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Response of Transmission Lines in Proximity to a Cylindrical Scatterer

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A wire is run very close to the surface of a highly conducting cylinder and parallel to its axis. The cylinder is longer and of much larger radius than the wire. Arbitrary values of impedance are connected between the ends of the wire and the cylinder. One objective of this study is to develop formulas for the currents in the load impedances when the transmission line formed by the wire and its image is driven by the nonuniform field consisting of the superposition of the incident plane-wave field and the backscattered field from the cylinder.

A second objective is to obtain formulas for the load currents in the terminating impedances of a transmission-line loop oriented parallel to and in the near-zone field of the cylinder. Coupling to the image two-wire line is taken into account.

Since the theory depends fundamentally on the field very close to an unloaded cylindrical receiving antenna of moderate diameter, a portion of the paper is devoted to a short-cut procedure for obtaining expressions for this field which satisfy the boundary conditions.

A brief tabulation of integral values required in the theory is supplied.

1. Introduction

Exposed unshielded cables are frequently used for interconnecting the electronics equipment in missiles. These cables ordinarily run parallel to the axis of the missile and are furrowed into its outside surface. The cables need not terminate at points equidistant from the ends of the missile, and cable runs of a fraction of a meter to somewhat over three-quarters of the missile length are common.

In this paper the missile is replaced by a perfectly conducting cylindrical tube of the same length and radius. Although the restrictions $\beta a \ll 1$ and $h \gg a$ are imposed at the outset in the theory, so that shadowing effect is ignored, the author strives only to achieve order of magnitude estimates of the amplitude of the currents which flow in the cable-terminating impedances when the incident plane-wave electric field is directed parallel to the axis of the cylinder. Here β is the radian wave number and h and a are the half-length and radius of the cylinder (scattering antenna), respectively.

In this study, two cable-cylinder models are chosen. In one model, a wire is run close to the surface and parallel to the axis of an isolated cylinder of much larger radius and somewhat longer length. Arbitrary values of impedance are connected from the ends of the wire to the cylinder, i.e., the cylinder acts as the return electrical path. Evidently, for this arrangement of wire and cylinder the resultant field, consisting of a superposition of the incident plane-wave and backscattered fields, excites the wire and its image. The other model consists of a terminated two-wire transmission line in juxtaposition to the cylinder. It is assumed that the orientation of the wires comprising the transmission line is such that their axes and the axis of the cylinder all lie in the same plane. Coupling to the image two-wire line is taken into account in the theory. For both cable-cylinder models, the objective is to develop formulas for the currents in the load impedances of the lines. With these formulas, it is possible to obtain some idea of the seriousness of the problem of RF interference pickup by exposed unshielded missile cabling for a specified incident plane-wave electric field. To facilitate studies of cable-cylinder receiving configurations, a brief tabulation of integral values required in the theory is presented.

2. Distribution of Current Along an Isolated Unloaded Scattering Antenna [1]

The integral equation governing the distribution of complex current $I_z(z)$ along a continuous perfect conductor of radius a extending from $z=-h$ to $z=h$, when the incident electric field $E_z^{\rm inc}$ is a plane wave directed parallel to the axis of the cylinder, is (King, 1956)

$$\int_{-h}^{h} I_z(z')K_d(z, z')dz' = j\frac{4\pi}{\zeta \cos \beta h}(U^i+U^s)F_{0z}. \quad (1)$$

Here $K_d(z, z') = K_a(z, z') - K_a(h, z')$

$$= K_{dR}(z, z') + jK_{dI}(z, z') \quad (2)$$

[1] The analysis of this section is based on a report by King (1964). See also, King and Wu (1965).

$$K_{dR}(z, z') = \frac{\cos \beta R}{R} - \frac{\cos \beta R_h}{R_h} \tag{3}$$

$$K_{dI}(z, z') = -\left(\frac{\sin \beta R}{R} - \frac{\sin \beta R_h}{R_h}\right) \tag{4}$$

$$R = \sqrt{(z-z')^2 + a^2}; \qquad R_h = \sqrt{(h-z')^2 + a^2}; \tag{5}$$

β is the radian wave number, $\beta = 2\pi/\lambda$. The value ζ is the characteristic resistance of space, $\zeta = 120\pi$ ohms;

$$U^i = -E_z^{\text{inc}}/\beta \tag{6}$$

$$U^s = -j\frac{\zeta}{4\pi}\int_{-h}^{h} I_z(z')K_a(h, z')dz' \tag{7}$$

$$F_{0z} = \cos \beta z - \cos \beta h. \tag{8}$$

The derivation of the one-dimensional integral eq (1) for the current in a cylindrical scattering antenna presupposes that the cylinder is illuminated uniformly around its circumference by the incident electric field. Hence, the backscattered and resultant (incident plus backscattered) fields in proximity to the scatterer must be rotationally symmetrical about the axis of the cylinder.

It has been shown (King, 1959; King and Wu, 1964) that the integral

$$\int_{-h}^{h} I_z(z')K_{dR}(z, z')dz'$$

varies in the same manner as $I_z(z)$, and that the integral

$$\int_{-h}^{h} I_z(z')K_{dI}(z, z')dz'$$

varies as
$$F'_{0z} = \cos\frac{\beta z}{2} - \cos\frac{\beta h}{2} \tag{9}$$

for reasonably slowly varying current distributions.[2] It is assumed that $\beta h \leq 5\pi/4$.

The foregoing remarks suggest that $I_z(z)$ may be approximated by

$$I_z(z) = I_U(z) + I_D(z), \tag{10}$$

$$I_U(z) = I_U F_{0z}; \qquad I_D(z) = I_D F'_{0z}, \tag{11}$$

where I_U and I_D are complex constants to be determined. Also, based on what has been said about the way in which the several integrals vary, one may write

$$\int_{-h}^{h} I_U(z')K_{dR}(z, z')dz' \doteq I_U(z)\Psi_{dUR}, \tag{12}$$

$$\int_{-h}^{h} I_D(z')K_d(z, z')dz' \doteq I_D(z)\Psi_{dD}, \tag{13}$$

$$\int_{-h}^{h} I_U(z')K_{dI}(z, z')dz' \doteq \frac{I_U}{I_D} I_D(z)\Psi_{dUI}. \tag{14}$$

The $\Psi(z)$ functions are determined when (11) is substituted into (12)–(14). Since these functions are substantially constant in value along the antenna when $\beta h \leq 5\pi/4$, the constant values $\Psi(0)$ are employed in the analysis.[3]

Substituting (10) into (7),

$$U^s = -j\frac{\zeta}{4\pi}[I_U\Psi_U(h) + I_D\Psi_D(h)]. \tag{15}$$

The Ψ constants introduced into the theory are defined appendix I.

Substituting (10) into (1) and making use of (11)–(14) leads to the relation

$$I_U\Psi_{dUR}F_{0z} + (jI_U\Psi_{dUI} + I_D\Psi_{dD})F'_{0z} = j\frac{4\pi}{\zeta \cos \beta h}(U^i + U^s)F_{0z}. \tag{16}$$

Equating coefficients of the functions F_{0z} and F'_{0z} leads to the simultaneous equations

$$I_U[\Psi_{dUR}\cos \beta h - \Psi_U(h)] + I_D[-\Psi_D(h)] = j\frac{4\pi U^i}{\zeta} \tag{17}$$

$$I_U[j\Psi_{dUI}] + I_D[\Psi_{dD}] = 0, \tag{18}$$

where in writing (17) use was made of (15). It follows that

$$I_U = j\frac{4\pi U^i}{\zeta}\frac{\Psi_{dD}}{Q}, \tag{19}$$

$$I_D = \frac{4\pi U^i}{\zeta}\frac{\Psi_{dUI}}{Q}. \tag{20}$$

[2] It is at this point that an improvement in the theory of the unloaded receiving and scattering antenna is effected. Heretofore it has been assumed that the integral

$$\int_{-h}^{h} I_z(z')K_{dI}(z, z')dz'$$

varies as F_{0z} instead of F'_{0z}.

[3] R. W. P. King furnished the writer plots of $\Psi(z)$ versus βz for $\beta h = \pi, 3\pi/4$, and $\pi/2$ in a private communication.

where

$$Q=\Psi_{dD}[\Psi_{dUR}\cos\beta h-\Psi_U(h)]+j\Psi_{dUI}\Psi_D(h). \tag{21}$$

Having determined the constants I_U and I_D, the current $I(z)$, as given by (10), is

$$I_z(z)=j\frac{4\pi U^i}{\zeta Q}\left[\Psi_{dD}(\cos\beta z-\cos\beta h)-j\Psi_{dUI}\left(\cos\frac{\beta z}{2}-\cos\frac{\beta h}{2}\right)\right]. \tag{22}$$

This is the final expression for the approximate distribution of current along an unloaded receiving and scattering antenna. The incident plane-wave electric field is directed parallel to the axis of the cylinder.

3. The Near Field of an Isolated Unloaded Scattering Antenna

The magnetic field $B_\theta(\rho, z)$ near the cylinder is

$$B_\theta(\rho,z)\doteq\frac{\mu_0}{2\pi\rho}I_z(z)\doteq j\frac{2U^i}{c\rho Q}\left[\Psi_{dD}(\cos\beta z-\cos\beta h)-j\Psi_{dUI}\left(\cos\frac{\beta z}{2}-\cos\frac{\beta h}{2}\right)\right]. \tag{23}$$

Here c is the velocity of light, $c=3\times10^8$ m/sec and μ_0 is the permeability of space; $\mu_0=4\pi\times10^{-7}$ H/m. Evidently, this result was obtained applying the generalized form of Ampere's law. It is apparent that the so-called displacement term, which involves the resultant (incident plus backscattered) electric field $E_z(\rho, z)$ contributes negligibly to the determination of $B_\theta(\rho, z)$, especially since $E_z(\rho, z)$ must be small near the skin of the missile. (On the surface of the missile, the boundary condition $E_z(a, z)=0$ applies.) Accordingly, (23) may be regarded as an approximate formula for *either* the *backscattered* or *resultant* magnetic field.

The radial component of the electric field near the cylinder is

$$E_\rho(\rho,z)\doteq\frac{q(z)}{2\pi\epsilon_0\rho}, \tag{24}$$

where ϵ_0 is the dielectric constant of space; $\epsilon_0=8.85\times10^{-12}$ F/m. But, from the equation of continuity,

$$q(z)=\frac{j}{\omega}\frac{dI_z(z)}{dz}; \tag{25}$$

hence,

$$E_\rho(\rho,z)=\frac{j}{2\pi\omega\epsilon_0\rho}\frac{dI_z(z)}{dz}$$

$$=\frac{2U^i}{\rho Q}\left(\Psi_{dD}\sin\beta z-j\frac{\Psi_{dUI}}{2}\sin\frac{\beta z}{2}\right). \tag{26}$$

Since there is no radial component of the incident field, the *resultant* and *backscattered* radial fields are *identical.*

The tangential component of electric field may be obtained from the Maxwell equation

$$\nabla x\mathbf{E}=-j\omega\mathbf{B} \tag{27}$$

or

$$\frac{\partial E_z(\rho,z)}{\partial\rho}=\frac{\partial E_\rho(\rho,z)}{\partial z}+j\omega B_\theta(\rho,z). \tag{28}$$

It follows that

$$\frac{\partial E_z(\rho,z)}{\partial\rho}\doteq\frac{1}{2\pi\rho}\left[\frac{1}{\epsilon_0}\frac{\partial q(z)}{\partial z}+j\omega\mu_0 I_z(z)\right]. \tag{29}$$

This expression may be integrated with respect to ρ from $\rho=a$ (where the resultant field $E_z(a, z)=0$) to ρ. The result is

$$E_z(\rho,z)\doteq\frac{1}{2\pi}\left[\frac{1}{\epsilon_0}\frac{\partial q(z)}{\partial z}+j\omega\mu_0 I_z(z)\right]\ln\left(\frac{\rho}{a}\right), \tag{30}$$

or

$$E_z(\rho,z)\doteq\frac{2\beta U^i}{Q}\left[\Psi_{dD}\cos\beta h+j\Psi_{dUI}\left(\frac{3}{4}\cos\frac{\beta z}{2}-\cos\frac{\beta h}{2}\right)\right]\ln\left(\frac{\rho}{a}\right). \tag{31}$$

This is the final formula for the *resultant* (incident plus backscattered) tangential field close to the skin of the missile. The $E_z(\rho, z)$ is the resultant field because $E_\rho(\rho, z)$ and $B_\theta(\rho, z)$ (which are themselves resultant fields) are used in constructing the expression for $E_z(\rho, z)$.. Note that the boundary condition $E_z(a, z)=0$ is satisfied by (31).

The formulas for $B_\theta(\rho, z)$, $E_\rho(\rho, z)$ and $E_z(\rho, z)$ are useful when $\rho\leqslant 10a$, $|z|$ is not too near h, and βh is not too near $\pi/2$.

4. Isolated Two-Wire Transmission Line Immersed in a Plane-Wave Electric Field

Consider a dissipationless transmission line of length s terminated in impedance Z_0 at $\xi=0$ and Z_s at

$\xi=s$, as shown in figure 1. The currents in Z_0 and Z_s are I_0 and I_s, respectively, and the characteristic impedance of the line is $Z_c=(\zeta/\pi)\ \ln\ (b/a_1)$, where b is the spacing between the centers of the wires of equal radius a_1. The line is driven by two arbitrarily positioned (but nonstaggered) impedanceless generators of the instantaneous polarity specified in the drawing and of voltage V/2.

Using elementary transmission-line formulas, it is a simple matter to show that

$$I_s=\frac{V}{D_1}\ [Z_c \cos\beta\xi+jZ_0 \sin\beta\xi], \tag{32}$$

and

$$I_0=\frac{V}{D_1}\ [Z_c \cos\beta(s-\xi)+jZ_s \sin\beta(s-\xi)], \tag{33}$$

where

$$D_1=Z_c(Z_0+Z_s)\cos\beta s+j(Z_0Z_s+Z_c^2)\sin\beta s. \tag{34}$$

Equations (32) and (33) are very important results because they permit determining the response of transmission lines excited by uniform as well as nonuniform fields.

Suppose that the transmission line is excited by a linearly polarized field E_ξ^{inc}, as shown in figure 2. If the reference for phase is taken midway between the centers of the conductors, and the azimuth angle Φ is measured from the positive x axis of a Cartesian coordinate system x, y, ξ, as shown, the fields $E_{\xi 1}$ and $E_{\xi 2}$ acting on conductors 1 and 2, respectively, are

$$E_{\xi 1}=E_\xi^{\text{inc}}e^{\,j\frac{\beta b}{2}\sin\Phi}, \tag{35}$$

$$E_{\xi 2}=E_\xi^{\text{inc}}e^{-j\frac{\beta b}{2}\sin\Phi}. \tag{36}$$

To resolve these fields into symmetrical and antisymmetrical components, one sets

$$E_{\xi 1}=E_\xi^s+E_\xi^a \tag{37}$$

$$E_{\xi 2}=E_\xi^s-E_\xi^a \tag{38}$$

so that

$$E_\xi^s=\frac{E_{\xi 1}+E_{\xi 2}}{2}=E_\xi^{\text{inc}}\cos\left(\frac{\beta b}{2}\sin\Phi\right) \tag{39}$$

and
$$E_\xi^a=\frac{E_{\xi 1}-E_{\xi 2}}{2}=jE_\xi^{\text{inc}}\sin\left(\frac{\beta b}{2}\sin\Phi\right). \tag{40}$$

The even field, E_ξ^s sets up the dipole mode in the structure, and E_ξ^a, the odd field, sets up the transmission-line mode. It is assumed that these modes do not

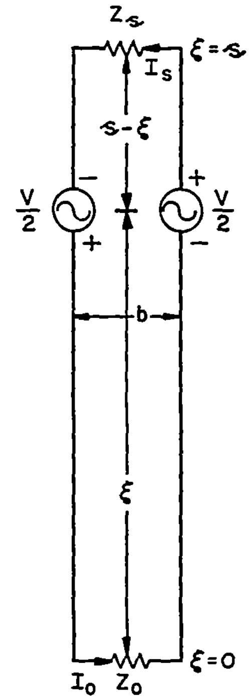

FIGURE 1. *Terminated transmission line driven by two nonstaggered series generators of voltage V/2.*

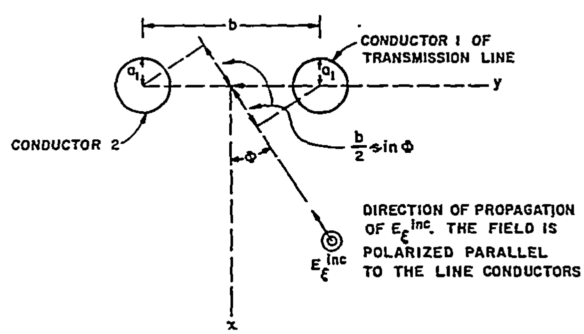

FIGURE 2. *Geometry of the incident electric field and conductors of the transmission line.*

couple. Since E_ξ^a, the differential electric field, acts at all points along the conductors, the cumulative effect may be obtained by integration. Evidently,

$$\frac{V}{2}=-E_\xi^a d\xi=-jE_\xi^{\text{inc}}\sin\left(\frac{\beta b}{2}\sin\Phi\right)d\xi. \tag{41}$$

Substituting (41) into (32) and (33), and integrating over the range $\xi=0$ to $\xi=s$, yields the results

$$I_s=-j\,\frac{2}{\beta D_1}\,E_\xi^{\rm inc}\,\sin\left(\frac{\beta b}{2}\sin\Phi\right)$$
$$[Z_c\sin\beta s+jZ_0(1-\cos\beta s)] \qquad (42)$$

and

$$I_0=-j\,\frac{2}{\beta D_1}\,E_\xi^{\rm inc}\,\sin\left(\frac{\beta b}{2}\sin\Phi\right)$$
$$[Z_c\sin\beta s+jZ_s(1-\cos\beta s)]. \qquad (43)$$

These are the final expressions for the currents in the loads of a transmission line when excited by a plane-wave electric field polarized parallel to the conductors. The same results may be obtained by employing the integral equation approach to folded receiving antenna theory (Harrison, 1965).

5. Terminated Wire Oriented Parallel to a Missile Axis

Figure 3 illustrates an isolated missile of radius a and length $2h$ with a wire of radius a_1 and length s run parallel to its axis at a distance $d-a$ from its skin. The coordinate axis displacement is given by $g=z-\xi$. It is assumed that the line does not terminate too close to either end of the missile where (31), the formula for the resultant field $E_z(\rho, z)$, may be inaccurate. Arbitrary values of impedance Z_0 and Z_s are connected

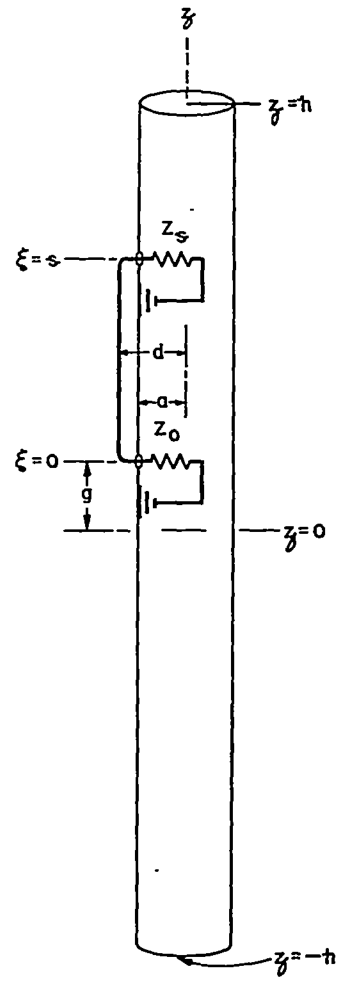

FIGURE 3. *Terminated wire oriented parallel to the axis of a cylinder.*

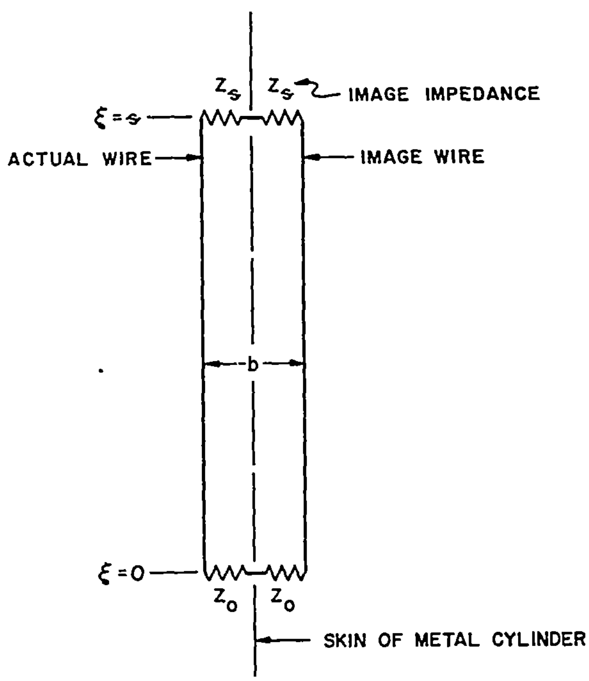

FIGURE 4. *Terminated single-wire line and its image.*

between the ends of the wire and missile at $\xi=0$ and $\xi=s$, respectively. The present purpose is to develop formulas for the currents in the load impedances when the transmission line formed by the wire and its image is driven by the nonuniform field $E_z(\rho, z)$.

Since it is assumed that $a \gg a_1$ and that $d \sim a (d>a)$, i.e., the wire just clears the missile skin, it is an excellent approximation to image the wire in an infinite plane. The coupling of the wire and its image is taken into account in the characteristic impedance of the line. In this case $Z_c=(\zeta/2\pi)\ \ln\ [2(d-a)/a_1]$. The configuration to be analyzed is shown in figure 4. The two-wire line, formed by the wire and its image, is terminated in impedances $2Z_0$ and $2Z_s$. Also observe that the *differential electric field* effective in driving the two-wire line in this case is the *resultant field* $E_z(\rho, z)$.

One proceeds as follows:
(a) Write (31) in the form

$$E_\xi(d, \xi) = -E_\xi^{\text{inc}}\left[K_1 + K_2 \cos\frac{\beta}{2}(g+\xi)\right], \quad (44)$$

where

$$K_1 = -j\frac{2}{Q}\left[\Psi_{dUI}\cos\frac{\beta h}{2} + j\Psi_{dD}\cos\beta h\right]\ln\left(\frac{d}{a}\right) \quad (45)$$

and

$$K_2 = j\frac{3}{2Q}\Psi_{dUI}\ln\left(\frac{d}{a}\right). \quad (46)$$

(Note that $E_z^{\text{inc}} = E_\xi^{\text{inc}}$ and $z = g+\xi$.)
(b) Observe that V, as it appears in (32) and (33), is

$$V = -2E_\xi(d, \xi)d\xi. \quad (47)$$

(c) Let

$$Z_0 \to 2Z_0 \quad (48)$$

and

$$Z_s \to 2Z_s \quad (49)$$

in (32) and (33) to take into account the image terminations.

Using (44) and (47)–(49) in (32)–(34) leads to the following integrals and results:

$$I_s = \frac{2E_\xi^{\text{inc}}}{D_2}\int_0^s\left[K_1 + K_2\cos\frac{\beta}{2}(g+\xi)\right]$$

$$[Z_c\cos\beta\xi + j2Z_0\sin\beta\xi]d\xi$$

$$= \frac{2E_\xi^{\text{inc}}}{D_2\beta}\left[K_1 Z_c\sin\beta s + j2K_1Z_0\right.$$

$$-j2K_1Z_0\cos\beta s + K_2Z_c\sin\frac{\beta}{2}(s-g)$$

$$+\frac{1}{3}K_2Z_c\sin\frac{\beta}{2}(3s+g) - j\frac{2}{3}K_2Z_0\cos\frac{\beta}{2}(3s+g)$$

$$-j2K_2Z_0\cos\frac{\beta}{2}(s-g) + \frac{2}{3}K_2Z_c\sin\frac{\beta g}{2}$$

$$\left.+j\frac{8}{3}K_2Z_0\cos\frac{\beta g}{2}\right]. \quad (50)$$

$$I_0 = \frac{2E^{\text{inc}}}{D_2\beta}\left[K_1Z_c\sin\beta s + j2K_1Z_s - j2K_1Z_s\cos\beta s\right.$$

$$+K_2Z_c\sin\frac{\beta}{2}(g+2s) + \frac{1}{3}K_2Z_c\sin\frac{\beta}{2}(2s-g)$$

$$-j\frac{2}{3}K_2Z_s\cos\frac{\beta}{2}(2s-g) - j2K_2Z_s\cos\frac{\beta}{2}(g+2s)$$

$$\left.-\frac{2}{3}K_2Z_c\sin\frac{\beta}{2}(g+s) + j\frac{8}{3}K_2Z_s\cos\frac{\beta}{2}(g+s)\right] \quad (51)$$

where

$$D_2 = 2Z_c(Z_0+Z_s)\cos\beta s + j(4Z_0Z_s + Z_c^2)\sin\beta s; \quad (52)$$

and

$$Z_c = \frac{\zeta}{2\pi}\ln\left[\frac{2(d-a)}{a_1}\right], \quad (53)$$

as mentioned earlier.

Note that if the line termination Z_0 is located in the region $z > 0$, g is positive; if Z_0 is located in the region $z < 0$, g is negative. This change of sign of g must be taken into account in applying (50) and (51).

The analysis of the terminated wire oriented parallel to the missile axis is now complete.

6. Terminated Transmission Line in the Near-Zone Field of an Isolated Missile

Figure 5 illustrates a terminated two-wire transmission line oriented parallel to the missile axis. The arrangement of all conductors is such that their axes lie in the same plane.

Evidently the differential electric field E_z^a responsible for the transmission-line mode in the line structure is

$$E_z^a = \frac{E_z(d+b, z) - E_z(d, z)}{2}. \quad (54)$$

With (31), (54) becomes

$$E_z^a \doteq \frac{\beta U^i}{Q}\left[\Psi_{dD}\cos\beta h + j\Psi_{dUI}\left(\frac{3}{4}\cos\frac{\beta z}{2}\right.\right.$$

$$\left.\left.-\cos\frac{\beta h}{2}\right)\right]\ln\left(\frac{b+d}{d}\right), \quad (55)$$

or

$$E_\xi^a = -E_\xi^{\text{inc}}\left[K_3 + K_4\cos\frac{\beta}{2}(g+\xi)\right], \quad (56)$$

where

$$K_3=-j\frac{1}{Q}\left[\Psi_{dUI}\cos\frac{\beta h}{2}+j\Psi_{dD}\cos\beta h\right]\ln\left(\frac{b+d}{d}\right), \tag{57}$$

$$K_4=j\frac{3}{4Q}\Psi_{dUI}\ln\left(\frac{b+d}{d}\right). \tag{58}$$

Expressions (56), (57), and (58) are analogous to (44), (45), and (46), respectively.

The currents I_s and I_0 may be obtained from (50) and (51), respectively, provided the following changes are made in these formulas:

(a) Let $K_1 \rightarrow K_3$ where K_3 is given by (57),
(b) Let $K_2 \rightarrow K_4$ where K_4 is given by (58),
(c) Let $D_2 \rightarrow D_1$ where D_1 is given by (34),
(d) Set

$$Z_c=\frac{\zeta}{2\pi}\ln\left[\frac{2b\sqrt{(d-a)(b+d-a)}}{a_1(2d-2a+b)}\right] \tag{59}$$

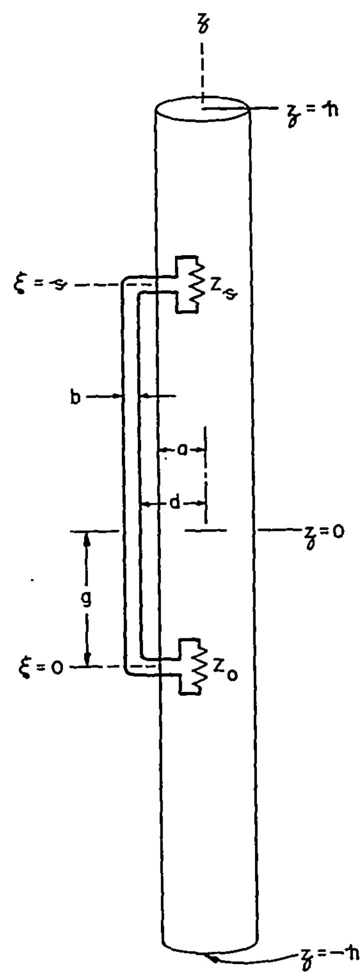

FIGURE 5. *Terminated two-wire transmission line excited by the scattered field in the vicinity of an isolated cylinder.*

in (34), (50), and (51). Here a_1 is the radius of each wire comprising the transmission-line loop. It is important to observe that coupling between the line and its image is considered, and this coupling is reflected in the theory in the form of a changed line characteristic impedance given by (59).

An analysis of the response of a terminated two-wire line in proximity to an isolated missile is now complete.

7. Conclusion

Two cable-cylinder receiving antenna configurations have been discussed in this paper. The theory is expected to yield order of magnitude results provided the transmission-line runs are very close to the cylinder skin, the cable terminations are not too close to the ends of the cylinder, and the radian half-length of the cylinder βh is not too near $\pi/2$. These restrictions are imposed primarily because of the shortcomings of the formula for $E_z(\rho, z)$ given by (31). Shadow effect is neglected in the theory as well as contributions to the currents in the transmission-line terminations due to the action of $E_\rho(\rho, z)$ (Taylor, Satterwhite, and Harrison, 1965).

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8. Appendix 1

The several Ψ functions introduced in the theory are defined as follows:

$$\Psi_{dUR}=(1-\cos\beta h)^{-1}\int_{-h}^{h}$$

$$(\cos\beta z'-\cos\beta h)\left(\frac{\cos\beta R_0}{R_0}-\frac{\cos\beta R_h}{R_h}\right)dz',$$

$$\Psi_{dD}=\left(1-\cos\frac{\beta h}{2}\right)^{-1}\int_{-h}^{h}$$

$$\left(\cos\frac{\beta z'}{2}-\cos\frac{\beta h}{2}\right)\left(\frac{e^{-j\beta R_0}}{R_0}-\frac{e^{-j\beta R_h}}{R_h}\right)dz',$$

$$\Psi_{dUI}=-\left(1-\cos\frac{\beta h}{2}\right)^{-1}\int_{-h}^{h}(\cos\beta z'-\cos\beta h)\left(\frac{\sin\beta R_0}{R_0}-\frac{\sin\beta R_h}{R_h}\right)dz',$$

$$\Psi_U(h)=\int_{-h}^{h}(\cos\beta z'-\cos\beta h)\frac{e^{-j\beta R_h}}{R_h}dz',$$

$$\Psi_D(h)=\int_{-h}^{h}\left(\cos\frac{\beta z'}{2}-\cos\frac{\beta h}{2}\right)\frac{e^{-j\beta R_h}}{R_h}dz'.$$

Here

$$R_0=\sqrt{z'^2+a^2}$$

$$R_h=\sqrt{(h-z')^2+a^2}.$$

Refer to table 1 for a tabulation of these integrals.

TABLE 1. *Unloaded receiving and scattering antenna integrals.*

$\Omega=7.0.\ \left(\frac{h}{a}=16.558\right)$

βh	Ψ_{dUR}	Ψ_{dUI}	Ψ_{dD}	$\Psi_U(h)$	$\Psi_D(h)$
0.25	4.1508	−0.0137	4.1505−j0.0034	0.0573−j0.0102	0.0144−j0.0026
0.50	4.2244	−0.1060	4.2233−j0.0270	0.2099−j0.0773	0.0536−j0.0197
0.75	4.3346	−0.3371	4.3325−j0.0879	0.4065−j0.2374	0.1065−j0.0619
1.00	4.4648	−0.7354	4.4619−j0.1983	0.5805−j0.4927	0.1583−j0.1329
1.25	4.5961	−1.2913	4.5921−j0.3634	0.6721−j0.8104	0.1937−j0.2284
1.50	4.7099	−1.9593	4.7033−j0.5812	0.6453−j1.1332	0.2017−j0.3373
π/2	4.7368	−2.1595	4.7289−j0.6514	0.6150−j1.2166	0.1981−j0.3685
1.75	4.7911	−2.6690	4.7783−j0.8429	0.4969−j1.3974	0.1776−j0.4447
2.00	4.8302	−3.3403	4.8044−j1.1347	0.2533−j1.5510	0.1243−j0.5356
2.25	4.8247	−3.8994	4.7747−j1.4393	−0.0411−j1.5663	0.0511−j0.5985
2.50	4.7791	−4.2923	4.6889−j1.7393	−0.3393−j1.4430	−0.0290−j0.6277
2.75	4.7036	−4.4923	4.5525−j2.0186	−0.6042−j1.2013	−0.1028−j0.6246
3.00	4.6125	−4.5009	4.3755−j2.2649	−0.8141−j0.8724	−0.1602−j0.5961
π	4.5599	−4.4294	4.2621−j2.3866	−0.9052−j0.6593	−0.1831−j0.5724
3.25	4.5214	−4.3425	4.1709−j2.4706	−0.9611−j0.4871	−0.1958−j0.5522
3.50	4.4457	−4.0553	3.9521−j2.6331	−1.0447−j0.0707	−0.2100−j0.5035
3.75	4.3982	−3.6815	3.7316−j2.7545	−1.0647+j0.3576	−0.2077−j0.4579
4.00	4.3895	−3.2587	3.5195−j2.8401	−1.0176+j0.7797	−0.1963−j0.4196

$\Omega=10.0.\ \left(\frac{h}{a}=74.207\right)$

βh	Ψ_{dUR}	Ψ_{dUI}	Ψ_{dD}	$\Psi_U(h)$	$\Psi_D(h)$
0.25	7.0537	−0.0137	7.0532−j0.0034	0.0600−j0.0102	0.0151−j0.0026
0.50	7.1291	−0.1060	7.1270−j0.0270	0.2203−j0.0773	0.0562−j0.0197
0.75	7.2423	−0.3371	7.2380−j0.0879	0.4288−j0.2374	0.1125−j0.0619
1.00	7.3768	−0.7357	7.3699−j0.1983	0.6179−j0.4930	0.1688−j0.1330
1.25	7.5138	−1.2920	7.5034−j0.3636	0.7257−j0.8112	0.2099−j0.2286
1.50	7.6347	−1.9609	7.6188−j0.5817	0.7144−j1.1350	0.2245−j0.3378
π/2	7.6638	−2.1615	7.6457−j0.6520	0.6879−j1.2187	0.2230−j0.3692
1.75	7.7244	−2.6721	7.6989−j0.8439	0.5785−j1.4006	0.2078−j0.4457
2.00	7.7736	−3.3456	7.7307−j1.1365	0.3424−j1.5560	0.1625−j0.5374
2.25	7.7799	−3.9074	7.7077−j1.4423	0.0486−j1.5736	0.0975−j0.6012
2.50	7.7478	−4.3034	7.6292−j1.7439	−0.2572−j1.4523	0.0256−j0.6317
2.75	7.6877	−4.5068	7.5008−j2.0253	−0.5385−j1.2124	−0.0404−j0.6301
3.00	7.6139	−4.5187	7.3325−j2.2742	−0.7735−j0.8844	−0.0905−j0.6031
π	7.5721	−4.4489	7.2242−j2.3977	−0.8824−j0.6714	−0.1098−j0.5804
3.25	7.5425	−4.3631	7.1370−j2.4831	−0.9534−j0.4990	−0.1198−j0.5609
3.50	7.4891	−4.0780	6.9279−j2.6493	−1.0761−j0.0810	−0.1288−j0.5139
3.75	7.4672	−3.7052	6.7176−j2.7749	−1.1390+j0.3503	−0.1227−j0.4701
4.00	7.4881	−3.2822	6.5160−j2.8651	−1.1355+j0.7773	−0.1091−j0.4334

9. Appendix 2. Terminated Coaxial Line in Contact With the Skin of a Missile, With Gap in the Shield

A terminated coaxial line run in contact with the skin of a missile, containing a break in the sheath at an arbitrary point, is illustrated by figure 6. An estimate of the effect of this incomplete shielding may be obtained as follows:

(a) Determine the input impedances of the line sections looking toward Z_s and Z_0 from the break in the sheath.

(b) The impedances determined from (a) are in series, forming an electrically small loop. The voltage induced into the loop is $e=-\frac{d\phi}{dt}=-j\omega AB_\theta(\rho, z)$, where $B_\theta(\rho, z)$ is given by (23) and A is the area of the loop; ρ is measured from the axis of the missile to the center of the loop.

(c) Using Ohm's law, determine line section input voltages and currents.

(d) Calculate the currents in the terminating impedances Z_0 and Z_s in terms of the sending end voltages and currents, using the conventional transmission-line equations.

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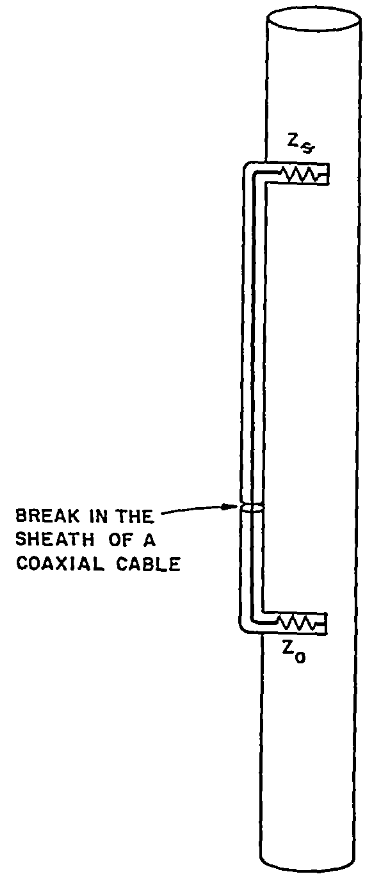

Figure 6. *Terminated coaxial line in contact with the skin of a metal cylinder, with gap in the shield.*