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Electromagnetic Scattering from Thin Wire Structures of Arbitrary Configuration

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ABSTRACT

A theoretical-numerical solution technique is presented for the treatment of the scattering from thin wire structures of arbitrary configuration. The formulation of the scattering problem leads in general to coupled integral equations for the current distributions. It is found that the arising integral equations are Fredholm integral equations of the first kind which are traditionally difficult to solve. In general, the number of equations is equal to the number of wires. The unknown current distributions are obtained by using piece-wise constant representations which are forced to satisfy the integral equations over a set of discrete points along with the appropriate boundary conditions. Thus, the integral equations are reduced to a system of linear equations allowing the problem to be solved by a high-speed digital computer.

General considerations are presented in the first part of this thesis. Then a detailed analysis is made for two perpendicular intersecting thin wires to determine the distributions of current induced in the wires by a plane-wave incident field. The wires are assumed sufficiently thin so that the phase change of the incident plane-wave across its diameter may be neglected.

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CHAPTER I

INTRODUCTION

In the present day missile studies it is necessary to know accurately the electromagnetic wave scattering characteristics of finite-length cylinders. Also with the defense of the country depending upon the radar detection of attacking aircraft and missiles, more than ever before a knowledge of electromagnetic wave scattering from various shaped objects is needed. To date the most fruitful theoretical approaches to these scattering problems have been the so called theoretical-numerical investigations that require the use of a high speed digital computer. The theoretical-numerical solution techniques are used in this thesis to treat the scattering from thin wire structures of arbitrary configuration.

In theoretical treatments of electrical circuits that are large in terms of wavelength, ordinarily the current distributions along the conductors comprising the circuit are unknown. The objective of this thesis is to find these distributions. The formulation of this type of boundary value problem, based on Maxwell's equations, almost inevitably leads to the integral equations which are found to be the Fredholm integral equations of the first kind. The unknown function is represented by using a piece-wise constant representation which is forced to satisfy the integral equations over a set of discrete points along with the appropriate boundary conditions. Thus the integral equations are reduced to a system of linear equations allowing the problem to be solved by a high-speed digital computer.

The first part of this thesis discusses general considerations. To illustrate the basic techniques, the scattering from an arbitrary single thin wire and from intersecting straight wires are treated. Then a detailed analysis is made later for two perpendicular intersecting thin wires to determine the distributions of current induced in the wires by a plane wave incident field. It is assumed that all the wires are perfect conductors, and the radii of the wires are small compared to their length and to the wavelengths of the monochromatic illuminating radiation.

Although the theoretical-numerical solution technique is quite simple, it may be used to treat relatively sophisticated problems. For example, it can be used to obtain the scattering from inhomogeneous cylinders,[1] resistive

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[1]Clayborne D. Taylor, "Electromagnetic Scattering by Thin Inhomogeneous Cylinders," Radio Science, vol. 2 (new series), no. 7, 729-738 (July 1967).

cylinders,[2] arrays of cylinders[3] and all types of straight wire antennas.[4] Modern high-speed digital computers have made possible by theoretical-numerical techniques the solution of many problems in electromagnetics that have traditionally been solvable only by experimental methods. With electromagnetic wave scattering problems formulated in terms of integral equations, the techniques described yield answers with an accuracy and completeness unobtainable by experimental methods in a small fraction of the time and at much less cost than by the experimental approach.

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[2]J. H. Richmond, "Scattering by Imperfectly conducting Wires," IEEE Transactions on Antennas and Propagation, AP-15, no. 6, 802-806 (November 1967).

[3]Ronald W. P. King, The Theory of Linear Antennas (Cambridge, Mass.: Harvard University Press, 1956).

[4]Clayborne D. Taylor, "Cylindrical Transmitting Antenna: Tapered Resistivity and Multiple Impedence Loadings" to appear in March 1968 issue of IEEE Transactions on Antennas and Propagation.

CHAPTER II

GENERAL CONSIDERATION

This chapter is concerned with the theoretical-numerical technique for treating electromagnetic scattering from any structure formed by thin wires. First the field equations are discussed, and then the vector and scalar potential of a perfectly conducting open-ended cylindrical wire are determined. Finally the boundary conditions for any abritrary configuration of straight wires are delineated. To illustrate the basic theoretical-numerical solution techniques, the scattering from two kinds of structures is treated; these are arbitrary thin wires and intersecting wires. The resulting integral equations must be solved by using a high speed digital computer. In principle, the theoretical-numerical procedure may be used to treat scattering from any arbitrary configuration of thin wires.

2.1 Field Equations.

A valid analytical determination of the scattering from a thin wire must proceed from the four Maxwell field equations (in MKS system):

$$\vec{\nabla} \times \vec{E} = -\frac{\partial}{\partial t} \vec{B} \quad , \qquad (1a)$$

$$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial}{\partial t} \vec{D} \ , \tag{1b}$$

$$\vec{\nabla} \cdot \vec{B} = 0 \ , \tag{1c}$$

and
$$\vec{\nabla} \cdot \vec{D} = \rho \ , \tag{1d}$$

where $\vec{B} = \mu_o \vec{H} \ , \quad \vec{D} = \epsilon_o \vec{E} \ .$ (2)

These equations define the electric and magnetic field $\vec{E}$ and $\vec{B}$ in terms of the density functions $\vec{J}$ and ρ and the two constitutive parameters: μ_o (the magnetic permeability) = $4\pi \times 10^{-7}$ henry per meter and ϵ_o (the fundamental electric permittivity) = 8.85×10^{-12} farad per meter. Consider that the fields have the suppressed harmonic time dependence exp ($j\omega t$); here ω is the angular frequency. Then Maxwell's equations may be written

$$\vec{\nabla} \times \vec{E} = -j\omega \vec{B} \ , \tag{3a}$$

$$\vec{\nabla} \times \vec{H} = \vec{J} + j\omega \epsilon_o \vec{E} \ , \tag{3b}$$

$$\vec{\nabla} \cdot \vec{B} = 0 \ , \tag{3c}$$

and
$$\vec{\nabla} \cdot \vec{D} = \rho \ . \tag{3d}$$

It is convenient to work with the potential functions $\vec{A}$ and ϕ, which are defined by the following equations:

$$-\vec{\nabla}\phi = \vec{E} + j\omega\vec{A}, \tag{4}$$

$$\vec{\nabla} \times \vec{A} = \vec{B}, \tag{5}$$

and

$$\vec{\nabla} \cdot \vec{A} = -j\frac{\omega}{c^2}\phi . \tag{6}$$

Here c and ω are given by

$$c = \frac{1}{\sqrt{\mu_0\epsilon_0}} = 3 \times 10^8 \text{ meters per second,} \tag{7}$$

and

$$\omega = 2\pi f = \frac{2\pi}{\lambda}c = K_0 c, \tag{8}$$

where K_0 = propagation constant for free space.

By using (4) and (5), the four Maxwell first order partial differential equations may be transformed into two wave equations of second order involving only the potential and density functions when the Lorentz gauge condition, (6), is applied.

$$\nabla^2\vec{A} + K_0^2\vec{A} = -\mu_0\vec{J}, \tag{9a}$$

and

$$\nabla^2\phi + K_0^2\phi = -\frac{1}{\epsilon_0}\rho . \tag{9b}$$

The explicit solutions of the foregoing wave equations which are the Helmholtz integrals yield $\vec{A}$ and ϕ in terms of the density functions.

On eliminating ϕ between (4) and (6), one obtains

$$\vec{E} = -j \frac{c^2}{\omega} \left[\vec{\nabla}\left(\vec{\nabla} \cdot \vec{A} \right) + K_o^2 \vec{A} \right]. \tag{10}$$

Equation (10) permits one to calculate the electric field at points in space exterior to the wire in terms of the vector potential and its derivatives at the same point.

In Figure 1, $\vec{A}(s,\rho)$ is the vector potential at any point p in space caused by the current $I_S(s')$ in a linear radiator which is of length ℓ and has its axis parallel to the s-axis. In order to obtain $\vec{A}(s,\rho)$, one has to use Green's function to solve differential equation (9a), i.e.,

$$\nabla^2 \vec{A}(s,\rho) + K_o^2 \vec{A}(s,\rho) = -\mu_o \vec{J}(s',\rho') \quad . \tag{11}$$

The current density $\vec{J}$ can be expressed by

$$\vec{J}(s',\rho') = \frac{1}{2\pi a} I_s(s') \delta(\rho'-a) \eta(\ell-s') \eta(s') \hat{s} \quad , \tag{12}$$

where a = the radius of the cylinder.

$\hat{s}$ = unit vector along the s-axis.

ρ' = the radial distance from the s-axis.

$\delta(\rho'-a)$ = the Dirac delta function.

$$\eta(x) \quad \text{(Heaviside function)} = \begin{cases} 1 & \text{when } x > 0 \\ 0 & \text{when } x < 0 \end{cases}$$

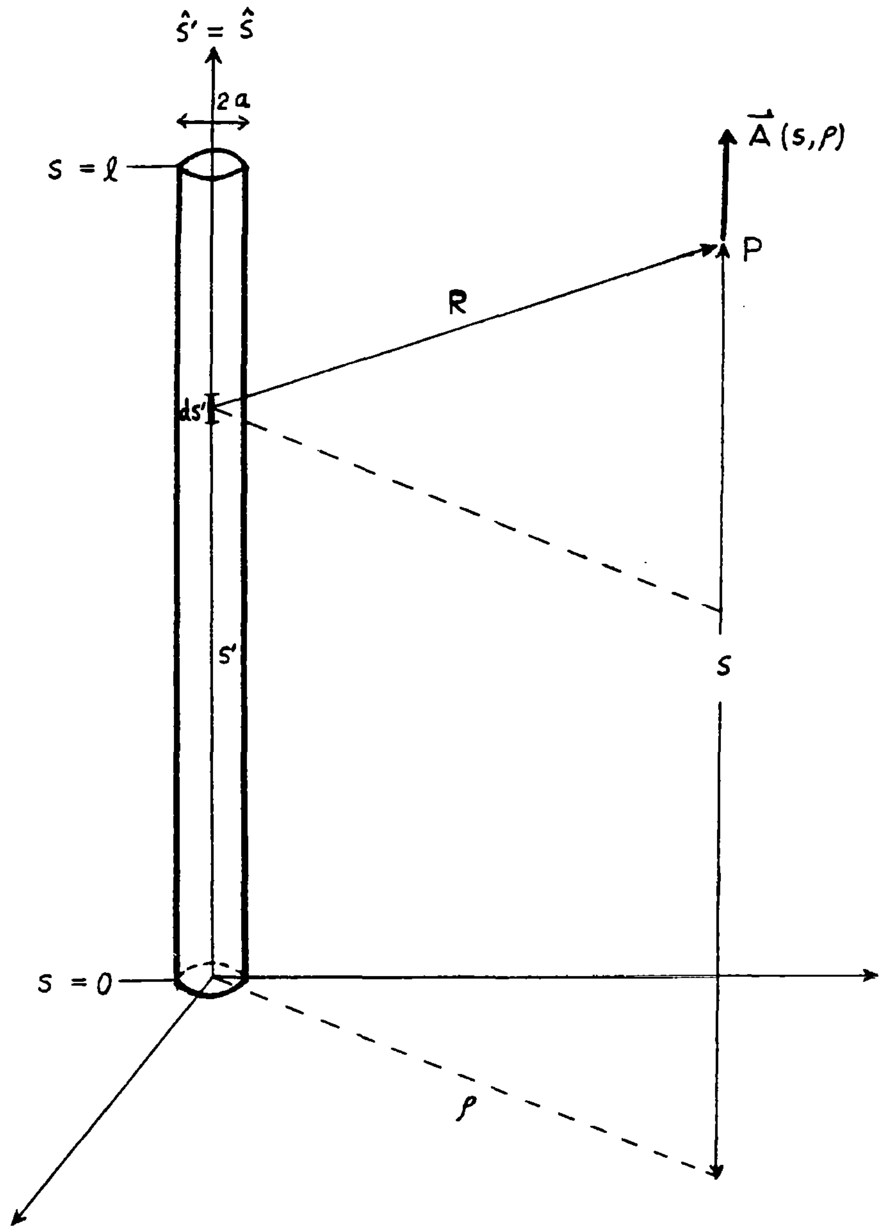

Figure 1. Geometry for Equation (16).

In the above the cylinder is considered sufficiently thin that azimuthal symmetry may be assumed in the current distribution, and that the contribution to the vector potential from the currents on the end faces is negligible. For thin cylinders the vector potential is essentially independent of the radius of the cylinder. For convenience we consider the current to be concentrated at a = 0.

Green's function G(R) is the solution of

$$\left(\nabla^2 + K_o^2 \right) G(R) = -4\pi\,\delta(R) \quad . \tag{13}$$

Here R is the distance from the current element at ds' to the point p, i.e., $R = \sqrt{(s-s')^2 + \rho^2}$. G(R) is found to be

$$G(R) = \frac{\exp(-jK_o R)}{R} \quad . \tag{14}$$

This is the Green's function for a point source. The vector potential can be therefore expressed in terms of G(R):

$$\vec{A}(s,\rho) = A_s(s,\rho)\,\hat{s} \quad , \tag{15}$$

where

$$A_s(s,\rho) = \frac{\mu_o}{4\pi} \int_c^{\ell} I_s(s') \frac{\exp(-jK_o R)}{R}\, d s' \quad . \tag{16}$$

The scalar potential $\phi(s,\rho)$ may be derived in an analogous manner. It is

$$\phi(s,\rho) = \frac{1}{4\pi\epsilon_o} \int_o^{\ell} q(s') \frac{\exp(-jK_o R)}{R}\, dS' \quad . \tag{17}$$

Here q(s') is the charge distribution on the conductor. The current $I_s(s')$ at a point on the conductor and charge q(s') at the same point are related by the equation of continuity

$$\frac{d I_s(s')}{d s'} + j \omega q(s') = 0 \quad . \tag{18}$$

Substituting equation (18) into equation (17) yields

$$\phi(s,\rho) = j \frac{c^2}{\omega}\left(\frac{\mu_o}{4\pi}\right)\int_0^{\ell} \frac{d I_s(s')}{d s'} \frac{\exp(-j K_o R)}{R} \, d s' \tag{19}$$

The Helmholtz integral equations (16) and (17) for $A_s(s,\rho)$ and $\phi(s,\rho)$ are the solutions of scalar Helmholtz wave equations for the vector and scalar potentials.

2.2 Boundary Conditions

For any arbitrary configuration of straight wire antennas, the following boundary conditions must be satisfied:[1]

(1) The tangential component of the electric field is zero on the surface of all wires. (Perfect conductors are assumed.)

(2) The current is zero at the open end of a wire.

(3) The sum of the currents that are directed forward the juncture of two wires is zero.

(4) The scalar potential at all points on the surface of the wires is continuous.

[1] C. W. Harrison, Jr., "Theory of Inverted L-Antenna with Image," Technical Memorandum SCTM 11-58(14), Sandia Corp., Albuquerque, New Mexico, (April 8, 1958).

2.3 Arbitrary Thin Wire.

In the subsequent development an integral equation is derived for the current distribution induced on an arbitrary thin wire antenna following the procedure set forth by K. K. Mei.[2] Figure 2 describes a curved cylindrical coordinate system, where s is the arc length measured from the origin, and $\hat{s}$ is the unit tangent vector at s.

In accord with the assumptions of a thin wire antenna, the tangential component of the vector potential and scalar potential on the antenna are given, respectively, as

$$A_s(s) = \frac{\mu_0}{4\pi}\int_L J(s')\, G(s,s')\, \hat{s}\cdot\hat{s}'\, ds', \tag{20}$$

and

$$\phi(s) = j\,\frac{c^2}{\omega}\left(\frac{\mu_0}{4\pi}\right)\int_L \frac{dJ(s')}{ds'}\, G(s,s')\, ds' . \tag{21}$$

A scalar function $\Phi(s)$ is defined by

$$\Phi(s) = -j\,\frac{\omega}{c^2}\int_0^s \phi(\xi)\, d\xi = \frac{\mu_0}{4\pi}\int_0^s\int_L \frac{dJ(s')}{ds'}\, G(\xi,s')\, ds'\, d\xi . \tag{22}$$

Integrating (22) by parts and considering J(s) to vanish at both ends yields

$$\Phi(s) = -\frac{\mu_0}{4\pi}\int_0^s\int_L J(s')\, \frac{\partial G(\xi,s')}{\partial s'}\, ds'\, d\xi . \tag{23}$$

For the s component of the electric field on the antenna to vanish, it is required that

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[2] K. K. Mei, "On the Integral Equations of Thin Wire Antennas," IEEE Transactions on Antennas and Propagation, AP-13, 374-378 (May 1965).

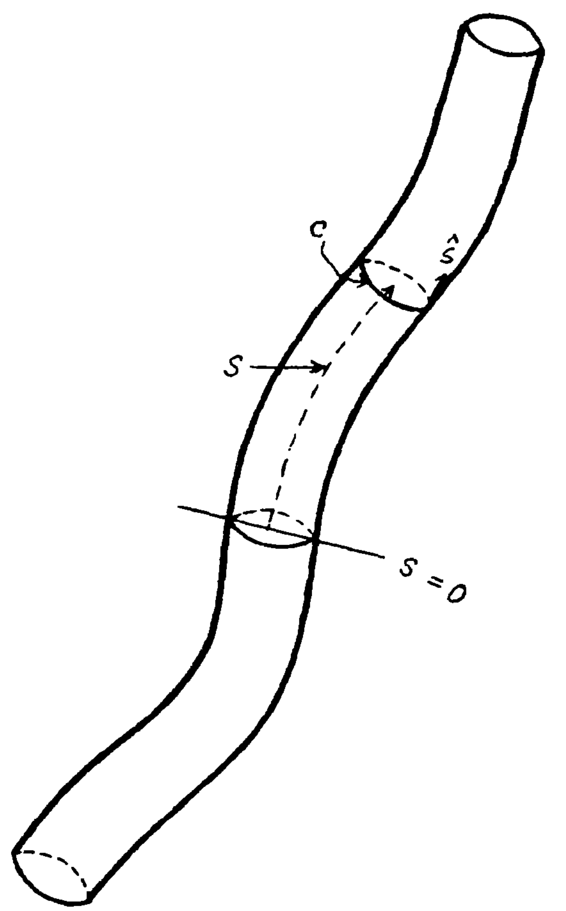

Figure 2. A curved cylindrical coordinate system.

$$E_s^s(s) + E_s^i(s) = 0 \quad , \tag{24}$$

where $E_s^s(s)$ is the s component of the scattered field at the surface of antenna; $E_s^i(s)$ is the s component of the incident electric field when the antenna is receiving, or it is the impressed field of the source if the antenna is transmitting.

From equation (4), the following is obtained

$$K_o^2 A_s(s) - j\frac{\omega}{c^2}\frac{d\Phi(s)}{ds} = -j\frac{\omega}{c^2} E_s^i(s) \quad , \tag{25}$$

or

$$\frac{d^2\Phi(s)}{ds^2} = -K_o^2 A_s(s) - j\frac{\omega}{c^2} E_s^i(s) \quad . \tag{26}$$

Adding $K_o^2 \Phi(s)$ to both sides of (26) yields

$$\frac{d\Phi^2(s)}{ds^2} + K_o^2 \Phi(s) = K_o^2\left[\Phi(s) - A_s(s)\right] - j\frac{\omega}{c^2} E_s^i(s) \, . \tag{27}$$

The solution of (27) for $s > 0$ is

$$\begin{aligned} \Phi(s) &= C \cos K_o s + D \sin K_o s \\ &\quad + K_o\int_0^s \left[\Phi(\xi) - A_\xi(\xi)\right] \sin K_o(s-\xi)\, d\xi \\ &\quad - \frac{j}{c}\int_0^s E_\xi^i(\xi) \sin K_o(s-\xi)\, d\xi \, . \end{aligned} \tag{28}$$

Using equations (20) and (23) in the third term of the right hand side of equation (28) becomes

$$\begin{aligned} &K_o\int_0^s \Phi(\xi) \sin K_o(s-\xi)\, d\xi \\ &= -K_o\frac{\mu_o}{4\pi}\int_0^s\int_0^\xi\int_L J(s')\frac{\partial G(\eta, s')}{\partial s'}\, ds'\, d\eta \sin K_o(s-\xi)\, d\xi \, , \end{aligned} \tag{29}$$

and $$K_0\int_0^s A_\xi(\xi)\sin K_0(s-\xi)\,d\xi$$

$$=K_0\frac{\mu_0}{4\pi}\int_0^s\int_L J(s')\,G(\xi,s')\hat{\xi}\cdot\hat{s}'\sin K_0(s-\xi)\,ds'\,d\xi\,. \qquad (30)$$

Substituting equations (23), (29) and (30) into (28), an integral equation is obtained for the current

$$\int_L J(s')\Pi(s,s')\,ds'=C\cos K_0 s+D\sin K_0 s$$

$$-j\frac{4\pi}{\mu_0 c}\int_0^s E_\xi^i(\xi)\sin K_0(s-\xi)\,d\xi\,, \qquad (31)$$

where

$$\Pi(s,s')=\int_0^s\Big[-\frac{\partial G(\xi,s')}{\partial s'}+K_0 G(\xi,s')\hat{\xi}\cdot\hat{s}\sin K_0(s-\xi)$$

$$+K_0\int_0^\xi\frac{\partial G(\eta,s')}{\partial s'}\sin K_0(s-\xi)\,d\eta\Big]\,d\xi\,. \qquad (32)$$

The integral equation (31) is the Fredholm integral equation of the first kind. The term D sin K_0s, represents the effect of a slice generator which is redundent when the integral of E_ξ^i is present. Indeed, if $E_\xi^i(\xi) = -V\delta(\xi)$, where V is the driving voltage and $\delta(\xi)$ is the Dirac delta function, then for $s > 0$

$$-j\frac{4\pi}{\mu_0 c}\int_0^s E_\xi^i(\xi)\sin K_0(s-\xi)\,d\xi = j\frac{4\pi V}{\mu_0 c}\sin K_0 s\,. \qquad (33)$$

For a dipole antenna, the source is assumed to be a slice generator. Notice that in this particular case

$$\frac{\partial G(\xi,s')}{\partial \xi} = -\frac{\partial G(\xi,s')}{\partial s'}\,, \qquad (34a)$$

and

$$\hat{\xi} \cdot \hat{s} = 1 \quad . \tag{34b}$$

Therefore equation (32) reduces to

$$\Pi(s,s') = G(s,s') - G(0,s') \cos K_0 s \quad . \tag{35}$$

Hence, equation (31) becomes

$$\int_L J(s')\, G(s,s')\, ds' = A \cos K_0 s + j \frac{4\pi V}{\mu_0 c} \sin K_0 s \quad . \tag{36}$$

Here $A = C + \int_L J(s')\, G(0,s')\, ds'$. (37)

For $s < 0$ an equation equivalent to (37) may be derived where only the sign of the sine term is changed.

The above equations are consistent with the integral equation of cylindrical dipole antenna.[3] It is clear that the integral equation (31) which describes an arbitrary thin wire antenna may be applied to dipole antennas, circular loop antennas and equiangular spiral antennas.

2.4 Arbitrary Configuration of Intersecting Wires.

In the previous section, a single wire antenna of simple structure is treated. In practice, most of the systems of radio communication are concerned with the intersecting wires. The object of this section is to describe

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[3]Ronold W. P. King, The Theory of Linear Antennas (Cambridge, Mass.: Harvard University Press, 1956), ch. 2.

the vector potential of this type of structure. Because the currents in the wires are no longer continuous as in the arbitrary thin wire antenna, and the vector potential on the surface of one wire is due to the currents in more than one wire, the treatment in this section differs from that in the previous section. More than one coordinate appears, and the two- and three-dimensional wave equation must be used. Solving for the total vector potential which has more than one component one obtains integral equations for the induced currents. Since the total vector potential on each wire has to be considered, the number of integral equations is generally equal to the number of wires. In principle, this approach may be applied to any physically realizable structure by using superposition.[4]

Figure 3 illustrates two perfectly conducting cylindrical wires (1) and (2) of lengths $2h_1$ and $h_2 + h_4$ and the same radius a. The assumptions regarding the radii of the wires are namely,

$$K_o a >> 1$$

and

$$a << h_1 , \quad a << h_2 , \quad a << h_4 .$$

Under these conditions it may be assumed that cross-sectional and axial distributions of current density in the conductors may be treated as independent of each other.

[4]<u>Ibid.</u>, ch. 6.

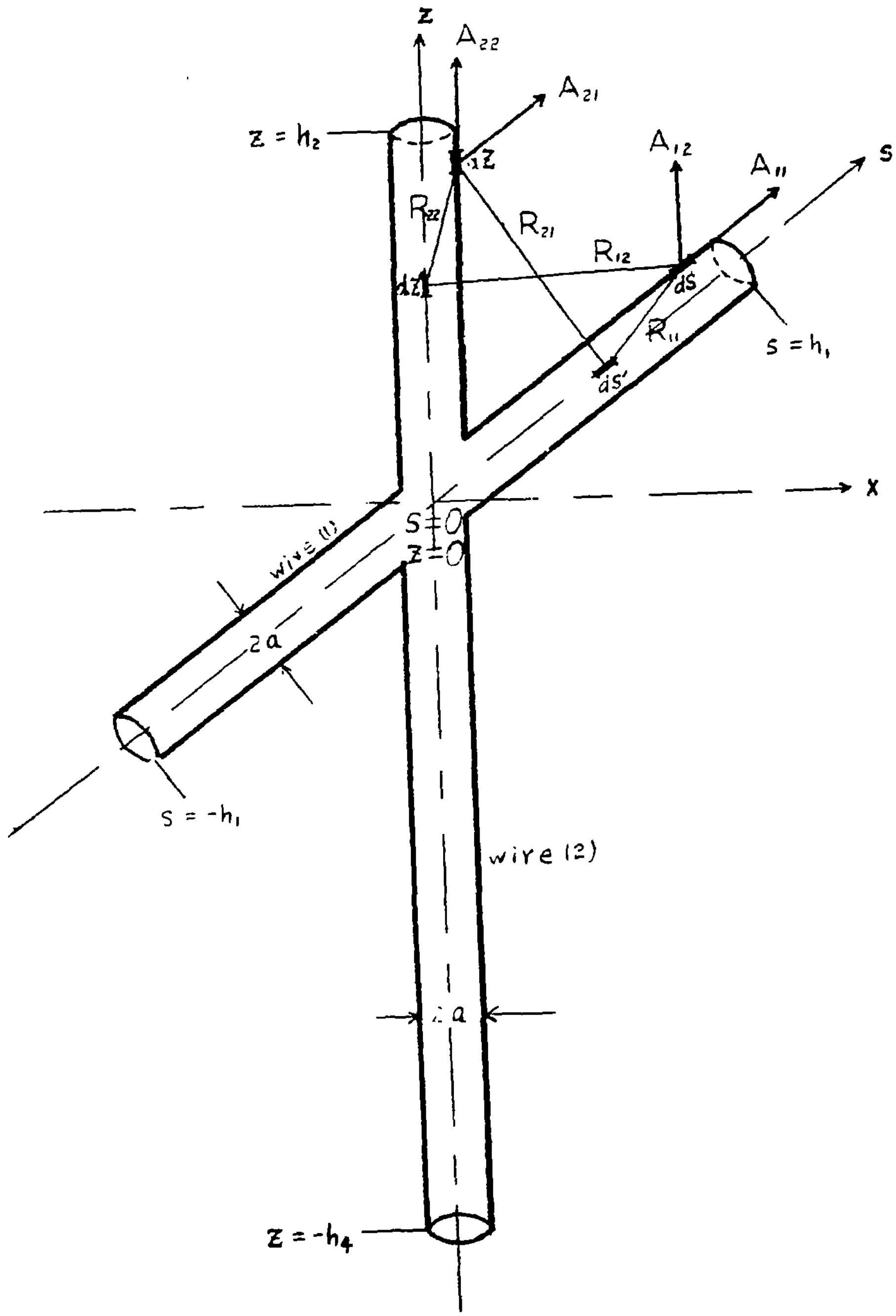

Figure 3. Intersecting Wires

The boundary conditions for the tangential component of the electric field on the surfaces of the two conductors are

$$\left(E_{1s}^{s} \right)_{r_1 = a} = - \left(E_{1s}^{i} \right)_{r_1 = a} , \tag{38a}$$

and

$$\left(E_{2z}^{s} \right)_{r_2 = a} = - \left(E_{2z}^{i} \right)_{r_2 = a} , \tag{38b}$$

where the axis of wire (1) is assumed to coincide with an s-axis, while the axis of wire (2) falls along the z-axis. The field just outside the conductors may be expressed in terms of scalar and vector potentials. Substituting equation (4) in (38a,b) and using appropriate components, one sees the results

$$\left(\frac{\partial \phi_1}{\partial s} + j\omega A_{1s} \right)_{r_1 = a} = \left(E_{1s}^{i} \right)_{r_1 = a} , \tag{39a}$$

and

$$\left(\frac{\partial \phi_2}{\partial z} + j\omega A_{2z} \right)_{r_2 = a} = \left(E_{2z}^{i} \right)_{r_2 = a} . \tag{39b}$$

Applying the equation of continuity [Eq. (6)] to (39a,b) leads to the following equations in which the subscript 1 or 2 on the operator $\vec{\nabla}$ refers to the variables r_1, s_1, or r_2, z_2 with respect to which the differentiation is performed:

$$\frac{\partial}{\partial s} \left(\vec{\nabla} \cdot \vec{A}_1 \right) + K_0^2 A_{1s} = - j \frac{\omega}{c^2} E_{1s}^{i} , \tag{40a}$$

and

$$\frac{\partial}{\partial z} \left(\vec{\nabla} \cdot \vec{A}_2 \right) + K_0^2 A_{2z} = - j \frac{\omega}{c^2} E_{2z}^{i} , \tag{40b}$$

where A_1 is the vector potential just outside the surface of wire (1) due to currents in both wires, and A_2 is the vector potential just outside the surface of wire (2) due to currents in both wires. Thus

$$\vec{A}_1 = \vec{A}_{11} + \vec{A}_{12} \tag{41a}$$

and

$$\vec{A}_2 = \vec{A}_{21} + \vec{A}_{22} , \tag{41b}$$

where the individual vector potentials are given by

$$\vec{A}_{11} = \hat{s} \frac{\mu_0}{4\pi} \int_{-h_1}^{h_1} I_{1s}(s') \frac{\exp(-j K_0 R_{11})}{R_{11}} ds' , \tag{42a}$$

$$\vec{A}_{12} = \hat{z} \frac{\mu_0}{4\pi} \int_{-h_4}^{h_2} I_{2z}(z') \frac{\exp(-j K_0 R_{12})}{R_{12}} dz' , \tag{42b}$$

$$\vec{A}_{21} = \hat{s} \frac{\mu_0}{4\pi} \int_{-h_1}^{h_1} I_{1s}(s') \frac{\exp(-j K_0 R_{21})}{R_{21}} ds' , \tag{42c}$$

and

$$\vec{A}_{22} = \hat{z} \frac{\mu_0}{4\pi} \int_{-h_4}^{h_2} I_{2z}(z') \frac{\exp(-j K_0 R_{22})}{R_{22}} dz' . \tag{42d}$$

In these equations R_{11}, R_{12}, R_{21} and R_{22} are the distances described in Figure 3. Since the current is discontinuous at the junction on both wires, it is convenient to define

$$I_{2z}^{-}(z) = I_{2z}(z) \qquad \text{for} \quad z < 0 , \tag{43a}$$

$$I_{2z}^{+}(z) = I_{2z}(z) \qquad \text{for} \quad z > 0 , \tag{43b}$$

$$I_{1s}^{-}(s) = I_{1s}(s) \qquad \text{for} \quad s < 0 , \tag{43c}$$

and

$$I_{1s}^{+}(s) = I_{1s}(s) \qquad \text{for} \quad s > 0 . \tag{43d}$$

These currents must satisfy the boundary condition

$$I_{1s}^{+}(0) - I_{1s}^{-}(0) = I_{2z}^{+}(0) - I_{2z}^{-}(0) . \tag{44}$$

Using (41a,b) in (40a,b) one obtains

$$\left(\frac{d^2}{ds^2} + K_0^2 \right) A_{1s} = - \frac{d}{ds} F_{1s} - j \frac{\omega}{c^2} E_{1s}^{i} \tag{45a}$$

and

$$\left(\frac{d^2}{dz^2} + K_0^2 \right) A_{2z} = - \frac{d}{dz} F_{2z} - j \frac{\omega}{c^2} E_{2z}^{i} , \tag{45b}$$

where $F_{1s} = \frac{\partial}{\partial z} A_{1z}$, $\qquad F_{2z} = \frac{\partial}{\partial s} A_{2s}$,

$$\begin{aligned} A_{1s} &= A_{11s} + A_{12s} , \quad A_{1z} = A_{11z} + A_{12z} , \\ A_{2z} &= A_{21z} + A_{22z} , \quad A_{2s} = A_{21s} + A_{22s} . \end{aligned} \tag{46}$$

Substitution of (42a,b,c,d) in (45a,b) gives two simultaneous integral equations in the currents I_{1s} and I_{2z} in the two wires. These coupled integral equations for the unknown current distributions may be reduced to a system of linear equations allowing the problem to be solved by a high-speed digital computer. Since the formulas obtained for the structure as shown in Figure 3 are formidable in general, to illustrate the suggested technique a special case is treated subsequently for which the axis of wire (1) coincides with the x-axis.

In Figure 4 are shown typical geometries that may be treated by the procedures set forth in this section.

2.5 Numerical Solution Technique.

The general form of the coupled Fredholm integral equations commonly appearing in the treatment of intersecting thin-wire structures may be expressed

$$\int_{L_2} dx'\, I_{2x}(x')\, K_{12}(x',z) + \int_{L_1} dz'\, I_{1z}(z')\, K_1(z-z') = S_1(z), \quad (47a)$$

and

$$\int_{L_2} dx'\, I_{2x}(x')\, K_2(x-x') + \int_{L_1} dz'\, I_{1z}(z')\, K_{21}(z',x) = S_2(x), \quad (47b)$$

where $I_{2x}(x')$ and $I_{1z}(z')$ are unknown current distributions, and $S_1(z)$, $S_2(x)$, $K_1(z-z')$, $K_2(x-x')$, $K_{12}(x',z)$, and $K_{21}(z',x)$ are known functions. The numerical solution of these integral equations may be effected by approximating the integrations with finite sums at N_1 and N_2 different points.[5] The unknown functions $I_{2x}(x')$ and $I_{1z}(z')$ may be represented by using piecewise constant representations as

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[5] E. A. Aronson and C. D. Taylor, "Matrix Methods for Solving Antenna Problems," IEEE Transactions on Antennas and Propagation, AP-15, no. 5, 696-697 (Sept. 1967).

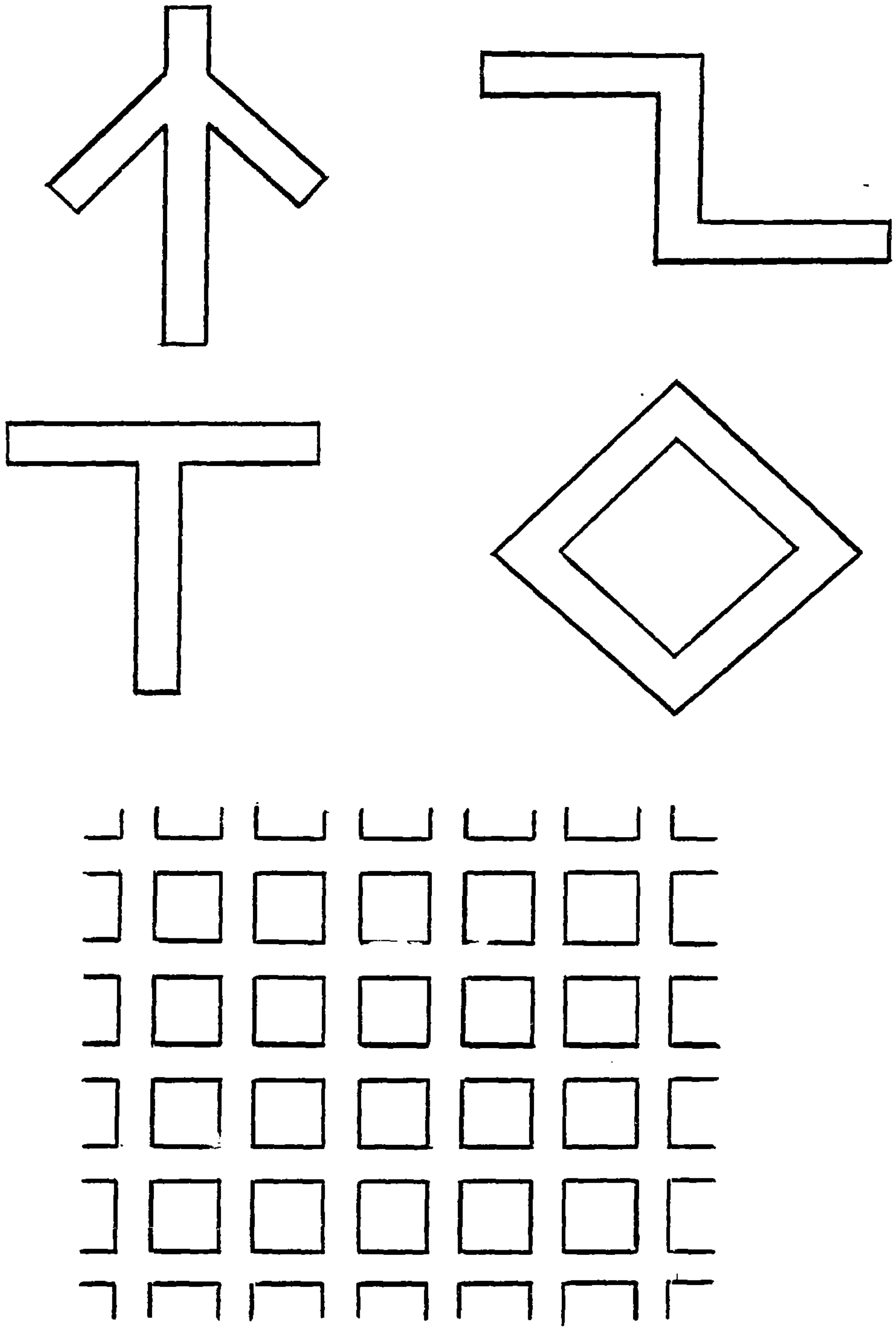

Figure 4. Typical geometries of antenna.

$$I_{2x}(x') = \sum_{n=1}^{N_2} g_n \, \chi\,(x_n, x_{n+1}) \,, \tag{48a}$$

and

$$I_{1z}(z') = \sum_{n=1}^{N_1} f_n \, \chi\,(z_n, z_{n+1}) \,, \tag{48b}$$

where

$$\chi\,(S_n, S_{n+1}) = \begin{cases} 1 & S_n \leqq S \leqq S_{n+1} \\ 0 & \text{elsewhere.} \end{cases} \tag{49}$$

Substitution of the above representations into the integral equations (47a,b) gives

$$\sum_{n=1}^{N_2} g_n \int_{\Delta x_n} K_{12}(x', z)\, dx' + \sum_{n=1}^{N_1} f_n \int_{\Delta z_n} K_1\,(z - z')\, dz' = S_1\,(z)\,, \tag{50a}$$

and

$$\sum_{n=1}^{N_2} g_n \int_{\Delta x_n} K_2\,(x - x')\, dx' + \sum_{n=1}^{N_1} f_n \int_{\Delta z_n} K_{21}(z', x)\, dz' = S_2(x)\,. \tag{50b}$$

In order to obtain a unique solution, the above equations are forced to be satisfied at N_1 points over the range of z and N_2 points over the range of x. For simplicity, equations (50a,b) may be combined into a single matrix equation

$$\sum_{n=1}^{N_1+N_2} I_n \, \Pi_{mn} = \Gamma_m \qquad , \; m = 1,2,\ldots,N_1+N_2, \tag{51}$$

noting that

$$I_n = f_n \qquad \text{for } n = 1,2,\ldots,N_1,$$

$$I_n = g_n \qquad \text{for } n = N_1+1,\ldots,N_2,$$

$$\Pi_{mn} = \int_{\Delta x_n} K_{12}(x', z_m)\, dx' \qquad \text{for } m = 1,2,\ldots,N_1$$
$$n = N_1+1,\ldots,N_2, \tag{52}$$
$$\Pi_{mn} = \int_{\Delta z_n} K_1\, (z_m - z')\, dz' \qquad \text{for } m = 1,2,\ldots,N_1$$
$$n = 1,2,\ldots,N_1,$$
$$\Pi_{mn} = \int_{\Delta x_n} K_2\, (x_m - x')\, dx' \qquad \text{for } m = N_1+1,\ldots,N_2$$
$$n = N_1+1,\ldots,N_2,$$

and

$$\Pi_{mn} = \int_{\Delta z_n} K_{21}(z', x_m)\, dz' \qquad \text{for } m = N_1+1,\ldots,N_2,$$
$$n = 1,2,\ldots,N_1.$$

It is clear that the matrix equation (51) includes N_1+N_2 linear equations for N_1+N_2 unknowns which may be solved by using a high speed digital computer.

If $I_{2x}(x')$ and $I_{1z}(z')$ in the coupled integral equations (47a,b) are discontinuous functions such as they must be for intersecting wires, the representations (48a,b) are no longer adequate, and a special technique is required. The discontinuity must be built into the proper representations. These are

$$I_{2x}^{-}(x') = \sum_{n=1}^{N_2^{-}} g_n^{-}\, \chi\, (x_n^{-}, x_{n+1}^{-})\ , \tag{53a}$$

$$I_{2x}^{+}(x') = \sum_{n=1}^{N_2^{+}} g_n^{+}\, \chi\, (x_n^{+}, x_{n+1}^{+})\ , \tag{53b}$$

$$I_{1z}^{-}(z') = \sum_{n=1}^{N_1^{-}} f_n^{-}\, \chi\, (z_n^{-}, z_{n+1}^{-})\ , \tag{53c}$$

and

$$I_{1z}^{+}(Z') = \sum_{n=1}^{N_1^+} f_n^+ \, \chi \, (Z_n^+, \; Z_{n+1}^+) \; , \tag{53d}$$

where $I_{2x}^{-}(X') = I_{2x}(X')$ for $X < 0$,

$$I_{2x}^{+}(X') = I_{2x}(X') \quad \text{for} \quad X > 0 \; ,$$

$$I_{1z}^{-}(Z') = I_{1z}(Z') \quad \text{for} \quad Z < 0 \; , \tag{54}$$

and

$$I_{1z}^{+}(Z') = I_{1z}(Z') \quad \text{for} \quad Z > 0 \; .$$

Note that two additional unknowns have been introduced. These require two additional equations which must come from boundary conditions. The appropriate boundary conditions are: the scalar potential at all points on the surface of the wires must be continuous, and the sum of the currents that are directed toward the juncture of two wires is zero. Then there are N_1+N_2+2 linear equations for the N_1+N_2+2 unknowns, and a solution may be obtained.

CHAPTER III

TREATMENT OF SCATTERING FROM INTERSECTING THIN WIRES

The technique for treating intersecting wire structures is illustrated in this chapter. For convenience the structure is considered to be formed by two straight wires that are perpendicular. Coupled Fredholm integral equations are derived for the currents induced in the wires by an incident plane wave field.

The intersecting wire configuration is constructed as shown in Figure 5. For convenience the structure is divided into two wires of circular cross-section, labeled (1) and (2). The axis of wore (1) coincides with the z-axis of a cartesian coordinate system and extends from $z = -\ell_1$ to ℓ_4. Wire (2) extends in the x-direction from $x = -\ell_2$ to ℓ_2. Both wires have radius a. Electrical continuity is maintained between wires (1) and (2) at $x = 0$, $z = 0$. The assumption is made that a is much smaller than ℓ_1, ℓ_2 or ℓ_4 and $k_0 a << 1$.

The procedure of analysis is to determine the distribution of currents in wires in terms of the incident electric field, to discuss the scalar potential and the incident

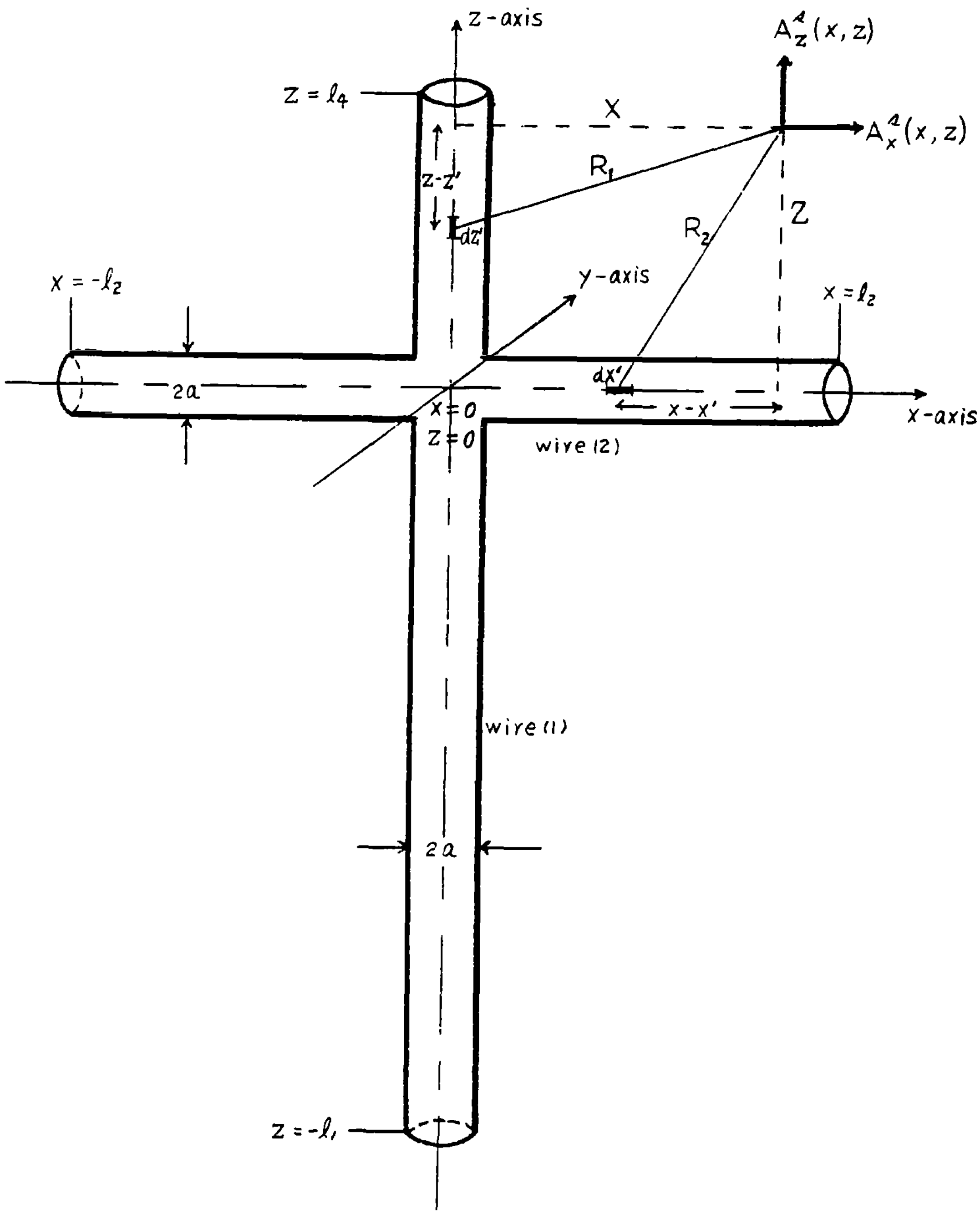

Figure 5. The intersecting thin wires.

electromagnetic plane wave and finally, to obtain numerical solution of the integral equations.

3.1 The Solution of the Differential Equation for the Vector Potential.

The total scattered vector potential $\vec{A}_1(x,z)$ on the surface of conductor (1) and $\vec{A}_2(x,z)$ on the surface of conductor (2) are given by

$$\vec{A}_1(x,z) = \hat{x}\left[A_x^s(x,z)\right]_{wire(1)} + \hat{z}\left[A_z^s(x,z)\right]_{wire(1)}\,, \qquad (55a)$$

and

$$\vec{A}_2(x,z) = \hat{x}\left[A_x^s(x,z)\right]_{wire(2)} + \hat{z}\left[A_z^s(x,z)\right]_{wire(2)}\,, \qquad (55b)$$

where $\hat{x}$ and $\hat{z}$ are the unit vectors in the x and z directions, respectively, $A_x^s(x,z)$ is the vector potential due to the current $I_{2x}(x)$ in the x-direction in wire (2), and $A_z^s(x,z)$ is the vector potential due to the current $I_{1z}(z)$ in the z-direction in wire (1). It may be assumed that no current exists in the structure in the y-direction. Accordingly, $A_y^s = 0$.

The components of the scattered vector potential $A_x^s(x,z)$ and $A_z^s(x,z)$ are given by

$$A_x^s(x,z) = \frac{\mu_0}{4\pi}\int_{-\ell_2}^{\ell_2} dx'\, I_{2x}(x')\, K_2(x-x';z)\,, \qquad (56)$$

where

$$K_2(x-x',z) = \frac{\exp\left[-jK_0\sqrt{(x-x')^2+z^2}\right]}{\sqrt{(x-x')^2+z^2}} \quad \text{when } z \geqq a\,; \qquad (57a)$$

$$K_2(x-x', z) = K_2(x-x', a) \qquad \text{when } z < a \quad , \tag{57b}$$

and

$$A_z^A(x,z) = \frac{\mu_0}{4\pi}\int_{-\ell_1}^{\ell_1} dZ' \, I_{1z}(z') \, K_1(z-z', x) \, , \tag{58}$$

where

$$K_1(z-z', x) = \frac{\exp\left[-jK_0\sqrt{(z-z')^2+x^2}\right]}{\sqrt{(z-z')^2+x^2}} \quad \text{when } x \geqq a; \tag{59a}$$

$$K_1(z-z', x) = K_1(z-z', a) \qquad \text{when } x < a \, . \tag{59b}$$

Now consider the field equation (10). The interpretation of this expression is that A is the vector potential associated with the electric field E. So take E to be the scattered field, then

$$E_{1z}^A(z) = -j\frac{c^2}{\omega}\left[\frac{\partial^2}{\partial z \partial x} A_x^A + \left(\frac{\partial^2}{\partial z^2} + K_0^2\right) A_z^A\right]_{wire(1)} , \tag{60a}$$

and

$$E_{2x}^A(x) = -j\frac{c^2}{\omega}\left[\frac{\partial^2}{\partial z \partial x} A_z^A + \left(\frac{\partial^2}{\partial x^2} + K_0^2\right) A_x^A\right]_{wire(2)} . \tag{60b}$$

Here $E_{1z}^A(z)$ is the z-component of the scattered field at the furface of wire (1), and $E_{2x}^A(x)$ is the x-component of the scattered field at the surface of wire (2). Define

$$A_{1z}(Z) = \left[A_z^A(x,z)\right]_{wire(1)} , \tag{61a}$$

$$F_{1x}(Z) = \left[\frac{\partial}{\partial x} A_x^A(x,z)\right]_{wire(1)} , \tag{61b}$$

$$A_{2x}(x) = \left[A_x^A(x,z) \right]_{wire(2)} , \tag{61c}$$

and

$$F_{2z}(x) = \left[\frac{\partial}{\partial z} A_z^A(x,z) \right]_{wire(2)} . \tag{61d}$$

Then (60a,b) may be written

$$\left(\frac{d^2}{dz^2} + K_o^2 \right) A_{1z}(z) = -\frac{d}{dz} F_{1x}(z) + j\frac{\omega}{c^2} E_{1z}^A(z) , \tag{62a}$$

and

$$\left(\frac{d^2}{dx^2} + K_o^2 \right) A_{2x}(x) = -\frac{d}{dx} F_{2z}(x) + j\frac{\omega}{c^2} E_{2x}^A(x) . \tag{62b}$$

Equation (62a) is a function of the variable z alone, and equation (62b) is a function of the variable x alone; therefore the use of the total derivative signs is in order.

The solution of the inhomogeneous equations (62a,b) may be written

$$A_{1z}(z) = C_1 \cos K_o z + C_2 \sin K_o z + \frac{1}{K_o}\int_0^z d\xi \; F_1(\xi) \sin K_o(z-\xi), \tag{63a}$$

and

$$A_{2x}(x) = C_3 \cos K_o x + C_4 \sin K_o x + \frac{1}{K_o}\int_0^x d\xi \; F_2(\xi) \sin K_o(x-\xi), \tag{63b}$$

where $F_1(\xi) = -\dfrac{d}{d\xi} F_{1x}(\xi) + j\dfrac{\omega}{c^2} E_{1z}^A(\xi)$, (64a)

$$F_2(\xi) = -\frac{d}{d\xi} F_{2z}(\xi) + j\frac{\omega}{c^2} E_{2x}^A(\xi) . \tag{64b}$$

Integration by parts yields

$$A_{1z}(Z) = C_1 \cos K_0 Z + C_2' \sin K_0 Z - \int_0^Z d\xi \; F_{1x}(\xi) \cos K_0 (Z - \xi)$$

$$+ j \frac{1}{c} \int_0^Z d\xi \; E_{1z}^A(\xi) \; \sin K_0 (Z - \xi) \quad , \qquad (65a)$$

and

$$A_{2x}(x) = C_3 \cos K_0 x + C_4' \sin K_0 x - \int_0^x d\xi \; F_{2z}(\xi) \cos K_0 (x - \xi)$$

$$+ j \frac{1}{c} \int_0^x d\xi \; E_{2x}^A(\xi) \; \sin K_0 (x - \xi) \quad . \qquad (65b)$$

Note that $$C_2' = C_2 + \frac{F_{1x}(0)}{K_0} \quad , \qquad (66a)$$

$$C_4' = C_4 + \frac{F_{2z}(0)}{K_0} \quad . \qquad (66b)$$

However the primes may be dropped now since C_2' and C_4' must be determined by the boundary conditions.

3.2 The Integral Equations for the Currents.

To arrive at the appropriate integral equations for the currents, it is best to consider

$$A_{1z}(Z) = \left[A_z^A (x,z) \right]_{wire(1)}$$

$$= \frac{\mu_0}{4\pi} \int_{-\ell_1}^{\ell_1} dZ' \; I_{1z}(Z') \; K_1 (z - z', a) \quad , \qquad (67a)$$

$$A_{2x}(x) = \left[A_x^A (x, z) \right]_{wire(2)}$$

$$= \frac{\mu_0}{4\pi} \int_{-\ell_2}^{\ell_2} dX' \; I_{2x}(x') \; K_2 (x - x', a) \quad , \qquad (67b)$$

$$F_{1x}(z) = \left[\frac{\partial}{\partial x} A_x^A(x,z) \right]_{wire(1)}$$

$$= -\frac{\mu_0}{4\pi}\int_{-\ell_2}^{\ell_2} dx'\, I_{2x}(x') \frac{\partial}{\partial x'} K_2(x',z) , \qquad (67c)$$

also $$F_{2z}(x) = \left[\frac{\partial}{\partial z} A_z^A(x,z) \right]_{wire(2)}$$

$$= -\frac{\mu_0}{4\pi}\int_{-\ell_1}^{\ell_4} dz'\, I_{1z}(z') \frac{\partial}{\partial z'} K_1(z',x) . \qquad (67d)$$

Note that

$$\int_{-\ell_2}^{\ell_2} dx'\, I_{2x}(x') \frac{\partial}{\partial x'} K_2(x',z)$$

$$= \left[I_{2x}(x') K_2(x',z) \right]_{-\ell_2}^{\ell_2} - \int_{-\ell_2}^{\ell_2} dx' \left[\frac{d}{dx'} I_{2x}(x') \right] K_2(x',z) . \qquad (68)$$

Using the boundary condition for the current

$$I_{2x}(\ell_2) = I_{2x}(-\ell_2) = 0 , \qquad (69)$$

one converts equation (68) into

$$\int_{-\ell_2}^{\ell_2} dx' \left[\frac{d}{dx'} I_{2x}(x') \right] K_2(x',z) = -\int_{-\ell_2}^{\ell_2} dx'\, I_{2x}(x') \frac{\partial}{\partial x'} K_2(x',z) . \qquad (70a)$$

Similarly it is found

$$\int_{-\ell_1}^{\ell_4} dz' \left[\frac{d}{dz'} I_{1z}(z') \right] K_1(z',x) = -\int_{-\ell_1}^{\ell_4} dz'\, I_{1z}(z') \frac{\partial}{\partial z'} K_1(z',x) . \qquad (70b)$$

then

$$F_{1x}(z) = \frac{\mu_0}{4\pi}\int_{-\ell_2}^{\ell_2} dx' \left[\frac{d}{dx'} I_{2x}(x') \right] K_2(x',z) , \qquad (71a)$$

and

$$F_{2z}(x) = \frac{\mu_0}{4\pi}\int_{-\ell_1}^{\ell_4} dZ' \left[\frac{d}{dZ'} \; I_{1z}(Z')\right] K_1(Z', x) \, . \tag{71b}$$

Define

$$\int_0^z d\xi \, F_{1x}(\xi) \; \cos K_0(z-\xi) = \frac{\mu_0}{4\pi}\int_{-\ell_2}^{\ell_2} dx' \left[\frac{d}{dx'} \; I_{2x}(x')\right] K_{12}(x', z) , \tag{72a}$$

and

$$\int_0^x d\xi \, F_{2z}(\xi) \; \cos K_0(x-\xi) = \frac{\mu_0}{4\pi}\int_{-\ell_1}^{\ell_4} dZ' \left[\frac{d}{dZ'} I_{1z}(Z')\right] K_{21}(Z', x) . \tag{72b}$$

So that

$$K_{12}(x', z) = \int_0^z d\xi \, K_2(x', \xi) \; \cos K_0(z-\xi) \, , \tag{73a}$$

and

$$K_{21}(Z', x) = \int_0^x d\xi \, K_1(Z', \xi) \; \cos K_0(x-\xi) \, . \tag{73b}$$

Inserting (72a,b) and (67a,b) into (65a,b) one sees that

$$\int_{-\ell_1}^{\ell_4} dZ' \, I_{1z}(Z') K_1(Z-Z', a) + \int_{-\ell_2}^{\ell_2} dx' \left[\frac{d}{dx'} I_{2x}(x')\right] K_{12}(x', Z)$$

$$= C_1'' \cos K_0 Z + C_2'' \sin K_0 Z + j \frac{4\pi}{\mu_0 c}\int_0^z d\xi \, E_{1z}^{A}(\xi) \; \sin K_0(z-\xi) , \tag{74a}$$

and

$$\int_{-\ell_2}^{\ell_2} dx' \, I_{2x}(x') K_2(x-x', a) + \int_{-\ell_1}^{\ell_4} dZ' \left[\frac{d}{dZ'} I_{1z}(Z')\right] K_{21}(Z', x)$$

$$= C_3'' \cos K_0 x + C_4'' \sin K_0 x + j \frac{4\pi}{\mu_0 c}\int_0^x d\xi \, E_{2x}^{A}(\xi) \; \sin K_0(x-\xi) . \tag{74b}$$

Note that in the above equations the arbitrary constants have been redefined for simplification.

The boundary condition for the electric field at the surface of the perfectly conducting wires may be used:

$$E^{i}_{1z}(\xi) + E^{s}_{1z}(\xi) = 0 \tag{75a}$$

$$E^{i}_{2x}(\xi) + E^{s}_{2x}(\xi) = 0 \tag{75b}$$

Here $E^{i}_{1z}(\xi)$ and $E^{i}_{2x}(\xi)$ are the tangential components of the incident electric field at the wires. Then equations (74a,b) become

$$\int_{-\ell_1}^{\ell_1} dz'\, I_{1z}(z') K_1(z-z',a) + \int_{-\ell_2}^{\ell_2} dx' \left[\frac{d}{dx'} I_{2x}(x')\right] K_{12}(x',z)$$

$$= C_1'' \cos K_0 z + C_2'' \sin K_0 z - j\frac{4\pi}{\mu_0 c}\int_0^z d\xi\, E^{i}_{1z}(\xi)\,\sin K_0(z-\xi) \tag{76a}$$

and

$$\int_{-\ell_2}^{\ell_2} dx'\, I_{2x}(x') K_2(x-x',a) + \int_{-\ell_1}^{\ell_1} dz' \left[\frac{d}{dz'} I_{1z}(z')\right] K_{21}(z',x)$$

$$= C_3'' \cos K_0 x + C_4'' \sin K_0 x - j\frac{4\pi}{\mu_0 c}\int_0^x d\xi\, E^{i}_{2x}(\xi)\,\sin K_0(x-\xi) \tag{76b}$$

where $$\mu_0 c = \sqrt{\frac{\mu_0}{\epsilon_0}} \equiv \zeta_0 \tag{77}$$

Here $\zeta_0 (\approx 120\pi\,\Omega)$ is the wave impedence of free space.

The coupled integral equations (76a,b) are to be solved simultaneously in order to obtain the induced current distribution.

3.3 Boundary Condition on the Scalar Potential.

The scalar potential is related to the vector potential through the Lorentz gauge condition, (6). It is

$$\phi(x,z) = j\frac{c^2}{\omega}\left[\frac{\partial}{\partial x}A_x^A(x,z) + \frac{\partial}{\partial z}A_z^A(x,z)\right]. \tag{78}$$

The scalar potential at the surface of wire (1), ,(z) is

$$\phi_1(z) = \left[\phi(x,z)\right]_{wire(1)}$$

$$= j\frac{c^2}{\omega}\left[F_{1x}(z) + \frac{d}{dz}A_{1z}(z)\right]. \tag{79a}$$

Similarly the scalar potential at the surface of wire (2), (x) is

$$\phi_2(x) = \left[\phi(x,z)\right]_{wire(2)}$$

$$= j\frac{c^2}{\omega}\left[\frac{d}{dx}A_{2x}(x) + F_{2z}(x)\right]. \tag{79b}$$

Continuity of the scalar potential at the juncture of the wires requires

$$\phi_2(0) = \phi_1(0) \quad . \tag{80}$$

From (65a) and (75a)

$$\begin{aligned}\frac{d}{dz}A_{1z}(z) = K_0\left[-C_1\sin K_0 z + C_2'\cos K_0 z\right] \\ -\frac{d}{dz}\int_0^z d\xi\, F_{1x}(\xi)\cos K_0(z-\xi) \\ -j\frac{1}{c}\frac{d}{dz}\int_0^z d\xi\, E_{1z}^i(\xi)\sin K_0(z-\xi)\,.\end{aligned} \tag{81}$$

Examination of the differential of an integral reveals

$$\frac{d}{dZ}\int_0^Z d\xi\ F_{1x}(\xi)\ \cos K_o(Z-\xi) = F_{1x}(Z) - K_o\int_0^Z d\xi\ F_{1x}(\xi)\ \sin K_o(Z-\xi), \quad (82a)$$

and

$$\frac{d}{dZ}\int_0^Z d\xi\ E_{1z}^{i}(\xi)\ \sin K_o(Z-\xi) = K_o\int_0^Z d\xi\ E_{1z}^{i}(\xi)\ \cos K_o(Z-\xi). \quad (82b)$$

Therefore

$$\frac{d}{dZ} A_{1z}(Z) + F_{1x}(Z) = K_o\Big[-C_1 \sin K_o Z + C_2' \cos K_o Z$$

$$+\int_0^Z d\xi\ F_{1x}(\xi)\ \sin K_o(Z-\xi)$$

$$-j\frac{1}{c}\int_0^Z d\xi\ E_{1z}^{i}(\xi)\ \cos K_o(Z-\xi)\Big]. \quad (85)$$

From (65b) and (75b)

$$\frac{d}{dZ} A_{2x}(X) = K_o\left[-C_3 \sin K_o X + C_4' \cos K_o X\right]$$

$$-\frac{d}{dX}\int_0^X d\xi\ F_{2z}(\xi)\ \cos K_o(X-\xi)$$

$$-j\frac{1}{c}\frac{d}{dX}\int_0^X d\xi\ E_{2x}^{i}(\xi)\ \sin K_o(X-\xi). \quad (84)$$

Also

$$\frac{d}{dX}\int_0^X d\xi\ F_{2z}(\xi)\ \cos K_o(X-\xi) = F_{2z}(X) - K_o\int_0^X d\xi\ F_{2z}(\xi)\ \sin K_o(X-\xi), \quad (85a)$$

and

$$\frac{d}{dX}\int_0^X d\xi\ E_{2x}^{i}(\xi)\ \sin K_o(X-\xi) = K_o\int_0^X d\xi\ E_{2x}^{i}(\xi)\ \cos K_o(X-\xi). \quad (85b)$$

Therefore

$$\frac{d}{dx} A_{2x}(x) + F_{2z}(x) = K_o\Big[-C_3 \sin K_o x + C_4' \cos K_o x + \int_0^x d\xi\; F_{2z}(\xi)\; \sin K_o(x-\xi) - j\frac{1}{c}\int_0^x d\xi\; E_{2x}^i(\xi)\; \cos K_o(x-\xi)\Big]. \tag{86}$$

Define

$$\int_0^z d\xi\; F_{1x}(\xi)\; \sin K_o(z-\xi) = \frac{\mu_o}{4\pi}\int_{-\ell_2}^{\ell_2} dx'\Big[\frac{d}{dx'} I_{2x}(x')\Big] H_{12}(x',z), \tag{87a}$$

and

$$\int_0^x d\xi\; F_{2z}(\xi)\; \sin K_o(x-\xi) = \frac{\mu_o}{4\pi}\int_{-\ell_1}^{\ell_1} dz'\Big[\frac{d}{dz'} I_{1z}(z')\Big] H_{21}(z',x). \tag{87b}$$

So

$$H_{12}(x',z) = \int_0^z d\xi\; K_2(x',\xi)\; \sin K_o(z-\xi), \tag{88a}$$

$$H_{21}(z',x) = \int_0^x d\xi\; K_1(z',\xi)\; \sin K_o(x-\xi). \tag{88b}$$

Then the scalar potentials at the surface of the wires are

$$\Phi_1(z) = jc\Big\{-C_1 \sin K_o z + C_2' \cos K_o z + \frac{\mu_o}{4\pi}\int_{-\ell_2}^{\ell_2} dx'\Big[\frac{d}{dx'} I_{2x}(x')\Big] H_{12}(x',z) - j\frac{1}{c}\int_0^z d\xi\; E_{1z}^i(\xi)\; \cos K_o(z-\xi)\Big\}, \tag{89a}$$

and

$$\Phi_2(x) = jc\Big\{-C_3 \sin K_o x + C_4' \cos K_o x + \frac{\mu_o}{4\pi}\int_{-\ell_1}^{\ell_1} dz'\Big[\frac{d}{dz'} I_{1z}(z')\Big] H_{21}(z',x) - j\frac{1}{c}\int_0^x d\xi\; E_{2x}^i(\xi)\; \cos K_o(x-\xi)\Big\}. \tag{89b}$$

The boundary condition on the scalar potential yields

$$C_2' = C_4' \tag{90}$$

3.4 Incident Electromagnetic Plane Wave.

To solve the coupled integral equations, the third terms or source terms on the right hand sides of (76a,b) must be discussed in this section. The general form of the electromagnetic plane wave equation is

$$\left(\nabla^2 + K^2 \right) \vec{E} = 0 \quad . \tag{91}$$

This wave equation has the well-known plane wave solution

$$\vec{E}^{i} = \hat{n} \; E_0 \; \exp\left[-j \, \vec{K} \cdot \vec{r} \right] \quad , \tag{92}$$

where $\hat{n}$ is the unit vector in the direction of $\vec{E}^{i}$, $\vec{K}$ is the direction of propagation and $\vec{r}$ is the radius vector to the field point. The incident electromagnetic wave is a plane wave, and the scattering wires are directed along the x and z axis of a cartesian coordinate system. On wire (1), the field is along the z-axis,

$$E_{1z}^{i} = \left[\vec{E}^{i} \cdot \hat{z} \right]_{x=y=0} = (\hat{n} \cdot \hat{z}) \, E_0 \; \exp\left[-j K_z z \right] \quad , \tag{93a}$$

while on wire (2), it is along the x-axis,

$$E_{2x}^{i} = \left[\vec{E}^{i} \cdot \hat{x} \right]_{y=z=0} = (\hat{n} \cdot \hat{x}) \, E_0 \; \exp\left[-j K_x x \right] \quad . \tag{93b}$$

For convenience, consider the incident electromagnetic plane wave for only two special cases: Case (a) the magnetic field associated with the plane wave is considered to be in the y-direction:

$$\hat{n} = (\hat{n}\cdot\hat{x})\ \hat{x} + (\hat{n}\cdot\hat{z})\hat{z}, \tag{94}$$

and

$$\vec{K} = K_x\ \hat{x} + K_z\ \hat{z}\ . \tag{95}$$

For plane waves

$$\vec{K}\cdot\hat{n} = 0\ ,\ \text{where}\ |\vec{K}| = K_o\ . \tag{96}$$

Therefore

$$K_z = (\hat{n}\cdot\hat{x})\ K_o\ , \tag{97a}$$

$$K_x = -(\hat{n}\cdot\hat{z})\ K_o\ . \tag{97b}$$

Equations (93a,b) become

$$E^i_{1z}(z) = \sin\alpha\ E_o\ \exp\left[-j K_o z \cos\alpha\right]\ , \tag{98a}$$

and

$$E^i_{2x}(x) = \cos\alpha\ E_o\ \exp\left[j K_o x \sin\alpha\right]\ . \tag{98b}$$

Note that $(\hat{n}\cdot\hat{z})$ and $(\hat{n}\cdot\hat{x})$ are the direction cosines of the incident electric field, i.e.,

$$\hat{n}\cdot\hat{x} = \cos\alpha \ , \tag{99a}$$

$$\hat{n}\cdot\hat{z} = \sin\alpha \ . \tag{99b}$$

Substitution of equations (98a,b) into the third terms of right hand sides of (76a,b) and integration yields

$$\int_0^z d\xi \ E^i_{1z}(\xi) \ \sin K_o(z-\xi)$$

$$= \frac{-E_o}{K_o \sin\alpha}\Big\{\exp\left[-jK_o z\cos\alpha\right]\left(\cos 2K_o z + j\cos\alpha \sin 2K_o z\right)$$

$$-\left(\cos K_o z + j\cos\alpha \ \sin K_o z\right)\Big\} , \tag{100a}$$

and

$$\int_0^x d\xi \ E^i_{2x}(\xi) \ \sin K_o(x-\xi)$$

$$= \frac{-E_o}{K_o \cos\alpha}\Big\{\exp\left[jK_o x \sin\alpha\right]\left(\cos 2K_o x - j\sin\alpha \ \sin 2K_o x\right)$$

$$-\left(\cos K_o x - j\sin\alpha \ \sin K_o x\right)\Big\} . \tag{100b}$$

Case (b) The direction of propagation is considered to be in the y-direction:

$$\vec{K} = K_y \ \hat{y} \ . \tag{101}$$

Therefore

$$E^i_{1z}(z) = (\hat{n}\cdot\hat{z}) \ E_o = E_o \ \sin\alpha \ , \tag{102a}$$

and

$$E_{2x}^{i}(x) = (\hat{n}\cdot\hat{x})\,E_o = E_o \cos\alpha \ . \tag{102b}$$

The third terms of right hand sides of (76a,b) become

$$\int_0^z d\xi\, E_{1z}^{i}(\xi)\, \sin K_o(z-\xi) = \frac{E_o}{K_o}\ \sin\alpha\left(1-\cos K_o z\right) , \tag{103a}$$

and

$$\int_0^x d\xi\, E_{2x}^{i}(\xi)\, \sin K_o(x-\xi) = \frac{E_o}{K_o}\ \cos\alpha\left(1-\cos K_o x\right) . \tag{103b}$$

3.5 Numerical Solution.

Since the currents are discontinuous at the junction between wires, they are redefined according to (54). Therefore

$$I_{2x}(x) = \eta(x)\, I_{2x}^{+}(x) + \left[1-\eta(x)\right] I_{2x}^{-}(x) , \tag{104a}$$

and

$$I_{1z}(z) = \eta(z)\, I_{1z}^{+}(z) + \left[1-\eta(z)\right] I_{1z}^{-}(z) . \tag{104b}$$

where

$$\eta(s) = \begin{cases} 1 & \text{for} \quad s > 0 \\ 0 & \text{for} \quad s < 0 \ . \end{cases} \tag{105}$$

The boundary condition on the current gives

$$I_{1z}^{-}(0) - I_{1z}^{+}(0) = I_{2x}^{-}(0) - I_{2x}^{+}(0) \ . \tag{106}$$

From (104a,b)

$$\frac{d}{dx} I_{2x}(x) = \delta(x)\left[I_{2x}^{+}(0) - I_{2x}^{-}(0) \right]$$

$$+\eta(x) \frac{d}{dx} I_{2x}^{+}(x) + \left[1-\eta(x)\right] \frac{d}{dx} I_{2x}^{-}(x) \;, \qquad (107a)$$

and

$$\frac{d}{dz} I_{1z}(z) = \delta(z)\left[I_{1z}^{+}(0) - I_{1z}^{-}(0) \right]$$

$$+\eta(z) \frac{d}{dz} I_{1z}^{+}(z) + \left[1-\eta(z)\right] \frac{d}{dz} I_{1z}^{-}(z) \;. \qquad (107b)$$

Therefore

$$\int_{-\ell_2}^{\ell_2} dx' \left[\frac{d}{dx'} I_{2x}(x')\right] K_{12}(x',z)$$

$$= \left[I_{2x}^{+}(0) - I_{2x}^{-}(0)\right] K_{12}(0,z) + \int_{0}^{\ell_2} dx' \left[\frac{d}{dx'} I_{2x}^{+}(x')\right] K_{12}(x',z)$$

$$+ \int_{-\ell_2}^{0} dx' \left[\frac{d}{dx'} I_{2x}^{-}(x')\right] \;, \qquad (108a)$$

$$\int_{-\ell_1}^{\ell_4} dz' \left[\frac{d}{dz'} I_{1z}(z')\right] K_{21}(z',x)$$

$$= \left[I_{1z}^{+}(0) - I_{1z}^{-}(0)\right] K_{21}(0,x) + \int_{0}^{\ell_4} dz' \left[\frac{d}{dz'} I_{1z}^{+}(z')\right] K_{21}(z',x)$$

$$+ \int_{-\ell_1}^{0} dz' \left[\frac{d}{dz'} I_{1z}^{-}(z')\right] \;, \qquad (108b)$$

$$\int_{-\ell_1}^{\ell_2} dx' I_{2x}(x') K_2(x-x',a) = \int_{0}^{\ell_2} dx' I_{2x}^{+}(x') K_2(x-x',a)$$

$$+ \int_{-\ell_2}^{0} dx' I_{2x}^{-}(x') K_2(x-x',a) \;, \qquad (108c)$$

and

$$\int_{-\ell_1}^{\ell_4} dz'\, I_{1z}(z') K_1(z-z',a) = \int_0^{\ell_4} dz'\, I_{1z}^{+}(z') K_1(z-z',a) + \int_{-\ell_1}^{0} dz'\, I_{1z}^{-}(z') K_1(z-z',a) . \tag{108d}$$

In order to solve the coupled integral equations piece wise constant representations are used for the unknown functions. These are given in (53). Then using that representation

$$\frac{d}{dx} I_{2x}^{-}(x) = \sum_{n=1}^{N_2^-} g_n^- \left[\delta(x-x_n^-) - \delta(x-x_{n+1}^-)\right] , \tag{109a}$$

$$\frac{d}{dx} I_{2x}^{+}(x) = \sum_{n=1}^{N_2^+} g_n^+ \left[\delta(x-x_n^+) - \delta(x-x_{n+1}^+)\right] , \tag{109b}$$

$$\frac{d}{dz} I_{1z}^{-}(z) = \sum_{n=1}^{N_1^-} f_n^- \left[\delta(z-z_n^-) - \delta(z-z_{n+1}^-)\right] , \tag{109c}$$

and

$$\frac{d}{dz} I_{1z}^{+}(z) = \sum_{n=1}^{N_1^+} f_n^+ \left[\delta(z-z_n^+) - \delta(z-z_{n+1}^+)\right] . \tag{109d}$$

Therefore

$$\int_{-\ell_2}^{\ell_2} dx' \left[\frac{d}{dx'} I_{2x}(x')\right] K_{12}(x',z)$$

$$= \left[g_1^+ - g_1^-\right] K_{12}(0,z)$$

$$+ \sum_{n=1}^{N_2^+} g_n^+ \left[K_{12}(x_n^+,z)/\epsilon_{n-1} - K_{12}(x_{n+1}^+,z)/\epsilon_{N_2^+-n}\right]$$

$$+ \sum_{n=1}^{N_2^-} g_n^- \left[K_{12}(x_n^-,z)/\epsilon_{n-1} - K_{12}(x_{n+1}^-,z)/\epsilon_{N_2^--n}\right] , \tag{110a}$$

$$\int_{-\ell_1}^{\ell_4} dZ' \left[\frac{d}{dZ'} I_{1z}(Z')\right] K_{21}(Z', X)$$

$$= \left[f_1^+ - f_1^- \right] K_{21}(0, X)$$

$$+ \sum_{n=1}^{N_1^+} f_n^+ \left[K_{21}(Z_n^+, X)/\epsilon_{n-1} - K_{21}(Z_{n+1}^+, X)/\epsilon_{N_1^+-n} \right]$$

$$+ \sum_{n=1}^{N_1^-} f_n^- \left[K_{21}(Z_n^-, X)/\epsilon_{n-1} - K_{21}(Z_{n+1}^-, X)/\epsilon_{N_1^--n} \right], \qquad (110b)$$

and

$$\int_{-\ell_2}^{\ell_2} dX' I_{2x}(X') K_2(X - X'; a) = \sum_{n=1}^{N_2^+} g_n^+ \int_{X_n^+}^{X_{n+1}^+} dX' \; K_2(X - X'; a)$$

$$+ \sum_{n=1}^{N_2^-} g_n^- \int_{X_n^-}^{X_{n+1}^-} dX' \; K_2(X - X', a), \qquad (110c)$$

$$\int_{-\ell_1}^{\ell_4} dZ' I_{1z}(Z') K_1(Z - Z'; a) = \sum_{n=1}^{N_1^+} f_n^+ \int_{Z_n^+}^{Z_{n+1}^+} dZ' K_1(Z - Z'; a)$$

$$+ \sum_{n=1}^{N_1^-} f_n^- \int_{Z_n^-}^{Z_{n+1}^-} dZ' \; K_1(Z - Z'; a), \qquad (110d)$$

where

$$\epsilon_n = \begin{cases} 2 & \text{for } n = 0 \\ 1 & \text{for otherwise.} \end{cases} \qquad (111)$$

Using (110a,b,c,d) and (90) in equations (76a,b) yields

$$\sum_{n=1}^{N_1^+} f_n^+ \int_{z_n^+}^{z_{n+1}^+} dz' \, K_1(z-z',a) + \sum_{n=1}^{N_1^-} f_n^- \int_{z_n^-}^{z_{n+1}^-} dz' \, K_1(z-z',a)$$

$$+ (g_1^+ - g_1^-) K_{12}(0,z) + \sum_{n=1}^{N_2^+} g_n^+ \left[K_{12}(x_n^+,z)/\epsilon_{n-1} - K_{12}(x_{n+1}^+,z)/\epsilon_{N_2^+-n} \right]$$

$$+ \sum_{n=1}^{N_2^-} g_n^- \left[K_{12}(x_n^-,z)/\epsilon_{n-1} - K_{12}(x_{n+1}^-,z)/\epsilon_{N_2^- -n} \right]$$

$$= C_1'' \cos K_0 z + C_2'' \sin K_0 z - j \frac{4\pi}{\mu_0 c} \int_0^z d\xi \, E_{1z}^i(\xi) \sin K_0(z-\xi), \quad (112a)$$

and

$$\sum_{n=1}^{N_2^+} g_n^+ \int_{x_n^+}^{x_{n+1}^+} dx' \, K_2(x-x',a) + \sum_{n=1}^{N_1^-} g_n^- \int_{x_n^-}^{x_{n+1}^-} dx' \, K_2(x-x',a)$$

$$+ (f_1^+ - f_1^-) K_{21}(0,x) + \sum_{n=1}^{N_1^+} f_n^+ \left[K_{21}(z_n^+,x)/\epsilon_{n-1} - K_{21}(z_{n+1}^+,x)/\epsilon_{N_1^+-n} \right]$$

$$+ \sum_{n=1}^{N_1^-} f_n^- \left[K_{21}(z_n^-,x)/\epsilon_{n-1} - K_{21}(z_{n+1}^-,x)/\epsilon_{N_1^- -n} \right]$$

$$= C_3'' \cos K_0 x - C_2'' \sin K_0 x - j \frac{4\pi}{\mu_0 c} \int_0^{-x} d\xi \, E_{2x}^i(\xi) \sin K_0(x-\xi). \quad (112b)$$

In order to get the relationship of and in the above coupled equations, the boundary conditions on the currents must be used. They are

$$I_{2x}^-(-\ell_2) = 0 \quad , \qquad (113a)$$

$$I_{2x}^+(\ell_2) = 0 \quad , \qquad (113b)$$

$$I_{1z}^-(-\ell_1) = 0 \quad , \qquad (113c)$$

$$I_{1z}^{+}(\ell_4) = 0 \ , \qquad (113d)$$

$$I_{2x}^{+}(0) - I_{2x}^{-}(0) = I_{1z}^{+}(0) - I_{1z}^{-}(0) \ . \qquad (106)$$

Consequently

$$g_{N_2^-}^{-} = 0 \ , \qquad (114a)$$

$$g_{N_2^+}^{+} = 0 \ , \qquad (114b)$$

$$f_{N_1^-}^{-} = 0 \ , \qquad (114c)$$

$$f_{N_1^+}^{+} = 0 \ , \qquad (114d)$$

$$g_1^{+} - g_1^{-} = f_1^{+} - f_1^{-} \ . \qquad (115)$$

Application of equations (114a,b,c,d) to the coupled integral equations (112a,b) obtains

$$\sum_{n=1}^{N_1^+-1} f_n^+ \int_{z_n^+}^{z_{n+1}^+} dz' \, K_1(z-z',a) + \sum_{n=1}^{N_1^--1} f_n^- \int_{z_n^-}^{z_{n+1}^-} dz' \, K_1(z-z',a)$$

$$\left(\tfrac{1}{2} g_1^+ - \tfrac{3}{2} g_1^-\right) K_{12}(0,z) + \sum_{n=1}^{N_2^+-1} g_n^+ \left[K_{12}(x_n^+, z) - K_{12}(x_{n+1}^+, z) \right]$$

$$+ \sum_{n=1}^{N_2^--1} g_n^- \left[K_{12}(x_n^-, z) - K_{12}(x_{n+1}^-, z) \right]$$

$$= C_1'' \cos K_0 z + C_2'' \sin K_0 z - j \frac{4\pi}{\mu_0 c} \int_0^z d\xi \, E_{1z}^i(\xi) \sin K_0 (z-\xi), \quad (116a)$$

and

$$\sum_{n=1}^{N_2^+-1} g_n^+ \int_{x_n^+}^{x_{n+1}^+} dx' \, K_2(x-x',a) + \sum_{n=1}^{N_2^--1} g_n^- \int_{x_n^-}^{x_{n+1}^-} dx' \, K_2(x-x',a)$$

$$\left(\tfrac{1}{2} f_1^+ - \tfrac{3}{2} f_1^-\right) K_{21}(0,x) + \sum_{n=1}^{N_1^+-1} f_n^+ \left[K_{21}(z_n^+, x) - K_{21}(z_{n+1}^+, x) \right]$$

$$+ \sum_{n=1}^{N_1^--1} f_n^- \left[K_{21}(z_n^-, x) - K_{21}(z_{n+1}^-, x) \right]$$

$$= C_3'' \cos K_0 x + C_2'' \sin K_0 x - j \frac{4\pi}{\mu_0 c} \int_0^x d\xi \, E_{2x}^i(\xi) \sin K_0 (x-\xi). \quad (116b)$$

Using equations (100a,b) in (116a,b) yields a system of linear equations for the current distributions when the magnetic field of the incident electromagnetic plane wave is directed along the y-azis.

$$\sum_{n=1}^{N_2^+-1} f_n^+ \alpha_n^+(z_m) + \sum_{n=1}^{N_2^--1} f_n^- \alpha_n^-(z_m) + \left(\tfrac{1}{2} g_1^+ - \tfrac{3}{2} g_1^-\right) K_{12}(0, z_m)$$

$$+ \sum_{n=1}^{N_2^+-1} g_n^+ \beta_n^+(z_m) + \sum_{n=1}^{N_2^--1} g_n^- \beta_n^-(z_m)$$

$$= C_1'' \cos K_0 z_m + C_2'' \sin K_0 z_m + j \frac{4\pi E_0}{\mu_0 K_0 c \sin\alpha} \gamma(z_m), \quad (117a)$$

and

$$\sum_{n=1}^{N_2^+-1} g_n^+ \Phi_n^+(X_m) + \sum_{n=1}^{N_2^--1} g_n^- \Phi_n^-(X_m) + \left(\tfrac{1}{2} f_1^+ - \tfrac{3}{2} f_1^-\right) K_{21}(0, X_m)$$

$$+ \sum_{n=1}^{N_1^+-1} f_n^+ \Psi_n^+(X_m) + \sum_{n=1}^{N_1^--1} f_n^- \Psi_n^-(X_m)$$

$$= C_3'' \cos K_o X_m + C_2'' \sin K_o X_m + j \frac{4\pi E_o}{\mu_o K_o c \cos\alpha} \theta(X_m). \qquad (117b)$$

Similarly, using equations (103a,b) in (116a,b) yields a system of linear equations for the current distributions when the propagation of the incident electromagnetic plane wave is in the y-direction.

$$\sum_{n=1}^{N_1^+-1} f_n^+ \alpha_n^+(Z_m) + \sum_{n=1}^{N_1^--1} f_n^- \alpha_n^-(Z_m) + \left(\tfrac{1}{2} g_1^+ - \tfrac{3}{2} g_1^-\right) K_{12}(0, Z_m)$$

$$+ \sum_{n=1}^{N_2^+-1} g_n^+ \beta_n^+(Z_m) + \sum_{n=1}^{N_2^--1} g_n^- \beta_n^-(Z_m)$$

$$= C_1'' \cos K_o Z_m + C_2'' \sin K_o Z_m - j \frac{4\pi E_o \sin\alpha}{\mu_o K_o c} \xi(Z_m), \qquad (118a)$$

and

$$\sum_{n=1}^{N_2^+-1} g_n^+ \Phi_n^+(X_m) + \sum_{n=1}^{N_2^--1} g_n^- \Phi_n^-(X_m) + \left(\tfrac{1}{2} f_1^+ - \tfrac{3}{2} f_1^-\right) K_{21}(0, X_m)$$

$$+ \sum_{n=1}^{N_1^+-1} f_n^+ \Psi_n^+(X_m) + \sum_{n=1}^{N_1^--1} f_n^- \Psi_n^-(X_m)$$

$$= C_3'' \cos K_o X_m + C_2'' \sin K_o X_m - j \frac{4\pi E_o \cos\alpha}{\mu_o K_o c} \xi(X_m). \qquad (118b)$$

In the foregoing the following definitions were used:

$$\alpha_n^+(Z_m) = \int_{Z_n^+}^{Z_{n+1}^+} dZ' \; K_1(Z_m - Z', a) \;,$$

$$\alpha_n^-(Z_m) = \int_{Z_n^-}^{Z_{n+1}^-} dZ' \; K_1(Z_m - Z', a) \;,$$

$$\beta_n^+(Z_m) = K_{12}(X_n^+, Z_m) - K_{12}(X_{n+1}^+, Z_m) \;,$$

$$\beta_n^-(Z_m) = K_{12}(X_n^-, Z_m) - K_{12}(X_{n+1}^-, Z_m) \;,$$

$$\gamma(Z_m) = \exp\left[-j K_0 Z_m \cos\alpha\right]\left\{\cos 2K_0 Z_m + j \cos\alpha \sin 2K_0 Z_m\right\}$$

$$- \left\{\cos K_0 Z_m + j \cos\alpha \sin K_0 Z_m\right\},$$

$$\theta(X_m) = \exp\left[j K_0 X_m \sin\alpha\right]\left\{\cos 2K_0 X_m - j \sin\alpha \sin 2K_0 X_m\right\}$$

$$- \left\{\cos K_0 X_m - j \sin\alpha \sin K_0 X_m\right\}, \tag{119}$$

$$\phi_n^+(X_m) = \int_{X_n^+}^{X_{n+1}^+} dX' \; K_2(X_m - X', a) \;,$$

$$\phi_n^-(X_m) = \int_{X_n^-}^{X_{n+1}^-} dX' \; K_2(X_m - X', a) \;,$$

$$\psi_n^+(X_m) = K_{21}(Z_n^+, X_m) - K_{21}(Z_{n+1}^+, X_m) \;,$$

$$\psi_n^-(X_m) = K_{21}(Z_n^-, X_m) - K_{21}(Z_{n+1}^-, X_m) \;,$$

$$\zeta(Z_m) = 1 - \cos K_0 Z_m \;,$$

$$\xi(X_m) = 1 - \cos K_0 X_m \;.$$

There are N_1+N_2-1 unknown for each of the coupled equations (117a,b) and (118a,b). Since there is only one additional boundary equation (115), N_1+N_2-2 more equations will be needed to obtain a unique solution. Therefore, let m take N_1-1 values for z, and N_2-1 values for x, i.e.,

$$z_1, z_2, \ldots\ldots\ldots, z_{N_1-1},$$

and

$$x_1, x_2, \ldots\ldots\ldots, x_{N_2-1},$$

and require the equations to be satisfied for these values of z and x.

CHAPTER IV

SUMMARY AND CONCLUSIONS

It is the aim of this thesis to present a theoretical-numerical solution technique for treating electromagnetic scattering from thin wire structures of arbitrary configuration. It is shown that the arising coupled integral equations for the induced current distributions may be reduced to a system of linear equations allowing the problem to be solved by a high-speed digital computer.

In Chapter II the basic solution technique for treating practically any configuration of wires is presented. The procedures are described briefly and applied to the problems of determining the current distributions induced on an arbitrary single thin wire configuration and on intersecting straight wires. It is shown that the treatment of intersecting wires requires special techniques not needed for the formulation of an arbitrary orientation of a single wire as presented by K. K. Mei.[1] In both cases integral equations are obtained which must be solved to obtain the induced current distribution.

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[1]K. K. Mei, "On the Integral Equations of Thin Wire Antennas," IEEE Transactions on Antennas and Propagation, AP-13, 374-378 (May 1965).

The problem chosen for solution in Chapter III is the determination of current distributions induced along two intersecting straight thin wires which are perpendicular. The exciting fields are chosen to be plane wave fields. And the problem is completely formulated into suitable form for programming digital computer.

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