SANDIA CORPORATION MONOGRAPH

MISSILE WITH ATTACHED UMBILICAL CABLE AS A RECEIVING ANTENNA

bу

C. W. Harrison, Jr.

February 1963

SUMMARY

In this paper principles of antenna analysis are applied to estimate the magnitude of undesired radio-frequency current along the skin of a rocket on the launching pad, with umbilical cable attached, due to a plane wave incident field. In the ready state, such systems are only partially shielded from the electromagnetic field environment, and form effective receiving antennas. Radio-frequency energy may be fed to sensitive electroexplosive initiating devices, resulting in spurious operation, or malfunctioning.

MISSILE WITH ATTACHED UMBILICAL CABLE AS A RECEIVING ANTENNA

Introduction

In this age of space exploration it has become necessary to assess the receiving characteristics of rockets to unwanted radio signals because they are fired by electroexplosive devices (EEDs). If the rocket on its launching pad, with umbilical cable attached, is unduly sensitive to the electromagnetic field environment, a premature launching might occur, or a degradation in performance result. The unwanted radio-frequency field environment may be caused by the operation of one or more radio transmitters in the vicinity of the launching pad, by local thunderstorm activity, or by a nuclear detonation in the neighborhood. 1-5

A similar problem exists in the general field of ordnance. Potter has written a good summary of the problem of radio-frequency hazards to ordnance. He says,

"The recent trend in radar and communications equipment toward greater effective radiated power has resulted in growing concern about RF hazards The most serious hazards stem from the use of sensitive electrically-initiated explosive elements, known as electro-explosive devices (EEDs), which can be spuriously initiated by induced radio frequency energy. EEDs are used extensively ... to activate control and arming devices and to initiate explosive trains. Hazards include both spurious functioning of the EED and degradation of the EED reliability or performance characteristics."

Radio-frequency energy is fed to the EEDs by the internal circuitry in the rocket. The wiring is activated by radio-frequency leaks through access doors (slots), anodized peripheral butts between sections, etc., and by direct connection (by way of the umbilical cable) to improperly shielded external circuitry.

The basic problem of radio-frequency hazards to ordnance consists in evaluating the response of a dipole receiving antenna within an imperfectly conducting cylinder of modest wall thickness and of finite length. Harrison and King⁷ have demonstrated that for any conceivable field amplitude there is no hazards problem at low frequencies, where the length of the cylinder and radius are small in terms of the wavelength. At higher frequencies skin effect affords sufficient protection. The theory presupposes no breaks in the rocket skin and no attached umbilical cable. However, King and Harrison⁶ have shown that the pickup of a coaxial cable of sufficient length to approach resonance at low frequencies is surprisingly large.

Harrison has discussed radio-frequency shielding of cables in a qualitative way, evaluated approximately the response of a loop antenna in a large imperfectly conducting cylinder, and determined the shielding properties of a circular grating of finite length. Work is currently under way at

the Sandia Laboratory to evaluate theoretically the transmission of radio energy through access doors. Dickinson has considered electromagnetic coupling to ordnance devices, and some 30 years ago King²³ obtained the shielding effect of imperfectly conducting spherical and infinite cylindrical shells at low frequencies. This work is now the classic in the field of electromagnetic shielding.

Radio-frequency hazards to ordnance problems relate to partially shielded receiving antennas of very general configuration. The solutions of these problems are often obtained by combining methods of antenna analysis with suitable arranged experiments. Thus theory complements experiment. If a correlation has been obtained in the Laboratory between some easily measured current in the rocket and EED current, and the former current can be estimated theoretically for a new radio-frequency field environment, the problem is solved.

In the present paper it is assumed that the relation between rocket EED current and total current in the shield of the umbilical cable at the point of attachment near the nose is known. The theoretical problem is that of estimating the umbilical cable current for a specified electromagnetic field in the vicinity. The physical configuration of the rocket on its launching pad, and the umbilical cable, resembles a folded monopole for reception. This paper lays the foundations of a very general theory of reception by folded dipoles. The analysis has many points in common with the earlier work of Harrison and King in which relations for mutual and self-impedance for identical folded antennas were determined, as well as the driving point impedance of folded dipoles and loops containing series impedances and reactive interconnections.

An area of electromagnetics research is unfolding where the objective is not the enhancement of the transmitting or receiving qualities of a given antenna but the evolvment of ways and means of reducing the signal pickup of extended circuits. In future rocket and ordnance design, as much attention must be paid to the radio-frequency environmental problem as is now given the effects of altitude, shock, temperature, humidity, etc.

Theoretical Considerations

The receiving characteristics of a symmetrically loaded dipole excited by plane waves may be deduced as follows: The antenna is driven by a generator at its center, and the current distribution I(z) found at all points along the dipole by solving the integral equation for the current. The current at the driving point I(o) determines the input admittance Y_o . The reciprocal of Y_o is the driving point impedance Z_o . Knowledge of the current distribution also permits calculation of the radiation field pattern $F\left(\theta,\beta h,\frac{a}{h}\right)$. As a consequence of the Rayleigh-Carson reciprocity theorem, $3h_e\left(\theta,\beta h,\frac{a}{h}\right)=\left(F(\theta,\beta h,\frac{a}{h})\right)$ for a two-terminal radiator. Here, $h_e\left(\theta,\beta h,\frac{a}{h}\right)$ is the effective half-length, β is the radian wave number, h is the half-length of the structure, h is the incident electric field strength in the plane of the receiving dipole, the open-circuit voltage at h is h in h is h current in the load impedance h is h is h and the current with load terminals short-circuited is h is h and h is h and h is h and the current with load terminals short-circuited is h and h is h and h is h and h and h is h and h and h and h and h are h and h and h are h and h and h are h and

It is apparent that if the receiving properties of an asymmetrically loaded dipole are desired, it is necessary to obtain a completely general expression for the current distribution for arbitrary generator position, so that the impedance and effective length of the structure may be obtained, referred to the terminal location. Evidentally, moving the generator alters both the current distribution and field pattern. While in principle the receiving properties of an antenna may be deduced from an accurate knowledge of the current distribution when the antenna is used for transmission, the writer is of the opinion that in some cases, especially for asymmetrically loaded folded dipoles, it is easier to work directly with the integral equations that apply to the structure when excited by an incident plane wave, than to determine the impedance and the effective length from the radiation field pattern utilizing the reciprocity theorem.

A folded receiving antenna consisting of two conductors of radii, a_1 and a_2 , and total length 2h, connected to equal impedances Z_L at the ends, is shown in Figure 1. The structure lies in the yz plane, and the wires are parallel to the z-axis. The axis of conductor 1 (of radius a_1) is located at x = 0, $y = \frac{b}{2}$, and $-h \le z \le h$. The axis of conductor 2 (of radius a_2) is located at x = 0, $y = -\frac{b}{2}$, and $-h \le z \le h$. For simplicity it is assumed that the incident electric field is linearly polarized parallel to the z-axis and arrives from the distant source at the azimuth angle Φ , measured from the positive x-axis. It is further assumed that the structure is so proportioned that the following inequalities apply: $a_1 << h$, $a_2 << h$, $(a_1 + a_2) < b$, $(3b)^2 << 1$.

The circuit of Figure 1 approximates a rocket of height h and radius a_1 , with umbilical cable of length h and radius a_2 attached, over a large perfectly conducting plane. It is assumed that there is a gap in the shield of the umbilical cable at the point of attachment to the rocket. The voltage developed across this gap by action of the incident field excites currents in the wires surrounded by the shield. The "equivalent impedance" of the internal circuitry is represented by the lumped loading impedance Z_1 .

The analysis begins with the equations

$$J_{d}(z) + I_{1}(z)r_{a1} + I_{2}(z)r_{b} = -j\frac{4\pi}{\varsigma} \left\{ C_{1}\cos\beta z + U_{1} \right\}$$
 (1)

$$J_{d}(z) + I_{1}(z)r_{b} + I_{2}(z)r_{a2} = -j\frac{4\pi}{\varsigma} \left\{ C_{2} \cos \beta z + U_{2} \right\}$$
 (2)

where (1) and (2) apply to conductors 1 and 2, respectively. The definitions of terms follow.

$$J_{d}(z) = \int_{-h}^{h} I_{T}(z') K_{d}(z, z') dz'$$
(3)

$$I_{T}(z) = I_{1}(z) + I_{2}(z); I_{T}(\pm h) = 0$$
 (4)

 $I_1(z)$ and $I_2(z)$ are the currents along conductors 1 and 2, respectively.

$$K_d(z, z') = \exp(-j\beta R_d)/R_d$$

Reference 16, Equation 18, p. 174.

$$R_{d} = \sqrt{(z - z')^{2} + d^{2}}$$
 (6)

$$r_{a1} = 2 \ln \frac{d}{a_1}$$
 (7)

$$r_{a2} = 2 \ln \frac{d}{a_2}$$
 (8)

$$r_{b} = 2 \ln \frac{d}{b}$$
 (9)

 $\zeta = 120\pi \text{ ohms}$

 C_1 and C_2 are constants of integration

$$U_{1} = U e^{j\frac{\beta h}{2} + in \Phi}$$

$$-j \frac{\beta h}{2} \sin \Phi$$

$$U_{2} = U e$$
(10)

$$U_2 = Ue \tag{11}$$

$$U = -\frac{E}{\beta}$$
 (12)

Multiplying (2) by a parameter m, and adding it to (1) gives

$$J_{d}(z) + I_{r}(z) \left[\frac{r_{a1} + mr_{b}}{m+1} \right] + I_{2}(z) \left[\frac{r_{b} - r_{a1} + m(r_{a2} - r_{b})}{m+1} \right] =$$

$$-j \frac{4\pi}{\zeta} \left[\frac{(C_{1} + mC_{2})}{m+1} \cos \beta z + \frac{(U_{1} + mU_{2})}{m+1} \right]$$
(13)

This expression may be reduced to the integral equation for the current along an unloaded solid conductor receiving dipole of length 2h and radius d by setting the coefficients of $I_{T}(z)$ and $I_{2}(z)$ equal to zero.

Thus,

$$r_{al} + mr_{b} = 0 \tag{14}$$

$$r_{b} - r_{a1} + m(r_{a2} - r_{b}) = 0 {(15)}$$

It follows that that

$$d = b^{\frac{1}{2}} (a_1 a_2)^{\frac{1}{4}} e^p$$
 (16)

where

$$p = \frac{\left(\ln \frac{a_1}{a_2}\right)^2}{4 \ln \frac{b}{a_1 a_2}}$$
 (17)

and

$$m = \frac{\ln \frac{b}{a_1}}{\ln \frac{b}{a_2}}$$
(18)

The antenna equation (13) now takes the form

$$J_{d}(z) = -j\frac{4\pi}{\zeta} \left[C_{d} \cos \beta z + U_{d} \right]$$
 (19)

where C_d is evaluated from the boundary condition $I_T(\pm h)$ = 0, and the source function U_d is given by

$$U_{d} = \frac{U_{1} + mU_{2}}{m+1} = -\frac{E}{\beta} \begin{bmatrix} \ln \frac{b}{a} e^{j\frac{\beta b}{2} \sin \phi} & -j\frac{\beta b}{2} \sin \phi \\ -\frac{E}{a} & -\frac{b}{a} e^{j\frac{\beta b}{2} \sin \phi} \\ -\frac{\ln \frac{b}{a} e^{j\frac{\beta b}{2} \sin \phi}}{\ln \frac{b^{2}}{a_{1} a_{2}}} \end{bmatrix}$$
(20)

Because (3b) $^2 <<$ 1, it is clear that $U_d \approx U = -\frac{E}{\beta}$.

Note that (19) is valid only for an electric field directed tangential to the wires.

 $I_T(z)$ is available from the work of King, and is considered known for the purposes of this paper. If only the current $I_T(0)$ is of interest, it may be obtained from the formula

$$I_{r}(0) = \frac{2h_{e}E}{Z_{c}}$$
 (21)

where the effective length $2h_e$ is obtained (by application of the reciprocity theorem) from the radiation field pattern of a dipole of length 2h and radius d, and Z_o is the impedance of the same dipole.

The transmission line equation

$$I_{1}(z) = I_{T}(z) \left[\frac{\ln \frac{b}{a_{2}}}{\ln \frac{b^{2}}{a_{1}a_{2}}} \right] - \frac{j}{Z_{c}} \left[C_{1} - C_{2} \right) \cos \beta z + j2U \sin \left(\frac{\beta b}{2} \sin \Phi \right) \right]$$

$$(22)$$

is obtained, fundamentally, by subtracting (2) from (1). Here $Z_{\bf c}$ is the characteristic impedance of the structure, given by

$$Z_{e} = 60 \ln \frac{b^{2}}{a_{1} a_{2}}$$
 (23)

The voltage drop across the load impedance $Z_{\mathbf{L}}$ is

$$I_1(h)Z_1 = \phi_1(h) - \phi_2(h)$$
 (24)

Here,

$$\phi_1(h) = j \frac{\omega}{\beta^2} \left[\frac{\partial A_1(z)}{\partial z} \right]_{xxy}$$
 (25)

$$\phi_2(h) = j \frac{\omega}{\beta^2} \left[\frac{\partial A_2(z)}{\partial z} \right]_{z=z}$$
 (26)

$$A_{1}(z) = -\frac{j}{c} \left\{ C_{1} \cos 3z + U_{1} \right\} \tag{27}$$

$$A_2(z) = -\frac{j}{c} \left\{ C_2 \cos \beta z + U_2 \right\} \tag{28}$$

The velocity of light is designated c, and ω is the radian frequency.

It follows that

$$C_1 - C_2 = -\frac{I_1}{\sin \beta h} \frac{(h) Z_L}{L}$$
 (29)

I (h) is found by substituting (29) into (22), with z = h. The result is

$$I_{1}(h) = \frac{2U \sin \left(\frac{\beta b}{2} \sin \Phi\right)}{Z_{c} - jZ_{c} \cot \beta h}$$
(30)

Equation (22) now becomes

$$I_{1}(z) = I_{T}(z) \left[\frac{\ln \frac{b}{a_{2}}}{\ln \frac{b^{2}}{a_{1}^{a_{2}}}} + \frac{2U}{Z_{c}} \sin \left(\frac{\beta b}{2} \sin \Phi \right) \right] \left[\frac{Z_{c} \sin \beta h + jZ_{L} (\cos \beta z - \cos \beta h)}{Z_{c} \sin \beta h - jZ_{L} \cos \beta h} \right]$$

$$(31)$$

This is a general formula for the current along conductor 1. A similar formula may be obtained for the current along conductor 2. Note that (31) is valid only when the electric field is directed parallel to the wires.

Alternative forms of (30) are

$$I_{1}(h) = 2U \sin\left(\frac{\beta b}{2} \sin \Phi\right) \begin{cases} \sin \beta h & \text{(32)} \\ Z_{1} \sin \beta h - jZ_{1} \cos \beta h & \text{(32)} \end{cases}$$

and

$$I_{1}(h) = j2U \sin\left(\frac{\beta b}{2} \sin \phi\right) \begin{cases} \frac{\tan \beta h}{R_{L} + j(Z_{c} + X_{L}) \tan \beta h} \end{cases}$$
(33)

Clearly from (32) $I_1(h) = 0$ when $\beta h = \pi$, $Z_L \neq 0$. Also, when $\beta h = \frac{\pi}{2}$,

$$I_1(h) = \frac{2U}{Z_c} \sin\left(\frac{\beta b}{2}\sin\phi\right) \approx -\frac{Eb\sin\phi}{Z_c}$$
 (34)

From (33) it is seen that whenever

$$Z_0 \tan \beta h \gg \left| R_L + j X_L \right| \tag{35}$$

(34) holds.

Readers are reminded that in the development of these equations radiation from the structure (in the transmission line mode) is ignored, transmission line losses are neglected, and no account is taken of proximity effect.

As a numerical illustration, let the current input to the nose of a rocket from a shielded umbilical cable be estimated under the conditions stated below. The structural dimensions are:

The incident plane wave E is vertically polarized and has a magnitude of 3 volts/m. The frequency is f = 1.85 mc/sec, or $\lambda = 162.2$ m. The shield is continuous, so that $Z_{\rm L} \approx 0$. It is assumed that this system can be represented satisfactorily by a two-conductor folded monopole over a perfectly conducting earth.

The characteristic impedance is

$$Z_c = 60 \ln \frac{b^2}{a^2} = 160.9 \text{ ohms}$$

Since $\beta h = \frac{2\pi}{\lambda} h = 0.6492$, $\tan \beta h = 0.7586$. It follows that $Z_c \tan \beta h >> |Z_L|$ since Z_L is assumed to be small. Hence the formula

$$\left|I_{1}(h)\right| = \frac{\text{Eb sin }\Phi}{Z_{c}}$$

is applicable. Substituting numbers in this formula gives

$$\left| I_1(h) \right| = \frac{3 \times 3.639}{160.9} = 67.85 \text{ ma}$$

if $\Phi = \frac{\pi}{2}$ radians.

If laboratory experiments reveal that considerably more than 100-ma nose current is required at 1.85 mc/sec before the most sensitive EED current exceeds the safe level, one may conclude in this instance that no hazards problem exists. As mentioned before, the firing circuits are energized by radio-frequency leaks in the umbilical cable shield and in the rocket skin when $Z_{\downarrow} \approx 0$.

It is to be emphasized that the solution of this problem is only approximate. The radius of the "equivalent" dipole is d = 1.904 m, so that Ω = 2 $\ln\frac{2h}{a}$ = 5.736. Accuracy may be obtained from existing methods of cylindrical antenna analysis for $\Omega \ge 7$.

If a break exists in the umbilical cable shield at the point it enters the missile, $Z_{\rm L} \neq 0$. This impedance, which appears across the gap in the shields, is a manifestation of the loading effect of the wires entering the missile from the umbilical cable. $Z_{\rm L}$ may be determined as follows: Measure the gap impedance $Z_{\rm G}$ (shield-to-shield). This impedance is the parallel combination of $Z_{\rm L}$ and $Z_{\rm T}$. $Z_{\rm T}$ is the impedance of a transmission line of length h and characteristic impedance $Z_{\rm C}$ (given by (23)) terminated in a short circuit (the ground plane). Then,

$$Z_{L} = \frac{Z_{G} Z_{T}}{Z_{T} - Z_{G}}.$$
(36)

The true current on the shield of the umbilical cable at the point of attachment to the missile is then computed from (30) using (36).

It is important to observe that a laboratory-determined relation between nose-cone input current and EED current (when the umbilical cable is disconnected) is no longer valid when $Z_{\rm L} \neq 0$, because the circuits in the missile are excited by the gap voltage. Ordinarily the currents flowing in the internal wiring induce larger currents in the firing circuits than the missile skin current, even when there are open access doors and anodized peripheral butts between sections. Indeed, it is possible that more correct results may be obtained by disregarding the shields and treating the internal wiring as a litz cable, in the form of a folded monopole. The total cable current is obtained theoretically for the specified RF environment. The RF hazards to ordinance problem is solved if the relation between the most sensitive EED current and total cable current has been obtained experimentally.

Discussion

A few remarks might be made relating to the use of (31) in solving conventional two-conductor folded receiving antenna problems. If conductor 1 is broken at point $z = \ell$, a voltage $V_{oc} = I_{sc}(\ell)Z(\ell)$ will appear across the break. Here $I_{sc}(\ell)$ is the short circuit current, and may be obtained from (31)

with z = l and $Z_1 = 0$. Z(l) is the driving point impedance of the structure, looking in at the break. For a two-conductor antenna, Z(l) may be obtained easily by superposition; if the number of conductors N > 2, more advanced techniques must be employed, as a general rule. Z(l) depends on the impedance of an asymmetrical dipole and of two transmission-line sections in series with short-circuited terminations. The sections of the equivalent dipole are of radius d, and the transmission lines are of length h + l and h - l.

The equivalent circuit of the receiving antenna consists of V_{oc} driving $Z(\P)$ connected in series with the load impedance Z_{\P} used.

When the electric field is directed tangential to the wires of the folded antenna only currents of even symmetry are excited. When the field is tilted with respect to the axes of the conductors, currents of even and odd symmetry flow. If the load is located in the middle of one of the conductors, the voltage drop across the load impedance is due only to current of even symmetry. (Note that this current is a function of the angle of wave tilt.) For a displaced load, i.e., an asymmetrically loaded structure (assuming nonparallel incidence of the field), currents of both symmetries flow in the load. These currents may be found individually by applying the integral equation technique. The total currents is then obtained by superposition.

Conclusions

Antenna theory is useful for solving problems relating to radio-frequency hazards to ordnance. In this paper a theory of folded conductor structures, as receiving antennas, is developed and applied to estimate the current in the shield of an umbilical cable at the point of attachment to a rocket on the launching pad. An experimentally obtained correlation between input current to the rocket and electro-explosive current permits solution of this RF hazards problem.

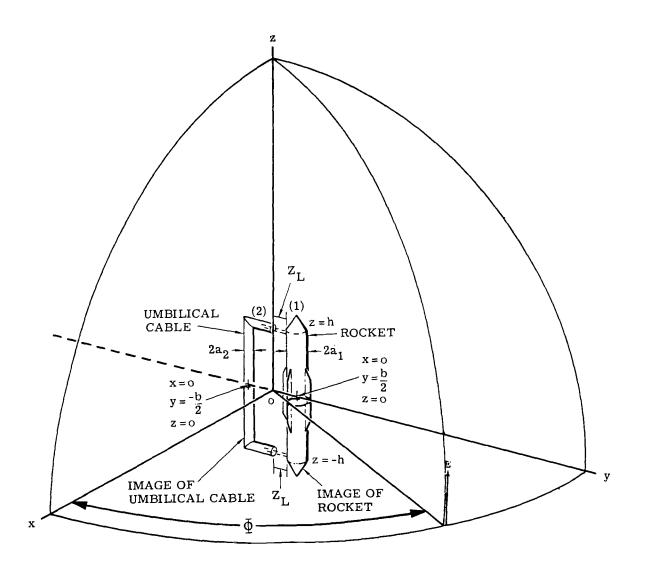


Figure 1. Rocket with attached umbilical cable, and image

REFERENCES

- 1. Lippman, B. A., Electromagnetic Signal from Nuclear Explosions, presented at URSI Spring 1962 Meeting at Georgetown University, Washington, D.C.
- 2. Suydam, B. R., Electromagnetic Signal from a Bomb Burst in Vacuo, Los Alamos Scientific Laboratory Report LAMS-2624 of November 3, 1961.
- 3. Kompaneets, A. S., "Radio Emission from an Atomic Explosion," <u>Soviet Physics JETP</u>, June 1959, Vol. 35(8), pp 1076-1080.
- 4. Karzas, W. J., and Latter, R., <u>Electromagnetic Radiation from a Nuclear Explosion in Space</u>, Rand-RM-2849-AFT, October 1961.
- 5. Karzas, W. J., The Electromagnetic Signal Due to the Exclusion of the Earth's Magnetic Field by Nuclear Explosions, Rand-RM-2890-PR, December 1961.
- 6. Potter, R. R., Weapon Design Requirements to Preclude Hazards from Environmental Electromagnetic Fields, U. S. Naval Weapons Laboratory (Dahlgren, Va.), Technical Memorandum No. W/2-61 of January 1961.
- 7. Harrison, C. W., Jr., and King, R. W. P., "Response of a Loaded Electric Dipole in an Imperfectly Conducting Cylinder of Finite Length," <u>Journal of Research of the National Bureau of Standards</u> D. Radio Propagation, May-June 1960, Vol. 64D, No. 3, pp 289-293.
- 8. King, R. W. P., and Harrison, C. W., Jr., "Cylindrical Shields," <u>IRE Transactions on Antennas</u> and Propagation, March 1961, Vol. AP-9, No. 2, pp 166-170.
- 9. Harrison, C. W., Jr., "Radio Frequency Shielding of Cables," <u>Journal of the American Society</u> of Naval Engineers, August 1961, Vol. 73, No. 3, pp 529-533.
- 10. Harrison, C. W., Jr., and Denton, D. H., Jr., Receiving Characteristics of Quasi-Shielded Antennas, Sandia Corporation Technical Memorandum 396-58(14) October 15, 1958.
- 11. Harrison, C. W., Jr., Radio Frequency Hazards to Ordnance Materials--Shielding Effect of a Circular Grating of Finite Length, Sandia Corporation Internal Memorandum, July 21, 1961.
- 12. Dickinson, W. T., A Method to Determine Electromagnetic Coupling to Ordnance Devices Aboard Ships, Technical Report No. 5456, June 15, 1961, Jansky & Bailey, Inc., Alexandria, Va. See also Dickinson, W. T. de Pian, Louis, and Glock, R., Jr., Electromagnetic Coupling to Ordnance Systems, Technical Report covering Phase I, June 15, 1960, Jansky & Bailey, Inc.

REFERENCES (continued)

- 13. King, L. V., "Electromagnetic Shielding at Radio Frequencies," Philosophical Magazine and Journal of Science, February 1933, Vol. 15, No. 97, pp 201-223. See also H. Kaden, "Die Elektromagnetische Schirmung in der Fernmelde and Hockfrequenztechnik," (Springer-Verlag, 1950).
- 14. Harrison, C. W., Jr., Folded Wire Structures as Receiving Antennas, Sandia Corporation Technical Memorandum 253-58(14), June 18, 1958.
- 15. Harrison, C. W., Jr., and King, R. W. P., "Theory of Coupled Folded Antennas," <u>IRE Transactions on Antennas and Propagation</u>, March 1960, Vol. AP-8, No. 2, pp 131-135.
- 16. Harrison, C. W., Jr., and King, R. W. P., "Folded Dipoles and Loops," IRE Transactions on Antennas and Propagation, March 1961, Vol. AP-9, No. 2, pp 171-187.
- 17. Harrison, C. W., Jr., "Antenna Coupling Error in Direction Finders," <u>Journal of Research of the National Bureau of Standards D. Radio Propagation</u>, July-August 1961, Vol. 65D, No. 4, pp 363-369, Equations (1)-(2).
- 18. King, R. W. P., Theory of Linear Antennas, Chapter 4, Harvard University Press, 1956.
- 19. King, R. W. P., Mimno, H. R., and Wing, A. H., Transmission Lines, Antennas, and Wave Guides, Section 45, Chapter 2, p. 224, McGraw-Hill Book Co., Inc., 1945.