SC-R-68-1747

THE RESPONSE OF A TERMINATED TWO-WIRE LINE BURIED IN THE EARTH AND EXCITED BY A PLANE-WAVE RF FIELD GENERATED IN FREE SPACE

by
Charles W. Harrison, Jr.
and
Margaret L. Houston
Sandia Laboratory
Albuquerque, New Mexico

May 1968

SUMMARY

A terminated two-wire transmission line is buried at constant depth near the earth-air interface with one conductor directly below the other. A plane-wave electromagnetic field, generated in free space, impinges upon the boundary where it undergoes partial reflection and transmission. The field transmitted into the earth excites the transmission line. The polarization of the electric field is chosen such that the field is directed parallel to the line conductors. The interaction of the line with the dispersive medium and the line losses are considered. The objective of the study is to determine the current in specified load impedances in terms of the amplitude of the incident electric field evaluated at the surface of the earth.

This work was supported by the U.S. Atomic Energy Commission.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

Introduction

The purpose of the present investigation is to determine the response of an impedance-loaded two-wire transmission line that is laid in a shallow trench in the earth, with one conductor directly below the other, and driven by that portion of the incident plane-wave electromagnetic field originating in free space that is transmitted into the earth. The electric field mid-way between the line conductors is assumed to be directed parallel to the wire axes. In the theoretical development the earth is treated as a semi-infinite homogeneous dissipative medium, and it is assumed that the ditch is filled with soil following installation of the line. It is desired to develop expressions for the load currents in the line terminations in terms of the amplitude of the incident electric field evaluated at the surface of the earth.

The arrangement of the transmission line with respect to the interface, and the polarization of the incident field is depicted graphically in Figure 1. The usual right-hand Cartesian coordinate system is employed. The direction of propagation of the incident electric field is along the positive x-axis; it is directed along the negative z-axis. The incident magnetic field is directed along the positive y-axis. The interface lies in the yz plane, and the transmission-line conductors are parallel to the z-axis. An imaginary line orthogonal to the earth-air boundary contains the centers of both wires.

The procedure used to solve the problem consists in (a) integrating the differential value of the transmitted electric field over the length of the wires to obtain the load currents in terms of the amplitude of the transmitted field in the dispersive medium mid-way between the line conductors, and (b) solving the elementary boundary value problem to express the field at this point in terms of the incident field. It is to be remembered that at the boundary the sum of the incident and reflected fields equals the transmitted field.

Line losses are taken into account in the theory, as well as the interaction of the conductors with the dissipative medium. As long as the distance to the boundary is several times the line spacing there is no interaction of the transmission line mode bi-directional currents with the interface. Only the transmission line currents flow through the terminating impedances; the unidirectional or antenna current on the wires does not. If the antenna current in the line were sought, multiple reflections between the wires and the earth-air boundary would have to be taken into account. There are two additional points the writers feel require mentioning: (a) The transmitted field is a plane wave (with coincident surfaces of constant amplitude and phase) because the incident field is directed tangential to the earth's surface; (b) the line terminations play no role in the response of the line since there is no component of electric field tangential to these loads.

Elementary Transmission-Line Theory

Consider a two-wire transmission line of length s terminated in impedance Z_s, as shown in Figure 2. The conductors are of radius a and are spaced a distance b apart. The line is lossy and located in an infinite dispersive medium designated region 1. The input current I_o is delivered by generator V_o. The loa current and load voltage are I_s and V_s, respectively.

Since

$$I_o = I_s \cosh \gamma s + \frac{V_s}{Z_c} \sinh \gamma s \tag{1}$$

and

$$V_s = I_s Z_s , \tag{2}$$

it follows that

$$I_s = \frac{I_o Z_c}{Z_c \cosh \gamma s + Z_s \sinh \gamma s} . \tag{3}$$

Also it can be shown easily that the driving-point impedance of the line is

$$Z_o = Z_c \left\{ \frac{Z_s + Z_c \tanh \gamma s}{Z_c + Z_s \tanh \gamma s} \right\} . \tag{4}$$

In the above*

$$\gamma = \alpha + j\beta = \sqrt{ZY} \tag{5}$$

$$Z_c = \sqrt{\frac{Z}{Y}} \tag{6}$$

$$Z = z^i + j\omega \ell^e \tag{7}$$

$$Y = j \frac{k_1^2}{\omega \ell^e} = j \frac{\omega \pi \xi_1}{\ell n \left(\frac{b}{a} \right)} \tag{8}$$

$$z^i = \frac{1}{\pi a} \sqrt{\frac{\omega \mu_c}{2\sigma_c}} (1 + j) \tag{9}$$

$$\left(\sqrt{\omega \sigma_c \mu_c} \geq 10 \right)$$

$$\ell^e = \frac{\mu_1}{\pi} \ell n \left(\frac{b}{a} \right) \tag{10}$$

---

*See Ronold W. P. King, "Transmission-Line Theory," McGraw Hill Book Co., Chapter I, 1955

$$k_1^2 = \omega^2 \mu_1 \xi_1 \tag{11}$$

$$\xi_1 = \epsilon_1 \left(1 - j \frac{\sigma_1}{\omega \epsilon_1}\right) . \tag{12}$$

μ_1, ϵ_1, and σ_1 are the absolute permeability, dielectric constant, and conductivity of the dissipative medium, respectively. μ_c and σ_c refer to the transmission line conductors.

Application to a Terminated Two-Conductor Transmission Line Driven by Two Generators

Equations (3) and (4), if properly interpreted, may be employed to find the load currents I(h) and I(-h) of Figure 3. This figure illustrates a transmission line of length 2h driven at point z, measured from the mid-point of the line, by two generators of voltage $V_z/2$ of the polarity shown, and terminated in impedances Z_h and Z_{-h}. These impedances need not be of equal value. For example, the line might be short-circuited or open-circuited at one end, and be terminated in a finite value of impedance at the other end. From Figure 4 it is clear that the current I_g, which corresponds to I_o in Equation (1), is given by the relation

$$I_g = \frac{V_z}{Z(h - z) + Z(h + z)} . \tag{13}$$

Evidently, from Equation (4)

$$Z(h - z) = Z_c \left\{ \frac{Z_h + Z_c \tanh \gamma(h - z)}{Z_c + Z_h \tanh \gamma(h - z)} \right\} \tag{14}$$

and

$$Z(h + z) = Z_c \left\{ \frac{Z_{-h} + Z_c \tanh \gamma(h + z)}{Z_c + Z_{-h} \tanh \gamma(h + z)} \right\} . \tag{15}$$

Also, from Equation (3),

$$I(h) = \frac{I_g Z_c}{Z_c \cosh \gamma(h - z) + Z_h \sinh \gamma(h - z)} \tag{16}$$

and

$$I(-h) = \frac{I_g Z_c}{Z_c \cosh \gamma(h + z) + Z_{-h} \sinh \gamma(h + z)} . \tag{17}$$

Substituting (14) and (15) into (13), and (13) into (16) and (17), it is found after a reasonable amount of algebraic manipulation that

$$I(h) = \frac{V_z}{D} \left\{ Z_c \cosh \gamma(h + z) + Z_{-h} \sinh \gamma(h + z) \right\} \tag{18}$$

and

$$I(-h) = \frac{V_z}{D} \left\{ Z_c \cosh \gamma(h - z) + Z_h \sinh \gamma(h - z) \right\} \tag{19}$$

where

$$D = Z_c (Z_h + Z_{-h}) \cosh 2\gamma h + \left(Z_h Z_{-h} + Z_c^2 \right) \sinh 2\gamma h . \tag{20}$$

The Excitation of the Transmission Line

Refer to Figure 1. Let the reference for phase be mid-way between the conductors at depth d below the earth-air boundary. The transmitted field at that point is designated E_d^t. Evidently the fields acting tangentially to the right-hand and left-hand conductors are

$$E_r = E_d^t \, e^{-j\frac{k_1 b}{2}} \tag{21}$$

and

$$E_\ell = E_d^t \, e^{j\frac{k_1 b}{2}} \quad , \tag{22}$$

respectively.

To resolve these fields into symmetrical and antisymmetrical components, one sets

$$E_r = E^s + E^a \tag{23}$$

$$E_\ell = E^s - E^a \ . \tag{24}$$

It follows that

$$E^s = \frac{E_r + E_\ell}{2} = E_d^t \cos\left(\frac{k_1 b}{2}\right) \tag{25}$$

and

$$E^a = \frac{E_r - E_\ell}{2} = jE_d^t \sin\left(\frac{k_1 b}{2}\right) \quad . \tag{26}$$

The even field E^s is responsible for the antenna currents in the transmission line, and E^a, the odd field, sets up the transmission-line currents in the structure. Since E^a, the differential electric field acts at all points along the conductors, the cumulative effect may be obtained by integration. One sets

$$\frac{V_z}{2} = -E^a dz = -jE_d^t \sin\left(\frac{k_1 b}{2}\right) dz \quad . \tag{27}$$

Substituting (27) into (18) and (19) gives

$$I_T(h) = -j\frac{2}{D} E_d^t \sin\left(\frac{k_1 b}{2}\right)\left[Z_c \int_{-h}^{h} dz \cosh \gamma(h + z) + Z_{-h} \int_{-h}^{h} dz \sinh \gamma(h + z)\right] \tag{28}$$

$$I_T(-h) = -j\frac{2}{D} E_d^t \sin\left(\frac{k_1 b}{2}\right)\left[Z_c \int_{-h}^{h} dz \cosh \gamma(h - z) + Z_h \int_{-h}^{h} dz \sinh \gamma(h - z)\right] \tag{29}$$

Integrating yields

$$I_T(h) = -j\frac{2}{\gamma D} E_d^t \sin\left(\frac{k_1 b}{2}\right)\left[Z_c \sinh 2\gamma h + Z_{-h}(\cosh 2\gamma h - 1)\right] \tag{30}$$

and

$$I_T(-h) = -j\frac{2}{\gamma D} E_d^t \sin\left(\frac{k_1 b}{2}\right)\left[Z_c \sinh 2\gamma h + Z_h(\cos 2\gamma h - 1)\right] \quad . \tag{31}$$

Here $I_T(h)$ and $I_T(-h)$ are the total currents in the load impedances Z_h and Z_{-h}, respectively, resulting from the field acting at all points along the line.

If $Z_{-h} = 0$, (30) with (20) becomes*

$$I_T(h) = -j \frac{2}{\gamma} E_d^t \sin\left(\frac{k_1 b}{2}\right)\left[\frac{\sinh 2\gamma h}{Z_h \cosh 2\gamma h + Z_c \sinh 2\gamma h}\right] \tag{32}$$

and if $Z_{-h} = \infty$, one obtains

$$I_T(h) = -j \frac{2}{\gamma} E_d^t \sin\left(\frac{k_1 b}{2}\right)\left[\frac{\cosh 2\gamma h - 1}{Z_c \cosh 2\gamma h + Z_h \sinh 2\gamma h}\right] . \tag{33}$$

The Electromagnetic Field Transmitted into a Semi-Infinite Dissipative Medium

Let a plane-wave electromagnetic field arriving from free space be incident upon the air-earth interface located at $x = 0$. The field is polarized parallel to the z-axis, and is propagated in the positive x direction as illustrated by Figure 1. The lossless medium (air) is designated region 0, and the dispersive medium (earth) is denoted region 1.

Now in any source-free media the wave equation

$$\nabla^2 \vec{E} + k^2 \vec{E} = 0 \tag{34}$$

---

*If upon investigation it is found that $2\alpha h \geq 5$, as is likely to occur for physically long transmission lines embedded in a moderately conducting earth, and the inequality $|k_1 b| \ll 1$ is satisfied, Equations (32) and (33) reduce to the same expression, namely

$$I_T(h) = -jE_d^t \frac{k_1 b}{\gamma} \left(\frac{1}{Z_c + Z_h}\right).$$

α is defined by (5). With (45),

$$I_T(h) = -j2E_o^i \frac{k_o k_1 b}{\gamma(k_o + k_1)} e^{-jk_1 d} \left(\frac{1}{Z_c + Z_h}\right).$$

applies. In the present application (34) reduces to

$$\frac{d^2 E_z}{dx^2} + k^2 E_z = 0 . \tag{35}$$

Accordingly, one may write

$$\left(E_z^i\right)_{air} = E_o^i \, e^{-jk_o x} \tag{36}$$

$$\left(E_z^r\right)_{air} = E_o^r \, e^{jk_o x} \tag{37}$$

$$\left(E_z^t\right)_{earth} = E_o^t \, e^{-jk_1 x} . \tag{38}$$

Here the superscripts i, r, and t mean incident, reflected, and transmitted, respectively. $k_o = 2\pi/\lambda_o$ is the free-space wave number.

The boundary conditions require continuity of the electric and magnetic fields at x = 0. Hence

$$E_o^i + E_o^r = E_o^t \tag{39}$$

and

$$k_o E_o^i - k_o E_o^r = k_1 E_o^t . \tag{40}$$

In writing (40), use has been made of the relation

$$\nabla \times \vec{E} = -j\omega\vec{B} = -j\omega\mu\vec{H} \tag{41}$$

so that in the present application,

$$-\nabla_y \times \vec{E} = \frac{\partial E_z}{\partial x} = j\omega\mu H_y \quad . \tag{42}$$

It follows that

$$E_o^t = \frac{2k_o E^i}{k_o + k_1} \quad . \tag{43}$$

Accordingly,

$$\left(E_z^t\right)_{earth} = \left(\frac{2k_o E_o^i}{k_o + k_1}\right) e^{-jk_1 x} \quad . \tag{44}$$

Specifically,

$$E_d^t = \left(\frac{2k_o E_o^i}{k_o + k_1}\right) e^{-jk_1 d} \quad . \tag{45}$$

Equation (45) gives the transmitted field at the mid-point of the line (a distance d below the earth-air boundary) in terms of the incident field E_o^i at $x = 0$. Evidently evaluation of the formula

$$db_{(loss)} = 10 \log_{10} \left| \frac{\dfrac{\left(E_o^i\right)^2}{\zeta_o}}{\dfrac{\left(E_d^t\right)^2}{\zeta_1}} \right| \tag{46}$$

is of great interest because this represents the diminution of the incident field resulting from reflection at the interface and propagation into the earth. In (46)

$$\zeta_o = 120\pi \text{ ohms} \tag{47}$$

is the characteristic resistance of free space, and

$$\zeta_1 = \frac{\mu_1\omega}{k_1} = \frac{\mu_1}{\left[\mu_1\epsilon_1\left(1 - j\,\dfrac{\sigma_1}{\omega\epsilon_1}\right)\right]^{1/2}} \tag{48}$$

is the characteristic impedance of medium 1.

The power density on the boundary in medium 0 is $S_o = (E_o^i)^2/\zeta_o$ and on the boundary in medium 1 it is $S_1 = (E_o^t)^2/\zeta_1$, where $\zeta_o = 120\pi$ ohms, and $\zeta_1 = \omega\mu/k_1$. Hence, as $\omega \longrightarrow \infty$, $k_1/k_o \longrightarrow \sqrt{\epsilon_{1r}}$. Note that $\epsilon_1 = \epsilon_{1r}\epsilon_o$ with $\epsilon_o = 8.85 \times 10^{-12}$ farads/m. Equation (43) furnishes the relationship between E_o^t and E^i at the earth-air interface. It follows that $S_1 = 4\sqrt{\epsilon_{1r}}\ S_o/(1 + 2\sqrt{\epsilon_{1r}} + \epsilon_{1r})$. Accordingly, if $\epsilon_{1r} \geq 1$, $S_1 \leq S_o$.

Determination of the Incident Field E_o^i in Terms of the Input Current and Power of a Dipole Source

The leading term in the correct expression for the far-zone electric field of a dipole source is

$$E_o^i = j60I_o\,\frac{e^{-jk_oR}}{R}\left[\frac{1 - \cos k_o\ell}{\sin k_o\ell}\right]. \tag{49}$$

Here I_o is the rms input current, R is the distance from the dipole to the earth-air boundary and ℓ is the half-length of the dipole.

The power supplied to the dipole source is*

$$P = I_o^2 R_o \tag{50}$$

where R_o is the real part of the driving-point impedance Z_o.

On the Determination of the Frequency at Which the Field Reaches a Maximum in a Dispersive Semi-Infinite Space that is Moderately Conducting

Referring to (45), it is evident that at $f = 0$, $E_d^t = 0$. But for $f > 0$, $E_d^t \neq 0$. As the frequency increases further a frequency will be reached where the exponential attenuation factor is controlling. The field at depth d then decreases with increasing frequency.

Assume that

$$\sigma_1 \gg \omega\epsilon_1 \tag{51}$$

then

$$k_1 = \sqrt{\frac{\omega\mu_1\sigma_1}{2}}\,(1 - j) = C_1(1 - j)\sqrt{f}. \tag{52}$$

Also let

$$k_o = \frac{\omega}{c} = C_2 f \tag{53}$$

---

* See Ronold W. P. King, "Theory of Linear Antennas," Harvard University Press, 1956, Chapter II.

where the velocity of light is

$$c = \frac{1}{\sqrt{\mu_o \epsilon_o}} = 3 \times 10^8 \text{ m/sec} . \tag{54}$$

Write

$$y = \frac{E_d^t}{2E_o^i} = \frac{k_o}{k_o + k_1} e^{-jk_1 d} = \frac{C_2 f \, e^{-C_1 d\sqrt{f}} \, e^{-jC_1 d\sqrt{f}}}{(C_2 f + C_1\sqrt{f}) - jC_1\sqrt{f}} . \tag{55}$$

Then

$$|y|^2 = \frac{C_2^2 f \, e^{-2C_1 d\sqrt{f}}}{\left(C_2\sqrt{f} + C_1\right)^2 + C_1^2} . \tag{56}$$

Performing the operation

$$\frac{d|y|^2}{df} = 0 \tag{57}$$

on (56), the following cubic equation is obtained:

$$dC_2^2 x^3 + 2dC_1C_2x^2 + \left(2dC_1^2 - C_2\right)x - 2C_1 = 0 . \tag{58}$$

Here

$$x = \sqrt{f} . \tag{59}$$

Since C_1 and C_2 are positive real constants it follows from a seriatim inspection of the terms in (58) that only one change in sign in (58) can occur. Using Descarte's rule of signs one may then conclude that a positive real root exists. The solution of (58), i.e., the determination of the turn-over frequency, is perhaps best determined by use of a computer.

On the Possible Range of Load Currents for a Dissipationless Transmission Line Located in Free Space Excited by a Plane-Wave Electric Field Directed Parallel to the Conductors

When a lossless transmission line is located in free space γ, as given by (5), becomes

$$\gamma = j\beta = jk_o \tag{60}$$

and k_1, as defined by (11) and (12) may be written

$$k_1 = \omega\sqrt{\mu_o \epsilon_o} = k_o = 2\pi/\lambda_o \ . \tag{61}$$

If now the approximation $k_o b \ll 1$ is made in (32) and (33),

$$I_T(h) = -jE_z^{inc} b \left(\frac{\sin 2k_o h}{Z_h \cos 2k_o h + jZ_c \sin 2k_o h} \right)_{Z_{-h} = 0} \tag{62}$$

and

$$I_T(h) = E_z^{inc} b \left(\frac{1 - \cos 2k_o h}{Z_c \cos 2k_o h + jZ_h \sin 2k_o h} \right)_{Z_{-h} = \infty} \ . \tag{63}$$

Here

$$Z_c = 120 \ln \frac{b}{a} . \tag{64}$$

By inspection of (62) it is observed that when $2k_o h = n\pi$, n = 0, 1, 2 ... $I_T(h) = 0$. Also, when $2k_o h = (2n + 1) \frac{\pi}{2}$,

$$\left| I_T(h) \right| = \left| \frac{E_z^{inc} b}{Z_c} \right| .$$

Hence, for the case $Z_{-h} = 0$,

$$0 \leq \left| I_T(h) \right| \leq \left| \frac{E_z^{inc} b}{Z_c} \right| . \tag{65}$$

On the other hand, it is evident from an inspection of (63) that when $2k_o h = (2n + 1)\pi$,

$$\left| I_T(h) \right| = \left| \frac{2E_z^{inc} b}{Z_c} \right| ,$$

but when $2k_o h = 2n\pi$, $I_T(h) = 0$. Also, if $2k_o h = (2n + 1) \frac{\pi}{2}$,

$$\left| I_T(h) \right| = \left| \frac{E_z^{inc} b}{Z_h} \right| .$$

It follows that for the case $Z_{-h} = \infty$,

$$0 \leq \left| I_T(h) \right| \leq \left| \frac{E_z^{inc} b}{Z_h} \right| . \tag{66}$$

In the problem under consideration $Z_h \ll Z_c$. Accordingly, one must expect larger variations in the load current when $Z_{-h} = \infty$ than when $Z_{-h} = 0$. But it is to be remembered that the above remarks apply with rigor only to a lossless line located in a vacuum.

On the Response of a Lossless Terminated Two-Wire Transmission Line Located in Free Space to an Electromagnetic Field of Arbitrary Polarization

It is incumbent upon the authors to ascertain if an enhanced load current is possible when the electric field of the incident plane wave is no longer polarized parallel to the transmission-line conductors. Evidently, in this case the electric field is no longer precisely cross-polarized with respect to the terminations, and in any rigorous analysis the contribution to the load current due to the component of electric field parallel to the terminations must be taken into account in the theory. However, to achieve brevity in the presentation it will suffice for present purposes to consider a lossless two-wire transmission line to be located in free space, and ignore the pickup of the terminations.

A study of Figure 5 reveals that when the incident field propagates along a line making an angle θ with respect to the z-axis, Equation (27) must be written in the form

$$-\frac{V_z}{2} = jE_o^{inc} \sin\theta \, e^{jk_o z \cos\theta} \sin\left(\frac{k_o b}{2} \sin\theta\right) dz \ . \tag{67}$$

But when $Z_{-h} = 0$,

$$I(h) = V_z \left[\frac{\cos k_o(h + z)}{Z_h \cos 2k_o h + jZ_c \sin 2k_o h}\right] \tag{68}$$

and when $Z_{-h} = \infty$,

$$I(h) = jV_z \left[\frac{\sin k_o(h + z)}{Z_c \cos 2k_o h + jZ_h \sin 2k_o h}\right] . \tag{69}$$

Accordingly,

$$I_T(h) = -j \left[\frac{2E_o^{inc} \sin\theta \sin\left(\frac{k_o b}{2} \sin\theta\right)}{Z_h \cos 2k_o h + jZ_c \sin 2k_o h}\right] \int_{-h}^{h} e^{jk_o z \cos\theta} \cos k_o(h + z) dz \tag{70}$$

for $Z_{-h} = 0$, and

$$I_T(h) = \left[\frac{2E_o^{inc} \sin\theta \sin\left(\frac{k_o b}{2} \sin\theta\right)}{Z_c \cos 2k_o h + jZ_h \sin 2k_o h}\right] \int_{-h}^{h} e^{jk_o z \cos\theta} \sin k_o(h + z) dz \ . \tag{71}$$

for $Z_{-h} = \infty$.

Whether or not the maximum possible magnitude of the load current is equal to or greater for some arbitrary angle of wave arrival θ than is possible for $\theta = 90$ degrees when $Z_{-h} = 0$ may be ascertained as follows: Let $E_z^i = E_o^{inc}$ and make the approximation $k_o b \ll 1$. Then from (62) and (70),

$$\left|\sin 2k_o h\right| \leq k_o \sin^2 \theta \left| \int_{-h}^{h} e^{jk_o z \cos \theta} \cos k_o (h + z) dz \right| . \tag{72}$$

Now $\left|\sin 2k_o h\right| = 1$ if $2k_o h = (2n + 1) \frac{\pi}{2}$ for $n = 0, 1, 2 \ldots$ Also, in carrying out the integration of (72) it is convenient to change variable by setting $z = -h + t$. It follows that

$$1 \leq \sqrt{1 + \cos^2\theta - 2 \cos \theta \sin\left[(2n + 1) \frac{\pi}{2} \cos \theta\right]} . \tag{73}$$

Hence for $Z_{-h} = 0$, a θ and n exists such that the maximum load current may be somewhat greater than can be obtained for the case $\theta = 90$ degrees.

The same procedure is used to compare the magnitudes of the maximum possible load currents when $Z_{-h} = \infty$, for an arbitrary angle of wave arrival θ, with the case $\theta = 90$ degrees, except that (63) and (71) are employed. Suppose that maximum response obtains when $\theta = 90$ degrees. The demonstration of the validity of this prognosis consists in proving that

$$\left|1 - \cos 2k_o h\right| \geq k_o \sin^2 \theta \left| \int_{-h}^{h} e^{jk_o z \cos \theta} \sin k_o (h + z) dz \right| . \tag{74}$$

Now $|1 - \cos 2k_o h| = 2$ when $2k_o h = (2n + 1)\pi$, $n = 0, 1, 2 \ldots$ It follows from (74) that

$$1 \geq \cos\left[(2n + 1)\,\frac{\pi}{2}\cos\theta\right] . \tag{75}$$

An inspection of (75) reveals that the response of the line for $Z_{-h} = \infty$ can never be increased over that obtained for $\theta = 90$ degrees. But attention is invited to the fact that a judicious adjustment in line length $2k_o h$ and θ may lead to more load current than can be obtained with θ fixed at 90 degrees and $2k_o h = (2n + 1)\pi$. This point could be investigated but bear in mind that different lines are then under consideration.

The analytical work presented in this section applies to a transmission line in air. But the same phenomena might be observed when the line is embedded in a dispersive medium. Again, it is to be mentioned that the pickup in the terminations has been ignored in the theoretical development.

Numerical Results

It was deemed of interest by the writers to determine the attenuation afforded by dry earth ($\sigma = 10^{-3}$, $\epsilon_r = 7$), damp earth ($\sigma = 12 \times 10^{-3}$, $\epsilon_r = 15$), and wet earth ($\sigma = 30 \times 10^{-3}$, $\epsilon_r = 30$) to the incident electric field when polarized parallel to the earth-air interface at several observation points depth d below the surface of the earth (refer to Figure 1). Figure 6 shows the dB loss in the frequency range 10^3 to 10^9 Hz for dry earth when $d = 0.01$ m. Similar data is presented in Figure 7 except that the frequency range is 10^6 to 10^9 Hz and $d = 1.0$ m. Corresponding data are presented in Figures 8 and 9 for damp earth, and in Figures 10 and 11 for wet earth.

A two-wire transmission line for which $2h = 100$ m, $b = 2.5$ mm, $a = 0.5$ mm, $\sigma_c = 5.8 \times 10^7$ mhos/m (copper), $Z_{-h} = 0$ or ∞ and $Z_h = 5$ ohms was selected for the numerical study. The line is buried an average depth d and is assumed to be oriented with respect to the incident field and earth-air boundary as illustrated in Figure 1. The objective is to determine the response of the line for an incident field strength of 1 volt/m, i.e., the current in the terminating impedance Z_h when the other end of the line is short-circuited or open-circuited ($Z_{-h} = 0$ or ∞, respectively).

Figure 12 presents graphically the current I(h) (in μA) in the load impedance $Z_h = 5$ ohms for $Z_{-h} = 0$ or ∞ when the line is buried $d = 0.01$ m in dry earth over the frequency range 10^3 to 10^9 Hz.

Note that the response of the line for $Z_{-h} = 0$ is somewhat greater than for $Z_{-h} = \infty$ when $f < 10^5$ Hz. Similar data are presented in Figure 13 except that the frequency range is 10^6 to 10^9 Hz and $d = 1.0$ m. Observe that the results are the same whether $Z_{-h} = 0$ or ∞. Corresponding data are presented in Figures 14 and 15 for damp earth, and in Figures 16 and 17 for wet earth. Computations were also made for the case $Z_h = 1$ ohm and $Z_{-h} = 0$ or ∞, but for the sake of brevity these results are omitted. It can be stated that I(h) cannot exceed by more than a factor of 2 the results reported here when $f < 10^5$ Hz. At higher frequencies the curves all merge whether or not $Z_{-h} = 0$ or ∞. Thus proof is presented that for a buried line it does not make any difference whether the line is open or short-circuited provided $f > 10^5$ Hz and $1 \leq Z_h \leq 5$ ohms.

Conclusions

Within the restrictions stated in the text of this paper an impeccable solution to the problem of the response of a two-wire transmission line buried in a homogeneous earth to a plane-wave electromagnetic field generated in free space has been achieved. The principal restrictions imposed in the theory are that the electric field arriving at the conductors is directed parallel to their axes; one conductor of the line lies directly below the other, and the dielectric insulation surrounding the wires has negligible effect on the electrical properties of the line. Line losses as well as coupling to the dispersive medium are taken into account. There is no significant interaction of the line with the earth-air boundary if the conductor spacing is small compared to the distance of the wires to the interface. Also, for the polarization of the incident field chosen there is no interaction of the field with the line terminations.

It has been demonstrated that the pickup of an isolated transmission line for appropriate changes in length and polarization of the electric field may exceed the response of the same line for parallel polarization. It is expected that the same phenomenon will be observed when the line is embedded near the surface of a semi-infinite dissipative medium.

Evidently, if the cord is twisted, i.e., one wire not directly below the other, the response to the incident radio frequency field will be reduced.

It has been shown that when $2\alpha h \geq 5$ and $|k_1 b| \ll 1$, the formulas for the load currents are identical and assume a singularly simple form. Accordingly, from the radio-frequency hazard point of view (spurious radio signal emissions or lightening) it is immaterial whether the transmission line is open or short-circuited prior to connection of the detonation battery by the road blasting crew.

Acknowledgment

The writers thank Barbra Ford for typing this paper and Roma Schramm for preparation of the drawings.

Supplementary References

C. W. Harrison, Jr., "Response of Balanced Two- and Four-Wire Transmission Lines Excited by Plane-Wave Electric Fields," IEEE Transactions on Antennas and Propagation, Vol. AP-13, No. 2, pp 319-321, March 1965.

C. D. Taylor, R. S. Satterwhite, and C. W. Harrison, Jr.,"The Response of a Terminated Two-Wire Transmission Line Excited by a Non-Uniform Electromagnetic Field," IEEE Transactions on Antennas and Propagation, Vol. AP-13, No. 6, pp 987-989, November 1965.

Appendix

The following papers relate to determing the location of buried wires utilizing their backscattering properties:

James R. Wait, "Conducting Sphere in a Time Varying Magnetic Field," Geophysics, 1951.

C. F. Casey, Manual, Roland F. Beers, Inc., Troy, New York, "Electromagnetic Methods: Ronka and Sharpe," Troy, New York, June 15, 1959.

"Electromagnetic Surveying Equipment," Aeromagnetic Surveys Limited, Manual of Operation for Horizontal Loop, Ronka Geophysical Instruments Ltd., 1450 O'Connor Drive, Toronto, Ontario, Canada, October 20, 1956.

S. H. Ward, W. O. Carter, H. A. Harvey, G. H. McLaughlin, and W. A. Robinson, "Prospecting by Use of Natural Alternating Magnetic Fields of Audio and Sub-Audio Frequencies," Annual General Meeting, Vancouver, April 1958, Transactions, Vol. LXI, 1958, pp 261-268.

"The SE-300 Portable Electromagnetic Unit," Manual, Sharpe Model SE 300 Dual Frequency Electromagnetic Transceiver, Sharpe Instruments of Canada, Toronto, Canada.

"The SE-100 Electromagnetic Survey Unit," Manual, Sharpe Model SE-100 Electromagnetic Survey Unit, Booklet "C", Sharpe Instruments Ltd., Toronto, Canada.

"Helicopter Electromagnetic Exploration, Data Interpretation," Report, Roland F. Beers, Inc., Troy, New York, Part I Interpretation, Part II Theory and Data.

"Fixed Wing Electromagnetic Exploration, Data Interpretation," Report, Roland F. Beers, Inc., Troy, New York.

H. S. Tuan, "Scattering from a Dipole Antenna Embedded in a Dissipative Half Space, Part II," Department of Electrical Sciences, New York University, Stonybrook, New York. (Estimated completion date - July 1967.)

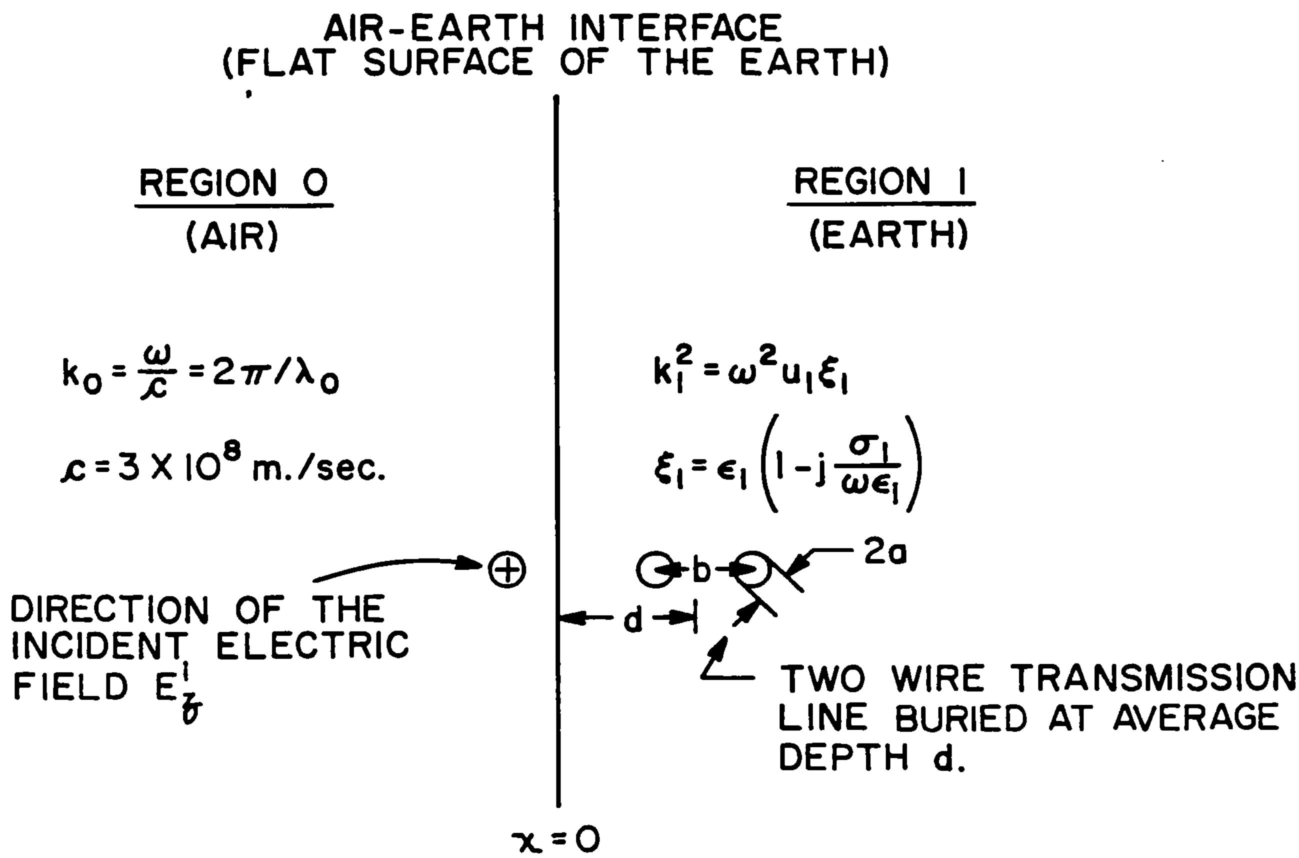

Figure 1. Two-Wire Transmission Line Buried in a Shallow Trench, and Positioned for Optimum Response to the Incident Electric Field Polarized as Indicated

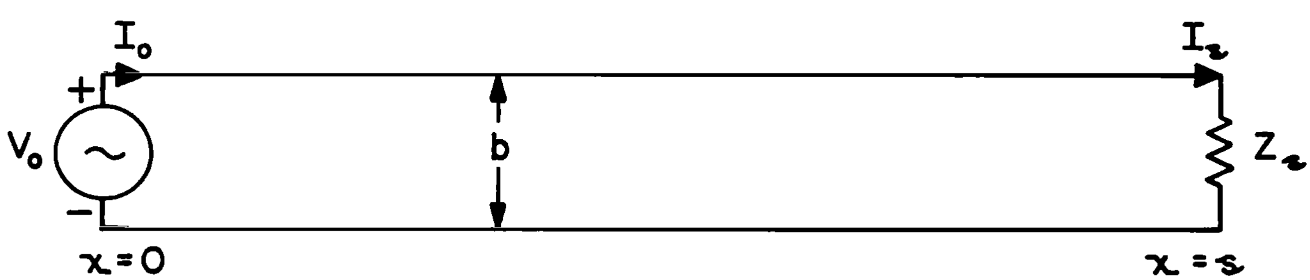

Figure 2. Transmission Line Driven by One Generator

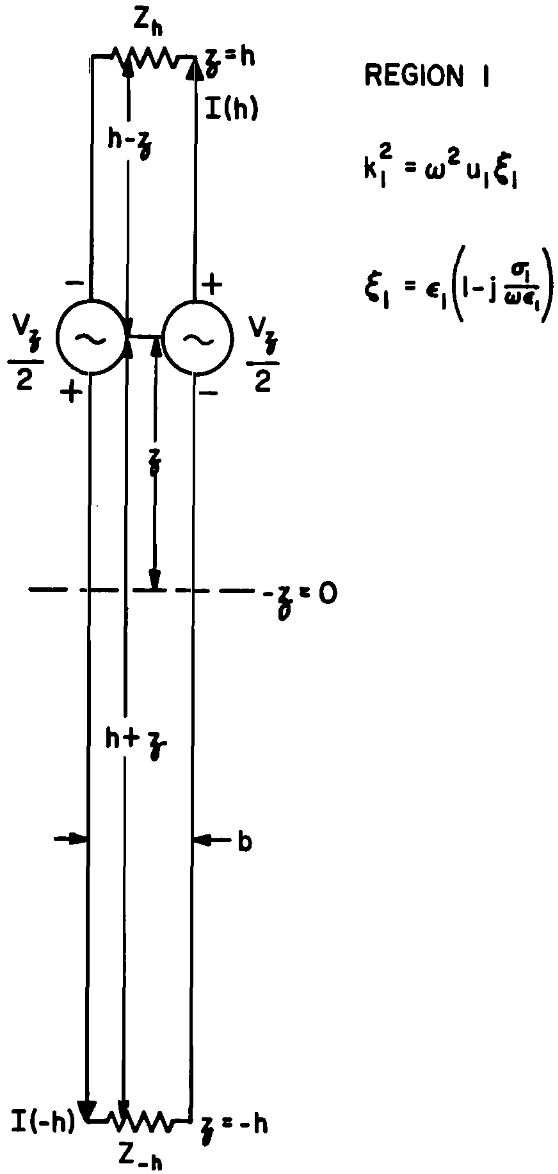

Figure 3. Lossy Transmission Line Embedded in a Dissipative Medium. The Line is Driven by Two Series Generators

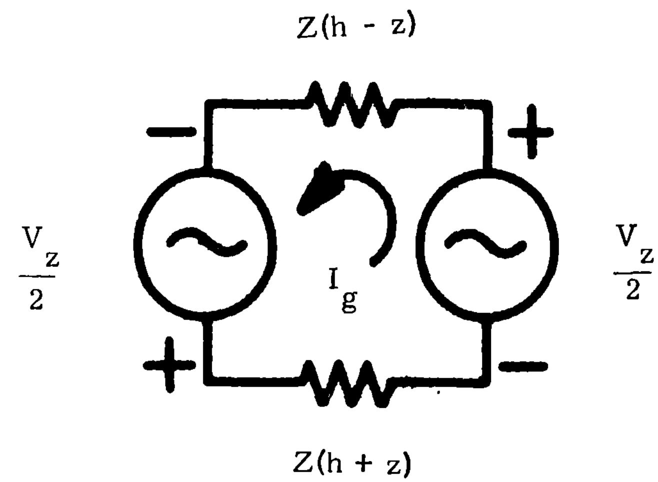

Figure 4. Generator Region of Figure 3. Z(h - z) and Z(h + z) are the Impedances Looking Toward the Loads Z_h and Z_{-h}, Respectively

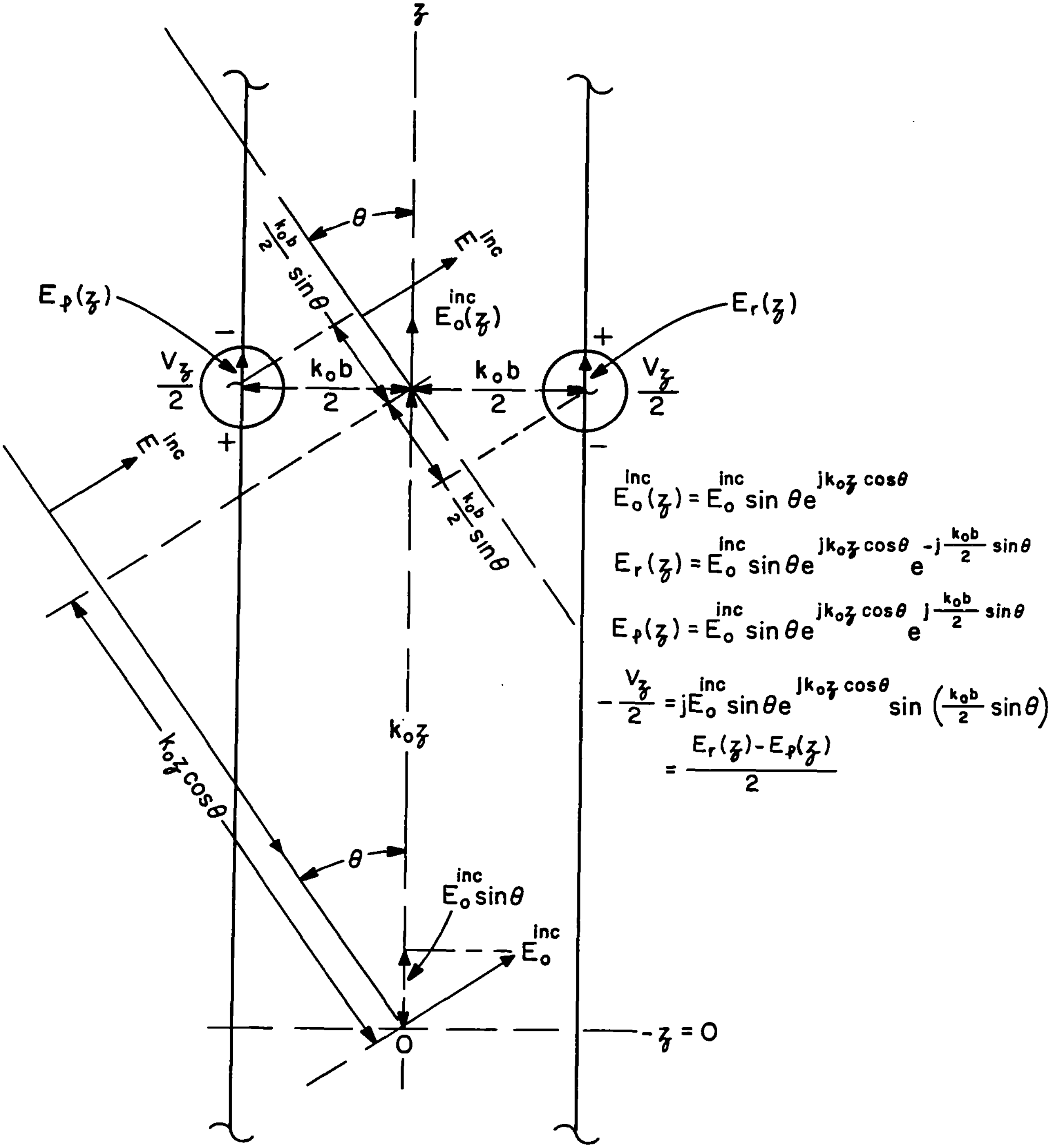

Figure 5. Construction for Obtaining $V_z/2$ in Terms of the Incident Field E_o^{inc} Referred in Amplitude and Phase to the Geometrical Center of the Transmission Line

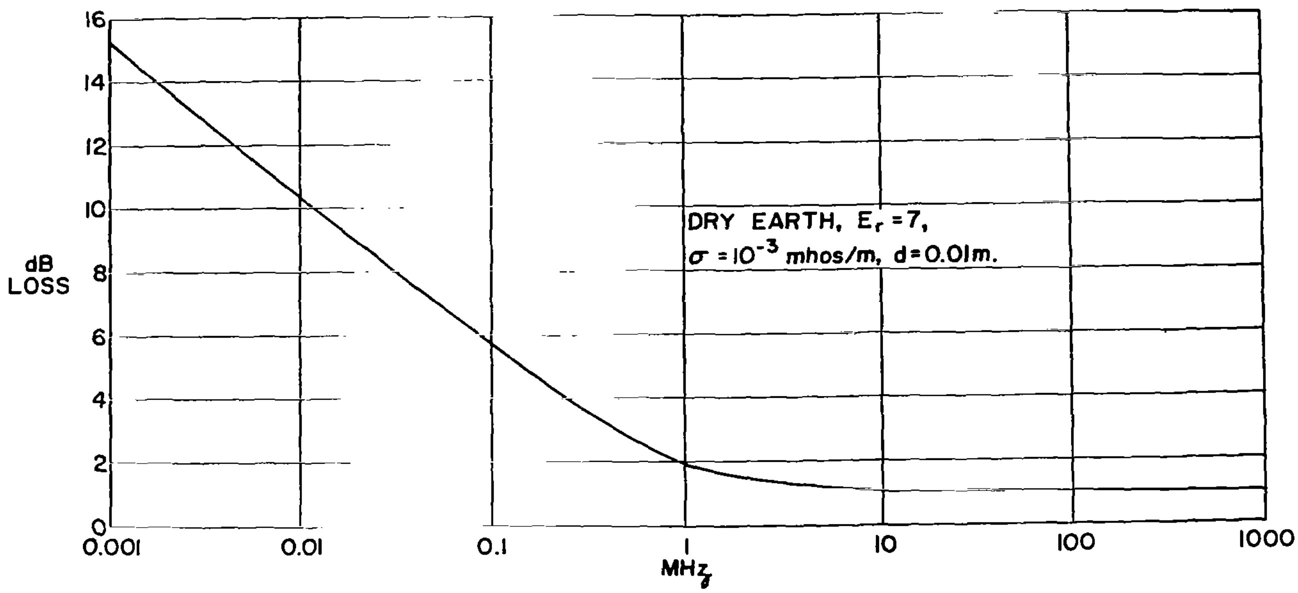

Figure 6. Dry Earth. Steady-State Transfer Characteristic Relating $\left[E_o^i(f)\right]^2 / \zeta_o$ to $\left[E_d^t(f)\right]^2 / \zeta_1(f)$ for d = 0.01 m

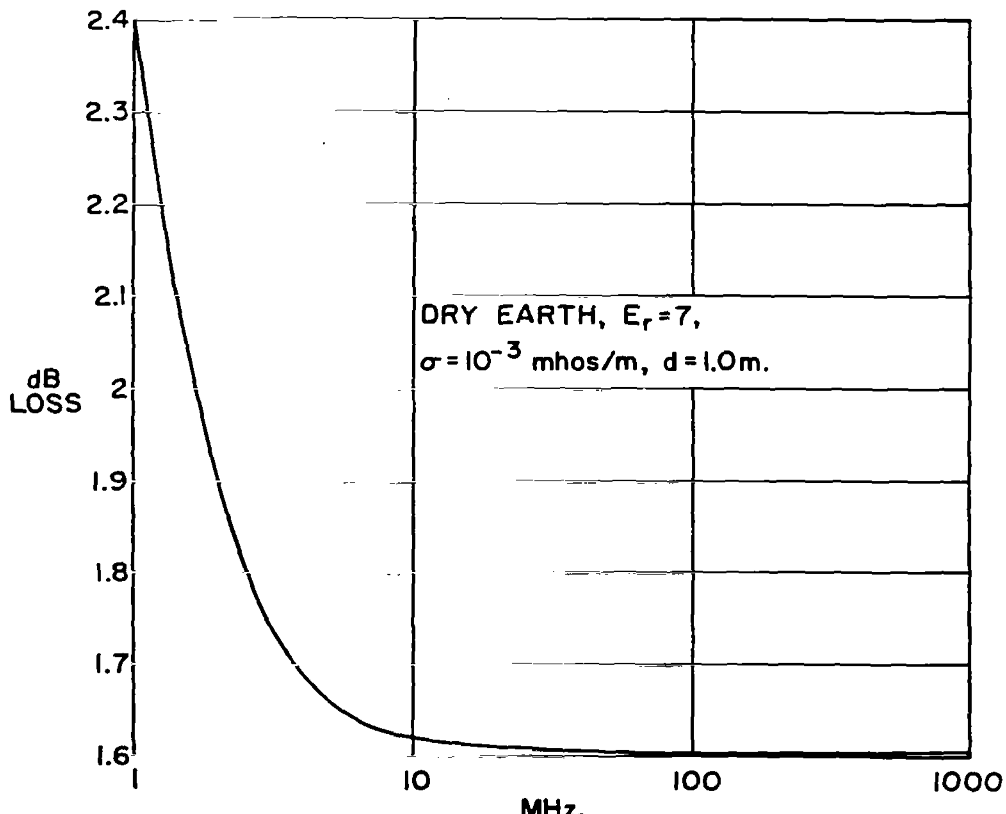

Figure 7. Dry Earth. Like Figure 6 Except for a Reduced Frequency Range and d = 1.0 m

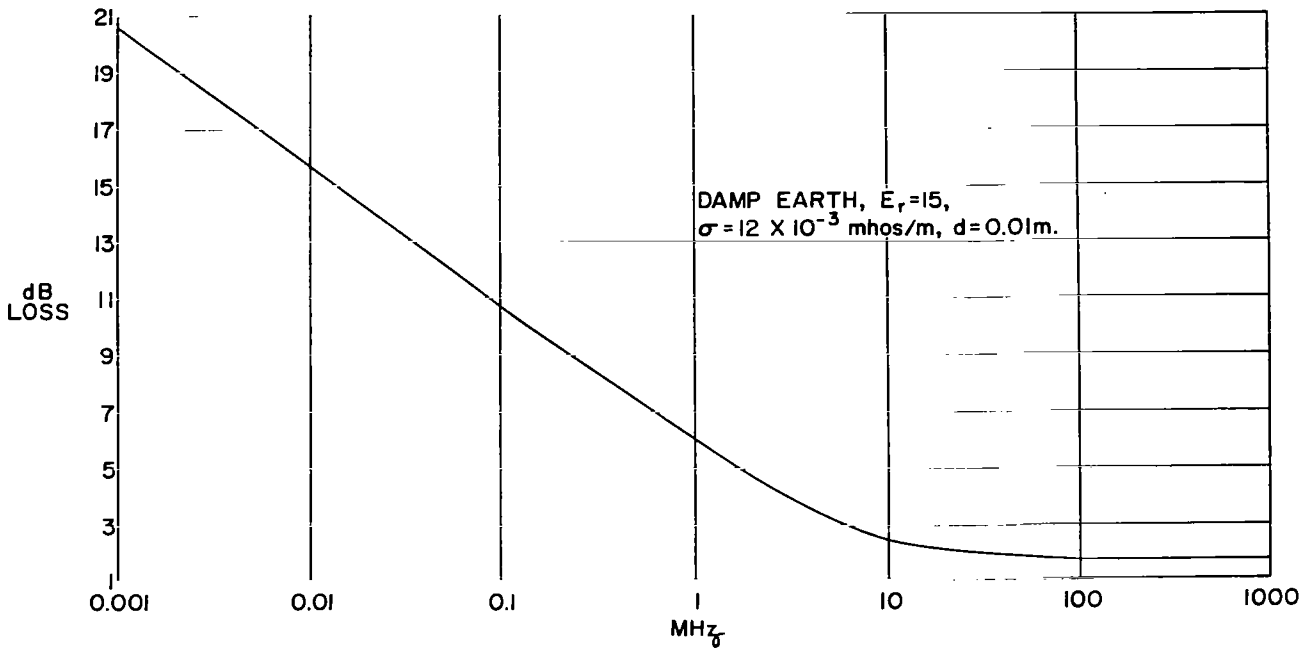

Figure 8. Damp Earth. Like Figure 6

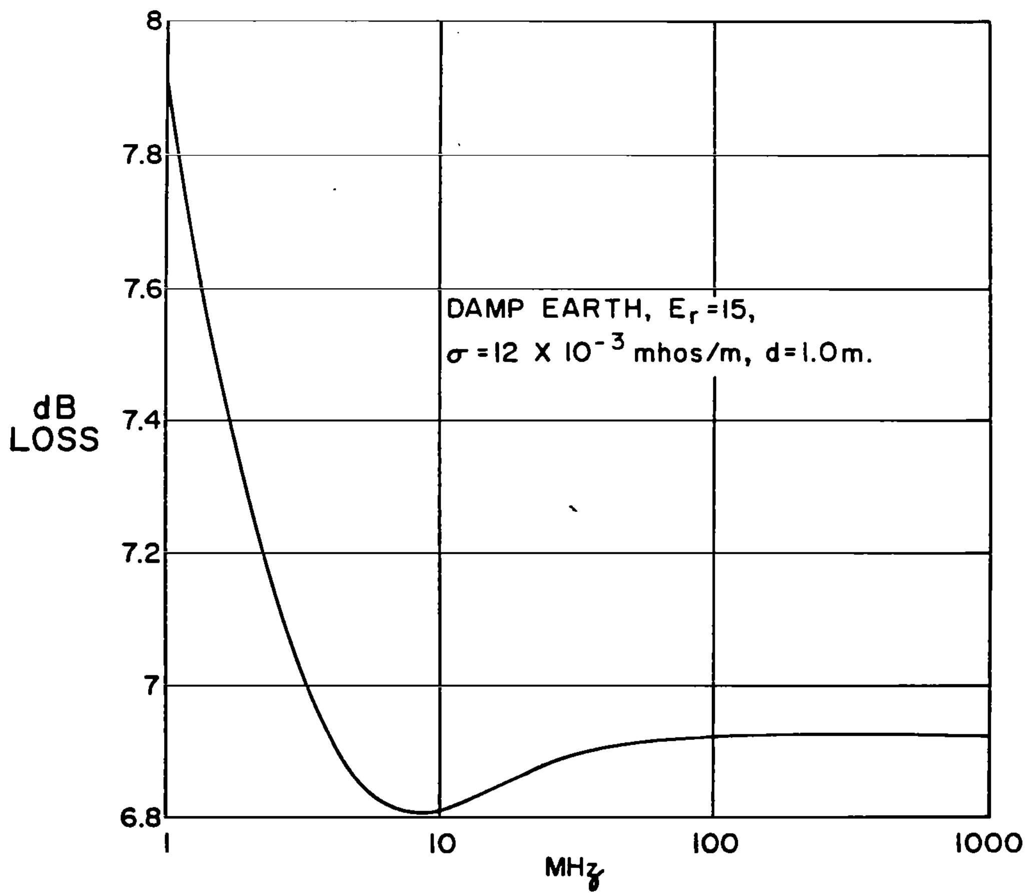

Figure 9. Damp Earth. Like Figure 7

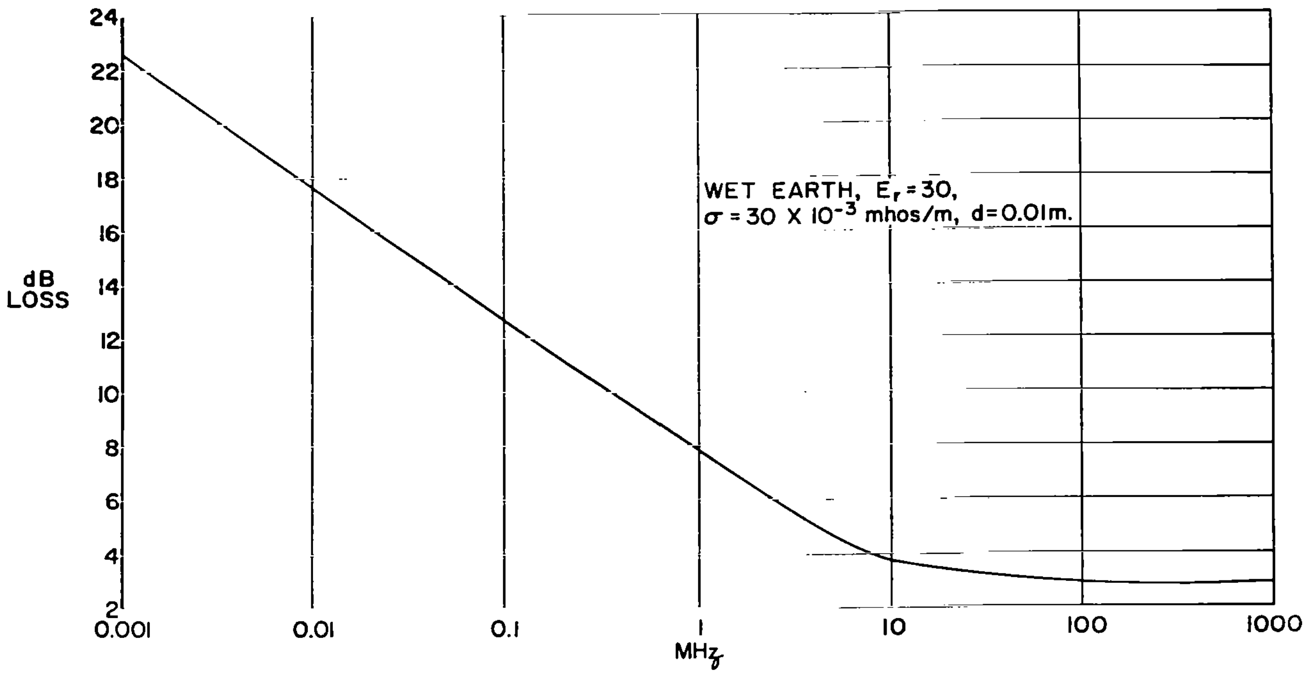

Figure 10. Wet Earth. Like Figure 6

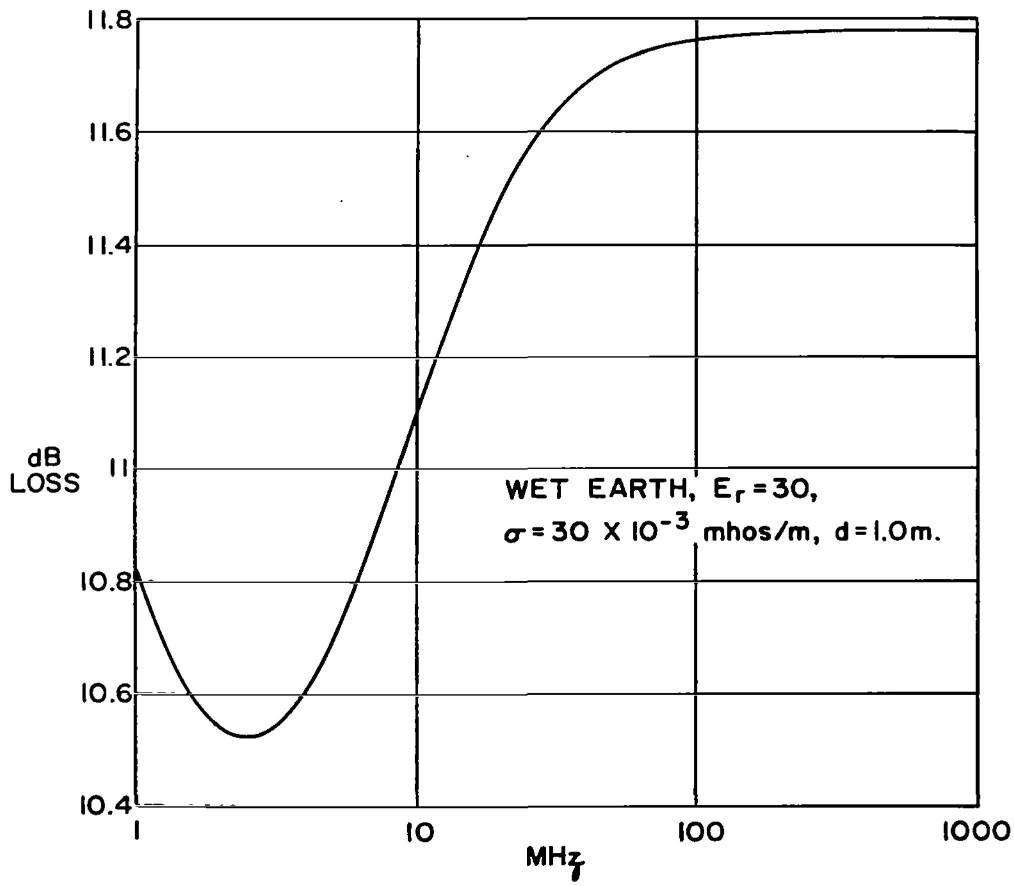

Figure 11. Wet Earth. Like Figure 7

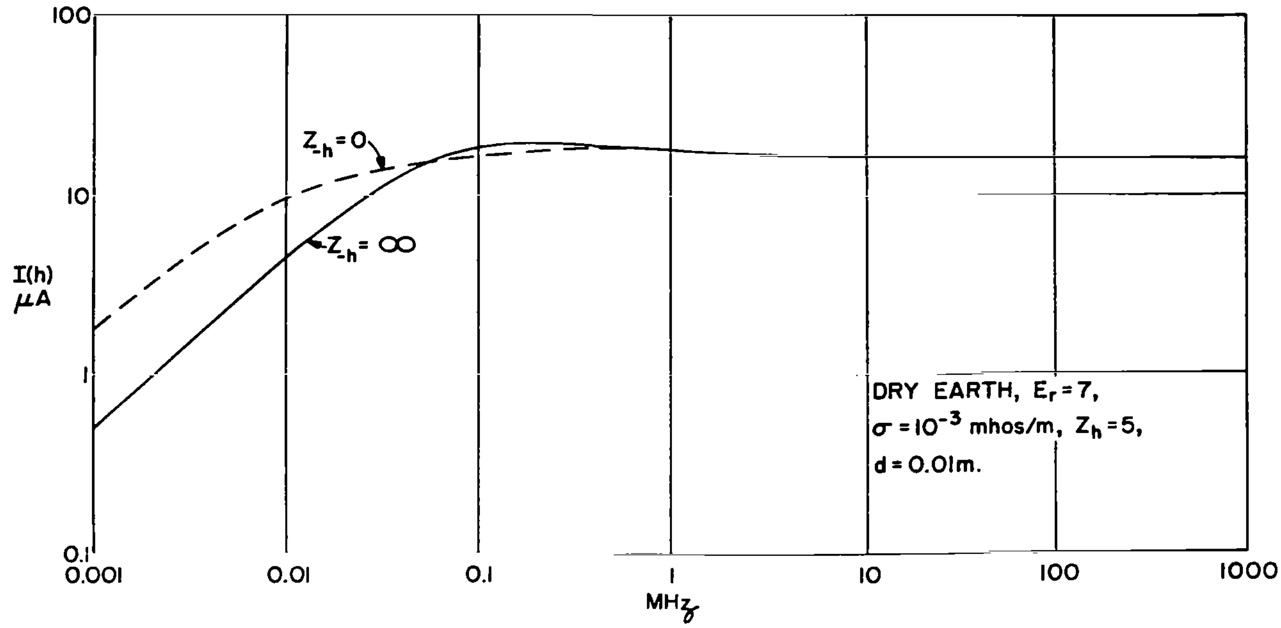

Figure 12. Dry Earth. I(h) in μA Against Frequency in HMz for $Z_{-h} = 0$ and ∞. d = 0.01 m

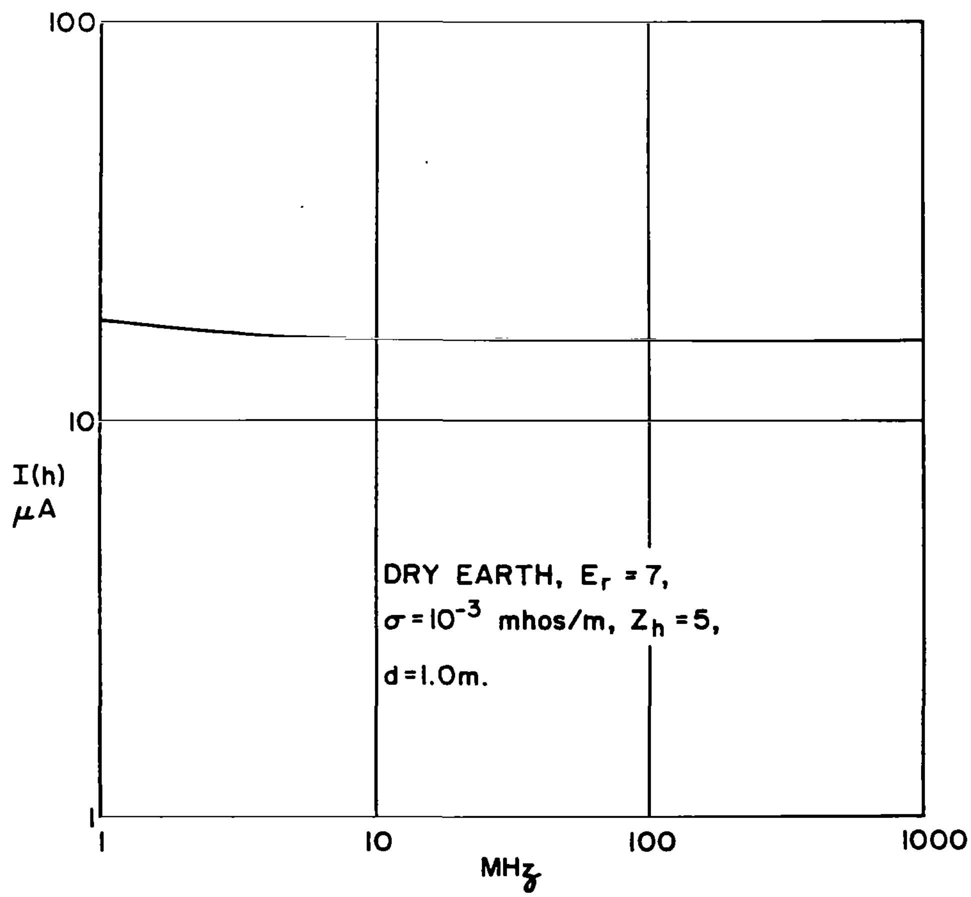

Figure 13. Dry Earth. Like Figure 12 Except for a Reduced Frequency Range and d = 1.0 m

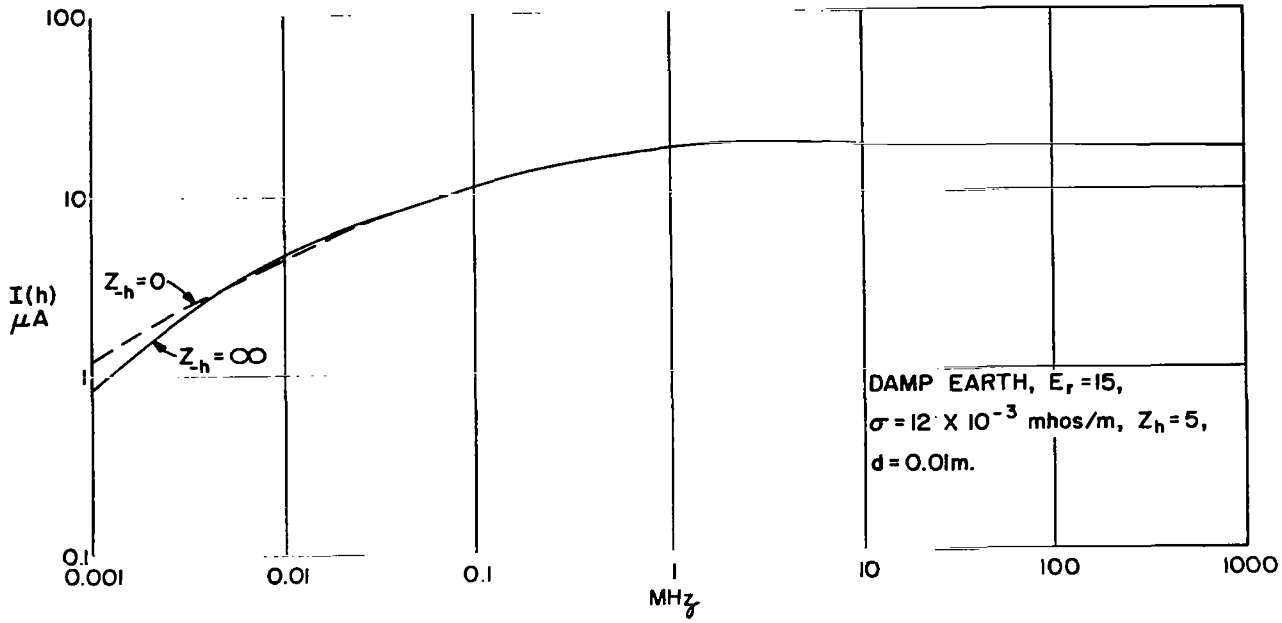

Figure 14. Damp Earth. Like Figure 12

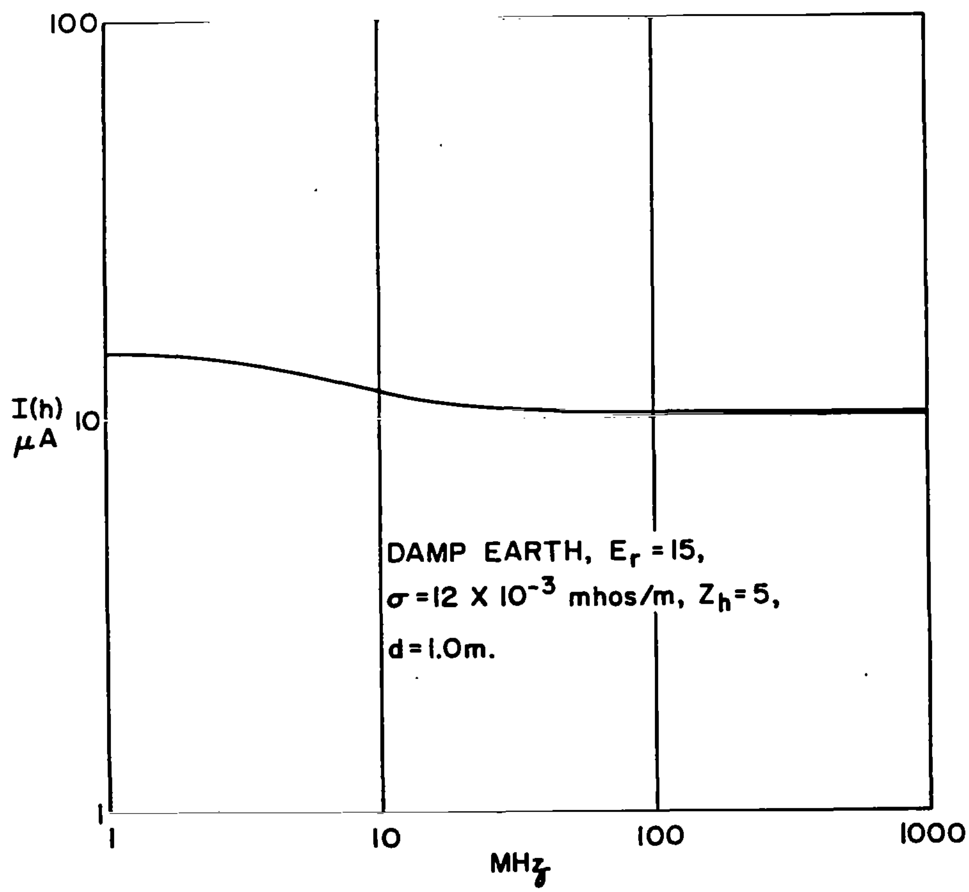

Figure 15. Damp Earth. Like Figure 13

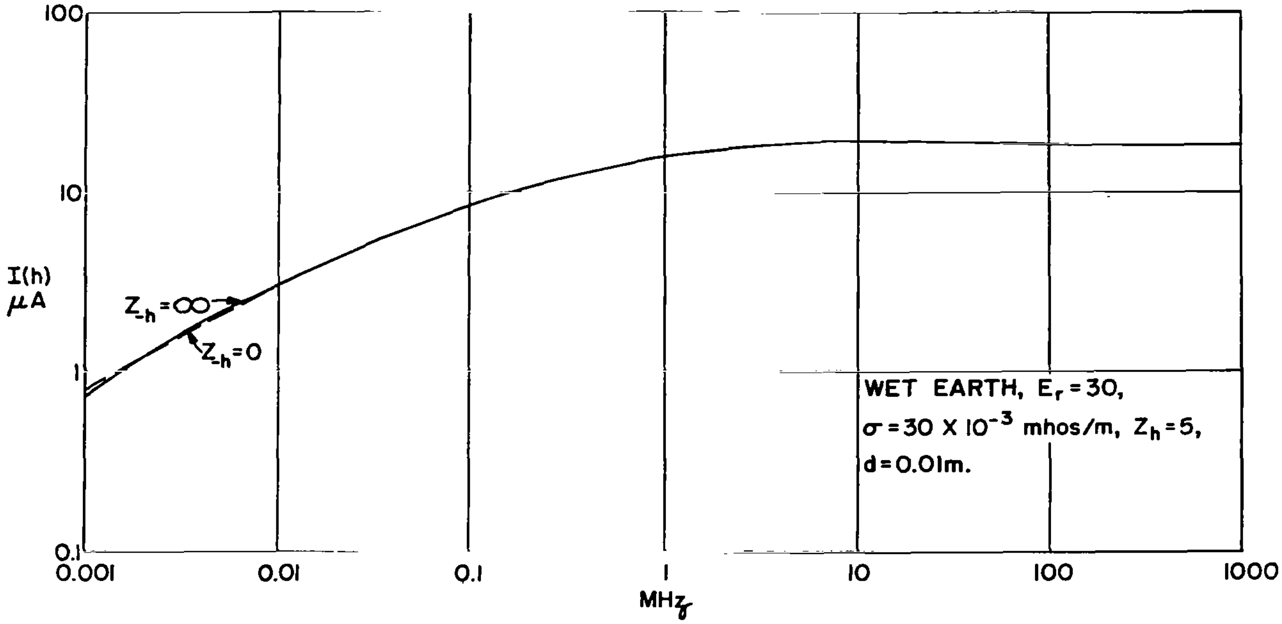

Figure 16. Wet Earth. Like Figure 12

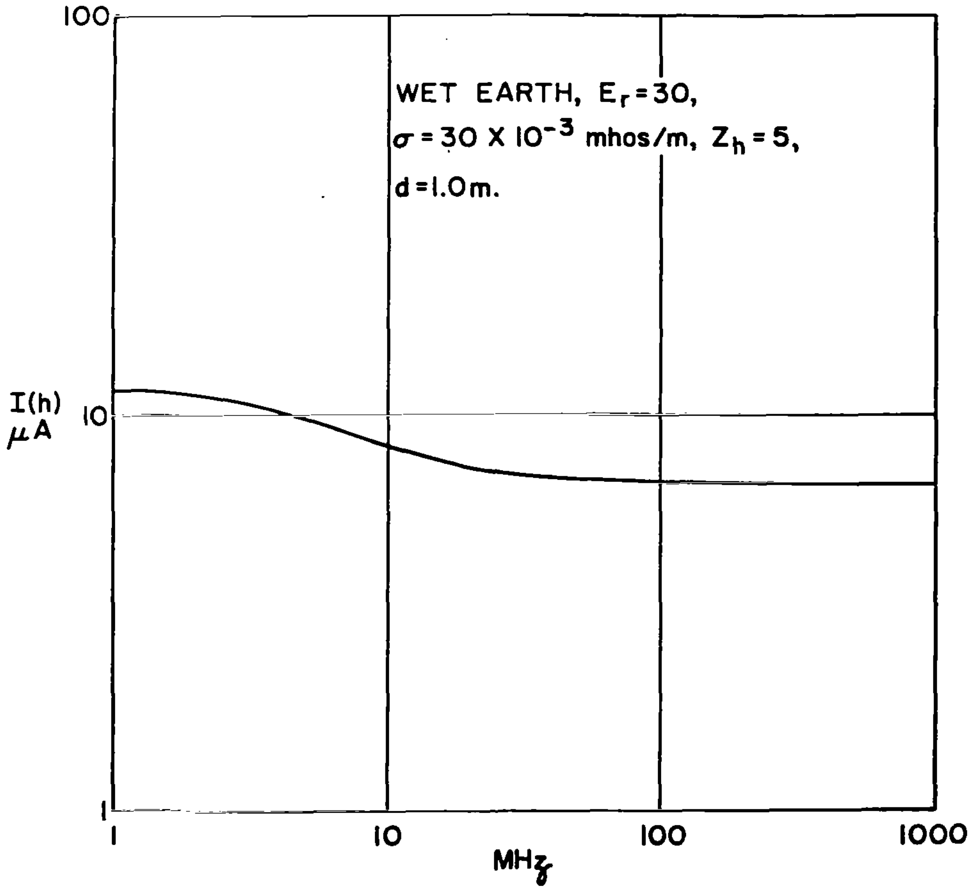

Figure 17. Wet Earth. Like Figure 13