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RECEIVING CHARACTERISTICS OF TWO-WIRE LINES EXCITED BY UNIFORM AND NON-UNIFORM ELECTRIC FIELDS

by

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SUMMARY

The receiving properties of an isolated two-conductor transmission line terminated at each end in arbitrary values of impedance are deduced by directly integrating over the length of the wires the differential value of the plane-wave incident electric field exciting the line. For identical configurations it is shown that this theory and the theory of folded receiving antennas are compatible. Finally, formulas for the load currents are obtained for a transmission-line loop oriented parallel to and in the near-zone field of a receiving and scattering antenna. The line and the antenna are not necessarily the same length.

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Introduction

In its present state of development folded receiving antenna theory is restricted in that the incident field must be of uniform amplitude and of the same phase over the length of the structure, and if terminating impedances are employed, they must be identical.[1] The purpose of the present paper is to remove these restrictions so that the load currents may be computed when the transmission line loop terminating impedances are different, and the line is excited by a non-uniform field such as that existing, for example, in proximity to a receiving and scattering antenna.

The Isolated Transmission-Line Loop Immersed in a Plane-Wave Electric Field

Consider a transmission line of length s terminated in impedance Z_s, as shown in Figure 1. The input current to the line I_o is delivered by generator V_o. The load current and load voltage are I_s and V_s, respectively. The characteristic impedance of the line is

$$Z_c = 60\,\ell n\left(\frac{d^2}{a_1 a_2}\right) \tag{1}$$

where a_1 and a_2 are the wire radii, and d is the separation distance between conductors, measured center-to-center.

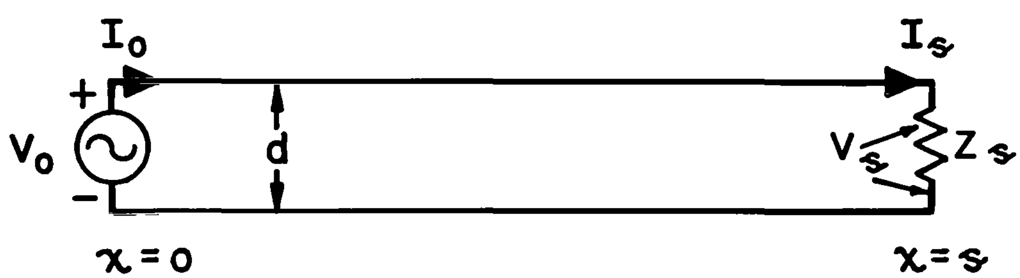

Figure 1. Transmission Line Driven by One Generator

Since

$$I_o = I_s \cosh \gamma s + \frac{V_s}{Z_c} \sinh \gamma s \tag{2}$$

[1] C. W. Harrison, Jr., "Missile with Attached Umbilical Cable as a Receiving Antenna," IEEE Transactions on Antennas and Propagation, Vol. AP-11, No. 5, pp 587-588, September 1963.

and

$$V_s = I_s Z_s , \tag{3}$$

it follows that

$$I_s = \frac{I_o Z_c}{Z_c \cosh \gamma s + Z_s \sinh \gamma s} \tag{4}$$

where

$$\gamma = \alpha + j\beta . \tag{5}$$

α and β are the attenuation and phase constants, respectively. Also, the driving point impedance of the line is

$$Z_{in} = Z_c \left\{ \frac{Z_s + Z_c \tanh \gamma s}{Z_c + Z_s \tanh \gamma s} \right\} . \tag{6}$$

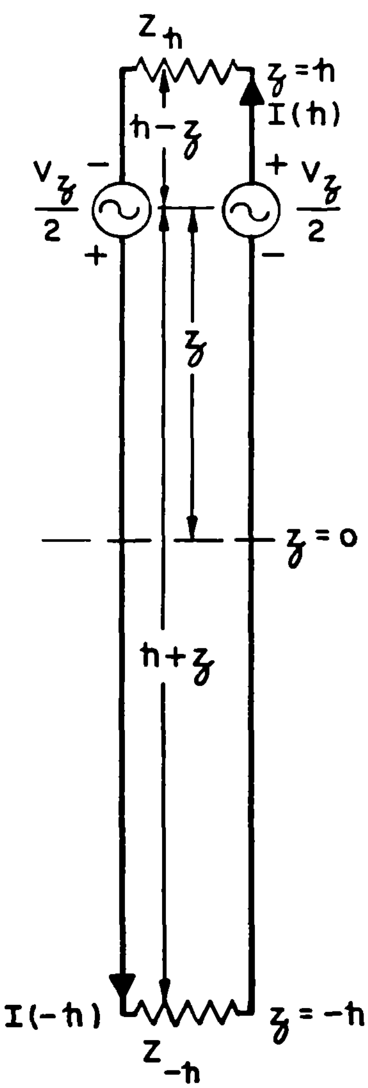

Figure 2. Transmission Line Driven by Two Series Generators

Equations (4) and (6), if interpreted properly, may be employed to find the load currents I(h) and I(−h) of Figure 2. This figure illustrates a transmission line of length 2h driven at point z, measured from the mid-point of the line, by two generators of voltage $V_z/2$ of the polarity shown, and terminated in impedances Z_h and $Z_{-h} (Z_h \neq Z_{-h})$.

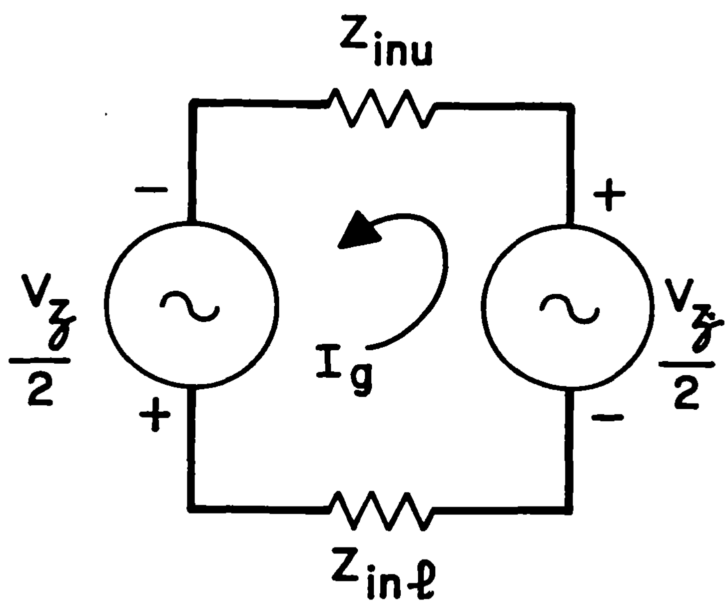

Figure 3. Generator Region of Figure 2. The Subscripts u and ℓ on Z_{in} Denote the Upper and Lower Impedances

From Figure 3 it is clear that the current I_g, which corresponds to I_o in Equation (4), is given by the relation

$$I_g = \frac{V_z}{Z_{inu} + Z_{in\ell}} . \tag{7}$$

Evidently, from Equation (6),

$$Z_{inu} = Z_c \left\{ \frac{Z_h + Z_c \tanh \gamma(h - z)}{Z_c + Z_h \tanh \gamma(h - z)} \right\}, \tag{8}$$

$$Z_{in\ell} = Z_c \left\{ \frac{Z_{-h} + Z_c \tanh \gamma(h + z)}{Z_c + Z_{-h} \tanh \gamma(h + z)} \right\}, \tag{9}$$

and

$$I(h) = \frac{I_g Z_c}{Z_c \cosh \gamma(h - z) + Z_h \sinh \gamma(h - z)} \tag{10}$$

$$I(-h) = \frac{I_g Z_c}{Z_c \cosh \gamma(h + z) + Z_{-h} \sinh \gamma(h + z)} . \tag{11}$$

Substituting (8) and (9) in (7), and (7) in (10) and (11), it is found after a reasonable amount of algebraic manipulation that[2]

$$I(h) = \frac{V_z \left[Z_c \cosh \gamma(h + z) + Z_{-h} \sinh \gamma(h + z) \right]}{Z_c \left(Z_h + Z_{-h} \right) \cosh 2\gamma h + \left(Z_h Z_{-h} + Z_c^2 \right) \sinh 2\gamma h} \tag{12}$$

and

$$I(-h) = \frac{V_z \left[Z_c \cosh \gamma(h - z) + Z_h \sinh \gamma(h - z) \right]}{Z_c \left(Z_h + Z_{-h} \right) \cosh 2\gamma h + \left(Z_h Z_{-h} + Z_c^2 \right) \sinh 2\gamma h} . \tag{13}$$

If α is sufficiently small, so that $\gamma = j\beta$, (12) and (13) become

$$I(h) = \frac{V_z}{D_1} \left[Z_c \cos \beta(h + z) + jZ_{-h} \sin \beta(h + z) \right] \tag{14}$$

$$I(-h) = \frac{V_z}{D_1} \left[Z_c \cos \beta(h - z) + jZ_h \sin \beta(h - z) \right] \tag{15}$$

where

$$D_1 = Z_c (Z_h + Z_{-h}) \cos 2\beta h + j\left(Z_h Z_{-h} + Z_c^2 \right) \sin 2\beta h. \tag{16}$$

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[2] It should be pointed out that general expressions for the current at any point along a transmission line driven by two generators, as in Figure 2 of this paper, may be obtained from Equations (5), page 245, and (7), page 246, of Ronold W. P. King, "Transmission-Line Theory," McGraw-Hill Book Co., Inc., 1955. Ordinarily the distribution of current along a transmission line is not needed.

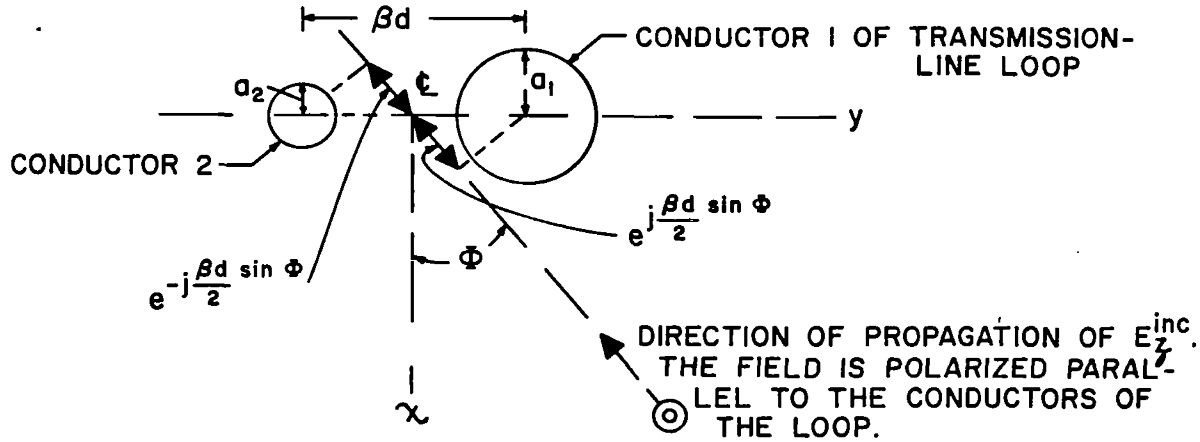

Figure 4. Geometry of the Incident Electric Field and Conductors of the Transmission-Line Loop

Suppose that the transmission-line loop is excited by a linearly polarized field E_z^{inc} as shown in Figure 4. If the reference for phase is taken midway between the centers of the conductors, and the azimuth angle Φ is measured from the positive x-axis, as shown, the fields E_{z1} and E_{z2} acting on conductors 1 and 2, respectively, are

$$E_{z1} = E_z^{inc} e^{j\frac{\beta d}{2}\sin\Phi} \tag{17}$$

$$E_{z2} = E_z^{inc} e^{-j\frac{\beta d}{2}\sin\Phi} . \tag{18}$$

To resolve these fields into symmetrical and antisymmetrical components, one sets

$$E_{z1} = E_{zd}^s + E_{zd}^a \tag{19}$$

$$E_{z2} = E_{zd}^s - E_{zd}^a \tag{20}$$

so that

$$E_{zd}^s = \frac{E_{z1} + E_{z2}}{2} = E_z^{inc} \cos\left(\frac{\beta d}{2}\sin\Phi\right) \tag{21}$$

$$E_{zd}^a = \frac{E_{z1} - E_{z2}}{2} = jE_z^{inc} \sin\left(\frac{\beta d}{2}\sin\Phi\right) \tag{22}$$

The subscript d on E_z^s and E_z^a refers to the direct field. The even field, E_{zd}^s sets up the dipole mode in the transmission-line loop and E_{zd}^a, the odd field, sets up the transmission-line mode in the structure. In all of the work reported in this paper it is assumed that these modes do not couple. Since E_{zd}^a, the differential electric field, acts at all points along the conductors, the cumulative effect may be obtained by integration. Evidently,

$$-\frac{V_z}{2} = E_{zd}^a dz = jE_z^{inc} \sin\left(\frac{\beta d}{2}\sin\Phi\right) dz. \tag{23}$$

Substituting (23) into (14) and (15), one obtains

$$I(h) = -j\frac{2}{D_1}E_z^{inc}\sin\left(\frac{\beta d}{2}\sin\Phi\right)\left[Z_c\int_{-h}^{h}\cos\beta(h+z)dz + jZ_{-h}\int_{-h}^{h}\sin\beta(h+z)dz\right]$$

$$= -j\frac{2}{\beta D_1}E_z^{inc}\sin\left(\frac{\beta d}{2}\sin\Phi\right)\left[Z_c\ \sin 2\beta h + jZ_{-h}(1-\cos 2\beta h)\right] \tag{24}$$

and

$$I(-h) = -j\frac{2}{\beta D_1}E_z^{inc}\sin\left(\frac{\beta d}{2}\sin\Phi\right)\left[Z_c\ \sin 2\beta h + jZ_h(1-\cos 2\beta h)\right]. \tag{25}$$

These are the final expressions for the currents in the loads of a transmission-line loop when excited by a plane-wave electric field polarized parallel to the conductors. Although only the pickup of a two-wire line is evaluated here, there is no reason why the method cannot be used, for example, to obtain the pickup of four-wire lines widely used in connecting receivers to their antennas. Also, it is apparent that Equations (14) and (15) are sufficiently general to permit calculation of the load currents when the differential electric field acting along the conductors is not constant. This case will be considered later in the paper.

Compatibility of Folded Receiving Antenna and Transmission-Line Loop Theory

Attention is now directed toward the development of a theory for folded receiving antennas when the terminations consist of unequal impedances. To accomplish this the principle of superposition is employed, as indicated in Figure 5.

According to the previously published theory,[3]

$$I_1(h) = I_1(-h) = \frac{2U}{Z_c}\sin\left(\frac{\beta d}{2}\sin\Phi\right) \tag{26}$$

with

$$U = -E_z^{inc}/\beta. \tag{27}$$

Evidently,

$$I_2(h) = -jV_h/Z_c\ \tan 2\beta h \tag{28}$$

$$I_2(-h) = I_2(h)/\cos 2\beta h = -jV_h/Z_c\ \sin 2\beta h \tag{29}$$

$$I_3(h) = -jV_{-h}/Z_c\ \sin 2\beta h \tag{30}$$

$$I_3(-h) = -jV_{-h}/Z_c\ \tan 2\beta h. \tag{31}$$

[3] See Reference 1, Equation 15, with $z = \pm h$, $I_T(\pm h) = 0$, and $Z_L = 0$.

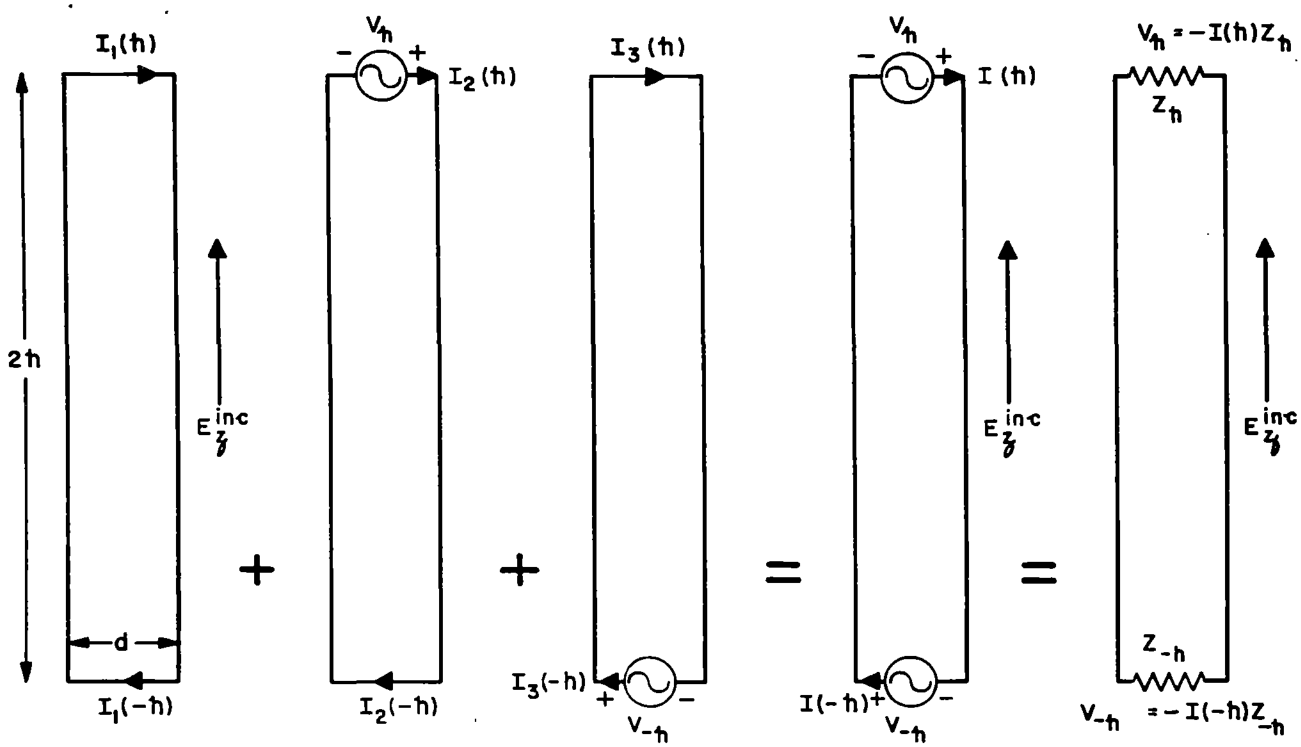

Figure 5. Use of the Superposition and Compensation Theorems in Developing the Theory of Folded Receiving Antennas Terminated in Unequal Impedances

By superposition,

$$I(h) = I_1(h) + I_2(h) + I_3(h) \tag{32}$$

$$I(-h) = I_1(-h) + I_2(-h) + I_3(-h). \tag{33}$$

Hence

$$I(h) = \frac{2U}{Z_c} \sin\left(\frac{\beta d}{2} \sin \Phi\right) - j \frac{V_h}{Z_c \tan 2\beta h} - j \frac{V_{-h}}{Z_c \sin 2\beta h} \tag{34}$$

$$I(-h) = \frac{2U}{Z_c} \sin\left(\frac{\beta d}{2} \sin \Phi\right) - j \frac{V_h}{Z_c \sin 2\beta h} - j \frac{V_{-h}}{Z_c \tan 2\beta h} \tag{35}$$

But by the compensation theorem,

$$V_h = -I(h)Z_h \tag{36}$$

$$V_{-h} = -I(-h)Z_{-h}. \tag{37}$$

Substituting (36) and (37) into (34) and (35), and solving these equations simultaneously for $I(h)$ and $I(-h)$ gives

$$I(h) = j \frac{2U}{D_1} \sin\left(\frac{\beta d}{2} \sin \Phi\right)\left[Z_c \sin 2\beta h + jZ_{-h}(1 - \cos 2\beta h)\right]. \tag{38}$$

$$I(-h) = j\frac{2U}{D_1}\sin\left(\frac{\beta d}{2}\sin\Phi\right)\left[Z_c\sin 2\beta h + jZ_h(1-\cos 2\beta h)\right]. \tag{39}$$

These expressions are the same as those derived previously in this paper by a different method. Although only the currents $I(h)$ and $I(-h)$ have been found, the method will yield $I_1(z)$ and $I_2(z)$ if needed. The current $I_T(z) = I_1(z) + I_2(z)$ is then involved. $I_T(z)$ may be determined from antenna theory, as set forth in Reference 1.

The Transmission-Line Loop Immersed in a Non-Uniform Field

Figure 6 illustrates a two-conductor transmission line loop in proximity to an unloaded receiving and scattering antenna. The antenna is of half-length h and radius a $(h \gg a,\ \beta a \ll 1)$. The spacing of the line wires is $d = c - b$, measured center-to-center. The inside wire clears the cylinder surface by the distance $(b - a)$. The line extends over the range $|z| \leq s$, but this restriction is not necessary in that the line may be positioned anywhere along the cylindrical receiving antenna not too close to its ends. The line terminations are designated Z_s and Z_{-s}. In the following analysis it is assumed that the presence of the transmission line does not significantly alter the current distribution along the receiving antenna. This assumption is the same as that made in all of the early developments of mutual and self-impedance relations for coupled antennas.

The z-component of the back-scattered electric field in the vicinity of an unloaded receiving antenna may be obtained as follows: One has

$$\nabla_\theta \times E = -j\omega\mu_o H_\theta \tag{40}$$

or

$$\frac{\partial E_r}{\partial z} - \frac{\partial E_z}{\partial r} = -j\omega\mu_o H_\theta . \tag{41}$$

If H_θ is calculated from the relation

$$\oint H_\theta \cdot d\ell = I(z) \tag{42}$$

one implicitly assumes that $\frac{\partial E_r}{\partial z} = 0$ since (42) is rigorously correct only for an infinitely long wire carrying a constant current. Accordingly,[4]

$$\frac{\partial E_z}{\partial r} = j\omega\mu_o \frac{I(z)}{2\pi r} \tag{43}$$

so that

$$E_z = j\frac{\omega\mu_o}{2\pi} I(z)\ell n\left(\frac{a+\delta}{a}\right). \tag{44}$$

[4] R. W. P. King, "Theory of Linear Antennas" Harvard University Press, 1956, Ch. 2, Section 26, pp. 127-141.

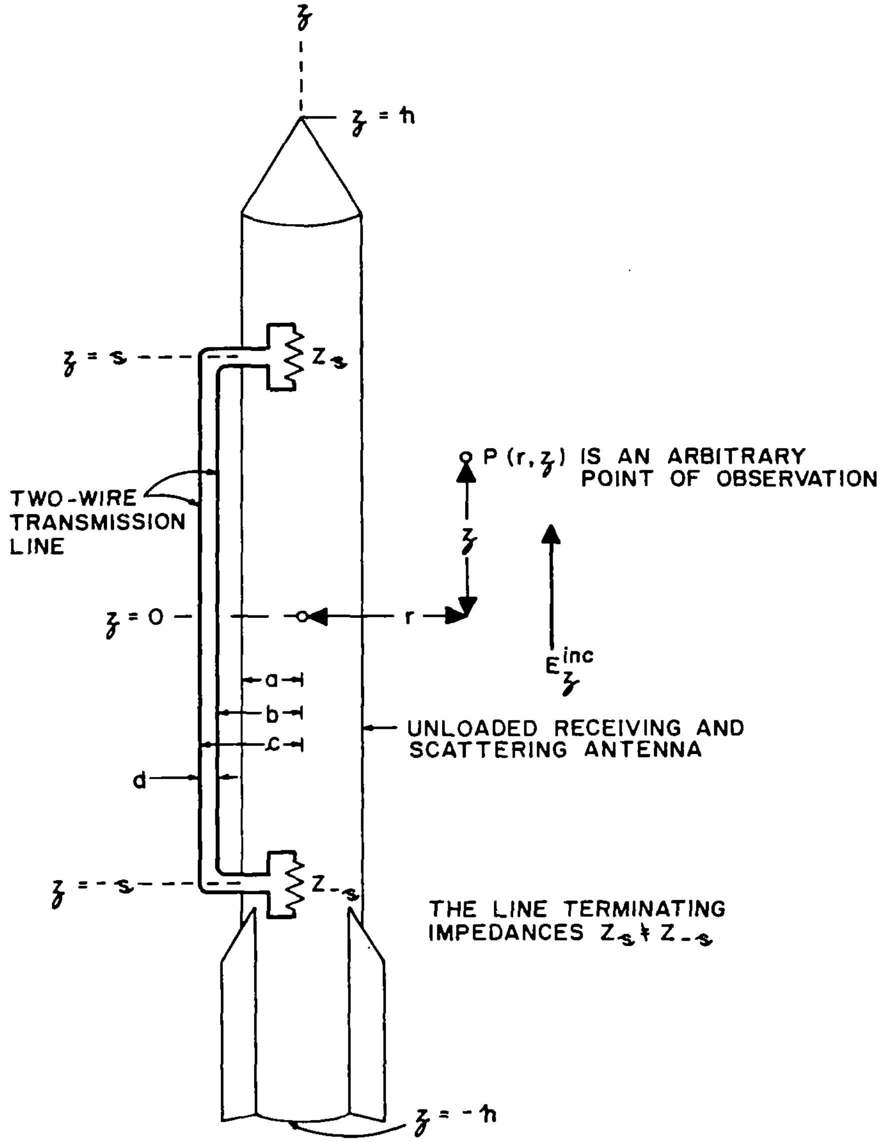

Figure 6. Terminated Transmission-Line in the Near Zone Field of an Isolated Unloaded Receiving and Scattering Antenna

Here $(a + \delta)$ is the distance from the center of the scattering antenna to the point where the electric field E_z is to be evaluated, and $I(z)$ is the current in the structure. Note that E_z is referred in phase to the axis of the scatterer.

If $\beta h \leq 5\pi/4$, it can be shown that the current $I(z)$ appearing in (44) is described quite accurately by the relation [5]

$$I(z) = -\frac{2h_e E_z^{inc}}{Z_o}\left(\frac{\cos\beta z - \cos\beta h}{1 - \cos\beta h}\right) \tag{45}$$

[5] C. W. Harrison, Jr., "The Radian Effective Half-Length of Cylindrical Antennas Less than 1.3 Wavelengths Long," IEEE Transactions on Antennas and Propagation, Vol. AP-11, No. 6, pp. 657-660, November, 1963.

where $2h_e$ is the effective length of the antenna, and Z_o is the driving point impedance at $z = 0$ (with E_z^{inc} suppressed). Note that both h_e and Z_o depend on the radius a of the receiving antenna.

Evidently the differential electric field acting on the line that is responsible for the transmission-line mode in the structure is analogous to (22). Thus,

$$E_{zs}^a = \frac{E_{zc} - E_{zb}}{2} \tag{46}$$

where the subscript s refers to the scattered field in contrast to the direct field, where the subscript d has been used. E_{zs}^a is obtained by substituting (45) into (44), and the result into (46). Hence,

$$-\frac{V_z}{2} = E_{zs}^a\, dz \sim -j \frac{\zeta_o \beta h_e E_z^{inc}}{2\pi Z_o(1 - \cos \beta h)} (\cos \beta z - \cos \beta h)\, \ell n\left(\frac{c}{b}\right) dz \tag{47}$$

where the relation $\omega\mu_o = \beta\zeta_o$ has been used and $\zeta_o = 120\pi$ ohms. Equation (47) applies when $2r/|h \pm z| << 1$.

To find the contribution of the scattered field to the transmission-line load currents, one inserts (47) into relations analogous to (14) and (15) that follow, and effects the integration over the range $-s \leq z \leq s$:

$$I(s) = \frac{V_z}{D_2}\left[Z_c \cos \beta(s + z) + j\, Z_{-s} \sin \beta(s + z)\right] \tag{48}$$

$$I(-s) = \frac{V_z}{D_2}\left[Z_c \cos \beta(s - z) + j\, Z_s \sin \beta(s - z)\right] \tag{49}$$

where

$$D_2 = Z_c(Z_s + Z_{-s})\cos 2\beta s + j(Z_s Z_{-s} + Z_c^2)\sin 2\beta s \tag{50}$$

To obtain the total current in the transmission-line impedances it is necessary to add the contribution of the direct field to (47). To do this reference is made to the phase diagram shown in Figure 7. Evidently,

$$E_{zd}^a = \frac{E_z^{inc}}{2}\left(e^{-j\beta c \sin \Phi} - e^{-j\beta b \sin \Phi}\right) \tag{51}$$

Since $\beta c \sim \beta b << 1$,

$$E_{zd}^a \sim -j\frac{E_z^{inc}}{2}(\beta d \sin \Phi) \tag{52}$$

In deriving (52), it is apparent that shadowing effects of the scattering antenna have been ignored.

Setting

$$E_{zT}^a = E_{zs}^a + E_{zd}^a \tag{53}$$

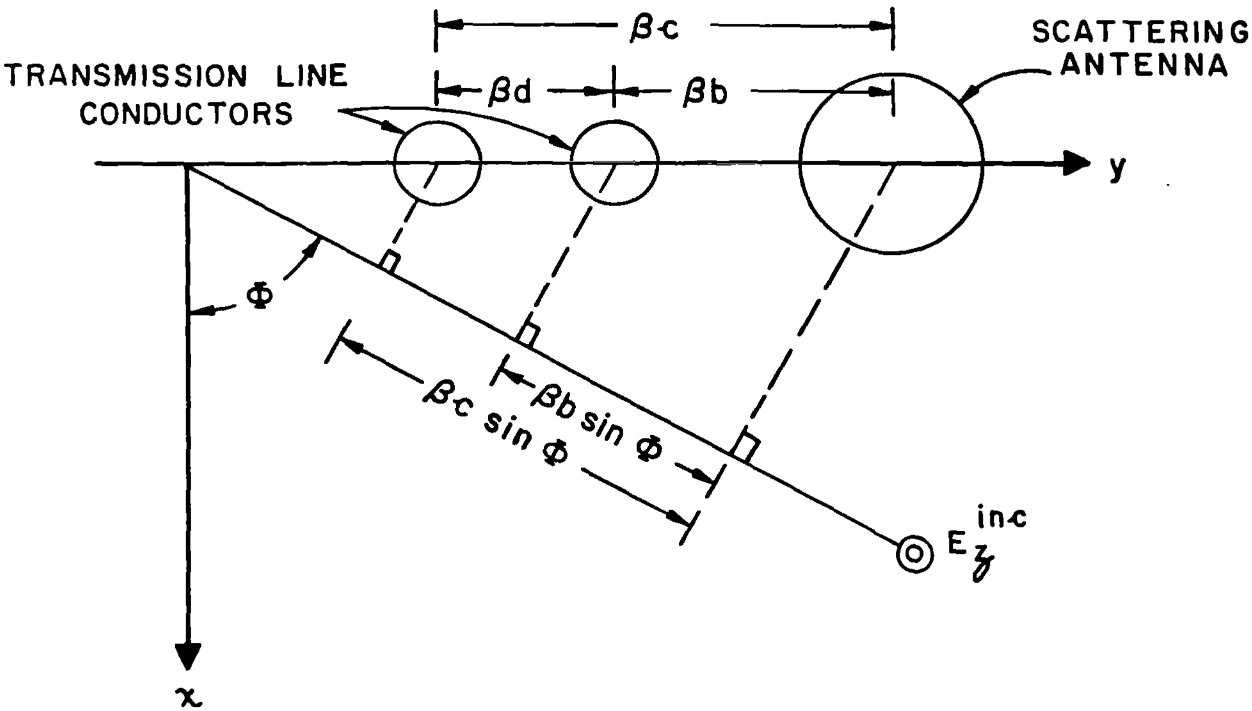

Figure 7. Diagram for Establishing the Phase Relationships Between the Scattering Antenna and the Transmission Line Conductors for the Incident Field

one obtains

$$-\frac{V_z}{2} = E^a_{zT}\,dz = -j\,\frac{E^{inc}_z}{2}\left\{C_1 \cos\beta z + C_2\right\} dz \tag{54}$$

where the dimensionless constants C_1 and C_2 are

$$C_1 = \frac{\zeta_o \beta h_e \ln\left(\frac{c}{b}\right)}{\pi Z_o(1 - \cos\beta h)} \tag{55}$$

and

$$C_2 = -\left[\frac{\zeta_o \beta h_e}{\pi Z_o(1 - \cos\beta h)} \ln\left(\frac{c}{b}\right)\cos\beta h - \beta d \sin\Phi\right]. \tag{56}$$

Substituting (54) into (48) and (49), and integrating over the range $-s \le z \le s$,

$$I(s) = j\,\frac{E^{inc}_z}{2\beta D_2}\left\{C_1(2\beta s + \sin 2\beta s)(Z_c \cos\beta s + jZ_{-s}\sin\beta s) + 2C_2\left[Z_c \sin 2\beta s + jZ_{-s}(1 - \cos 2\beta s)\right]\right\} \tag{57}$$

$$I(-s) = j\,\frac{E^{inc}_z}{2\beta D_2}\left\{C_1(2\beta s + \sin 2\beta s)(Z_c \cos\beta s + jZ_s\sin\beta s) + 2C_2\left[Z_c \sin 2\beta s + jZ_s(1 - \cos 2\beta s)\right]\right\}. \tag{58}$$

Equations (57) and (58) are the final expressions for the load currents I(s) and I(−s) resulting from the action of the total field (incident plus back-scattered) driving the transmission line. The constants C_1 and C_2 are defined by (55) and (56), respectively. D_2 is given by (50).

Illustrative Example

For the receiving and scattering antenna let $\beta h = \frac{\pi}{2}$ and $\Omega = 2\,\ell n\left(\frac{2h}{a}\right) = 7$. For the transmission line let $\beta s = \frac{\pi}{4}$, $Z_{-s} = Z_s = Z_c$, and $\Phi = 0^\circ$. Assume that the spacing of the line $d = 0.5$ cm, and that the line is constructed of A.W.G. No. 18 wire (40.30 mils in diameter). Further, assume that the inside wire of the line clears the cylinder surface by 0.5 cm. Let the incident electric field E_z^{inc} be 1 volt/m at a frequency of $f = 8.485$ mc/sec.

From these data it follows that $C_2 = 0$, $\sin 2\beta s = 1$, and $\cos 2\beta s = 0$. Also D_2, as given by (50), becomes $D_2 = j2Z_c^2$. With these simplifications Equation (57) reduces to

$$I(s) = I(-s) = \frac{\zeta_o E_z^{inc}\lambda(1 + j)\left(1 + \frac{\pi}{2}\right)\beta h_e}{8\pi^2\sqrt{2}Z_c Z_o}\,\ell n\left(\frac{c}{b}\right). \tag{59}$$

Evidently,

$$Z_c = 120\,\ell n\left(\frac{0.5}{0.05118}\right) = 273.5 \text{ ohms}$$

and $\lambda = 35.356$ m, so that $2h = 17.678$ m $= 58$ feet. Also $a = h/16.56 = 53.376$ cm. For the cylinder,[6] $\beta h_e \sim 1.238 - j0.13$ and $Z_o \sim 94 + j33.7$ ohms. From the foregoing it follows that $b = 53.876$ cm and $c = 54.376$ cm (Figure 6) so that $c/b = 1.00928$. Now $\ell n\,(1 + x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 + \cdots$ for $x^2 < 1$. Therefore, $\ell n\,(c/b) \sim 9.28 \times 10^{-3}$. Substituting these data (in sequence) into (59) gives

$$I(s) = \frac{(120\pi)(1)(35.356)(1 + j)(1 + \frac{\pi}{2})(1.238 - j0.13)(9.28)10^{-3}}{8\pi^2\sqrt{2}(273.5)(94 + j33.7)}$$

so that $|I(s)| = 0.1836$ ma for an incident electric field of 1 volt/m. For this field the voltage across $Z_s = Z_c$ is $|V_s| = |I(s)|\,Z_s = 0.0502$ volts.

Conclusions

Formulas for the load currents of a two-wire transmission-line loop with unequal terminating impedances have been derived, and compatibility with folded antenna theory exhibited when the structures are excited by plane-wave electric fields polarized parallel to the conductors. The z-component of the near-zone electric field of a receiving and scattering antenna was obtained and used in predicting the response of a transmission-line loop located close to the scatterer.

[6] Ibid., p. 12.