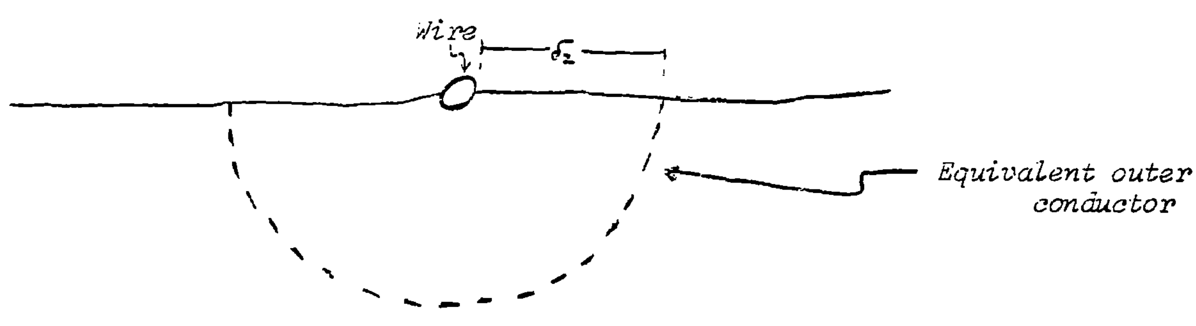
Ground Magnetic Permeability = $4\pi \times 10^{-7}$ henry/meter

$\epsilon_2 \cong 0.$

Frequency	$\ln\frac{b}{b+\delta_2}$	$\ln\frac{\gamma\lambda_2 b}{2i}$ Real	$\ln\frac{\gamma\lambda_2 b}{2i}$ Imaginary
10^2	-9.56	-10.63	-1.24
10^3	-8.41	-10.02	-1.24
10^4	-7.26	- 9.42	-1.24
10^5	-6.11	- 8.81	-1.25
10^6	-4.96	- 8.20	-1.25

In many cases of interest, the transmission line model gives quite good accuracy, as will be seen from reports by AFWL and SRI to be published in the near future. When greater accuracy is desired, $\ln\frac{b}{b+\delta_2}$ is a good first approximation to use in the iteration by which $\ln\frac{\gamma\lambda_2 b}{2i}$ is found.

Perhaps the greatest value in the transmission line model lies in the ease with which more complicated configurations can be treated. For example, if the long conductor lies on the surface of the ground, then the "semi-coaxial" transmission line is the appropriate model. (See below)



The effect of termination of long lines is also simple to treat via transmission line theory. If the conductor is of length d ($d \gg \lambda_2$), and has terminal impedances Z_1 and Z_2, then the solution to (30) is

$$I_z(z) = K e^{-\Gamma z} + L e^{\Gamma z} + \frac{1}{2Z_o} \int_0^d E_o(\nu)\, e^{-\Gamma |z-\nu|}\, d\nu$$

where

$$\Gamma = \sqrt{\frac{(Z_L + Z_s)}{Z_T}}$$

$$Z_o = \sqrt{Z_T(Z_L + Z_s)}$$

$$\text{If } F(Z) = \frac{1}{2Z_o} \int_0^d E_o(\nu)\, e^{-\Gamma |z-\nu|}\, d\nu$$

the reflection terms are

$$K = \frac{(Z_2+Z_o)(Z_o-Z_1)\, e^{\Gamma d} F(o) + (Z_1-Z_o)(Z_2-Z_o)\, F(d)}{(Z_1+Z_o)(Z_2+Z_o)\, e^{\Gamma d} - (Z_1-Z_o)(Z_2-Z_o) e^{-\Gamma d}}$$

$$L = \frac{(Z_2-Z_o)(Z_o-Z_1)\, e^{-\Gamma d} F(o) + (Z_1+Z_o)(Z_2-Z_o)\, F(d)}{(Z_1-Z_o)(Z_2-Z_o)\, e^{-\Gamma d} - (Z_1+Z_o)(Z_2+Z_o)\, e^{\Gamma d}}$$

This theory should be applicable to curved conductors as well as straight conductors as long as the radius of curvature is large compared with δ_o. For the case of an electric field not parallel to the conductor, only the parallel component of the field need be considered, since the normal component will generate only highly damped modes.

The case of an insulated conductor can be treated by considering the solvable problem of a coaxial transmission line in which the center conductor is surrounded by successive dielectric cylinders of different properties.

V. <u>Conclusions.</u>

In this note, an accurate form and a simpler approximation for the current induced in long conductors have been derived for the case of CW excitation. The next step is to consider the pulse problem. One way to treat a pulsed field is to make a Fourrier decomposition of the field at each space point, solve the resulting CW problem, and then transform the answer back into the time domain. It is hoped, however, that some simpler and more elegant way of treating the pulse problem will be found.

I would like to thank Mr. E. Vance of SRI for first suggesting the transmission line model and to Lt D. Marston for many useful discussions and calculational assistance.