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DIRECT GAMMA-INDUCED CURRENTS IN BURIED CABLES

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ABSTRACT

This report outlines a method, based on transmission line theory, for estimating currents and voltages induced in buried insulated cables by the gamma flux from a nuclear weapon detonated in the near vicinity of the cable. The driving function is modeled as a delta function in time and a square function of distance along the cable. Numerical examples are given for postulated characteristics of the driving function.

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1. INTRODUCTION

The problem considered herein was suggested by Bob Parker of Sandia Corporation in a meeting with Harold Price of Kaman Nuclear.

If a nuclear weapon is detonated near a buried cable, two mechanisms are available to introduce currents and voltages on the cable. The more familiar problem is the interaction with the cable of the intense electromagnetic fields generated by the burst. The second mechanism occurs when the gamma rays from the bomb strike the cable, liberating compton and photo-electrons. A replacement current, originating in the cable, will surge to replace the charge knocked out of the cable. It is the second mechanism with which the present work is concerned.

The problem consists of two parts:

(a) Determination of the photon flux at the cable as a function of time after burst and distance from the source.

(b) Determination of the transient voltages and currents in the cable caused by the photon flux.

The photon transport, part (a), is by no means a trivial task. It is complicated by the various gamma producing neutron interactions with the soil, and the time-dependent transport of these photons to the cable. Even obtaining order of magnitude estimates of the photon flux is a complex and time consuming chore.

Because of the uncertainty of the outcome of a gamma ray transport calculation, it has been decided that for the present time an upper bound on the problem should be obtained in as simple a way as possible. To this end, the authors have estimated that the maximum dose that the cable could receive from a one megaton bomb is 10^{11} rads over a 10 meter length. This upper bound would have to be increased, perhaps as much as two orders of magnitude, if the depth of cable burial were reduced from 6 ft to 4 ft. For the purpose of this calculation, the pulse shape was assumed to be a delta function in time. It is not clear whether or not this assumption corresponds to the worst case.

According to Reference 1, the charge scattered from a wire by gamma rays is roughly 10^{-13} coulomb per cm (of length) per rad deposited in the cable. Thus, for a dose of 10^{11} rads, and converting centimeters to meters, the upper bound estimate of the charge is 1 coulomb/meter, deposited instantly. The next section provides a solution for the current and voltage for this type excitation.

2. TRANSMISSION LINE THEORY FOR A CURRENT DRIVING SOURCE

For the present analysis it is assumed that the transmission line analogy for the buried cable provides adequate solutions. (Reference 4). The transmission line equations for arbitrary driving conditions are, for $e^{i\omega t}$ time dependence, the following:

$$\frac{dI}{dx} = - VY - J(x) \tag{1}$$

$$\frac{dV}{dx} = - ZI + E \tag{2}$$

where

- I = current in the line
- V = voltage induced across the line
- x = distance along the line from the origin (in meters)
- Z = line impedance
- Y = admittance
- J = injection current per unit length
- E = electric field parallel to the wire

For the problem under consideration $E(x) = 0$, and $J(x)$ is a square function, unity between $-h$ and h and zero outside $|x| = h$. (Figure 1)

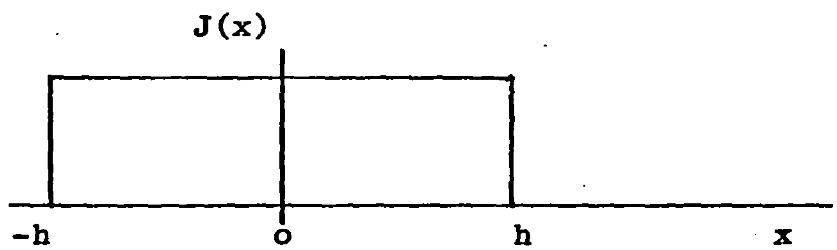

Figure 1 - Current Driving Function

In the present problem, $h = 5$ meters, as the gamma pulse was assumed to irradiate a 10 meter section of the cable.

Given the solution to (1) and (2) for $I(x,\omega)$ and $V(x,\omega)$, for $e^{i\omega t}$ time dependence, the time dependent solutions $I(x,t)$ and $V(x,t)$ can be obtained by Fourier transform theory.

$$I(x,t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} I(x,\omega)\ e^{i\omega t}\ d\omega \tag{3}$$

$$V(x,t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} V(x,\omega)\ e^{i\omega t}\ d\omega \tag{4}$$

Equations (3) and (4) are simplified by the assumption of a delta function driving current, the Fourier transform of which is unity for all frequencies. For an arbitrary driving function $J(x,t)$, the integrands in (3) and (4) would have to be multiplied by $J(x,\omega)$ where

$$J(x,\omega) = \int_{0}^{\infty} J(x,t)\ e^{-i\omega t}\ dt \tag{5}$$

The driving function $(J(x)$, as shown in Figure 1 can be expressed analytically by

$$J(x) = \eta(x+h) - \eta(x-h) \tag{6}$$

where η is the Heaviside function

$$\eta(x) = 0 \quad -\infty < x < 0$$
$$= 1 \quad 0 \le h < \infty . \tag{7}$$

The Heaviside function is related to the delta function by

$$\frac{d}{dx}\eta(x) = \delta(x). \tag{8}$$

Differentiating (1) and using (2), (6) and (8), it is shown that the differential equation for I is

$$\frac{d^2I}{dx^2} - ZYI = \delta(x-h) - \delta(x+h). \qquad (9)$$

Similarly,

$$\frac{d^2V}{dx^2} - ZYV = Z[\eta(x+h) - \eta(x-h)] . \qquad (10)$$

By following the usual procedure and defining

$$\gamma = \sqrt{YZ} , \qquad (11)$$

equations (9) and (10) become

$$\frac{d^2I}{dx^2} - \gamma^2 I = \delta(x-h) - \delta(x+h) \qquad (12)$$

$$\frac{d^2V}{dx^2} - \gamma^2 V = Z[\eta(x+h) - \eta(x-h)] \qquad (13)$$

The solutions of (12) and (13), subject to the appropriate boundary conditions, form the frequency domain current and voltage on the cable. Before returning to the time domain vis (3) and (4), one must obtain γ and Z as functions of frequency.

It can be shown that the general solutions to (12) and (13) are

$$I(x) = Ae^{-\gamma x} + Be^{\gamma x} + \frac{1}{2\gamma} \sinh[\gamma(x-h)]\eta(x-h) - \frac{1}{2\gamma} \sinh[\gamma(x+h)]\eta(x+h) \quad (14)$$

$$V(x) = \frac{1}{Y} \Big[\gamma Ae^{-\gamma x} - \gamma Be^{\gamma x} + \{1 - \tfrac{1}{2} \cosh[\gamma(x-h)]\} \eta(x-h) - \{1 - \tfrac{1}{2} \cosh[\gamma(x+h)]\} \eta(x+h) \Big] \quad (15)$$

For the present study it is sufficient to consider an infinite line with no discontinuities. There are three regions of interest:

1. $-\infty < x < -h$
2. $-h < x < h$
3. $h < x < \infty$

The boundary conditions are:

(a) as $x \to -\infty$, $I(x) \sim e^{+\gamma x}$

(b) at $x = -h$, $I(x)$ is continuous

(c) at $x = 0$, $I(x) = 0$

(d) at $x = +h$, $I(x)$ is continuous

(e) as $x \to +\infty$, $I(x) \sim e^{-\gamma x}$

(f) as $x \to \pm\infty$, $V(x) \to 0$

The solutions for the current which match the boundary conditions and satisfy the differential equations are:

$$I(x) = \frac{1}{2\gamma} [e^{-\gamma(h+x)} - e^{-\gamma(h-x)}] \qquad |x| < h \qquad (16)$$

$$I(x) = \frac{1}{2\gamma} (e^{-\gamma h} - e^{\gamma h}) \, e^{-\gamma x} \qquad x > h \qquad (17)$$

Equation (17) represents the current at distances away from the burst for a harmonic time dependence. In order to obtain the current in the time domain, it is necessary to determine the attenuation constant γ as a function of frequency.

3. DETERMINATION OF THE ATTENUATION AND PROPAGATION CONSTANTS

Experience of other Kaman personnel[2] has indicated that the values of γ as given by ordinary transmission line theory are not adequate for the SANGUINE study. The reason for this is that the soil conductivity at the Sanguine site is too low for the earth to be considered a good conductor. An exact expression for γ (for a cable buried in an infinite earth) can be obtained from the roots of the so-called determinental equation[3]

$$\frac{N_o(\lambda_2 a_1) - \dfrac{\mu_1 \lambda_1 k_2^2}{\mu_2 \lambda_2 k_1^2} \dfrac{J_o(\lambda_1 a_1)}{J_1(\lambda_1 a_1)} N_1(\lambda_2 a_1)}{J_o(\lambda_2 a_1) - \dfrac{\mu_1 \lambda_1 k_2^2}{\mu_2 \lambda_2 k_1^2} \dfrac{J_o(\lambda_1 a_1)}{J_1(\lambda_1 a_1} J_1(\lambda_2 a_1)}$$

$$= \frac{N_o(\lambda_2 a_2) - \dfrac{\mu_3 \lambda_3 k_2^2}{\mu_2 \lambda_2 k_3^2} \dfrac{H_o^{(1)}(\lambda_3 a_2)}{H_1^{(1)}(\lambda_3 a_2)} N_1(\lambda_2 a_2)}{J_o(\lambda_2 a_2) - \dfrac{\mu_3 \lambda_3 k_2^2}{\mu_2 \lambda_2 k_3^2} \dfrac{H_o^{(1)}(\lambda_3 a_2)}{H_1^{(1)}(\lambda_3 a_2)} J_1(\lambda_2 a_2)} \qquad (18)$$

In equation (18), the subscripts 1, 2 and 3 denote values of variables in the cable conductor, dielectric, and soil, respectively. J_o, N_o, and $H_o^{(1)}$ are, respectively, the Bessel function, Neumann function and Hankel function of the first kind. a_1 is the radius of the conductor and a_2 is the outer radius of the dielectric. (see Figure 2)

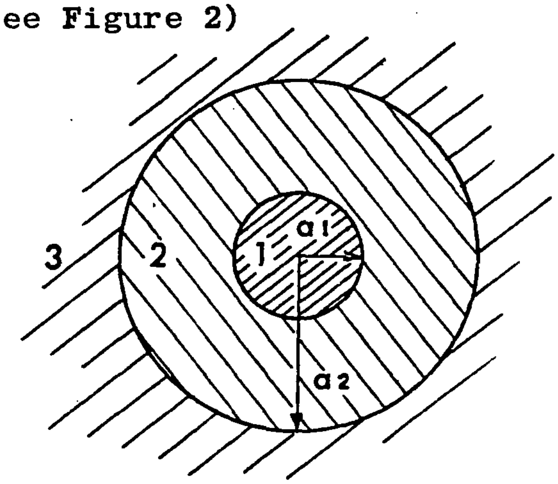

Figure 2 - Cable Geometry

λ is defined by:

$$\lambda^2 = k^2 + \gamma^2 \tag{19}$$

$$k^2 = \omega^2\mu\epsilon - i\omega\mu\sigma \tag{20}$$

The solutions of (18) for γ as a function of frequency determine the propagation and attenuation coefficients. Unfortunately, the solution of (18) is a very complicated and time-consuming process which to date has been accomplished only for frequencies below 40 kHz. For an upper bound calculation, as is presented in this memorandum, it was considered sufficient to approximate γ by the plane-wave propagation constant in soil

$$\gamma = -i\,(\omega^2\mu\epsilon - i\omega\mu\sigma)^{\frac{1}{2}} \tag{21}$$

for frequencies greater than 40 kHz.

Figure 3 presents plots of real and imaginary parts of γ as functions of frequency. For $0 < f < 40$ kHz the roots of equation (18) were used for γ ; for $f > 40$ kHz, equation (21) was used. The discontinuity in γ at 40 kHz was smoothed in by hand. It is felt that for present purposes the values of γ as given in Figure 3 are adequate.

4. GAP VOLTAGES

Also of interest is the voltage across the dielectric. It is seen in equation (15) that the evaluation of voltage requires an expression for the line admittance Y. As remarked earlier, the transmission line theoretical expressions for line parameters, such as admittance, impedance, etc., are not sufficiently accurate, even for the present upper bound study. An expression which does seem to provide reasonable answers is given by Stratton[3],

$$V(X) = \frac{i\omega\mu\gamma}{2\pi k_2^2} \log_e \left(\frac{a_2}{a_1}\right) \cdot I(X) \; .$$

Using (17) for I(X),

$$V(X) = \frac{i \log({}^{a_2}/a_1)\,[e^{-\gamma h} - e^{\gamma h}]\; e^{-\gamma x}}{4\pi\;\omega\;\epsilon_2} \tag{22}$$

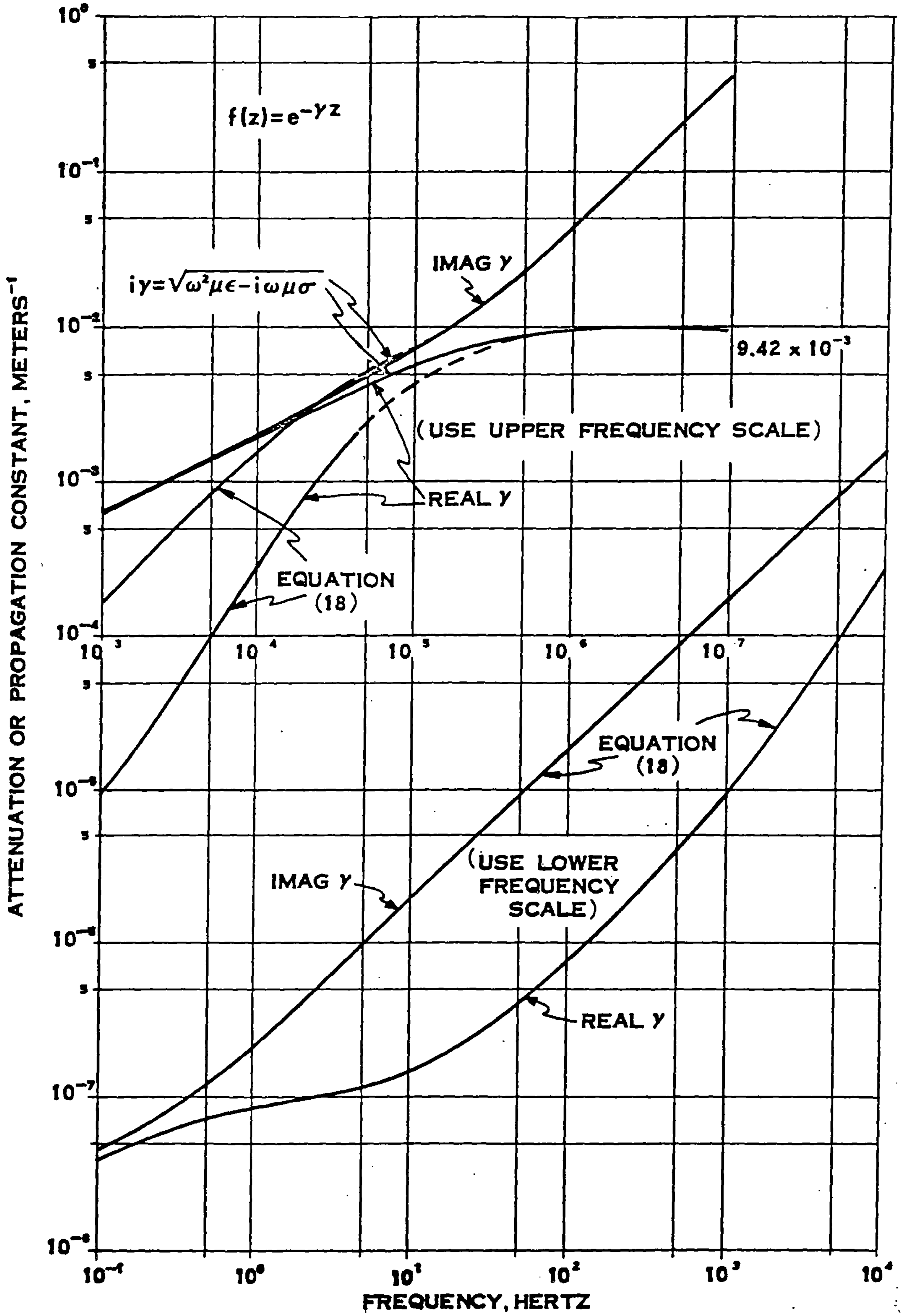

FIGURE 3

PROPAGATION AND ATTENUATION CONSTANTS VS FREQUENCY

5. RESULTS

Expressions (17) and (22), along with the values given in Figure 3 for γ, were inserted into equations (3) and (4) for $I(x,t)$ and $V(x,t)$. Qualitatively, it is easy to explain the expected behavior. The problem considered is the response of an infinitely long transmission line after a large amount of charge has been deposited upon it. The charge will redistribute itself along the line. Since the signal cannot propagate faster than the speed of light, no current should be observed at point X at least until time $t \cong X/C$. For positive charge deposition, at $t \cong X/C$ the current should swing positive until it peaks and then it should decay to zero. It should never become negative, and the integral

$$\int_0^\infty I(t)\,dt \qquad (23)$$

should be half the charge deposited on the line. For the case considered here, the total charge is

$$Q = 10 \text{ coulombs},$$

so the integral (23) should equal 5.

Because not all frequencies will propagate at the same speed, the pulse will broaden as shown in Figure 4.

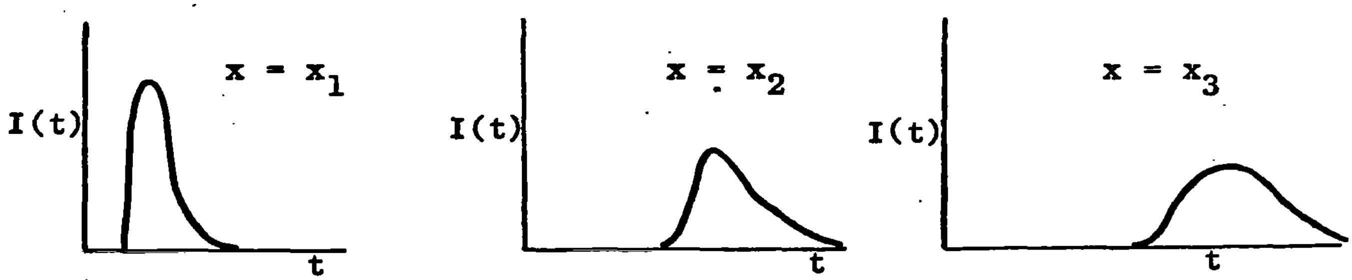

Figure 4 - Expected Behavior of $I(t)$ vs Time for Three Positions $x_1 < x_2 < x_3$. In each case $\int I(t)dt = 5$ coulombs

The voltage-time curves should behave similarly, except that there is no physical restriction on $\int_{-\infty}^{\infty} V(t)dt$.

Equations (3) and (4) were integrated numerically for three values of X; 500, $5x10^3$ and $5x10^4$ meters. The results are shown in Figures 5 through 10. Figures 5, 6 and 7 give the cable current as a function of time for X = 500, $5x10^3$ and $5x10^4$ meters from the burst. The current behaves as was predicted in Figure 4. In each case the time integral of the current was within one percent of 5 coulombs. Figures 8, 9 and 10 show the calculated gap voltage vs time after burst. Again, the calculations provided no surprises, except perhaps that the peak currents and gap voltages at large distances are indeed appreciable.

Although the current and voltage should start at zero, rise quickly, and decay monotonically to zero, some wiggles are seen in Figures 5 through 10 both prior to the main pulse and during the decay. These oscillations are artificial and are due primarily to truncation of the integrals in equations (3) and (4) at finite frequencies, and to inexact propagation constants at high frequencies. It is felt that the characteristics of the main pulse are generally correct, within the model.

Table 1 gives the peak current and gap voltage at the three distances -- x = 500, $5x10^3$ and $5x10^4$ meters.

Table 1

Peak Cable Currents and Gap Voltages

	Peak Current (amps)	Peak Voltage
500 meters	$3.6x10^5$	$2.5x10^7$
5 kilometers	7×10^4	6×10^6
50 kilometers	10^4	8×10^5

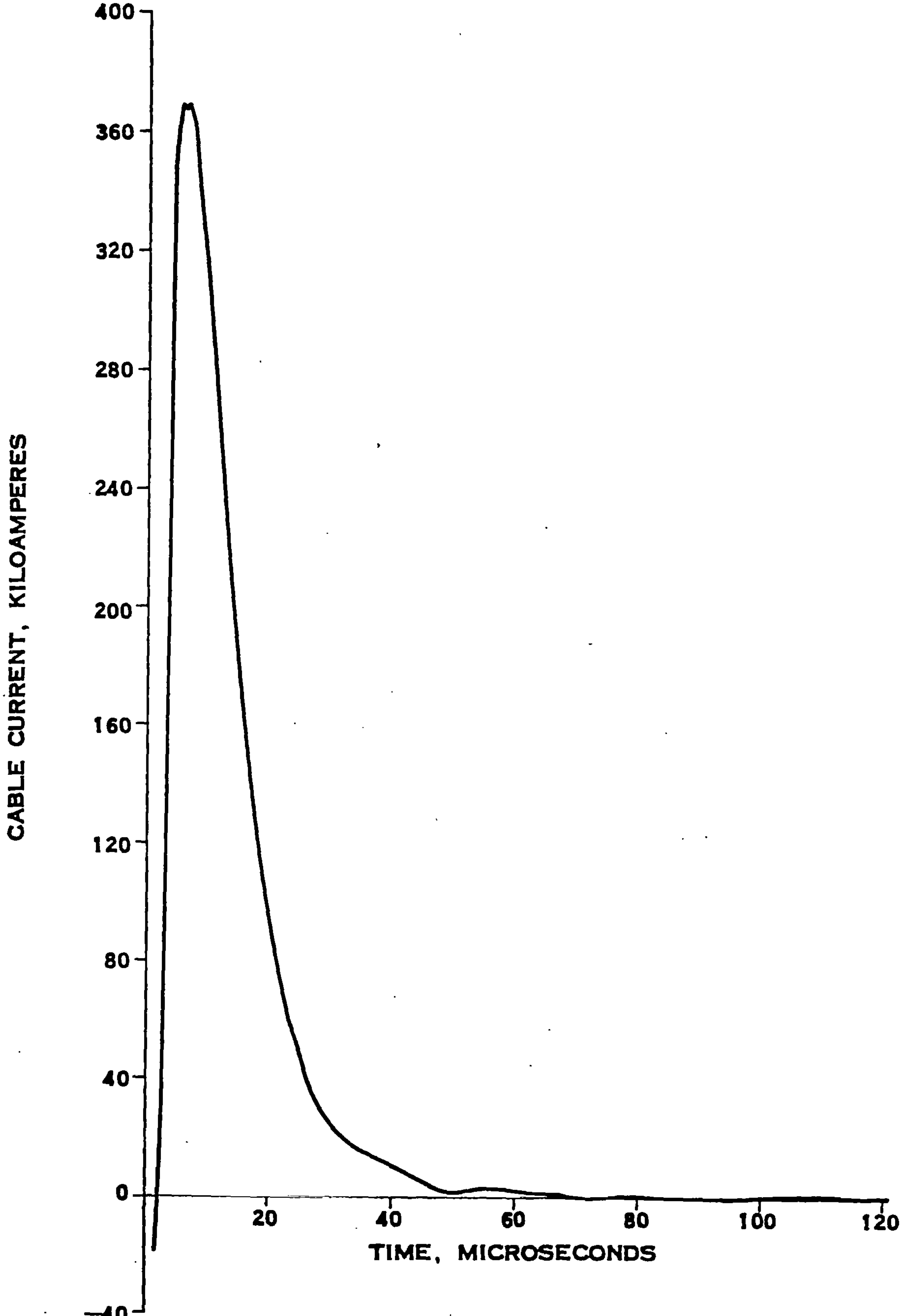

FIGURE 5

CABLE CURRENT VS. TIME AT X = 500 METERS

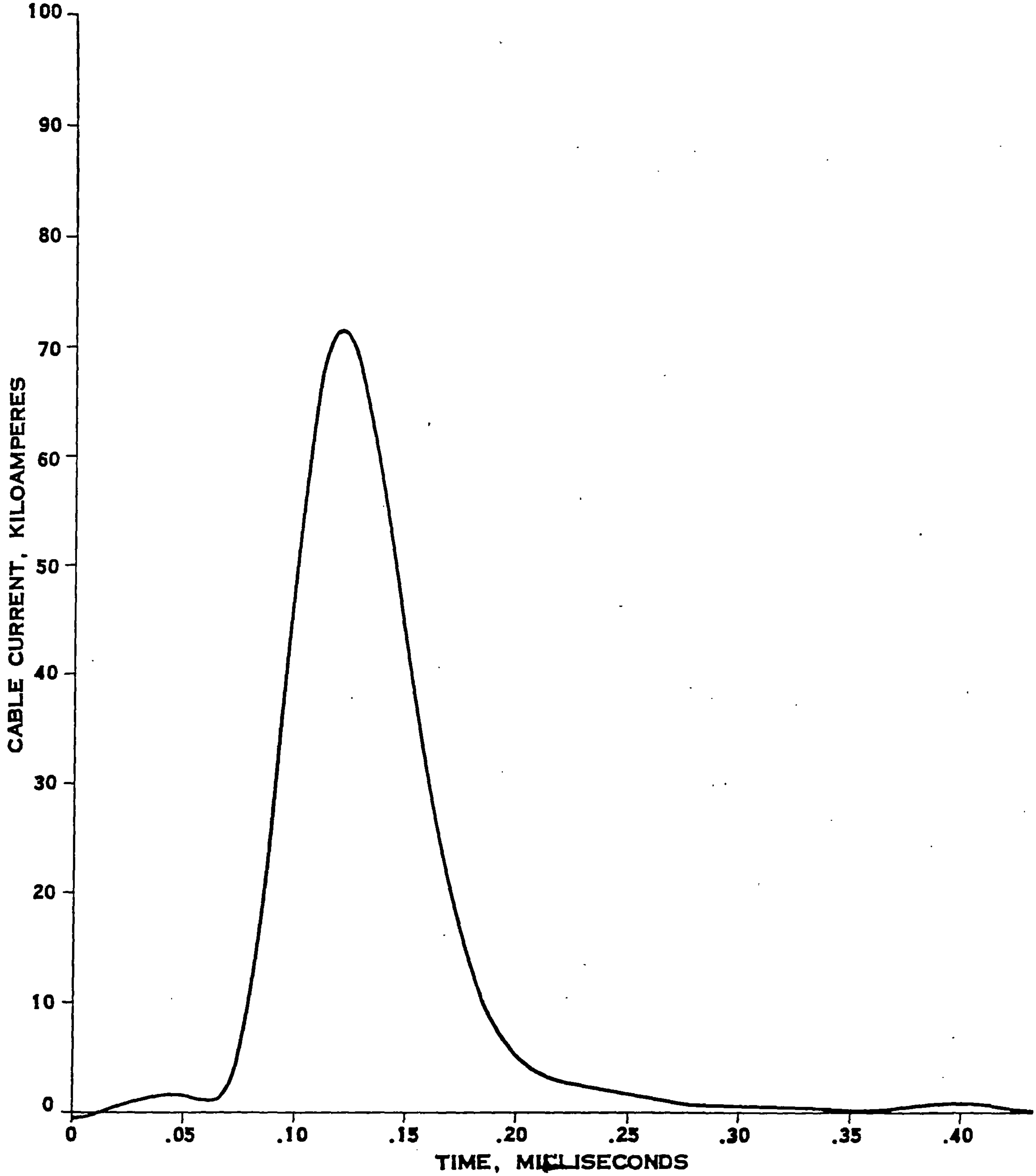

FIGURE 6

CABLE CURRENT VS. TIME AT X = 5 KILOMETERS

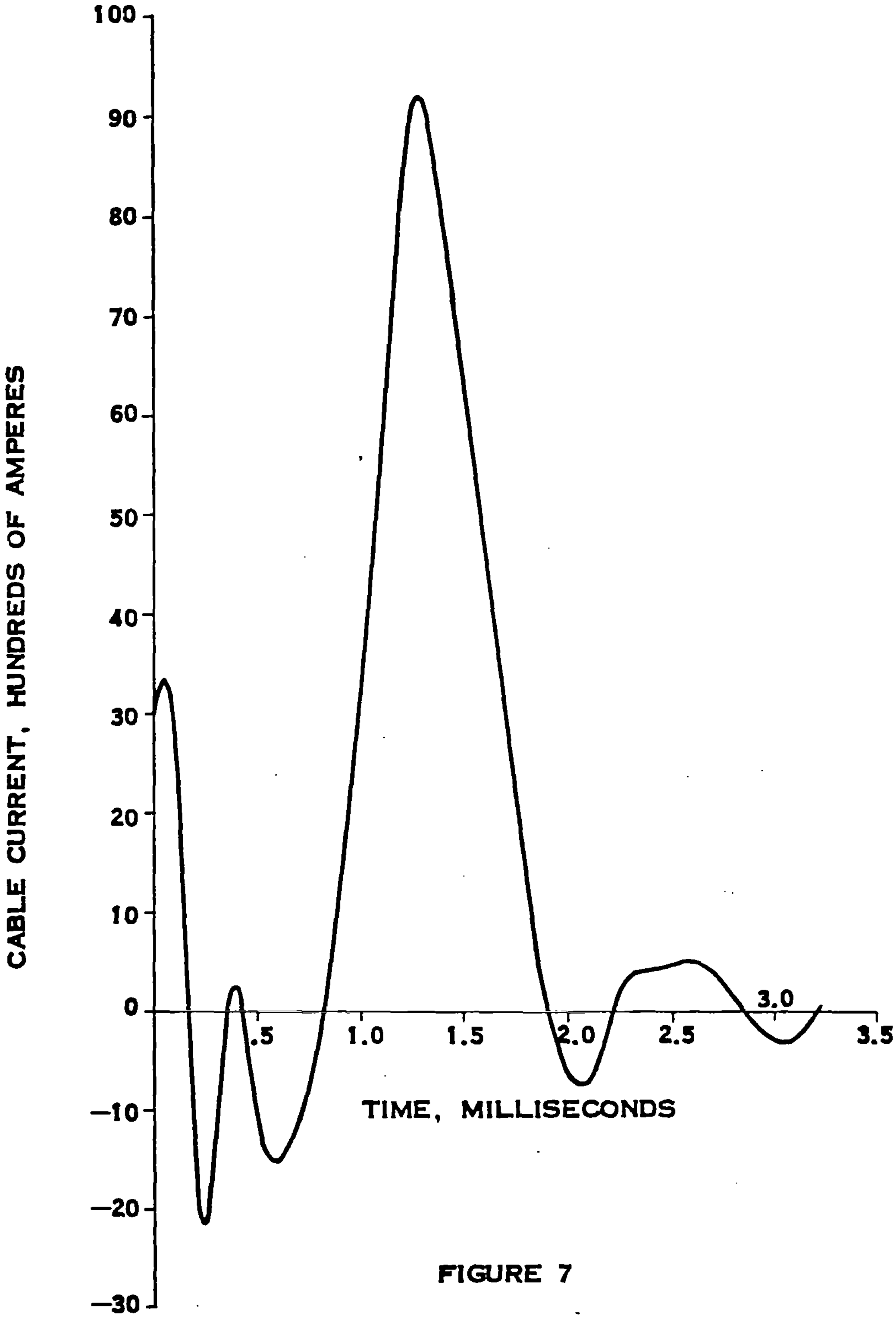

FIGURE 7

CABLE CURRENT VS. TIME AT X = 50 KILOMETERS

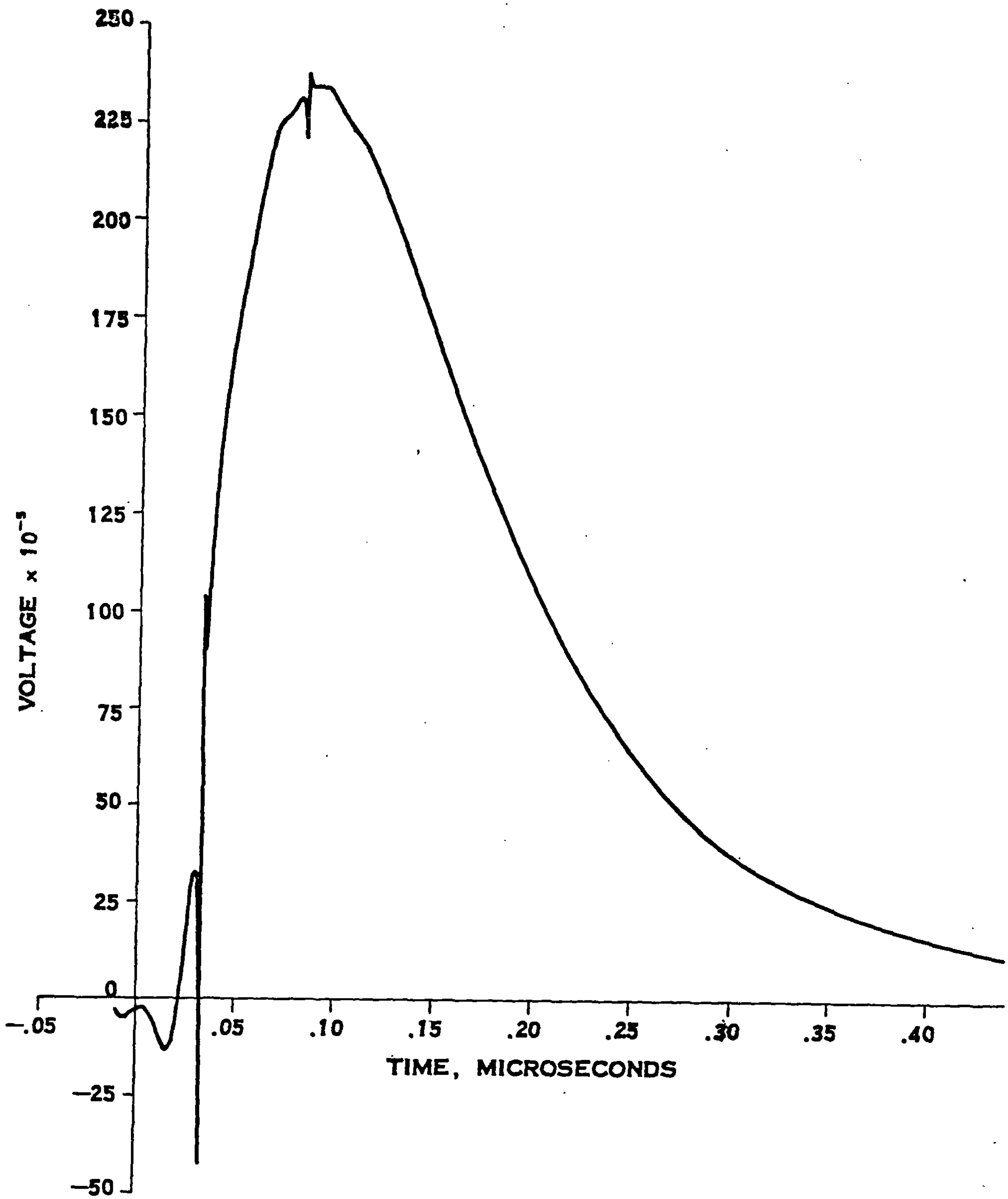

FIGURE 8

GAP VOLTAGE VS. TIME AT X = 500 METERS

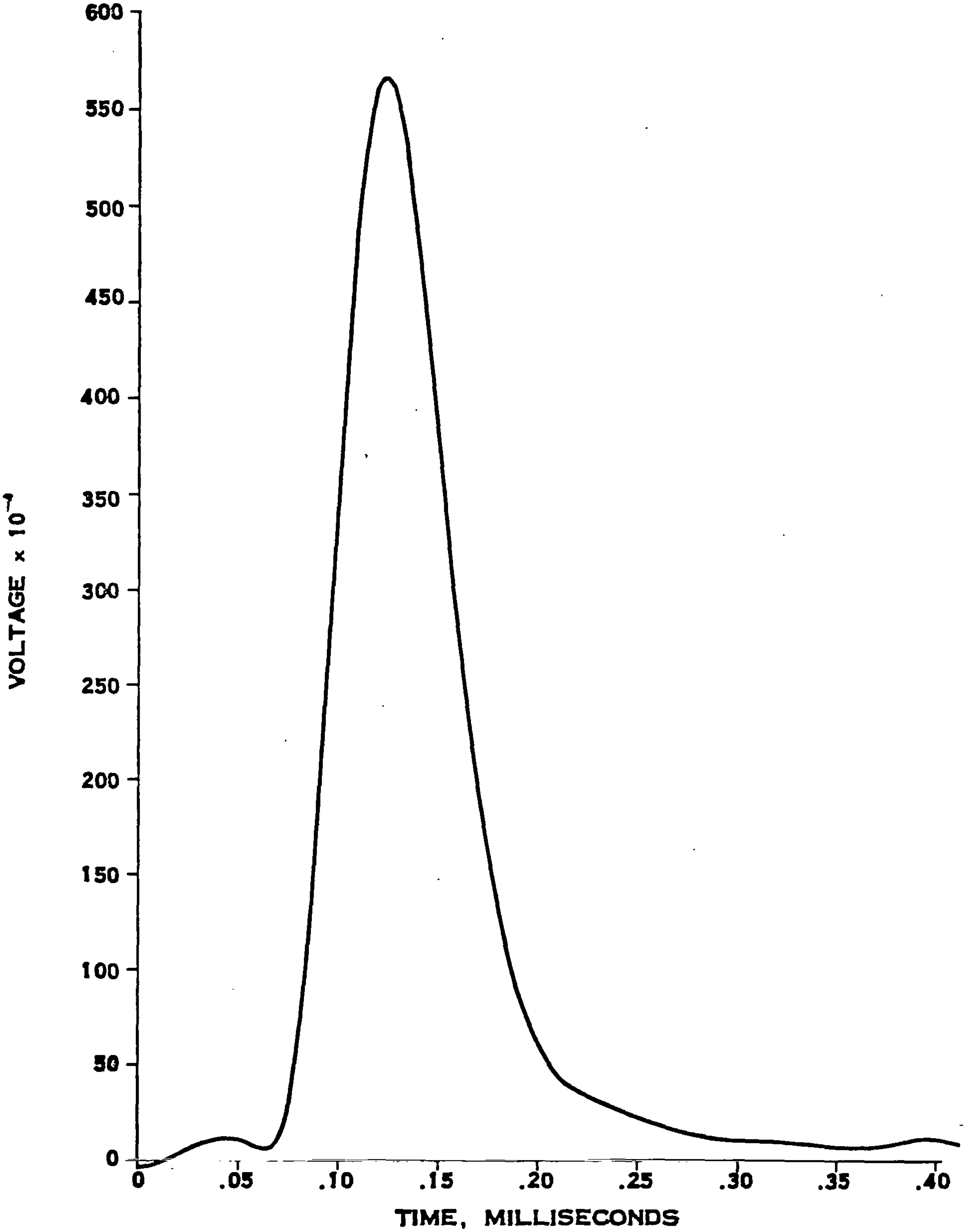

FIGURE 9

GAP VOLTAGE VS. TIME AT X = 5 KILOMETERS

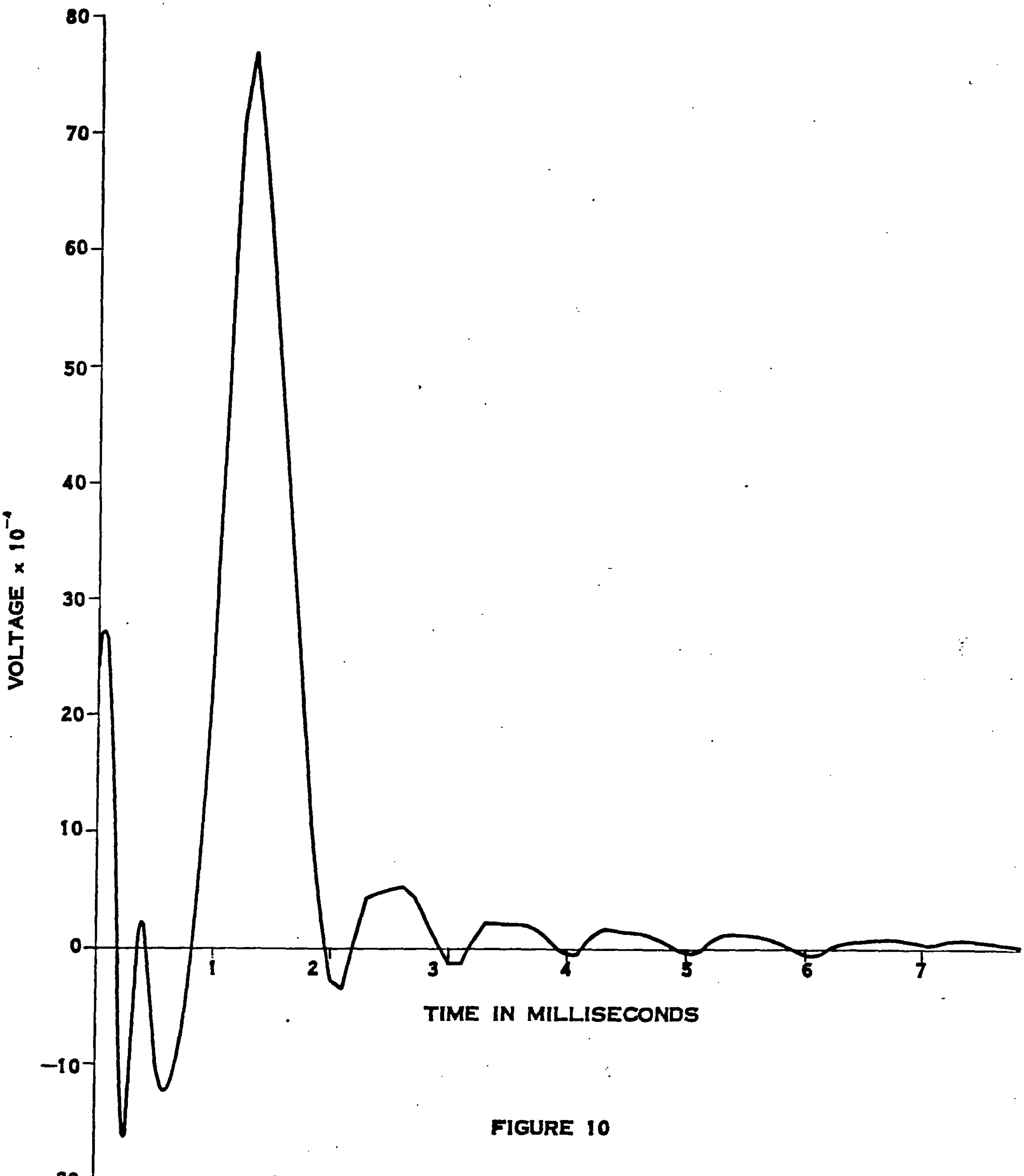

FIGURE 10

GAP VOLTAGE VS. TIME AT X = 50 KILOMETERS

6. DISCUSSION

The calculations for the current for this assumed model are more accurate than is the calculation of the voltage. The reason for this is that the voltage calculation not only relies on the current, but included a further approximation which essentially assumes a good conducting earth. This may tend to overestimate the gap voltages.

It should be stressed that the calculations in this memo are based upon a very crude model, and to improve the quality of the calculations would require better estimates of the gamma ray flux at the cable. The most sensitive parameter is the depth of burial of the cable; burial at 4 feet rather than 6 feet would increase the peak current and voltages (as calculated above) at least by a factor 10^2.

Another question arises as to the overall importance of this effect when viewed from the systems vulnerability standpoint, viz., what is the probability of a warhead landing near enough to the cable to induce these effects. It is, therefore, recommended that the following two efforts be undertaken:

(a) a thorough photon transport calculation should be initiated to determine the dose rate as a function of time and distance from the burst point,

and (b) a system analysis study should be performed to determine the vulnerability of the system to this effect, taking into account the sensitivity to transverse displacement of the detonation point with respect to the cable in light of the CEP of the threat.

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