SCTM 457-58(14)

RESPONSE OF AN IMPEDANCE-LOADED ELECTRIC DIPOLE SYMMETRICALLY ORIENTED WITHIN AN IMPERFECTLY CONDUCTING CYLINDER - VLF CASE

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ABSTRACT

Analytical relationships are developed which permit calculation of the power in the load impedance of an electric probe, symmetrically located within an imperfectly conducting cylinder of small radius compared to the wavelength, in terms of the electric field incident upon the cylinder.

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Introduction

The problem to be solved is that of determining the power in the load impedance of a short dipole receiving antenna located within an imperfectly conducting shield of cylindrical shape in terms of the electric field existing in the vicinity of the shield. It is assumed that the radius of the shield and length of the probe are small in terms of the wavelength. The solution may be effected logically by subdividing the problem into the following parts:

a. Determination of the ratio of the electric field on the outside surface to the field on the inside of the imperfectly conducting cylindrical shield.

b. Establishment of the relationship existing between the electric field on the outside surface of the hollow cylinder to the incident electric field.

c. Determination of the effective height and driving-point impedance of a partially shielded electric probe.

d. Calculation of the power in the load impedance of the probe in terms of the electric field existing inside of the cylindrical shield.

Relation Between the Electric Field on the Outside Surface of an Imperfectly Conducting Shield to the Field Inside

The first problem associated with determining the operational behavior of an electric probe within a tubular conductor is to find the field inside the tube in terms of the field on its exterior surface.[1]

[1] A somewhat different analysis from that presented here for the field inside a conducting tube is given by King, Ronold, Electromagnetic Engineering, McGraw-Hill Book Co., Inc., 1945, pp. 350-359.

Figure 1 illustrates a tube of inner radius a_1 and outer radius a_2. The metal annulus is designated region 1, the outside and inside of the tube being denoted regions 2 and 3, respectively. Let the axis of the cylindrical shield be along the z coordinate of a system of cylindrical coordinates r, θ, and z. If the length of the tube is at least 10 times its diameter and $a_2 << \lambda$, it is a sufficiently good approximation for the purposes of this paper to consider the problem as one in the dimension r. The field on the outside surface of the cylinder is assumed to be directed parallel to its axis, i.e., in the z direction.

The governing wave equation in region 1 is

$$\nabla^2 E = j\omega\sigma_1\mu_1 E. \tag{1}$$

Here

E is the electric field.

$\omega = 2\pi f$, where f is the frequency.

σ_1 is the conductivity of the metal used in region 1.

μ_1 is the permeability of the metal used in region 1.

∇^2 is an operator. When applied to the vector E it has the significance

$\nabla^2 E$ = grad div E - curl curl E.

A time dependence of the form $\exp(j\omega t)$ is assumed.

Since there is no θ and z dependence, Equation 1 may be written

$$\frac{\partial^2 E_z(r)}{\partial_r^2} + \frac{1}{r}\frac{\partial E_z(r)}{\partial r} + k_m^2 E_z(r) = 0. \tag{2}$$

The solution of Equation 1 is

$$E_z(r) = A_1 J_o(k_m r) + B_1 N_o(k_m r) \tag{3}$$

$$a_1 \leq r \leq a_2$$

where

$$k_m = (1 - j)\sqrt{\frac{\omega\sigma_1\mu_1}{2}}. \tag{4}$$

In region 3 the free space wave equation

$$\nabla^2 E = -\omega^2 \mu_o \epsilon_o E \tag{5}$$

applies. In this expression μ_o is the permeability of space and ϵ_o is the dielectric constant of space. These constants have the numerical values $4\pi \times 10^{-7}$ henrys per meter and

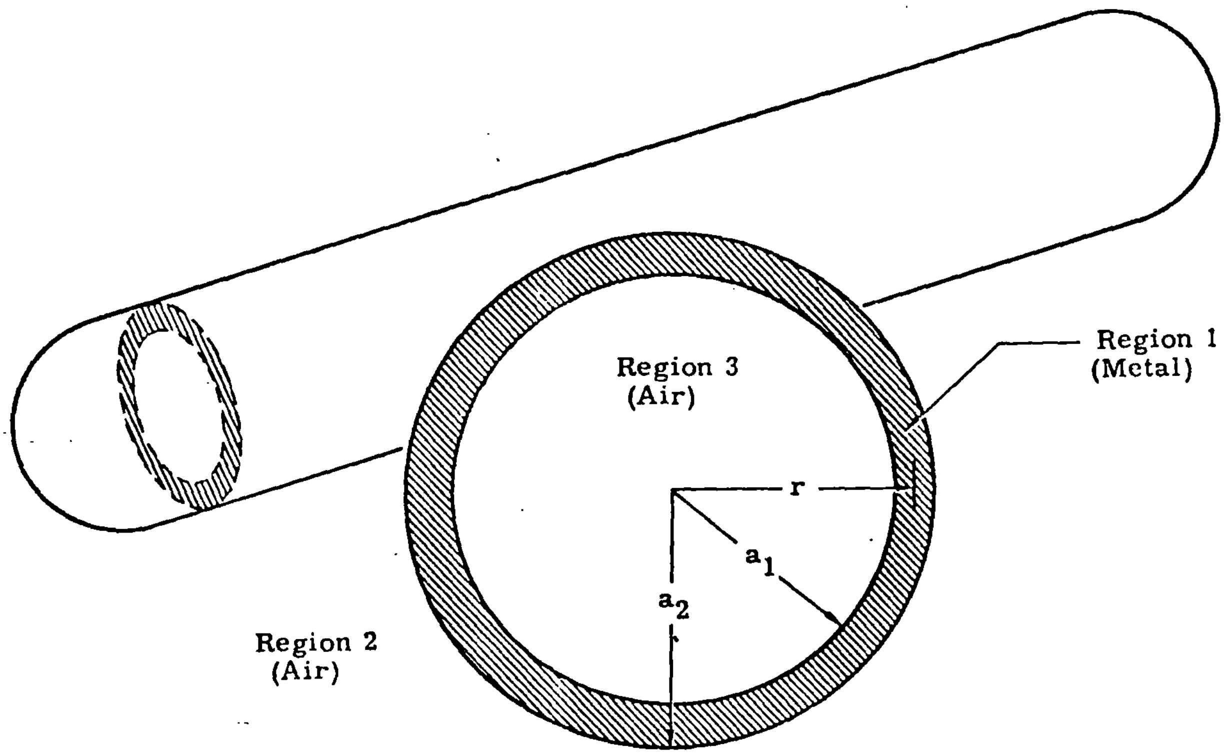

Figure 1. Cross section of hollow conductor

.85 x 10^{-12} farads per meter, respectively. The solution of Equation 5, paralleling Equation 4, is

$$E_z(r) = A_3 J_o(k_s r) + B_3 N_o(k_s r) \tag{6}$$

$$0 \leq r \leq a_1$$

where

$$k_s = \omega \sqrt{\mu_o \epsilon_o} \; . \tag{7}$$

Because of the fact that $N_o(k_s r) \rightarrow \infty$ at $r = 0$, the constant $B_3 = 0$. Accordingly

$$E_z(r) = A_3 J_o(k_s r) \tag{8}$$

$$0 \leq r \leq a_1$$

and

$$E_z(a_1) = A_3 J_o(k_s a_1). \tag{9}$$

Now if $(k_s a_1)^2 << 1$, $J_o(k_s r) = 1$.

Then $E_z(r) = A_3$ = const. $(0 \leq r \leq a_1)$

so that

$$\frac{\partial E_z(r)}{\partial r} = 0. \tag{10}$$

$$(0 \leq r \leq a_1)$$

Now

$$\frac{d}{dx} J_o(u) = -J_1(u) \frac{du}{dx} \tag{11}$$

and

$$\frac{d}{dx} N_o(u) = -N_1(u) \frac{du}{dx} \, . \tag{12}$$

Applying Equation 10 to Equation 4, using Equations 11 and 12, one obtains

$$\frac{\partial}{\partial r} \left\{ A_1 J_o(k_m r) + B_1 N_o(k_m r) \right\}_{r = a_1} = -k_m \left\{ A_1 J_1(k_m r) + B_1 N_1(k_m r) \right\}_{r = a_1} .$$

Thus

$$-k_m \left\{ A_1 J_1(k_m a_1) + B_1 N_1(k_m a_1) \right\} = 0 \tag{13}$$

or

$$B_1 = -A_1 \frac{J_1(k_m a_1)}{N_1(k_m a_1)} . \tag{14}$$

Now at $r = a_2$, Equation 4 becomes

$$E_z(a_2) = A_1 J_o(k_m a_2) + B_1 N_o(k_m a_2) . \tag{15}$$

Forming the ratio of $E_z(r)/E_z(a_2)$ using Equations 4 and 15 and eliminating the constants A_1 and B_1 by the employment of Equation 14 result in the expression

$$\frac{E_z(r)}{E_z(a_2)} = \frac{J_o(k_m r)\, N_1(k_m a_1) - N_o(k_m r)\, J_1(k_m a_1)}{J_o(k_m a_2)\, N_1(k_m a_1) - N_o(k_m a_2)\, J_1(k_m a_1)} . \tag{16}$$

$$a_1 \leq r \leq a_2$$

When $r = a_1$

$$\frac{E_z(a_1)}{E_z(a_2)} = \frac{J_o(k_m a_1)\, N_1(k_m a_1) - N_o(k_m a_1)\, J_1(k_m a_1)}{J_o(k_m a_2)\, N_1(k_m a_1) - N_o(k_m a_2)\, J_1(k_m a_1)} . \tag{17}$$

Equation 17 establishes the relationship between the field inside the imperfectly conducting hollow cylinder to the field on its outside surface.

If the relation

$$a_1 \sqrt{\omega \mu_1 \sigma_1} \geq 10^{*} \tag{18}$$

holds, it is possible to use the asymptotic forms of the Bessel functions to effect a simplification in Equation 17. These are:

$$\left.\begin{aligned} J_o(x) &= \sqrt{\frac{2}{\pi x}} \cos\left(x - \frac{\pi}{4}\right) \\ J_1(x) &= \sqrt{\frac{2}{\pi x}} \cos\left(x - \frac{3\pi}{4}\right) \\ N_o(x) &= \sqrt{\frac{2}{\pi x}} \sin\left(x - \frac{\pi}{4}\right) \\ N_1(x) &= \sqrt{\frac{2}{\pi x}} \sin\left(x - \frac{3\pi}{4}\right) \end{aligned}\right\} . \tag{19}$$

*At 15 kcs an aluminum cylinder must be at least 0.447 cm in radius; an iron shield could be smaller.

hen Equation 19 is substituted into Equation 17, one obtains

$$\frac{E_z(a_1)}{E_z(a_2)} = \sqrt{\frac{a_2}{a_1}} \left\{\frac{1}{\cos\left[k_m(a_2 - a_1)\right]}\right\} . \tag{20}$$

Now

$$\cos\left[(1 - j)\gamma\right] = \cos\gamma \cosh\gamma + j \sin\gamma \sinh\gamma \tag{21}$$

and

$$\left|\cos\left[(1 - j)\gamma\right]\right| = \sqrt{\cos^2\gamma + \sinh^2\gamma} . \tag{22}$$

Accordingly,

$$\frac{E_z(a_1)}{E_z(a_2)} = \sqrt{\frac{a_2}{a_1}} \left\{\frac{\cos\gamma \cosh\gamma - j \sin\gamma \sinh\gamma}{\cos^2\gamma + \sinh^2\gamma}\right\} \tag{23}$$

and

$$\frac{\left|E_z(a_1)\right|}{\left|E_z(a_2)\right|} = \sqrt{\frac{a_2}{a_1}} \left\{\frac{1}{\sqrt{\cos^2\gamma + \sinh^2\gamma}}\right\} . \tag{24}$$

Here

$$\gamma = (a_2 - a_1)\sqrt{\frac{\omega\sigma_1\mu_1}{2}} . \tag{25}$$

Equation 23 must be used when the complex ratio of $E_z(a_1)/E_z(a_2)$ is required. When the ratio of the magnitudes suffices, Equation 24 is valid. In either case it is necessary for the inequality, Equation 18, to be satisfied.

Relation Between the Field Incident Upon an Imperfectly Conducting Hollow Cylinder and the Field on its Outside Surface

Equation 16 established the relationship between the electric field $E_z(r)$ and $E_z(a_2)$. Because

$$i_z(r) = \sigma_1 E_z(r) , \tag{26}$$

$$\frac{E_z(r)}{E_z(a_2)} = \frac{i_z(r)}{i_z(a_2)} . \tag{27}$$

Here $i_z(r)$ is the volume density of current at radius r, $(a_1 \leq r \leq a_2)$; $i_z(a_2)$ is the volume density of current extrapolated to the surface $r = a_2$. At the surface of the cylinder the relation

$$i_z(a_2) = \sigma_1 E_z(a_2) \tag{28}$$

holds because surface cells may be disregarded in describing tangential effects.

The total current flowing in the cylinder is given by the formula

$$I(0) = \int_{a_1}^{a_2} i_z(r)\, 2\pi r\, dr. \tag{29}$$

Replacing $E_z(r)/E_z(a_2)$ by $i_z(r)/i_z(a_2)$ in Equation 16 and integrating it as specified by Equation 29 results in the expression

$$i_z(a_2) = \frac{I(0)}{\pi a_2^2} \left(\frac{k_m a_2}{2}\right) \left\{\frac{J_o(k_m a_2)\, N_1(k_m a_1) - N_o(k_m a_2)\, J_1(k_m a_1)}{J_1(k_m a_2)\, N_1(k_m a_1) - N_1(k_m a_2)\, J_1(k_m a_1)}\right\}. \tag{30}$$

The integration is accomplished by use of the relations

$$\left.\begin{aligned} \int x\, J_o(ax)\, dx &= \frac{x}{a} J_1(ax) \\ \int x\, N_o(ax)\, dx &= \frac{x}{a} N_1(ax) \end{aligned}\right\}. \tag{31}$$

Using the asymptotic forms of the Bessel functions (assuming $a_1 \sqrt{\omega \mu_1 \sigma_1} \geq 10$) it is easy to demonstrate that

$$i_z(a_2) = \frac{I(0)}{\pi a_2^2} \left(\frac{k_m a_2}{2}\right) \cot\left[k_m(a_2 - a_1)\right]. \tag{32}$$

Observe that

$$\cot\left[(1 - j)\gamma\right] = \frac{\cos\gamma \cosh\gamma + j \sin\gamma \sinh\gamma}{\sin\gamma \cosh\gamma - j \cos\gamma \sinh\gamma}. \tag{33}$$

On substituting Equation 28 into Equation 32, one obtains

$$E_z(a_2) = \frac{I(0)}{\pi \sigma_1 a_2^2} \left(\frac{k_m a_2}{2}\right) \cot\left[k_m(a_2 - a_1)\right]. \tag{34}$$

This expression relates the electric field on the outside surface of the hollow cylinder to the total current I(0) flowing in the cylinder.

The current I(0) must be found using antenna theory. For this purpose one considers the cylindrical shield to be a receiving antenna without load (refer to Figure 2). The current at its center is I(0). This is the short-circuit current, and is available from the formula:

$$I(0) = \frac{V_{oc}}{Z_{in}} = \frac{2h_e E_z^i}{Z_{in}} \quad . \tag{35}$$

V_{oc} is the induced voltage. It equals the effective height of the cylinder $2h_e$ multiplied by the incident tangentially directed electric field, E_z^i. Z_{in} is the driving-point impedance of a symmetrical center-driven antenna having dimensions identical to those of the shield. That Equation 35 is correct is immediately verified by sketching the equivalent circuit of the receiving antenna, omitting the usual load impedance.

$2h_e$, as used in Equation 35, is

$$2h_e = 0.958\,\ell\left\{1 + 0.061\,(k_s\ell)^2\right\} \tag{36}$$

where ℓ is the half-length of the shield, and $k_s = 2\pi/\lambda$. λ is the wavelength. It is assumed that $k_s\ell \leq 0.5$.

The radiation impedance of a short antenna for which $k_s\ell \leq 0.5$ is given by the formula

$$Z_o^r = -j\,\frac{\eta}{2\pi k_s\ell}\left\{(\Omega - 2 - 2\ln 2)\left(1 - j\,\frac{k_s\ell}{3(\Omega - 2 - 2\ln 2)}\right)\right\}. \tag{37}$$

In Equation 37

$$\eta = \sqrt{\frac{\mu_o}{\epsilon_o}} \approx 120\pi \tag{38}$$

and

$$\Omega = 2\ln\left(\frac{2\ell}{a_2}\right) \quad . \tag{39}$$

Now

$$Z_{in} = Z_o^r + R_o^\ell \tag{40}$$

where R_o^ℓ is the ohmic loss resistance referred to the input terminals.

R_o^ℓ is evaluated as follows:

The current distribution along an electrically short transmitting antenna is triangular in shape. Hence the average current along the structure in terms of the driving-point current is

$$I_{av} = \frac{I(0)}{2} \quad . \tag{41}$$

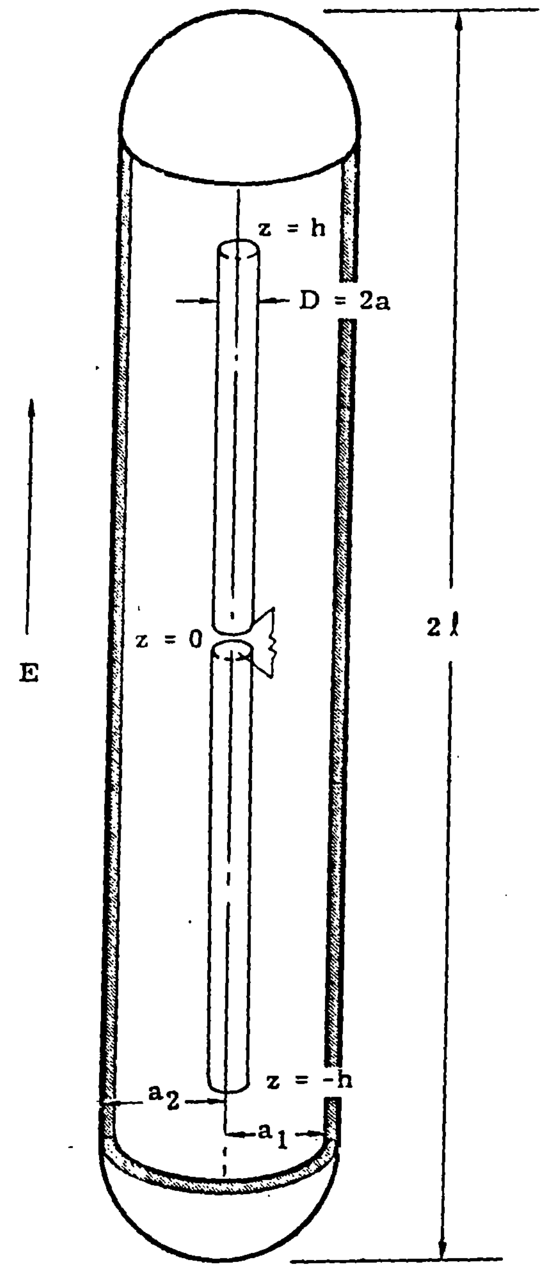

Figure 2. Electric probe symmetrically oriented along the axis of an imperfectly conducting cylinder

A formula for the resistance of the tube of length 2ℓ has been derived by King.[2] It is

$$R^t = \frac{\ell k_m}{\pi a_2 \sigma_1} \left(\sqrt{\frac{\cosh 2\gamma + \cos 2\gamma}{\cosh 2\gamma - \cos 2\gamma}} \right) \cos\left(\psi_r - \psi_c + \frac{\pi}{4}\right) \tag{42}$$

where

$$\tan \psi_r = -\tanh \gamma \tan \gamma \tag{43}$$

and

$$\tan \psi_c = \frac{\tanh \gamma}{\tan \gamma} , \tag{44}$$

provided $a_1 \sqrt{\omega \mu_1 \sigma_1} \geq 10$. The power lost in heating the antenna is

$$P_L = I^2(0) R_o^{\ell} = \left(\frac{I(0)}{2}\right)^2 R^t. \tag{45}$$

Hence

$$R_o^{\ell} = \frac{R^t}{4} . \tag{46}$$

Accordingly

$$Z_{in} = -j \frac{\eta}{2\pi k_s \ell} \left\{ (\Omega - 2 - 2\ell n 2)\left(1 - j \frac{k_s \ell}{3(\Omega - 2 - 2\ell n 2)}\right) \right\} + \frac{R^t}{4} . \tag{47}$$

Substituting Equation 35 into Equation 34,

$$E_z(a_2) = \frac{2 h_e E_z^i}{\pi Z_{in} \sigma_1 a_2^2} \left(\frac{k_m a_2}{2}\right) \cot \left[k_m (a_2 - a_1)\right] . \tag{48}$$

Thus, if one knows the incident field E_z^i in the vicinity of the cylinder, Equation 48 may be used to find the field $E_z(a_2)$ on the outside surface. The field within the cylinder may then be calculated, using Equation 23.

Effective Height and Driving-Point Impedance of an Electric Probe within an Imperfectly Conducting Cylinder

To determine the behavior of a center-loaded electric probe symmetrically located along the axis of an imperfectly conducting cylinder requires not only a knowledge of the electric field inside the cylinder in terms of the incident electric field, but additionally one must know the effective length and driving-point impedance of the probe.

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[2] King, Ronold, op. cit., p. 357, Equation 4.

The effective length of a probe encased within an imperfectly conducting cylinder is the same as its effective length when isolated. The effective length is defined in terms of the field maintained along the probe by all currents other than in the probe itself. This is precisely the field $E_z(a_2)$ maintained in the cylinder by the distant signal source and the current in the imperfectly conducting shield. (It is to be remembered that the field in the interior of the tube is uniform.) The only approximation involved is the assumption that the current in the probe does not react on currents in the shield to change them significantly. The required effective length $2h_e$ may be calculated using Equation 36 provided h is written for ℓ throughout (h is the half-length of the electric probe).

The driving-point impedance of the electric probe encased in the imperfectly conducting cylinder lies between the impedance of an identical isolated probe and the impedance of two perfectly conducting coaxial transmission-line sections in series. The line sections are open-circuited, and of length equal to the half-length of the probe. Capacitive end effects are probably small enough to neglect. In terms of the electromagnetic field set up by the probe antenna, when used for transmission, three situations must be considered:

a. When the probe is isolated, i.e., is outside the metal cylinder, the distant field is due entirely to the currents in the antenna.

b. When the probe is in a perfectly conducting cylinder, the field maintained by the currents in the antenna is exactly cancelled by the currents set up in the shield.

c. When the probe is in an imperfectly conducting cylinder, currents are induced in the shield which maintain a field that partially cancels the field of the probe, but this is not complete.

Thus the current in the probe can be separated into two parts. One part is equal and opposite to the current in the shield, designated I_L, because it is a transmission-line current. The other part is the true antenna current, designated I_A. The total short-circuit current I_t in the probe is then

$$I_t = I_A + I_L \,. \tag{49}$$

The open-circuit voltage of the probe is

$$V_{oc} = I_t Z_o = I_A Z_A + I_L Z_L = I_t \left\{ Z_L - \frac{I_A}{I_t} \left(Z_L - Z_A \right) \right\} . \tag{50}$$

Here

Z_o is the impedance of the probe when encased in the imperfectly conducting cylinder.

Z_L is the impedance of two perfectly conducting open-circuited coaxial line sections of equal length connected in series. Z_L may be computed from the formula

$$Z_L = -j\ 276 \log_{10}\left(\frac{a_1}{a}\right) \cot k_s h \tag{51}$$

where h is the half-length of the probe and a is its radius.

Z_A is the self-impedance of the probe, when isolated. It is given by Equation 40, or by Equation 37, if losses are negligible.

From Equation 50,

$$Z_o = \frac{V_{oc}}{I_t} = Z_L - \frac{I_A}{I_t}(Z_L - Z_A). \tag{52}$$

The current I_A is the current in the probe required to maintain the field $E_z(a_2)$ just outside the shield, if the shield were absent. I_t is the current required to maintain the field $E_z(a_1)$ just inside the shield. Since the thickness of the shield is small compared with any radial .istance over which E_z could vary significantly in amplitude, it follows that to a good approximation

$$\frac{I_A}{I_t} = \frac{E_z(a_2)}{E_z(a_1)}\ . \tag{53}$$

This complex ratio is given by Equation 23, when inverted. On substituting Equation 53 into Equation 52, one obtains

$$Z_o = Z_L - \frac{E_z(a_2)}{E_z(a_1)}\left\{Z_L - Z_A\right\}. \tag{54}$$

This is the final formula for the driving-point impedance of an electric probe symmetrically situated within an imperfectly conducting cylinder.

It is important to observe that the ratio $E_z(a_2)/E_z(a_1)$ occurring in Equation 54 is small. When the probe is driven $E_z(a_2)$, the field on the outside of the shield is small, but the field on the inside of the shield, $E_z(a_1)$, is large. On the other hand in the receiving case $E_z(a_2)$ is large and $E_z(a_1)$ is small. Thus

$$\underbrace{\left\{\frac{E_z(a_2)}{E_z(a_1)}\right\}}_{\text{Driven probe}} = \underbrace{\left\{\frac{E_z(a_1)}{E_z(a_2)}\right\}}_{\text{Receiving probe}}\ . \tag{55}$$

Since the ratio $E_z(a_2)/E_z(a_1)$ is small when the dipole is driven (assuming that the cylinder is a moderately good conductor),

$$Z_o \approx Z_L . \tag{56}$$

This result is dictated by common sense. On the other hand when the ratio $E_z(a_2)/E_z(a_1) \rightarrow 1$, as is the case when the cylindrical shield is nonexistent,

$$Z_o \approx Z_A . \tag{57}$$

Equations 56 and 57 serve as limiting checks on the validity of Equation 54.

The equivalent receiving circuit of the probe consists of a voltage equal to $2h_e E_z(a_1)$ driving a circuit consisting of Z_o in series with the load impedance. The power in the load is thus readily calculated in terms of the field $E_z(a_1)$.

The impedance of an electric probe is essentially a capacitive reactance, the resistive component being only a fraction of an ohm. To maximize the power in a given load, one would tune the probe to resonance, using series inductors. If this is done, the full induced voltage appears across the load, if all losses are neglected. Z_o does not appear in the equivalent circuit of the probe when it is tuned to resonance, and under this circumstance, Equation 54 is not needed.

Conclusion

A method has been presented for calculating the power in the load of an electric probe encased in a partially conducting cylindrical shield. The analysis has been carried out for sinusoidal signals. If the signal source emits repetitive pulses, it is necessary to make a Fourier analysis of the incoming wave and solve the problem for the fundamental and several harmonic frequencies. The total power in the load impedance is then the sum of the powers absorbed at each frequency considered. Again it is assumed that the circuit dimensions are small in terms of the shortest wavelength of the signal component that contributes significantly to the power in the load.

The field incident upon the cylindrical shield need not be linearly polarized, as assumed in the analysis. An elliptically polarized electric field may be decomposed into two components which are fixed in space and which differ in phase, magnitude, and direction. In particular, the field may be resolved into two mutually perpendicular components which remain stationary in space. One of these components may be chosen parallel to the cylindrical shield, and the other perpendicular to it. Both vary periodically in time, but the latter contributes nothing to the induced voltage; its magnitude and phase are of no significance whatsoever, and

It may be ignored. Accordingly, maximum power is delivered to the load impedance of the probe when the hollow cylinder is oriented in space so that the major axis of the elliptical contour of the electric field is directed parallel to the axis of the cylindrical shield.

REFERENCE

The following reference to the literature dealing with the problem of electromagnetic shielding was brought to the attention of the writer after SCTM 396-58(14) and SCTM 457-58 (14) had been completed:

King, Louis V., "Electromagnetic Shielding at Radio Frequencies," Philosophical Magazine, Vol. 15, No. 97, February 1933, pp. 201-223.

This is reported to be an excellent article on the subject.

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APPENDIX

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