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CYLINDRICAL SHIELDS

by

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Cylindrical Shields*

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Summary—**The effectiveness of an imperfectly-conducting cylindrical shield of small cross section depends on both the attenuation through the metal wall of the externally maintained field and the amplitude of the current that is induced in the cylinder. When the length of the cylinder, which behaves like a linear scattering antenna, approaches a resonant value, the currents induced in the walls and the field inside the tube are relatively large. Under these conditions, large currents may be induced in a thin dipole placed coaxially within the shield.**

Introduction

In a recent paper,[1] a general theory of shielding by imperfectly conducting walls enclosing a region of finite size was formulated and applied specifically to the response of a dipole probe in a metal cylinder that is electrically small, both in cross section and in length. It is the purpose of this investigation to consider shielding by cylinders that are electrically small in cross section but may be up to a wavelength long. Two problems are considered. First, the determination of the axial electric field in a metal cylinder of length $2l$, inner radius b and outer radius c, when immersed in an incident field E^i parallel to its axis, is outlined. Second, the response of a thin dipole antenna of length $2h$ and radius a, placed coaxially within the shield, is investigated.

In order to permit the application of the theory of cylindrical antennas, the following conditions are imposed:

$$c \ll l, \qquad \beta_0 c \ll 1, \tag{1}$$

$$a \ll h, \qquad \beta_0 a \ll 1, \tag{2}$$

where $\beta_0 = \omega\sqrt{\mu_0\epsilon_0} = 2\pi/\lambda_0$ is the free space wave number. The conductivity of the metal walls and of the dipole[2] is σ, the permeability $\mu = \mu_0\mu_r$. The complex propagation constant in the metal is

$$k = j^{-1/2}\beta, \qquad \beta = \sqrt{\omega\mu\sigma}. \tag{3}$$

In most cases it may be assumed that

$$\beta b \geqq 10, \tag{4}$$

so that the simpler asymptotic forms of the Bessel functions may be used. Formally, the general case may be carried through without difficulty. Note that the inequalities (1) and (4) are compatible since

$$(\beta/\beta_0) = \sqrt{\mu_r\sigma/\omega\epsilon_0} \gg 1, \tag{5}$$

by definition of a good conductor.

The coaxial cylinders to be analyzed are shown in Fig. 1. They are equivalent to a coaxial line with two electrically open ends but with a completely closed

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[1] C. W. Harrison, Jr. and R. W. P. King, "Response of a loaded dipole in an imperfectly conducting cylinder of finite length," *J. Res. NBS*, sect. D, vol. 64D, pp. 289–293; May–June, 1960.

[2] For simplicity, the conductivities of the dipole and the shield are assumed to be the same, since the numerical value of the conductivity of the dipole is unimportant as long as it is in the range of metals used as electrical conductors. If desired, separate values may be used with no difficulty.

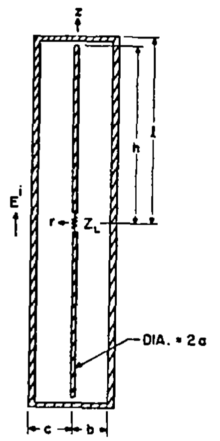

Fig. 1—Dipole in cylindrical shield.

shield. The electrically open ends are obtained by means of gaps between the ends of the inner conductor and the metal disks that close the ends of the shield. A lumped load Z_L is shown connected in series with the inner conductor at its center. The electric field E^i parallel to the axis of the cylinders is maintained by an external source.

For completeness, a brief summary of the relevant parts of the earlier work[1] is given together with the appropriately generalized formulas.

The Electric Field in a Closed Tubular Conductor

The first problem is to determine the ratio of the axial field inside the shield to the incident field when there is no inner conductor.

Since the tubular shield satisfies the conditions (1) of linear antenna theory, it may be treated as an unloaded receiving antenna in a uniform field. The total axial current $I_c(z)$ has the following leading term,[3]

$$I_c(z) = I_c(0)\left[\frac{\cos\beta_0 z - \cos\beta_0 l}{1 - \cos\beta_0 l}\right], \tag{6}$$

where

$$I_c(0) = 2l_e E^i / Z_{cin}. \tag{7}$$

In (7), $2l_e$ is the effective length of an antenna of actual length $2l$ and Z_{cin} is the input impedance of the antenna when cut in two at the center and driven by a delta-function generator. Curves of l_e/λ are in King;[3] an approximate formula when $\beta_0 l < \pi$ is

$$\beta_0 l_e = \tan(\beta_0 l/2). \tag{8}$$

The input impedance of an imperfectly conducting antenna is given by

$$Z_{cin} = Z_{c0} + Z_c^i, \tag{9}$$

where Z_{c0}, the impedance of a perfectly conducting antenna, is given in tabular and graphical form in King,[4] and

$$Z_c^i = (z_c^i/\beta_0)(\beta_0 l \csc^2\beta_0 l - \cot\beta_0 l) \tag{10}$$

is the contribution from ohmic resistance.[5] The internal impedance z_c^i per unit length of a tube of outer radius c is given below. In general, Z_c^i is negligible compared with Z_{c0}.

The general formula for the internal impedance per unit length of a tubular conductor of outer radius c and inner radius b is[6]

$$z_c^i = \frac{E(c,z)}{I_c(z)} = \frac{k}{2\pi c\sigma}\left[\frac{J_0(kc)N_1(kb) - N_0(kc)J_1(kb)}{J_1(kc)N_1(kb) - N_1(kc)J_1(kb)}\right]. \tag{11a}$$

When (4) is satisfied, (11a) reduces to

$$z_c^i = \frac{k}{2\pi c\sigma}\sqrt{\frac{\cosh 2A_c + \cos 2A_c}{\cosh 2A_c - \cos 2A_c}}\, e^{j(\Phi_c + \psi_c - \pi/4)}. \tag{11b}$$

If the additional condition, $A_c \geqq 4$, is imposed (which is equivalent to requiring the wall thickness to be at least four times the skin depth) the following simple formula is obtained:

$$z_c^i = \frac{\beta}{2\pi c\sigma} e^{j\pi/4}. \tag{11c}$$

In the above formulas:

$$\tan\Phi_c = \tanh A_c \tan A_c \tag{12a}$$

$$\tan\psi_c = \tanh A_c \cot A_c \tag{12b}$$

and

$$A_c = (c - b)/d_s \tag{13}$$

where

$$d_s = \sqrt{2/\omega\mu\sigma} \tag{14}$$

is the skin depth.

With (11) and (7), it follows that,

$$E(c,0)/E^i = 2l_e z_c^i / Z_{cin}. \tag{15}$$

This is the fundamental relation between the field at the outer surface of the tube and the incident field.

The relation between the tangential field at the inner surface ($r = b$) to that at the outer surface ($r = c$) has been given in the literature.[1,6] It is,

$$\frac{E(b,z)}{E(c,z)} = \frac{J_0(kb)N_1(kb) - N_0(kb)J_1(kb)}{J_0(kc)N_1(kb) - N_0(kc)J_1(kb)}. \tag{16a}$$

[3] R. W. P. King, "Theory of Linear Antennas," Harvard University Press, Cambridge Mass., ch. 4, p. 469, (18) and p. 492; 1956.

[4] Ibid., pp. 154–179.

[5] Ibid., p. 147, (11) and (12).

[6] R. W. P. King, "Electromagnetic Engineering," McGraw-Hill Book Co., Inc., New York, N. Y., p. 356; 1945.

Subject to (4) this becomes,

$$\frac{E(b, z)}{E(c, z)} = \sqrt{\frac{c}{b}}\left(\frac{e^{-j\Phi_c}}{\sqrt{\frac{1}{2}[\cosh 2A_c + \cos 2A_c]}}\right) \quad (16b)$$

with the additional restriction, $A_c \geqq 4$, the following simple result is obtained;

$$\frac{E(b, z)}{E(c, z)} = 2\sqrt{\frac{c}{b}}\, e^{-A_c(1+j)}. \quad (16c)$$

The condition $\beta_0 b \ll 1$, which is implied in (1), has as a consequence the relation

$$\frac{E(r, z)}{E(b, z)} = \frac{J_0(\beta_0 r)}{J_0(\beta_0 b)} \doteq 1 \quad \text{for} \quad r \leqq b. \quad (17)$$

The combination of (17) with (16) and (15) and the use of (11) leads to the following formulas for the ratio of the axial electric field in the air within the conducting tube to the incident field outside the tube: $(r \leqq b)$

$$\delta = \frac{E(r, 0)}{E^i}$$
$$= \frac{l_e k}{\pi c \sigma Z_{cin}}\left[\frac{J_0(kb)N_1(kb) - N_0(kb)J_1(kb)}{J_1(kc)N_1(kb) - N_1(kc)J_1(kb)}\right], \quad (18a)$$

when $\beta b \geqq 10$,

$$\delta = \frac{E(r, 0)}{E^i} = \frac{l_e \beta}{\pi\sigma\sqrt{bc}\, Z_{cin}} \frac{e^{j(\Psi_c - \pi/4)}}{\sqrt{\frac{1}{2}[\cosh 2A_c - \cos 2A_c]}} \cdot \quad (18b)$$

When, in addition, $A_c \geqq 4$,[7]

$$\delta = \frac{E(r, 0)}{E^i} = \frac{2l_e\beta}{\pi\sigma\sqrt{bc}\, Z_{cin}} e^{-A_c(1+j)} e^{j\pi/4}; \quad (18c)$$

where $A_c = (c-b)/d_s$. Except near the ends at $z = \pm l$, the axial distribution of the electric field is obtained from the relation $E = z^i I$ with (6). It is approximately

$$E(r, z) = E(r, 0)\left[\frac{\cos\beta_0 z - \cos\beta_0 l}{1 - \cos\beta_0 l}\right], \quad (19)$$

when $\beta_0 l < 2\pi$. This completes the determination of the axial electric field in the interior of the imperfectly conducting tube when it contains no inner conductor.

The Current in a Dipole within the Shield

When a conducting dipole is placed along the axis in the air within the shield, the axial field that exits there induces currents in the dipole. These currents in turn set up a field that induces additional currents in the shield and, since the shield is imperfectly conducting, a field is also maintained outside the shield. It will be assumed that this is much smaller than the incident field as, indeed, it will be if the ratio in (18) is small. Let the field maintained by the current in the dipole be denoted by $E'(r, z)$. At the outer surface of the shield it is $E'(c, z)$. It follows that the current that must be maintained on the shield in order to satisfy the boundary condition for the continuity of the tangential component of the electric field at $r = c$ is given by (6) and (7) with $E^i + E'(c, 0)$ substituted for E^i in (7). Since it is assumed that

$$\left|\frac{E'(c, 0)}{E^i}\right| \ll 1, \quad (20)$$

no serious error is made by replacing $E'(c, z)$ by the constant value $E'(c, 0)$ at the center. However, if (20) is satisfied the resulting change in the small ratio δ is of higher order and may be neglected. In other words, it is assumed that the attenuation through the shield is sufficiently great so that what might be described as multiple reflections from the dipole back through the shield may be neglected.

Subject to (20), the field $E(a, z)$ given by (19) with (18) and $r = a$ at the surface of the dipole is independent of the current in the dipole. This latter is then given by

$$I_a(z) = I_a(0)\frac{\cos\beta_0 z - \cos\beta_0 h}{1 - \cos\beta_0 h} \quad (21)$$

with

$$I_a(0) = 2h_e E(a, 0)/(Z_{ain} + Z_L), \quad (22)$$

if the approximation is made of replacing $E(a, z)$ by $E(a, 0)$. Since for dipoles of length $2h < \lambda$ the large currents are all induced near the center, no large error is involved in this simplification. In (22) $2h_e$ is the effective length of the dipole as if it were in free space, Z_{ain} is its input impedance, and Z_L is a load that may be connected in series at its center.

The input impedance of the dipole in the imperfectly conducting shield is not the same as it would be if it were either in a perfectly conducting shield or in free space. Its approximate value may be obtained as follows. Consider the dipole center driven by a delta-function voltage V that maintains a current $I(z)$. This current may be separated into two parts,

$$I(z) = I_A(z) + I_T(z), \quad (23)$$

where $I_T(z)$ is the part for which an equal and opposite current is induced in the shield, and $I_A(z)$ is the part for which no current is induced in the shield. If the shield were perfectly conducting, $I_A(z)$ would be zero. In other words, $I_A(z)$ is the algebraic sum of the currents in the dipole and the induced currents in the shield. Since the electromagnetic field outside the shield due to $I_T(z)$ and the current induced in the shield is zero, the entire field outside the shield is that maintained by $I_A(z)$, just as if there were no shield and $I_A(z)$ were the total current on the dipole. It follows that

$$V = I(0)Z_{ain} = I_A(0)Z_{a0} + I_T(0)Z_T, \quad (24)$$

where Z_{a0} is the input impedance of the dipole in free

[7] For example, with an aluminum shield ($\sigma = 3.54 \times 10^7$ 1/ohm-m) at a frequency of 10 kc, this condition leads to $c - b \geqq 4d_s = 4\sqrt{2/\omega\mu\sigma} = 0.338$ cm.

space and Z_T is the input impedance of two sections of transmission line in series. That is,

$$Z_T = 2R_c \coth\left[\alpha h + j(\beta_0 h + \Phi_h)\right], \tag{25}$$

where R_c is the characteristic impedance of the coaxial line, $\alpha = 2r_a{}^i/R_c$ is the attenuation constant, and $\Phi_h \doteq 0$ is the terminal phase function of the ends which are open-circuited. With (23), the input impedance of the dipole in the shield may be expressed in the form

$$Z_{ain} = Z_T + \frac{I_A(0)}{I(0)}(Z_{a0} - Z_T). \tag{26}$$

The ratio of currents in (26) may be replaced by a ratio of fields in the following manner. The field $E'(c, 0)$ maintained on the outer surface of the shield is due entirely to $I_A(z)$ and is, in fact, proportional to it. The field $E^i(b, 0)$ incident on the inner surface of the shield is proportional to the total current $I(z)$ in the dipole. Both fields are those that would be maintained in the absence of the shield. Since the wall thickness $(c-b)$ is a very small fraction of a free-space wavelength, it follows that

$$\frac{I_A(0)}{I(0)} = \frac{E'(c, 0)}{E^i(b, 0)} = \delta'. \tag{27}$$

It may now be argued that the attenuation through the shield of a field that is incident from the outside must be essentially the same as the attenuation through the same shield of a field incident from the inside, provided βb is sufficiently great to satisfy (4). That is,

$$\delta' = \delta. \tag{28}$$

With (27) and (28) it follows that

$$Z_{ain} = Z_T + \delta(Z_{a0} - Z_T), \tag{29}$$

where δ is given by (18b) or (18c). Note that if the walls of the shield are perfectly conducting $\delta = 0$ and $Z_{ain} = Z_T$; similarly, when the shield is absent, $\delta = 1$ and $Z_{ain} = Z_{a0}$.

The final expression for the current at the center of the dipole when enclosed by the imperfectly conducting cylindrical shield may now be expressed as follows,

$$\frac{I_a(0)}{E^i} = \frac{2h_e\delta}{Z_T + \delta(Z_{a0} - Z_T) + Z_L}, \tag{30}$$

where δ is given by (18b) or (18c) and it is assumed that (4) is satisfied together with (20).

Numerical Examples

In order to obtain a quantitative estimate of the magnitude of the ratio δ of the field in a cylindrical shield of finite length and of the current that may be induced in a conductor along the axis of such a shield, consider the following numerical examples. These involve an aluminum shield of given wall thickness and cross-sectional size but two different lengths, the one very short compared with the wavelength, the other a half wavelength long.

The subscripts 1 and 2 appearing on the various parameters entering the problem refer to the short and half-wavelength long shields, respectively. The cylinder and inner conductor are made of aluminum. $\sigma = 3.54 \times 10^7$ (ohm-meter)$^{-1}$. In both illustrations, $f = 10$ kc, $\omega = 2\pi f = 6.283 \times 10^4$, $\lambda_0 = 3 \times 10^4$ meter, $\beta_0 = 2\pi/\lambda_0 = 2.094 \times 10^{-4}$ meter^{-1}.

The Cylinder as an Antenna

$$c = 6.8 \text{ cm},\ b = 6.7 \text{ cm},\ (c - b) = 1 \text{ mm}$$

$$l_1 = 5m,\ \beta_0 l_1 = 1.047 \times 10^{-3},\ \Omega_{1c} = 2 \ln \frac{2l_1}{c} = 9.98$$

$$l_2 = 7.5 \text{ km},\ \beta_0 l_2 = 1.571,\ \Omega_{2c} = 2 \ln \frac{2l_2}{c} = 24.61$$

$$\beta = \sqrt{\omega\mu\sigma} = 1.672 \times 10^3 \text{m}^{-1}$$

$$\mu = 4\pi \times 10^{-7} \text{ henry} - \text{m}^{-1}$$

$$\beta b = 112.02,\ d_s = \sqrt{\frac{2}{\omega\mu\sigma}} = 0.846 \text{ mm}$$

$$A_c = \frac{(c - b)}{d_s} = 1.182$$

$Z_{c\,in\,1} = -j3.78 \times 10^5$ ohms[8]

$Z_{c\,in\,2} \approx 77.3 + j43.6 = 88.75e^{j0.514}$ ohms[9]

$l_{e1} \approx \frac{1}{2}l_1 = 2.5$m[10]

$l_{e2} = 5.04 \times 10^3$m[11]

Computed Field Ratio

$$\delta_1 = \frac{E(a, 0)}{E^i} = j0.845 \times 10^{-9} e^{-j0.458}$$

$$= (0.374 + j0.758)10^{-9} \qquad \text{[from (18b)]}$$

$$\delta_2 = \frac{E(a, 0)}{E^i} = 7.25 \times 10^{-3} e^{-j0.972}$$

$$= (4.08 - j5.98)10^{-3} \qquad \text{[from (18b)]}$$

Interior Data

Center Conductor:

$$a = 2 \times 10^{-3}\text{m},\ h_1 = 5\text{m},\ \beta_0 h_1 = 1.047 \times 10^{-3},$$

$$\Omega_1 = 2 \ln \frac{2h_1}{a} = 17.03,$$

$$h_2 = 7.5 \text{ km},\ \beta_0 h_2 = 1.571,\ \Omega_2 = 2 \ln \frac{2h_2}{a} = 31.66$$

$h_{e1} \approx \frac{1}{2}h_1 = 2.5$m[10]

$h_{e2} \approx 4.77 \times 10^3$m[12]

$Z_{0a1} = -j7.817 \times 10^5$ ohms[13]

[8] King, *op. cit.*, footnote 3, p. 192 (46b).
[9] *Ibid.*, by extrapolation of Table 30.1, p. 168.
[10] *Ibid.*, p. 496.
[11] *Ibid.*, p. 492, Fig. 9.6b; $h_e/\lambda_0 = 0.168$ for $\Omega_{2c} = 24.61$.
[12] *Ibid.*, p. 492, Fig. 9.6b; $h_e/\lambda \approx 0.159$ for $\Omega \approx \infty$.
[13] *Ibid.*, p. 184, (6c) with $\Omega = 17.03$.

$$Z_{0a2} \approx 74 + j42.5 = 85.3e^{j0.52114}$$

$$Z_{T1} = 2R_c \coth(\alpha + j\beta_0)h_1 \approx \frac{-j2R_c}{\beta_0 h_1}$$

$$R_c = 60 \ln \frac{b}{a} = 210.7 \text{ ohms. Hence}$$

$$Z_{T1} = -j4.03 \times 10^5 \text{ ohms.}$$

$$Z_{T2} = 2R_c \tanh \alpha h_2 = 2R_c \alpha h_2 \approx r^i h_2$$

$$= (r_a{}^i + r_c{}^i)h_2, \text{ since } \alpha = \frac{r^i}{2R_c}.$$

Also[15]

$$r^i = \frac{\beta a}{2\pi a^2 \sigma} \frac{M_0(\beta a)}{M_1(\beta a)} \cos\left[\frac{3\pi}{4} + \theta_0(\beta a) - \theta_1(\beta a)\right]$$

$$\beta a = 3.3$$

$$M_0(3.3) = 2.301, \theta_0(3.3) = 109°.25$$ [16]

$$M_1(3.3) = 2.124, \theta_1(3.3) = 206°.83$$ [17]

$$r_a{}^i = 3.23 \times 10^{-3}$$ [18]

$$r_c{}^i = 9.59 \times 10^{-5} \, [(11b) \text{ with } (12a) \text{ and } (12b)]$$

$$r^i = r_a{}^i + r_c{}^i = 3.33 \times 10^{-3}$$

$$Z_{T2} = r^i h_2 = 24.95 \text{ ohms.}$$

Ratio of the Current at the Middle of the Unloaded Center Conductor to the Incident Field

$$\frac{I_{a1}(0)}{E^i} = 1.05 \times 10^{-14} e^{j2.683} \, [(30) \text{ with } Z_L = 0]$$

$$\frac{I_{a2}(0)}{E^i} = 2.72 e^{-j0.967} \, [(30) \text{ with } Z_L = 0]$$

Thus, for an incident field E^i of 10 volts/meter the magnitude of the current at the center of the inner conductor is only 1.05×10^{-13} amperes for the case of the short cylinder. However, for the half-wave cylinder the current is 27.2 amperes for the same incident field.

[14] *Ibid.*, p. 168, by extrapolation with Table 30.1, noting that for $\Omega \approx \infty$, $Z = 73.1 + j\,42.5$.
[15] King. *op, cit.*, footnote 6, p. 346, (2).
[16] *Ibid.*, p. 523.
[17] *Ibid.*, p. 524.
[18] *Ibid.*, p. 346, (2).

Conclusion

The field in the interior of a cylindrical shield depends on the attenuation through the shield and on the amplitude of the current that can be induced on it by the external field. When a closed cylindrical shield is very short compared with the wavelength, the effective length is small and the impedance is enormous. It follows that the induced current is very small, and with it the field in the interior of the shield. When the shield approaches a resonant length, the effective length is relatively large and the impedance quite small. It follows that a rather large current is induced in the cylinder and a correspondingly large field is maintained in the interior.

If a dipole is placed inside the shield with its ends not connected to the metal end surfaces of the shield, the current induced in it depends on its length and on the magnitude of the surrounding field. When the field in the shield is small and the length of the dipole is far from a resonant value, the current induced in the dipole is extremely small. On the other hand, when the field in the shield is more intense and the dipole has a resonant length, surprisingly large induced currents are possible.

Similar results may be expected if the dipole in a shield is replaced by a conventional coaxial line with a load at one end and a generator at the other. If the length of the line is resonant at the frequency of an external field and the wall thickness is not very great compared with the skin depth, significant fields may be maintained within the shield. These may induce relatively large currents in the coaxial line, particularly if a resonant condition obtains. Evidently, all of the conclusions reached for solid metal shields also apply to the practically important case of braided shields of comparable thickness.

Although somewhat idealized, the assumed numerical values are not physically unreasonable under special circumstances. The short cylinder in a uniform field is obviously realizable. A cable 15 km long in a uniform field is not easily realized. However, a long cable on dry sand near a high-powered VLF transmitter might not be very different in its response.