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Electromagnetic Pulse Excitation of a Perfectly Conducting Cylinder in a Lossy Medium

by

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ABSTRACT

A general formulation is presented for obtaining the current induced on a perfectly conducting cylinder in a lossy medium by an incident plane wave. Numerical results are presented showing the frequency dependence of the induced current for various conductivities.

INTRODUCTION

The general problem of the pulse excitation of a perfectly conducting cylinder in lossy media is formulated and solved using a high speed digital computer. In the formulation the cylinder is considered to be sufficiently thin that the exciting fields are essentially uniform about the axis of the cylinder. However outside this restriction the formulation is completely general.

By deriving and solving an integral equation, an expression is obtained for the current distribution induced on the cylinder. The solution presented requires that the current be represented by a finite Fourier series; the expansion coefficients are obtained by forcing the series representation to satisfy the original integral equation. First the steady state currents are obtained which are then superimposed to obtain the appropriate time history for the specific pulse excitation.

Numerical results are presented which demonstrate the solution technique. Also the general effects of lossy media upon pulse excitation are delineated.

ANALYSIS

Integral Equation for the Current Distribution

Consider a cylinder of length 2h extending from z = -h to z = h.

Provided that the cylinder is sufficiently thin, the illumination may be considered to be rotationally symmetric about its surface. It is easily shown that the vector potential associated with the scattered fields, $A_z^s(a,z)$, at the surface of the cylinder (radius a) satisfies the differential equation

$$\left[\frac{d^2}{dz^2} + k^2\right] A_z^s(a,z) = \frac{jk^2}{\omega} \left[E_z^t(a,z) - E_z^i(a,z)\right] \tag{1}$$

where the assumed (but suppressed) time dependence is $\exp(j\omega t)$; $E_z^t(a,z)$ is the total electric field at point z; and $E_z^i(a,z)$ is the incident field at point z. The propagation constant for a lossy medium is

$$k^2 = \omega^2\mu\varepsilon - j\omega\mu\sigma \tag{2}$$

where the constitutive properties of the surrounding medium are ε (permittivity), μ (permeability) and σ (conductivity).

If the cylinder is highly conducting then

$$E_z^t(a,z) = o \tag{3}$$

and for an electrically thin cylinder

$$A_z^s(a,z) = \frac{\mu}{4\pi} \int_{-h}^{h} dz' \; I_z(z') \; K_a(z-z') \tag{4}$$

where $I_z(z)$ is the total induced axial current and

$$K_a(z-z') = \exp\left[-jk\sqrt{(z-z)^2 + a^2}\right] / \sqrt{(z-z')^2 + a^2} \tag{5}$$

using (3) and (4) in (1) yields

$$\int_{-h}^{h} dz' \; I_z(z') \; \hat{K}(z-z') = -j \; \frac{4\pi k}{\zeta} \; E_z^i(a,z) \tag{6}$$

where

$$\zeta = \sqrt{\mu/(\varepsilon - j\sigma/\omega)} \tag{7}$$

The foregoing integral equation is to be solved for the induced current distribution, $I_z(z)$.

<u>Solution for the current Distribution</u>

It is observed that the Kernel is an even function of (z-z') and therefore may be represented

$$\hat{K}(z-z') = k^2 \sum_{n=o}^{\infty} K_n \cos\left[\frac{n\pi}{2h} (z-z')\right] \tag{8}$$

where

$$K_n = \frac{k^{-2} \; K'(2h)}{kh\varepsilon n} \; (-1)^n + \frac{1 - \left(\frac{n\pi}{2kh}\right)^2}{kh\varepsilon n} \int_o^{2h} d\xi \; K(\xi) \; \cos \frac{n\pi}{2h} \xi$$

$$k^{-2} \; K'(2h) = - \frac{(1 + jkR)}{(kR)} \; (2kh) e^{-jkR}$$

$$R = \sqrt{4h^2 + a^2}$$

$$\varepsilon_n = 2 \quad n = o$$

$$= 1 \quad n > o$$

A general representation for the current distribution that satisfies the boundary conditions $I_z(h) = I_z(-h) = 0$, is

$$I_z(z) = -j \; \frac{4\pi U}{\zeta} \sum_{m=0}^{\infty} \left[I_m^s \cos \; \frac{(2m+1\pi)}{2h} \; z + I_m^a \sin \frac{m\pi}{h} \; z \right] \tag{9}$$

The constant U (in volts) will be chosen later to simplify the mathematics.

Substituting (8) and (9) into (6), multiplying the result by $\cos \frac{p\pi}{h} z$, and integrating over z yields

$$\sum_{m=o}^{\infty} \left[\varepsilon_p K_{2p} + K_{2m+1}\right] \gamma^{s}_{pm} I^{s}_{m} = S^{s}_{p} \tag{10}$$

where

$$\gamma^{s}_{pm} = \frac{4(2m+1)\ (-1)^{m+p}}{\pi\left[(2m+1)^2 - 4p^2\right]} \tag{11}$$

$$S^{s}_{p} = \frac{1}{U(kh)^2} \int_{-h}^{h} dz\ E^{i}_{z}(a,z) \cos \frac{p\pi}{h} z \tag{12}$$

Also, using $\sin \frac{(2p+1)}{2h} z$ instead of $\cos \frac{p\pi}{h} z$ in the foregoing procedure yields

$$\sum_{m=o}^{\infty} \left[\varepsilon_m K_{2m} + K_{2p+1}\right] \gamma^{a}_{pm} I^{a}_{m} = S^{a}_{p} \tag{13}$$

where

$$\gamma^{a}_{pm} = \frac{8m\ (-1)^{m+p}}{\pi\ (2p+1)^2 - 4m^2} \tag{14}$$

$$S^{a}_{p} = \frac{1}{(kh)^2 U} \int_{-h}^{h} dz\ E^{i}_{z}\ (a,z) \sin \frac{(2p+1)\pi}{2h} z \tag{15}$$

Because the current distribution is expected to be a well behaved function, the infinite series expansion given by (9) may be truncated at some high order N, where $N^2 \gg (k_o h/\pi)^2$ and $k_o = \omega\sqrt{\mu\varepsilon}$, and yet maintain reasonable accuracy. The direct effect of this truncation is the reduction of the infinite system of equations (10) and (13) to finite systems suited for solution by a high speed digital computer.

Consider that the cylinder is irradiated by a plane wave where $k_o a << 1$. The z component of the incident electric field may be represented in the general form

$$E_z^i(a,z) = E_o \sin \Psi \sin \Theta \exp\left[-jkz \cos \Theta\right] \tag{16}$$

where the direction of propagation is at angle Θ with the positive z axis, and the electric field is directed at an angle Ψ with the normal to the plane determined by the z axis and the direction of propagation. Then from using (16) in (12)

$$S_p^s = \frac{2}{(kh)^2\cos\Theta} \frac{\sin (kh\cos\Theta)}{1 - (p\pi/kh\cos\Theta)^2} (-1)^p \tag{17}$$

but if $\Theta = \frac{\pi}{2}$

$$S_p^s = \frac{2}{kh} S_{po} \tag{18}$$

Here the constant U is defined

$$U = \frac{1}{k} \sin \Psi \sin \Theta E_o \tag{19}$$

Similarly using (16) in (15) yield

$$S_p^a = j\frac{2}{(kh)^2\cos\Theta} \frac{\cos(kh\cos\Theta)}{1 - (2p+1\pi/2kh\cos\Theta)^2} (-1)^p \tag{20}$$

but if $\Theta = \frac{\pi}{2}$, $S_p^a = 0$.

Electromagnetic Pulse Excitation

In the foregoing development harmonic time dependence is assumed. However the results may be extended to arbitrary time dependence for the exciting field. It is well known that the Fourier time transforms of the field components satisfy the same equations as the field components with harmonic time dependence. Since the current distribution induced on the cylinder is simply the tangential component of the magnetic field, the current distribution obtained in the foregoing development is the Fourier transform of the current distribution with arbitrary time dependence. The specific time dependence is determined by the time variation of the incident electric field, i.e. the electric field component appearing in (6) and (16) is the Fourier time transform of the incident field component.

If the expression for the current distribution is rewritten with explicit frequency dependence (9) becomes

$$\hat{I}(z,\omega) = -j \frac{4\pi U(\omega)}{\zeta(\omega)} \sum_{m=o}^{\infty} \left[I_m^s(\omega) \cos \frac{(2m+1)}{2h} z + I_m^a(\omega) \sin \frac{m\pi}{h} z \right] \tag{21}$$

The time history of the current is

$$I(z,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} d\omega \, \hat{I}(z,\omega) \, e^{j\omega t} \tag{22}$$

But if I(z,t) is real, $\hat{I}(z,\omega) = \hat{I}^*(z,-\omega)$ and

$$I(z,t) = \sqrt{\frac{2}{\pi}} \int_{o}^{\infty} d\omega \, \mathrm{Re} \left[\hat{I}(z,\omega) e^{\omega t} \right] \tag{23}$$

The foregoing Fourier transform may be carried out numerically by using a high speed digital computer. Considering 20 terms for the sum in (21) and using a CDC 6600 computer it takes approximately 15 seconds to obtain a solution for the current as shown in (21). And to obtain a time history of the current in (23) requires only about 25 minutes overall but additional time histories may be obtained in a few seconds for different pulse excitations.

Numerical Results

The difficult part in obtaining the time history of the induced current is the solution for the frequency spectrum $\hat{I}(z,\omega)$. This frequency spectrum will depend upon the conductivity of the surrounding medium. To demonstrate this frequency dependence figures (1) through (5) show the magnitude of the center current, $|\hat{I}(o,\omega)|$, versus frequency for increasing conductivities. Here E_o = 1 volt/meter and $\Theta = \pi/2$, i.e., the direction of propagation is normal to the antenna axis. It is noted that as the conductivity increases the frequency spectrum becomes more flat and the magnitude of the current increases. The latter effect is a result of the energy density of the incident field increasing with increasing conductivity. The effect on the frequency spectrum may be explained by the damping of the current waves produced on the cylinder tending to eliminate resonant conditions.

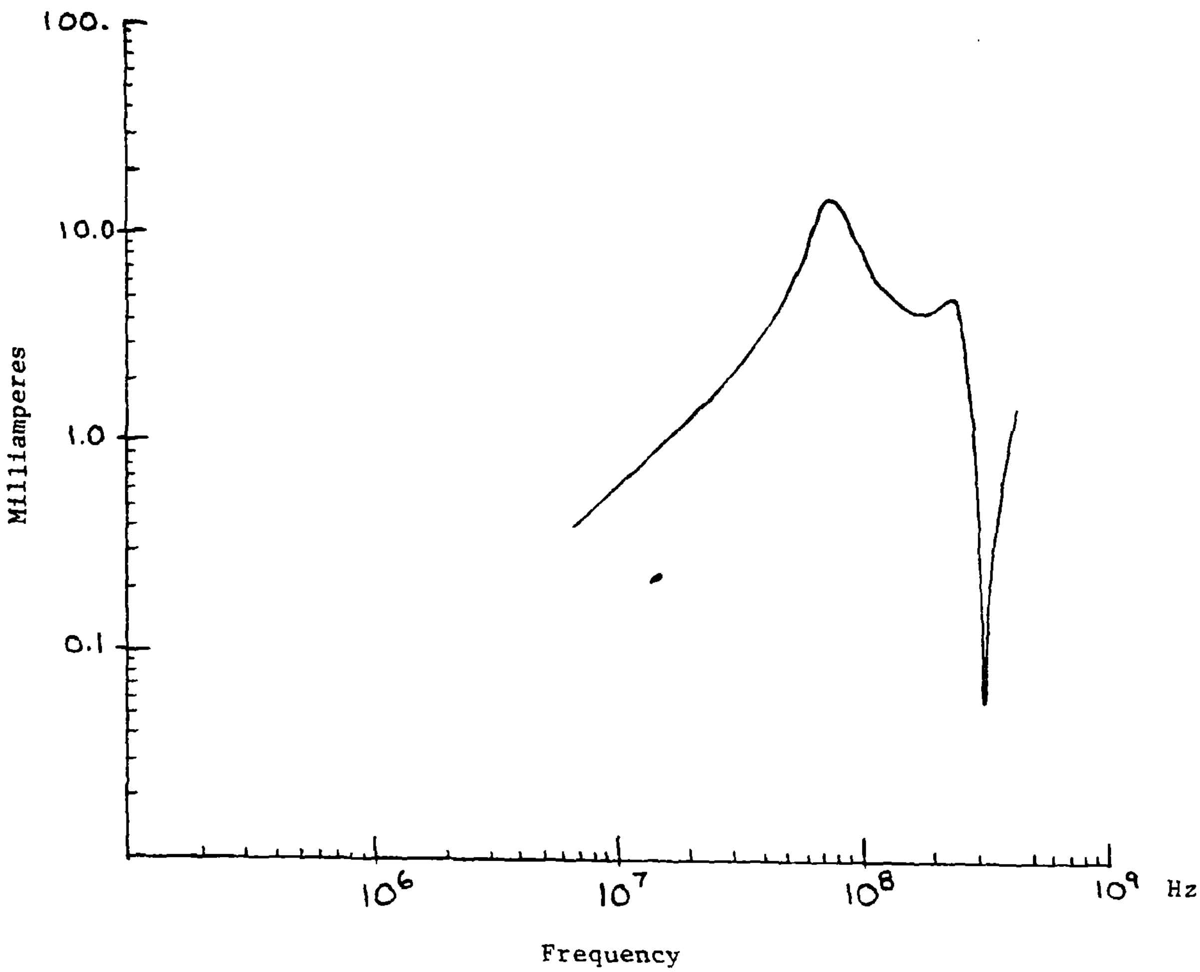

Figure 1: Center current versus frequency for broadside incidence. $\Omega = 2 \ln(2h/a) = 6.0$, $h = 0.761$ m, $\sigma = 0$.

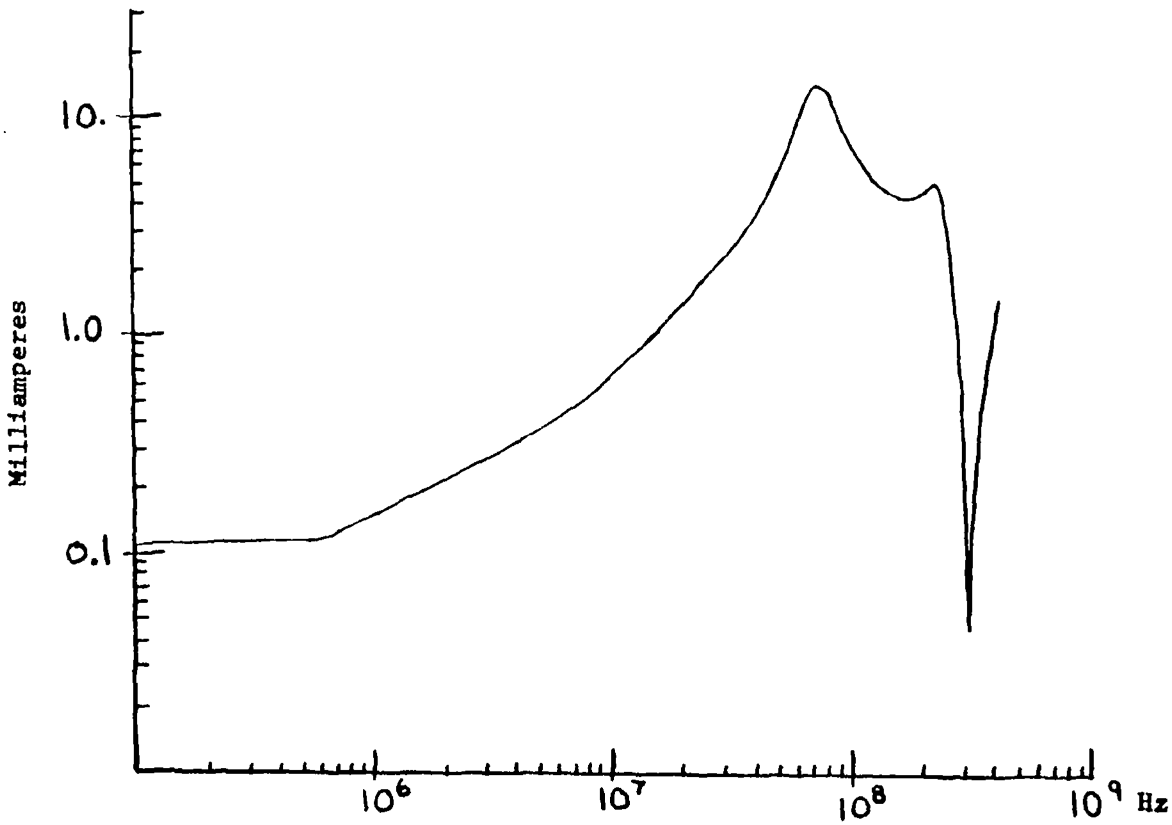

Figure 2: Center current versus frequency for broadside incidence.
$\Omega = 2 \ln (2h/a) = 6.0$, $h = 0.761$ m, $\sigma = 0.0001$ mho/meter.

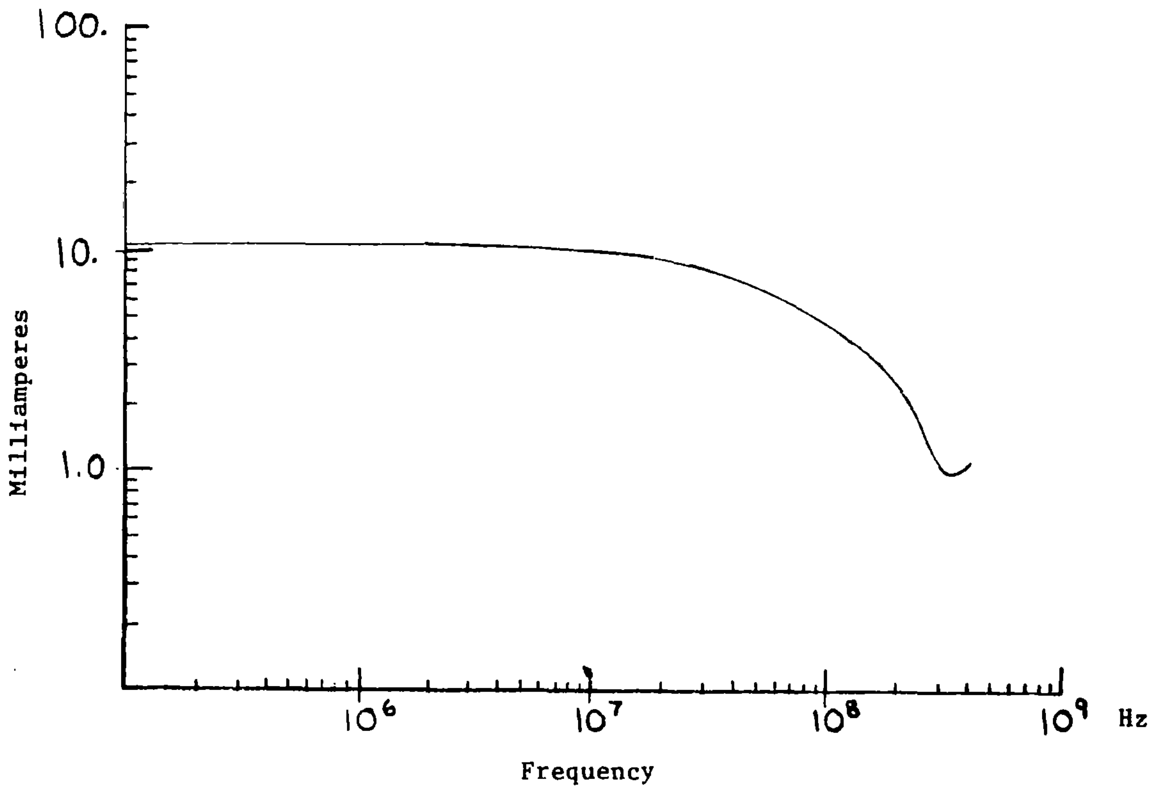

Figure 3: Center current versus frequency for broadside incidence. $\Omega = 2 \ln (2h/a) = 6.0$, $h = 0.761$ m, $\sigma = 0.01$ mho/meter.

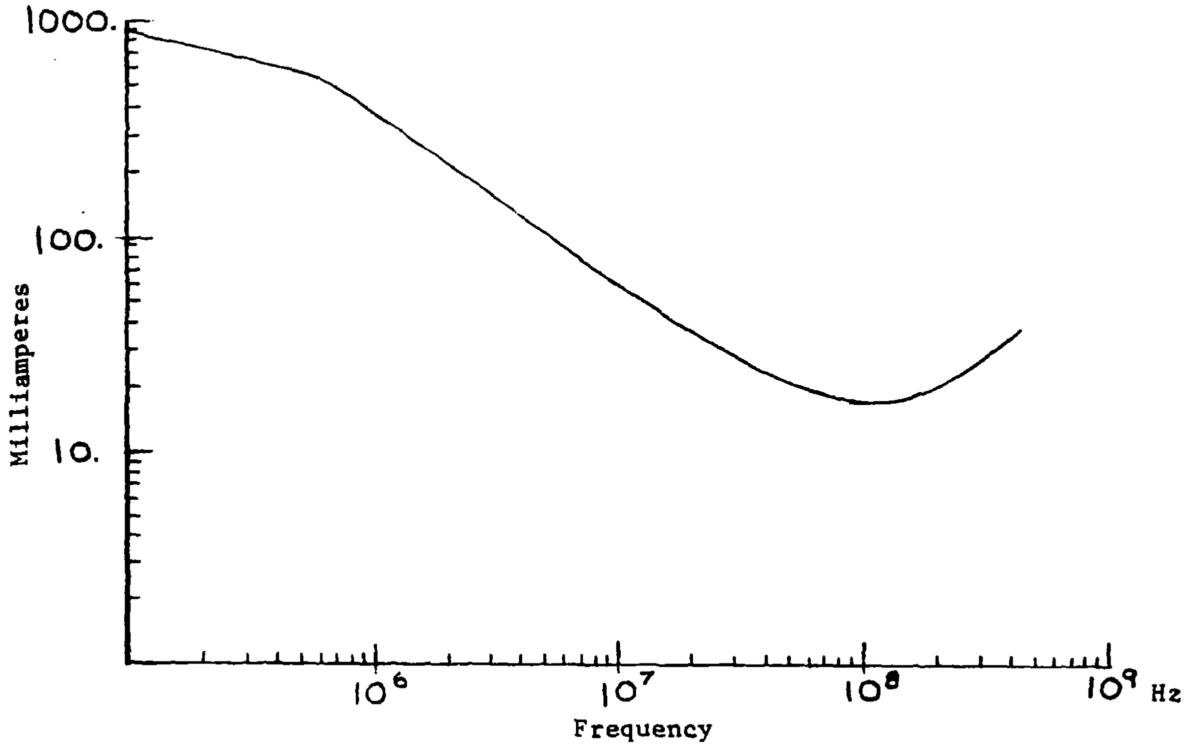

Figure 5: Center current versus frequency for broadside incidence. $\Omega = 2 \ln (2h/a) = 6.0$, $h = 0.761$ m, $\sigma = 1$ mho/meter.

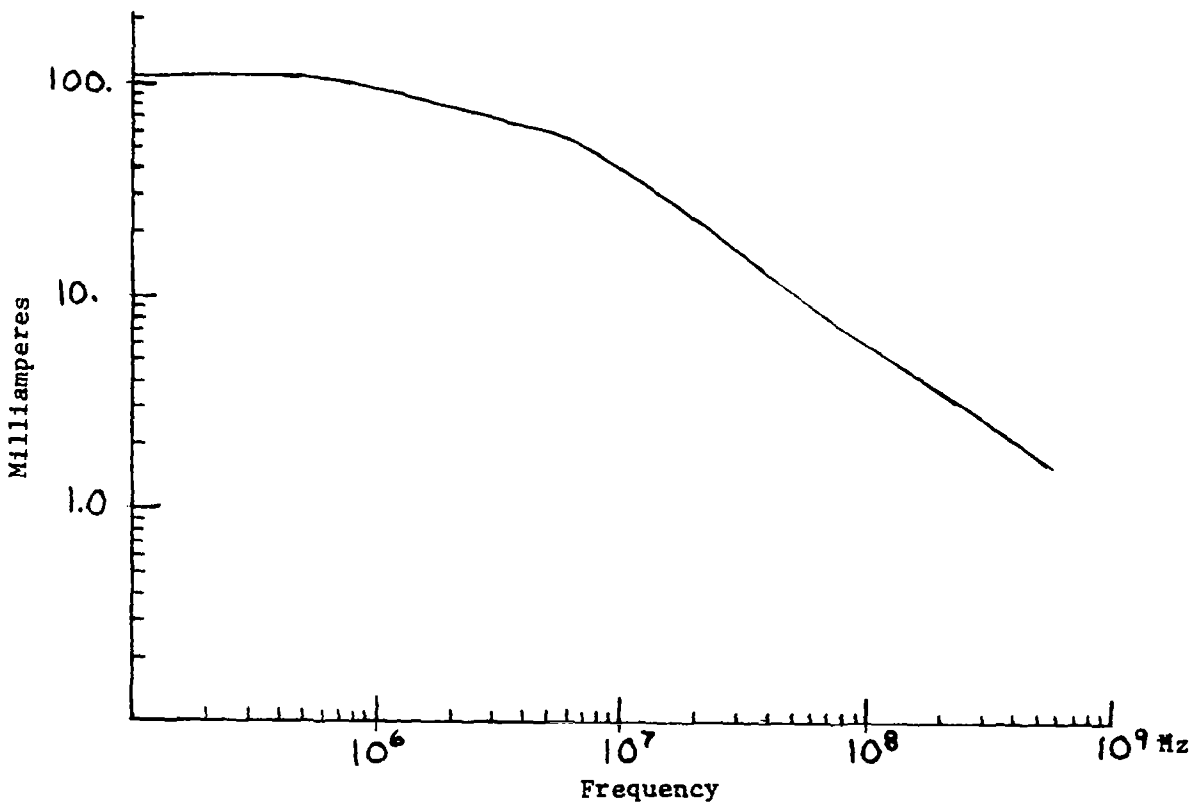

Figure 4: Center current versus frequency for broadside incidence. $\Omega = 2 \ln (2h/a) = 6.0$, $h = 0.761$ m, $\sigma = 0.1$ mho/meter.

REFERENCES

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