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PRELIMINARY REPORT ON THE ATTENUATION OF TRANSIENT FIELDS BY IMPERFECTLY CONDUCTING CYLINDRICAL SHELLS

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SUMMARY

The radiation incident upon an infinitely long imperfectly conducting cylindrical shell is a plane wave of longitudinal or transverse polarization. Formulas for the steady-state electric and magnetic fields in the cavity are derived for both polarizations of the incident field. The Fourier integral is then employed to obtain the time histories of the fields on axis of the cylinder when the incident electromagnetic field is a pulse of Gaussian shape. Numerical information relating to the effectiveness of the shell as a shield is provided in the final report for several pulse durations and shell radii.

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Introduction

This study was undertaken to determine the shielding characteristics of infinite cylindrical shells of arbitrary radii and wall thicknesses under steady-state and transient conditions for longitudinal and transverse polarization of the incident electric field. Shells made of steel and aluminum with no slots are considered. Specifically, the time histories of the electric and magnetic fields on axis of the cylinders are computed, employing the steady-state transfer functions, when the incident electric field is a plane wave of longitudinal or transverse polarization with an amplitude distribution in the shape of a Gaussian pulse. In the numerical work, pulses of several time derivations and a number of shell dimensions (radii and wall thicknesses) are used.

An infinite cylindrical shell is not physically realizable. The results obtained from the analysis, using the infinite model, cannot be expected to yield quantitative information on the magnitude of the field in a finite cylindrical shield when resonances occur since the infinite cylindrical shield can exhibit no longitudinal resonances. The general significance of partial reflection and transmission on the one hand, and of the skin effect and attenuation on the other, should be the same in finite and infinite shields.

The electric-field shielding ratio, under steady-state or transient conditions, is defined to be the ratio of the peak field at a selected point within the shield to the amplitude of the incident field. i.e., the field that would exist at the same point with the shield removed. The shielding ratio for the magnetic field is defined in the same way. A different shielding ratio is obtained if it is defined in terms of the field inside and outside the shield, since the field outside the shield is the resultant of the incident and diffracted fields.

In the first part of this paper the steady-state transfer functions for use in the Fourier integral are developed in general terms. In the latter part of the paper the use of the Fourier integral to obtain the time histories of the electric and magnetic fields on axis of the shield is explained briefly, and an approximate form is developed for evaluation by a high-speed digital computer.

Preliminary Remarks

Figure 1 illustrates a homogeneous imperfectly conducting cylindrical shell of outer radius a and inner radius b characterized by permeability μ_1, dielectric constant ϵ_1, and conductivity σ_1. It is embedded in an infinite homogeneous medium with constitutive parameters μ_2, ϵ_2, and $\sigma_2 = 0$. The inner and outer regions of the shell are assumed to possess the same electrical properties. The center of the cylindrical shell is the origin of superimposed Cartesian and cylindrical coordinate systems. The unit vectors in these systems are $\hat{x}$, $\hat{y}$, and $\hat{z}$; and $\hat{\rho}$, $\hat{\phi}$, and $\hat{z}$, respectively. The axis of the cylinder is coincident with the z-axis of both coordinate systems. ϕ is the angle between $\hat{x}$ and $\hat{\rho}$ measured in a counterclockwise sense, and ρ is measured from the origin. The plane-wave incident electric field is assumed to propagate in the positive x-direction, and in one case is polarized parallel to the z-axis (longitudinal polarization) and in the other case parallel to the y-axis (transverse polarization).

Analytical Representation of the Electromagnetic Fields

1. Longitudinal Polarization of the Electric Field

When the incident electric field is given by $\underline{E}^i = \hat{z}E_o e^{-jk_2x} = \hat{z}E_o e^{-jk_2\rho \cos \phi}$ the expansions in cylindrical wave functions for the incident, reflected, shell, and cavity electric fields are:[1]

$$E_z^{i||} = E_o \sum_{n=-\infty}^{\infty} j^{-n} J_n(k_2\rho) e^{jn\phi} \tag{1}$$

$$E_z^{r||} = E_o \sum_{n=-\infty}^{\infty} j^{-n} a_n^{||} H_n^{(2)}(k_2\rho) e^{jn\phi} \tag{2}$$

$$E_z^{s||} = E_o \sum_{n=-\infty}^{\infty} j^{-n} \left[b_n^{||} J_n(k_1\rho) + c_n^{||} N_n(k_1\rho) \right] e^{jn\phi} \tag{3}$$

$$E_z^{c||} = E_o \sum_{n=-\infty}^{\infty} j^{-n} d_n^{||} J_n(k_2\rho) e^{jn\phi} \ . \tag{4}$$

Since,

$$\underline{\nabla} \times \underline{E} = -j\omega\mu \underline{H} \tag{5}$$

and $\underline{E} = \hat{z} E_z$, it follows that

$$\underline{H} = \frac{j}{\omega\mu} \left\{ \frac{1}{\rho} \frac{\partial}{\partial\phi} E_z \hat{\rho} - \frac{\partial}{\partial\rho} E_z \hat{\phi} \right\}. \tag{6}$$

Accordingly,

$$\underline{H}^{i||} = - \frac{E_o}{\omega\mu_2} \frac{1}{\rho} \sum_{n=-\infty}^{\infty} n j^{-n} J_n(k_2\rho) e^{jn\phi} \hat{\rho} - j \frac{E_o}{\omega\mu_2} k_2 \sum_{n=-\infty}^{\infty} j^{-n} J_n'(k_2\rho) e^{jn\phi} \hat{\phi} \tag{7}$$

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[1] R. F. Harrington, "Time-Harmonic Electromagnetic Fields," McGraw-Hill Book Co., Inc. pp 232-238 (1961).

$$\underline{H}^{r\parallel} = -\frac{E_o}{\omega\mu_2}\frac{1}{\rho}\sum_{n=-\infty}^{\infty} n a_n^{\parallel} j^{-n} H_n^{(2)}(k_2\rho)e^{jn\phi}\,\hat{\rho}$$

$$- j\frac{E_o}{\omega\mu_2}k_2\sum_{n=-\infty}^{\infty} a_n^{\parallel} j^{-n} H_n^{(2)'}(k_2\rho)e^{jn\phi}\,\hat{\phi} \tag{8}$$

$$\underline{H}^{s\parallel} = -\frac{E_o}{\omega\mu_1}\frac{1}{\rho}\sum_{n=-\infty}^{\infty} n j^{-n}\left[b_n^{\parallel} J_n(k_1\rho) + c_n^{\parallel} N_n(k_1\rho)\right]e^{jn\phi}\,\hat{\rho}$$

$$- j\frac{E_o}{\omega\mu_1}k_1\sum_{n=-\infty}^{\infty} j^{-n}\left[b_n^{\parallel} J_n'(k_1\rho) + c_n^{\parallel} N_n'(k_1\rho)\right]e^{jn\phi}\,\hat{\phi} \tag{9}$$

$$\underline{H}^{c\parallel} = -\frac{E_o}{\omega\mu_2}\frac{1}{\rho}\sum_{n=-\infty}^{\infty} n j^{-n} d_n^{\parallel} J_n(k_2\rho)e^{jn\phi}\,\hat{\rho} - j\frac{E_o}{\omega\mu_2}k_2\sum_{n=-\infty}^{\infty} j^{-n} d_n^{\parallel} J_n'(k_2\rho)e^{jn\phi}\,\hat{\phi}\ . \tag{10}$$

In writing the relations for $\underline{E}$ and $\underline{H}$ the time dependence assumed (but suppressed) is $\exp(j\omega t)$. E_o is the amplitude of the incident electric field, and the superscripts i, r, s, and c on E_z and $\underline{H}$ represent incident, reflected, shell, and cavity. The superscript "$\parallel$" on the electromagnetic fields and on the constants a_n, b_n, c_n, and d_n denotes that the case of longitudinal polarization of the electric field is being considered. The propagation constants in media 1 and 2 are

$$k_1 = (1 - j)\sqrt{\frac{\omega\mu_1\sigma_1}{2}} \tag{11}$$

(provided $\sigma_1 >> \omega\epsilon_1$), and

$$k_2 = \omega\sqrt{\mu_2\epsilon_2} \tag{12}$$

respectively.

2. Transverse Polarization of the Electric Field

When the incident magnetic field is given by $\underline{H}^i = \hat{z}H_o e^{-jk_2 x} = \hat{z}H_o e^{-jk_2\rho\cos\phi}$ the expansions in cylindrical wave functions for the incident, reflected, shell, and cavity magnetic fields are

$$H_z^{i\perp} = H_o \sum_{n=-\infty}^{\infty} j^{-n} J_n(k_2\rho) e^{jn\phi} \tag{13}$$

$$H_z^{r\perp} = H_o \sum_{n=-\infty}^{\infty} j^{-n} a_n^{\perp} H_n^{(2)}(k_2\rho) e^{jn\phi} \tag{14}$$

$$H_z^{s\perp} = H_o \sum_{n=-\infty}^{\infty} j^{-n} \left[b_n^{\perp} J_n(k_1\rho) + c_n^{\perp} N_n(k_1\rho) \right] e^{jn\phi} \tag{15}$$

$$H_z^{c\perp} = H_o \sum_{n=-\infty}^{\infty} j^{-n} d_n^{\perp} J_n(k_2\rho) e^{jn\phi} , \tag{16}$$

where H_o is the amplitude of the incident magnetic field. Since,

$$\underline{\nabla} \times \underline{H} = j\omega\epsilon\underline{E} \tag{17}$$

and $\underline{H} = \hat{z}H_z$, it follows that

$$\underline{E} = -\frac{j}{\omega\epsilon}\left\{ \frac{1}{\rho}\frac{\partial}{\partial\phi} H_z\hat{\rho} - \frac{\partial}{\partial\rho} H_z \hat{\phi} \right\}. \tag{18}$$

Accordingly,

$$\underline{E}^{i\perp} = \frac{H_o}{\omega\epsilon_2}\frac{1}{\rho}\sum_{n=-\infty}^{\infty} n j^{-n} J_n(k_2\rho) e^{jn\phi}\hat{\rho} + j\frac{E_o}{\omega\epsilon_2} k_2 \sum_{n=-\infty}^{\infty} j^{-n} J_n'(k_2\rho) e^{jn\phi}\hat{\phi} \tag{19}$$

$$\underline{E}^{r\perp} = \frac{H_o}{\omega\epsilon_2}\frac{1}{\rho}\sum_{n=-\infty}^{\infty} a_n^{\perp} j^{-n} H_n^{(2)}(k_2\rho) e^{jn\phi}\hat{\rho} + j\frac{E_o}{\omega\epsilon_2} k_2 \sum_{n=-\infty}^{\infty} a_n^{\perp} j^{-n} H_n^{(2)'}(k_2\rho) e^{jn\phi}\hat{\phi} \tag{20}$$

$$\underline{E}^{s\perp} = \frac{H_o}{\omega\epsilon_1}\frac{1}{\rho}\sum_{n=-\infty}^{\infty} n j^{-n}\left[b_n^{\perp} J_n(k_1\rho) + c_n^{\perp} N_n(k_1\rho)\right] e^{jn\phi}\hat{\rho}$$

$$+ j\frac{E_o}{\omega\epsilon_1} k_1 \sum_{n=-\infty}^{\infty} j^{-n}\left[b_n^{\perp} J_n'(k_1\rho) + c_n^{\perp} N_n'(k_1\rho)\right] e^{jn\phi}\hat{\phi} \tag{21}$$

$$\underline{E}^{c\perp} = \frac{H_o}{\omega\epsilon_2}\frac{1}{\rho}\sum_{n=-\infty}^{\infty} n j^{-n} d_n^{\perp} J_n(k_2\rho) e^{jn\phi}\hat{\rho}$$

$$+ j\frac{H_o}{\omega\epsilon_2} k_2 \sum_{n=-\infty}^{\infty} j^{-n} d_n^{\perp} J_n'(k_2\rho) e^{jn\phi}\hat{\phi} . \tag{22}$$

where the superscript "$\perp$" on the electromagnetic fields and on the expansion constants denotes transverse polarization of the incident electric field.

The Boundary Equations

The boundary conditions on the fields to be satisfied at $\rho = a$ and $\rho = b$ are

$$\left.\begin{array}{l} \hat{\rho} \times \left[\underline{E}^i + \underline{E}^r\right] = \hat{\rho} \times \underline{E}^s \\ \hat{\rho} \times \left[\underline{H}^i + \underline{H}^r\right] = \hat{\rho} \times \underline{H}^s \end{array}\right\} \quad \rho = a \tag{23a}$$

$$\left.\begin{aligned} \hat{\rho} \times \underline{E}^s &= \hat{\rho} \times \underline{E}^c \\ \hat{\rho} \times \underline{H}^s &= \hat{\rho} \times \underline{H}^c \end{aligned}\right\} \quad \rho = b \tag{23b}$$

$$\left.\begin{aligned} \hat{\rho} \cdot \epsilon_2\left[\underline{E}^i + \underline{E}^r\right] &= \hat{\rho} \cdot \epsilon_1\underline{E}^s \\ \hat{\rho} \cdot \mu_2\left[\underline{H}^i + \underline{H}^r\right] &= \hat{\rho} \cdot \mu_1\underline{E}^s \end{aligned}\right\} \quad \rho = a \tag{24a}$$

$$\left.\begin{aligned} \hat{\rho} \cdot \epsilon_1\underline{E}^s &= \hat{\rho} \cdot \epsilon_2\underline{E}^c \\ \hat{\rho} \cdot \mu_1\underline{H}^s &= \hat{\rho} \cdot \mu_2\underline{H}^c \end{aligned}\right\} \quad \rho = b \tag{24b}$$

All the constants appearing in the field expansions are fixed by (23) only. Let the following notation be introduced

$$\left.\begin{aligned} A_1 &= H_n^{(2)}(k_2a) \\ A_2 &= kH_n^{(2)'}(k_2a) \\ B_1 &= J_n(k_1a) \\ B_2 &= \eta'' J_n'(k_1a) \\ C_1 &= N_n(k_1a) \\ C_2 &= \eta'' N_n'(k_1a) \\ D_1 &= J_n(k_2b) \\ D_2 &= kJ_n'(k_2b) \\ F_1 &= J_n(k_2a) \\ F_2 &= kJ_n'(k_2a) \\ G_1 &= N_n(k_1b) \\ G_2 &= \eta'' N_n'(k_1b) \\ H_1 &= J_n(k_1b) \\ H_2 &= \eta'' J_n'(k_1b) \end{aligned}\right\} \tag{25}$$

where $\eta^{\shortparallel} = \dfrac{\mu_2}{\mu_1}$ and $k = \dfrac{k_2}{k_1}$. It can then be shown, using (23), that the boundary equations take the following form for the case of longitudinal polarization.*

$$\left.\begin{aligned} -A_1 a_n^{\shortparallel} + B_1 b_n^{\shortparallel} + C_1 c_n^{\shortparallel} &= F_1 \\ -A_2 a_n^{\shortparallel} + B_2 b_n^{\shortparallel} + C_2 c_n^{\shortparallel} &= F_2 \\ H_1 b_n^{\shortparallel} + G_1 c_n^{\shortparallel} - D_1 d_n^{\shortparallel} &= 0 \\ H_2 b_n^{\shortparallel} + G_2 c_n^{\shortparallel} - D_2 d_n^{\shortparallel} &= 0 \end{aligned}\right\} \qquad (26)$$

The solution of (26) for $d_n^{\shortparallel}$ is

$$d_n^{\shortparallel} = \frac{(G_1H_2 - G_2H_1)(F_1A_2 - A_1F_2)}{(H_1D_2 - D_1H_2)(A_1C_2 - C_1A_2) + (G_1D_2 - D_1G_2)(B_1A_2 - A_1B_2)} \, . \qquad (27)$$

Equation (27) can be simplified by use of the Wronskian relation for Bessel functions:

$$J_n(z)N_n'(z) - J_n'(z)N_n(z) = \frac{2}{\pi z} \, . \qquad (28)$$

The result is

$$d_n^{\shortparallel} = j\,\frac{\left(\dfrac{2}{\pi k_1}\right)^2 \dfrac{\eta^{\shortparallel}}{ab}}{(H_1D_2 - D_1H_2)(A_1C_2 - C_1A_2) + (G_1D_2 - D_1G_2)(B_1A_2 - A_1B_2)} \, . \qquad (29)$$

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*It should be pointed out that although the boundary conditions only give the equality of infinite sums, one also must have term-by-term equality because of the orthogonality property of the $\exp(jn\phi)$ functions.

In the interest of brevity the boundary equations for the case of transverse polarization of the electric field are omitted. However, $d_n^{\perp}$ may be obtained from (29) by replacing $\eta^{\parallel} = \frac{\mu_2}{\mu_1}$ by $\eta^{\perp} = \frac{\epsilon_2}{\epsilon_1}$.

The Electromagnetic Fields on Axis of the Cylinder

The cavity fields are given by (4), (10), (16), and (22). To obtain relations for the fields at $\rho = 0$, i.e., on the axis of the cylinder, the following relations are employed:

$$\lim_{z \to 0} J_n(z) \longrightarrow \frac{1}{n!}\left(\frac{z}{2}\right)^n \tag{30}$$

$$Z_{-n}(z) = (-1)^n Z_n(z) \quad . \tag{31}$$

It follows that

$$\underline{E}^{c\parallel}\Big]_{\rho=0} = d_o^{\parallel} E_o \hat{z} \quad . \tag{32}$$

$$\underline{H}^{c\parallel}\Big]_{\rho=0} = \frac{j}{\omega\mu_2}\,\frac{k_2 E_o}{2}\left\{d_1^{\parallel}(e^{j\phi}\hat{\rho} + je^{j\phi}\hat{\phi}) - d_{-1}^{\parallel}(e^{-j\phi}\hat{\rho} - je^{-j\phi}\hat{\phi})\right\} \tag{33}$$

$$\underline{E}^{c\perp}\Big]_{\rho=0} = -\frac{j}{\omega\epsilon_2}\,\frac{k_2 H_o}{2}\left\{d_1^{\perp}(e^{j\phi}\hat{\rho} + je^{j\phi}\hat{\phi}) - d_{-1}^{\perp}(e^{-j\phi}\hat{\rho} - je^{-j\phi}\hat{\phi})\right\} \tag{34}$$

$$\underline{H}^{c\perp}\Big]_{\rho=0} = d_o^{\perp} H_o \hat{z} \, . \tag{35}$$

Evidently, an infinite sum is not required to express the fields on the axis of the infinite cylindrical shell.

Since,

$$\left.\begin{aligned}\hat{x} &= \cos\phi\hat{\rho} - \sin\phi\hat{\phi}\\ \hat{y} &= \sin\phi\hat{\rho} + \cos\phi\hat{\phi}\end{aligned}\right\} \tag{36}$$

it follows from (33) and (34) that

$$\underline{H}^{c\parallel}\Big]_{\rho=0} = \frac{j}{\omega\mu_2}\,\frac{k_2E_o}{2}\left\{\left(d_1^{\parallel} - d_{-1}^{\parallel}\right)\hat{x} + j\left(d_1^{\parallel} + d_{-1}^{\parallel}\right)\hat{y}\right\} \tag{37}$$

$$\underline{E}^{c\perp}\Big]_{\rho=0} = -\frac{j}{\omega\epsilon_2}\,\frac{k_2H_o}{2}\left\{\left(d_1^{\perp} - d_{-1}^{\perp}\right)\hat{x} + j\left(d_1^{\perp} + d_{-1}^{\perp}\right)\hat{y}\right\}. \tag{38}$$

It is evident from (29) using (25) and (31) that $d_{-n}^{\parallel} = d_n^{\parallel}$ and $d_{-n}^{\perp} = d_n^{\perp}$. Hence (37) and (38) become

$$\underline{H}^{c\parallel}\Big]_{\rho=0} = -\frac{k_2}{\omega\mu_2}\,d_1^{\parallel}\,E_o\hat{y} \tag{39}$$

$$\underline{E}^{c\perp}\Big]_{\rho=0} = \frac{k_2}{\omega\epsilon_2}\,d_1^{\perp}H_o\hat{y}\ , \tag{40}$$

respectively, and

$$\left.\frac{\left|\underline{E}^{c\parallel}\right|}{\left|\underline{H}^{c\parallel}\right|}\right]_{\rho=0} = \zeta\cdot\frac{\left|d_o^{\parallel}\right|}{\left|d_1^{\parallel}\right|} \tag{41}$$

$$\left.\frac{\left|\underline{E}^{c\perp}\right|}{\left|\underline{H}^{c\perp}\right|}\right]_{\rho=0} = \zeta\,\frac{\left|d_1^{\perp}\right|}{\left|d_o^{\perp}\right|} \tag{42}$$

where $\zeta = \omega\mu_2/k_2$.

The Form of the Integrals to be Evaluated by a Computer

The functional form of the time dependence of the incident field pulse assumed is

$$e_o(t) = Ae^{-\frac{t^2}{2t_1^2}} , \qquad (43)$$

where A is the value $e_o(0)$ in volts per meter, t is the time, and t_1 is a measure of the pulse width. The frequency spectrum of the pulse is

$$E_o(f) = At_1\sqrt{2\pi}\, e^{-\frac{f^2}{2f_1^2}} . \qquad (44)$$

Here $f_1 = \frac{1}{2\pi t_1}$.

Let $G(f) = G_R(f) + jG_I(f)$ represent one of the desired steady-state shielding ratios, such as $\left(E^c(f)\right)_{\rho=0} \big/ E_o(f)$. The time development of the electric field on the axis of the cylindrical shell is then

$$e_c(t) = \int_{-\infty}^{+\infty} G(f)E_o(f)e^{j2\pi ft}\, df$$

$$\simeq 2At_1\sqrt{2\pi}\int_0^{f_c} \left[G_R(f)\cos 2\pi ft - G_I(f)\sin 2\pi ft\right] e^{-\frac{f^2}{2f_1^2}}\, df , \qquad (45)$$

where in obtaining the second part of (45) use has been made of the relation $G^*(f) = G(-f)$. For computations of the electric field the constant A in (43) was taken to be 1 volt/m; for computations of the magnetic field h and H are substituted in (43) and (45) in place of e and E, and A is set equal to 1 amp/m. In this paper the highest significant frequency contained in a Gaussian pulse is considered to be $f_c = 2.6f_1$. The "significant" base width of the time function is $2 \times 2.6t_1 = 5.2t_1$. The half amplitude of a Gaussian pulse is $2.355t_1$.

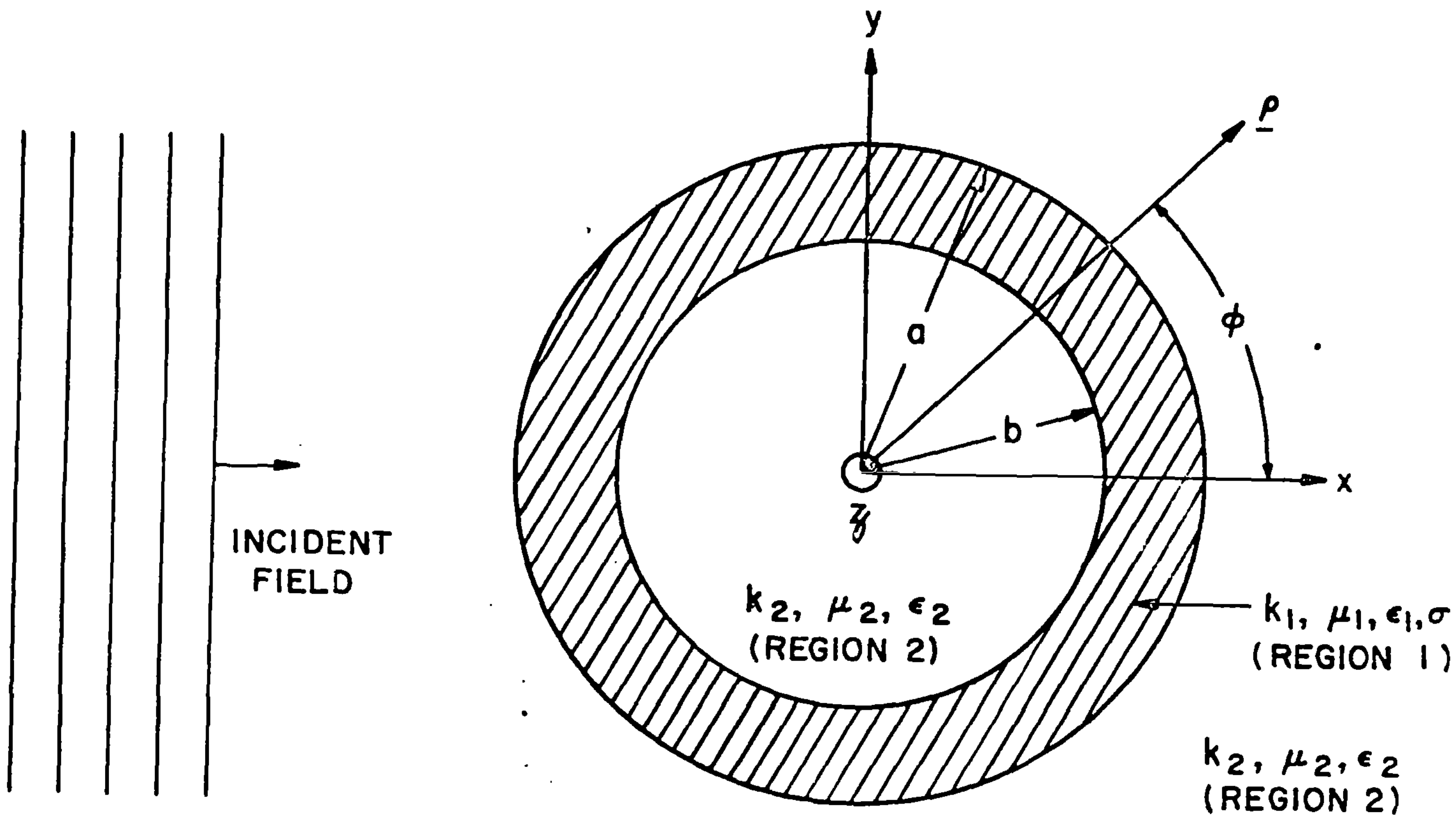

FIGURE 1. CYLINDRICAL SHIELD WITH THE INCIDENT PLANE-WAVE FIELD.