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ANTENNA THEORY CALCULATIONS OF THE EFFECTS OF MISSILE EXHAUST PLUMES ON MISSILE SKIN CURRENTS (U)

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1. INTRODUCTION

This memo presents some unclassified results from our loaded antenna model of the missile with an exhaust plume. Specifically, three things are of interest:

1) The skin current response of the missile to EMP with no exhaust plume.

2) The response of the missile with the exhaust plume considered.

3) The effect of the missile image in a perfectly conducting ground plane on skin current, and the comparison of the missile image with the exhaust plume.

2. ANALYSIS

The problem of a missile with a conducting exhaust can be treated as a receiving antenna with a resistive load at the end. The exhaust gases will be relatively poor conductors, compared to the metallic components of the missile, so the entire structure will be considered to be a perfectly conducting cylindrical antenna with a fairly resistive, inhomogeneous load at the end.

The inhomogeneous nature of the exhaust can be accounted for by dividing it into n pieces of length s_i, each with impedance

$$Z_i = z_i s_i \qquad (1)$$

where z_i is the impedance per unit length of each division. The problem to be considered is shown in Figure 1.

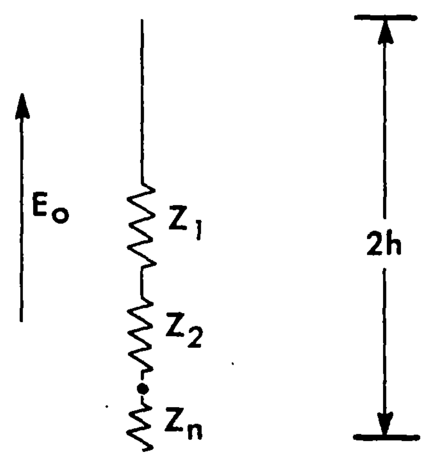


FIGURE 1

PLANE WAVE INCIDENT UPON LOADED ANTENNA

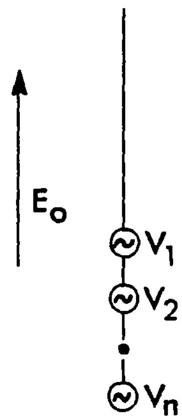
Two features of electromagnetic theory now allow the problem to be handled analytically. The first is the Compensation Theorem, which states that in any electrical circuit an impedance Z may be replaced by a generator whose voltage is given by

$$V = -IZ$$

where I is the current through the load impedance Z. Thus the structure shown in Figure 1 may be replaced by that shown in Figure 2.

FIGURE 2

CIRCUIT EQUIVALENT TO FIGURE 1



where

$$V_1 = -I(\ell_1)Z_1$$

$$V_2 = -I(\ell_2)Z_2$$

$$V_3 = -I(\ell_3)Z_3$$

$$V_4 = -I(\ell_4)Z_4$$

. . . .

$$V_n = -I(\ell_n)Z_n$$

and ℓ_i is the location of the i^{th} impedance Z_i. Now let $I_R(z)$ be the current distribution on an antenna of length 2h which had no generators, and I_{ai} be the current distribution of an antenna which had a unit voltage generator situated at ℓ_i. According to the principle of superposition

$$I(z) = I_R(z) + I_{a1}(z)V_1 + I_{a2}(z)V_2 + \ldots + I_{an}(z)V_n \quad (2)$$

where I(z) is the current distribution on the loaded antenna. Equation (2) is demonstrated symbolically in Figure 3.

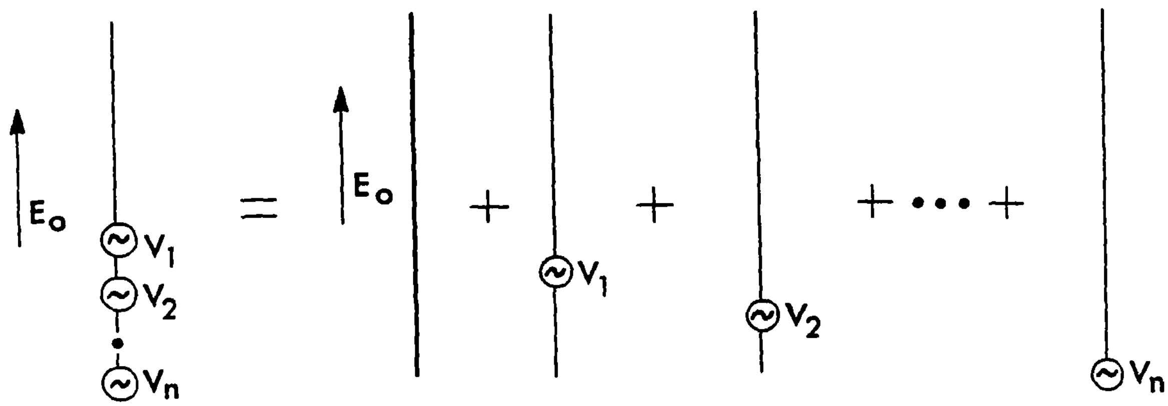


FIGURE 3

ILLUSTRATION OF THE PRINCIPLE OF SUPERPOSITION

The voltages V_i, i = 1, n are determined from the conditions

$$V_i = -I(\ell_i)Z_i. \tag{3}$$

Thus, substituting (2) into (3),

$$\begin{aligned}
-V_1 &= I_R(\ell_1)Z_1 + I_{a1}(\ell_1)V_1Z_1 + \ldots + I_{zn}(\ell_1)V_nZ_1 \\
-V_2 &= I_R(\ell_2)Z_2 + I_{a1}(\ell_2)V_1Z_2 + \ldots + I_{an}(\ell_2)V_nZ_2 \\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
-V_n &+ I_R(\ell_n)Z_n + I_{a1}(\ell_n)V_1Z_n + \ldots + I_{an}(\ell_n)V_nZ_n
\end{aligned} \tag{4}$$

Rearranging (4) results in the matrix equation

$$\begin{pmatrix} I_{a1}(\ell_1) + 1/Z_1 & I_{a2}(\ell_1) & \ldots & I_{an}(\ell_1) \\ I_{a1}(\ell_2) & I_{a2}(\ell_2)+1/Z_2 & \ldots & I_{an}(\ell_2) \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ I_{a1}(\ell_n) & I_{a2}(\ell_n) & \ldots & I_{an}(\ell_n)+1/Z_n \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ \cdot\cdot \\ \cdot\cdot \\ V_n \end{pmatrix} = - \begin{pmatrix} I_R(\ell_1) \\ I_R(\ell_2) \\ \cdots \\ \cdots \\ I_R(\ell_n) \end{pmatrix} \tag{5}$$

The solution of equation (5) provides the V_i's which, when substituted into (2) gives the current distribution on the loaded antenna. The current on the asymmetrically driven antennas, I_{ai}, $i = 1, 2, \ldots, n$ is given by King and Wu.[1]

The impedance per unit length, z, can be given in terms of the conductivity of the exhaust as follows:

$$z = \frac{k}{2\pi a \sigma} \quad \frac{J_0(ka)}{J_1(ka)} \tag{6}$$

where

$k = (1-i) \quad \frac{\omega\mu\sigma}{2}$

σ = conductivity of plume

ω = radial frequency of signal

$\mu = 4\pi \times 10^{-7}$

a = radius of exhaust

$i = \sqrt{-1}$

Further investigation of the effect of the plume on the missile current distribution has been hampered by a lack of precise information concerning the electrical characteristics of the exhaust. Taylor[2] has assumed that the conductivity of the plume varies exponentially as

$$\begin{aligned} \sigma(z) &= \sigma_o\, e^{-\alpha z} \qquad && 0 < z < h_p \\ \sigma &= 0 && z > h_p \end{aligned} \qquad (7)$$

where σ_o is the value of the conductivity at the base of the missile and h_p is the length of the plume. In Taylor's work, σ_o ranges from .001 to .1 mhos/meter. α varied between 0 and .5393. Taylor's solution consisted of a very complex Fourier Series analysis, the application of which is severely limited by the time required for computation.

Figures 4 and 5 compare the results of the present theory with Taylor's results for a missile length of 17.08 meters, plume length of 17.08 meters, frequency of 2.8 MHz, σ_o=0.1, and more precise theory of Taylor is gratifying. The dotted line on the figures shows the current that would exist for the same conditions if the plume were not present.

3. TRANSIENT RESPONSE

Thus far the theory has assumed that the antenna comprised of the missile and its exhaust was illuminated with a CW signal of frequency $f = \omega/2\pi$. If the antenna is exposed to a transient electric field E(t), the time-domain solution can be obtained by Fourier analysis.

Let

$$E(\omega) = \int_0^\infty E(t)\, e^{i\omega t} dt \qquad (8)$$

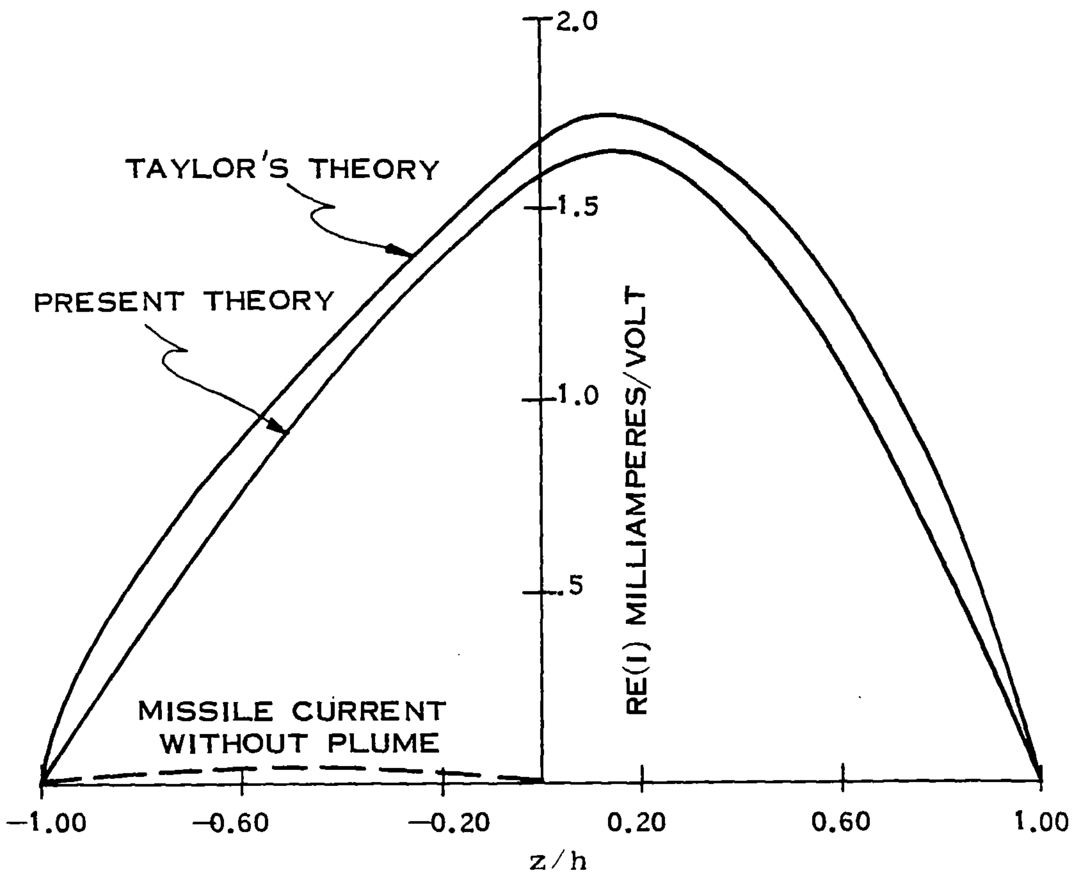


FIGURE 4(A)

COMPARISON OF PRESENT THEORY WITH TAYLOR'S FOURIER ANALYSIS. (A) REAL PART OF CURRENT VS DISTANCE FROM CENTER. MISSILE LENGTH EQUAL TO PLUME LENGTH. $a = 0$

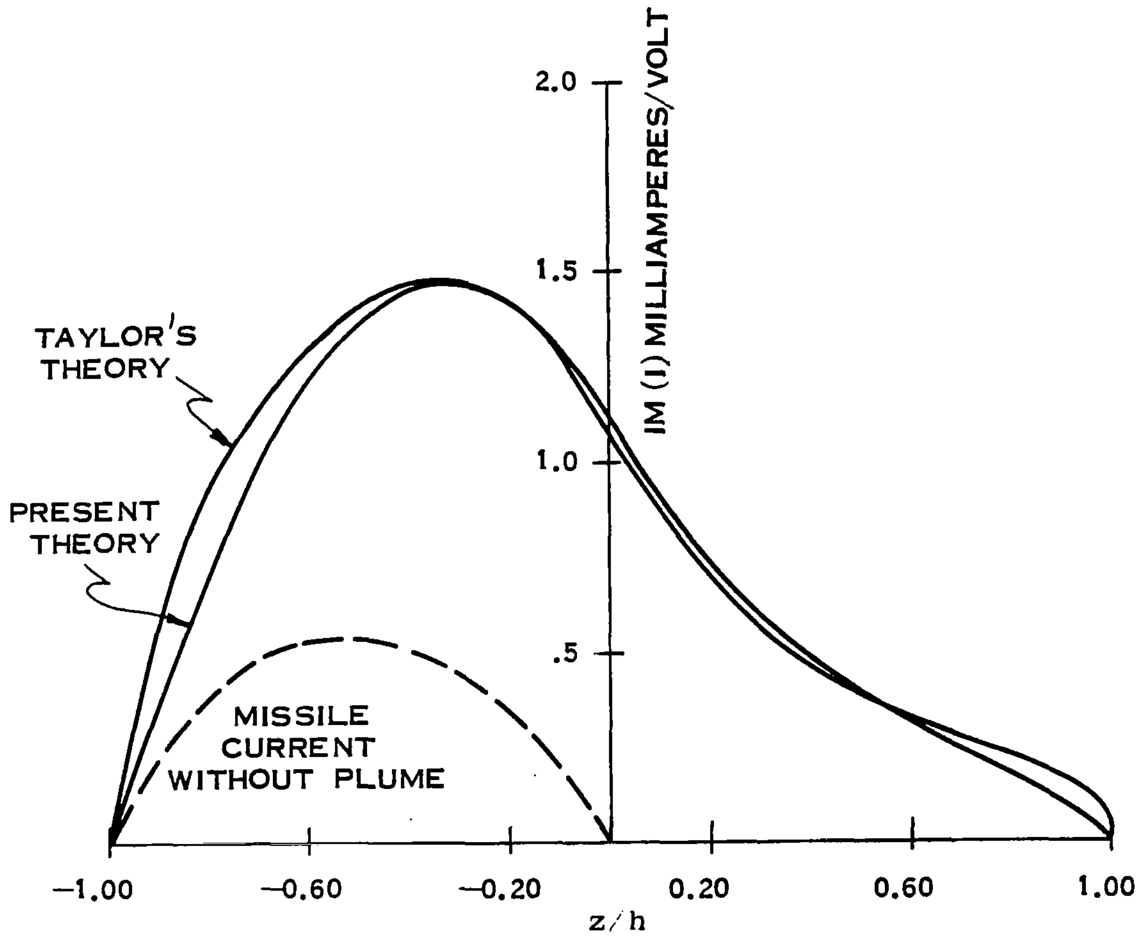


FIGURE 4(B)

COMPARISON OF PRESENT THEORY WITH TAYLOR'S FOURIER ANALYSIS. (B) IMAGINARY PART OF CURRENT VS DISTANCE FROM CENTER. $\alpha = 0$

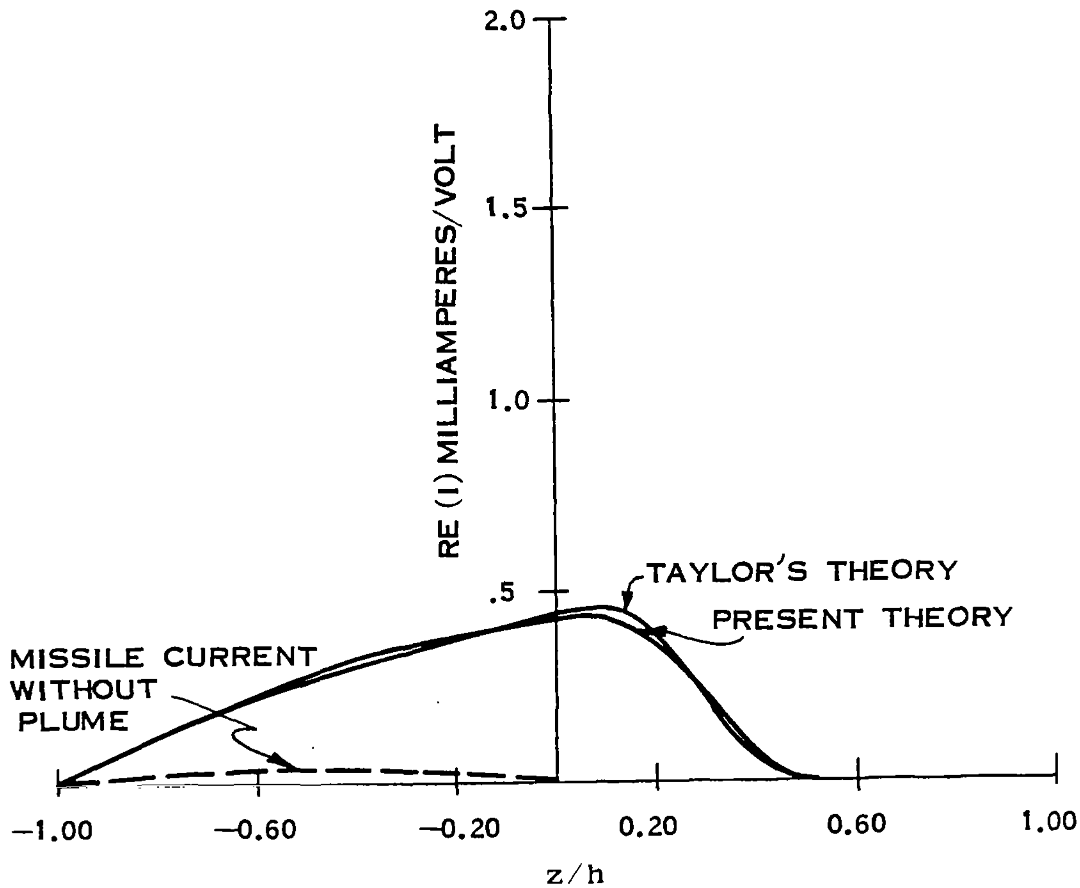


FIGURE 5(A)

COMPARISON OF PRESENT THEORY WITH TAYLOR'S FOURIER ANALYSIS. (A) REAL PART OF CURRENT VS DISTANCE FROM CENTER. MISSILE LENGTH EQUAL TO PLUME LENGTH. $\alpha = .5393$

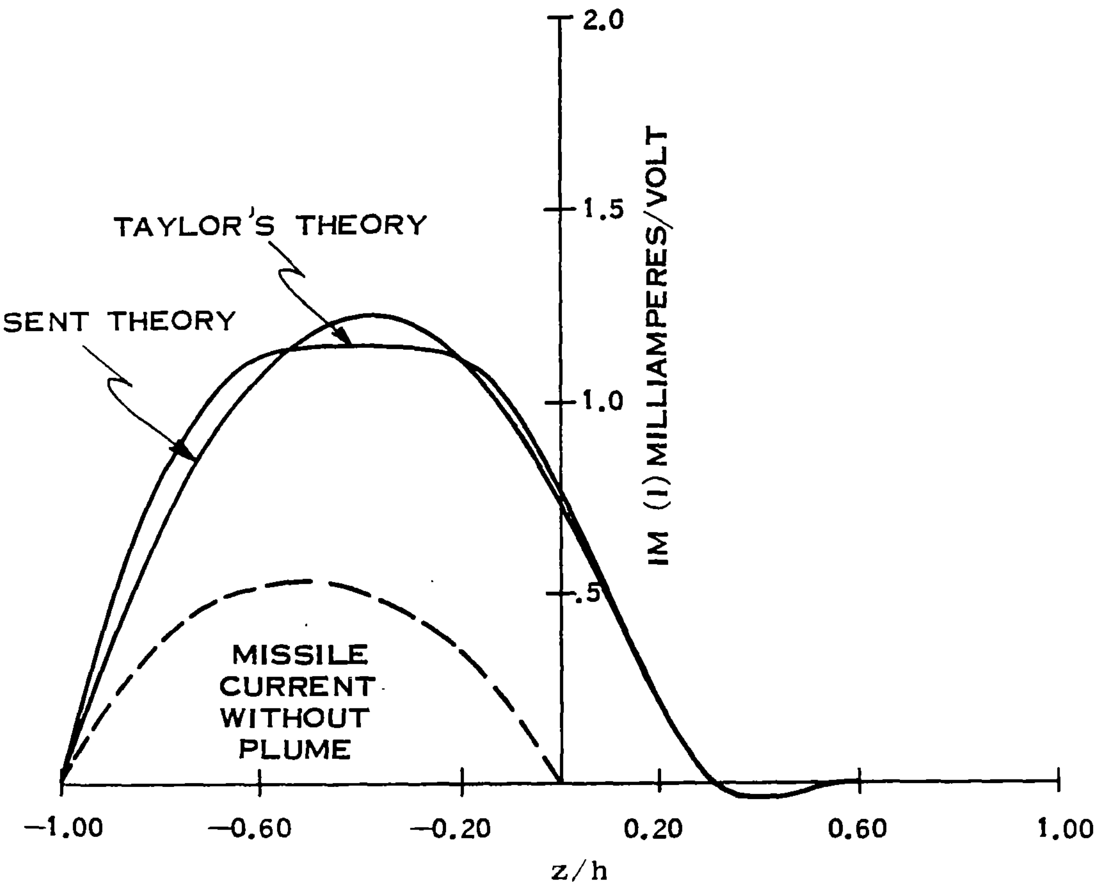


FIGURE 5(B)

COMPARISON OF PRESENT THEORY WITH TAYLOR'S FOURIER ANALYSIS. (B) IMAGINARY PART OF CURRENT VS DISTANCE FROM CENTER. MISSILE LENGTH EQUAL TO PLUME LENGTH. $a = .5393$

and let I (ω,z) be the current at point z on the antenna induced by a harmonic electric field of unit magnitude and angular frequency ω.

The current induced by the pulse E(t) is then given by

$$I(t,z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} E(\omega) I(\omega,z) e^{-i\omega t} d\omega. \qquad (9)$$

As an example of an antenna in a transient electric field, the missile considered earlier (Figures 4 and 5) was assumed to be irradiated by a unit step field, i.e.,

$$E(t) = 0, \qquad -\infty < t < o \qquad (10)$$
$$= 1, \qquad o < t < \infty$$

For which

$$E(\omega) = - \frac{1}{i\omega} \quad . \qquad (11)$$

Figure 6 shows the current at the center of the missile as a function of time for the unit step driving function. The response is the characteristic damped sine wave, peaking at about .03 amperes, and ringing with a resonant frequency of about 8 mHz. No exhaust plume was considered in Figure 6.

The effects of the exhaust plume on the transient response of the missile were examined for the plume model described above with $\sigma_o = .1$, $\alpha = 0$ and $\alpha = .5393$. The plume length was assumed to be the same as the missile length for these calculations (see Figure 7).

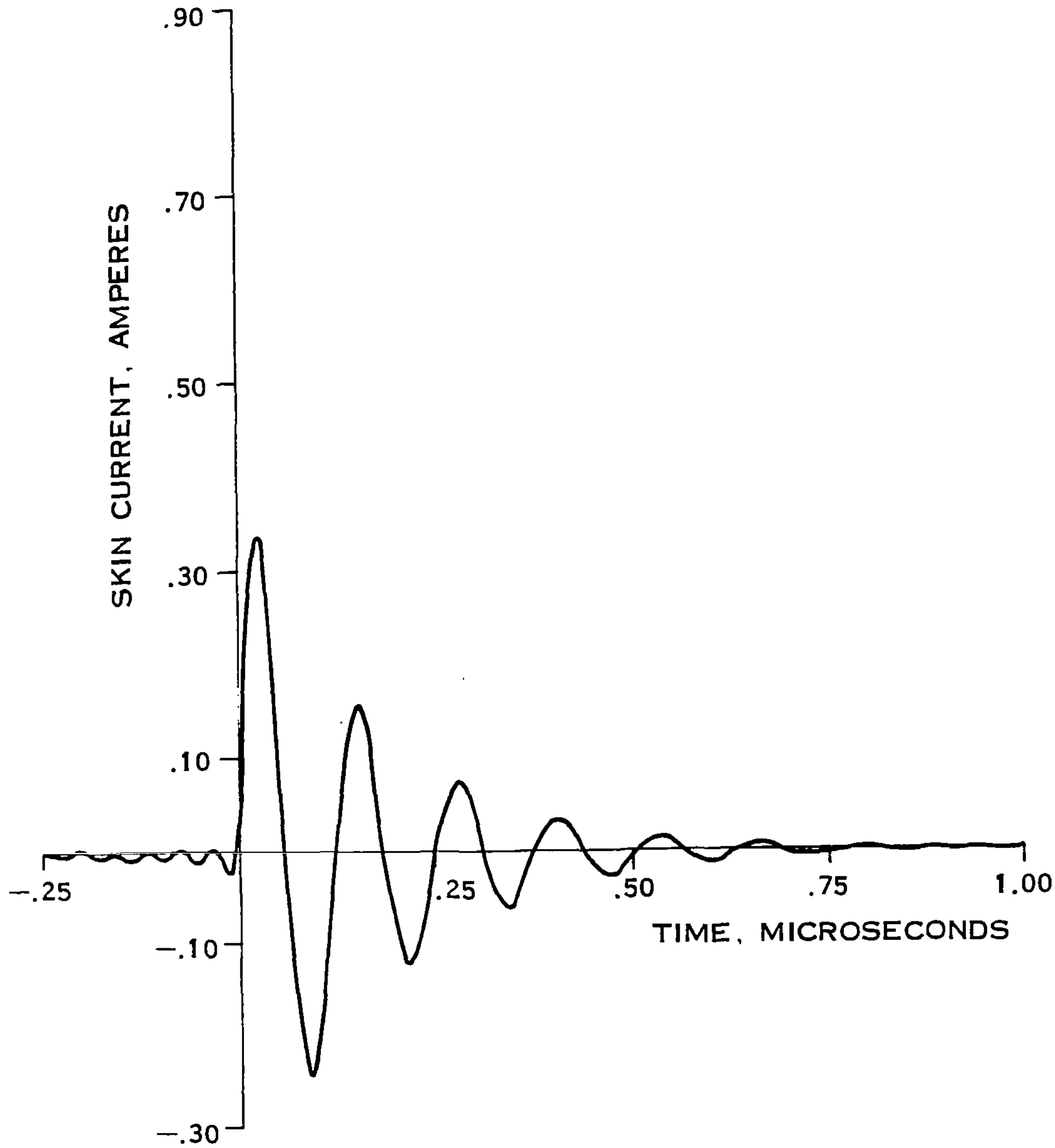


FIGURE 6

SKIN CURRENT VS TIME AT CENTER OF MISSILE WITH NO EXHAUST PLUME. UNIT STEP DRIVING FUNCTION.

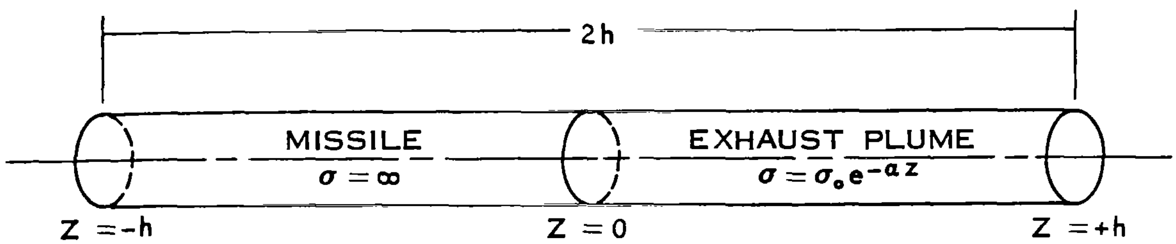


FIGURE 7

LOADED DIPOLE MODEL OF MISSILE WITH EXHAUST PLUME

Figures 8 through 15 show the current as a function of time at various points along the missile and plume. In Figures 8 through 11, $\alpha = .5393$, and in Figures 12 through 15, $\alpha = 0$.

Figures 8 and 12 give the current vs time at the center of the missile, and it is of interest to compare them with Figure 6, which shows the current at the same point if no plume exists. It is seen that the plume does not significantly affect the peak current at the center of the missile for the model assumed here. The principle influence of the plume has been to decrease the resonant frequency and ringing time.

Figures 10 and 14 give the skin current at the missile nozzle as functions of time. In Figure 14, ($\alpha = 0$) the peak current is about twice that in Figure 10 ($\alpha = .5393$). However, in Figure 14 the current pulse seems to be much more highly damped than in Figure 10. If there were no plume, of course, the model would predict no current at the end of the missile.

It has been suggested that for simulation purposes, a missile could be electrically connected to the ground plane, and that the image current would simulate the exhaust plume. Figure 16 shows the current that would be induced at the base of the missile in such a configuration.

Compared with Figures 10 and 14, it appears that there are significant differences in the response of the grounded monopole and the loaded dipole. The current on the grounded monopole (Figure 16) peaks at about .06 amps and rings at 4 mHz. For the loaded dipoles, Figures 10 and 14, the peak current is .018 and .03 amps respectively, and the ringing frequency is about 6 mHz. Also, the grounded monopole is much less critically damped than the loaded dipole.

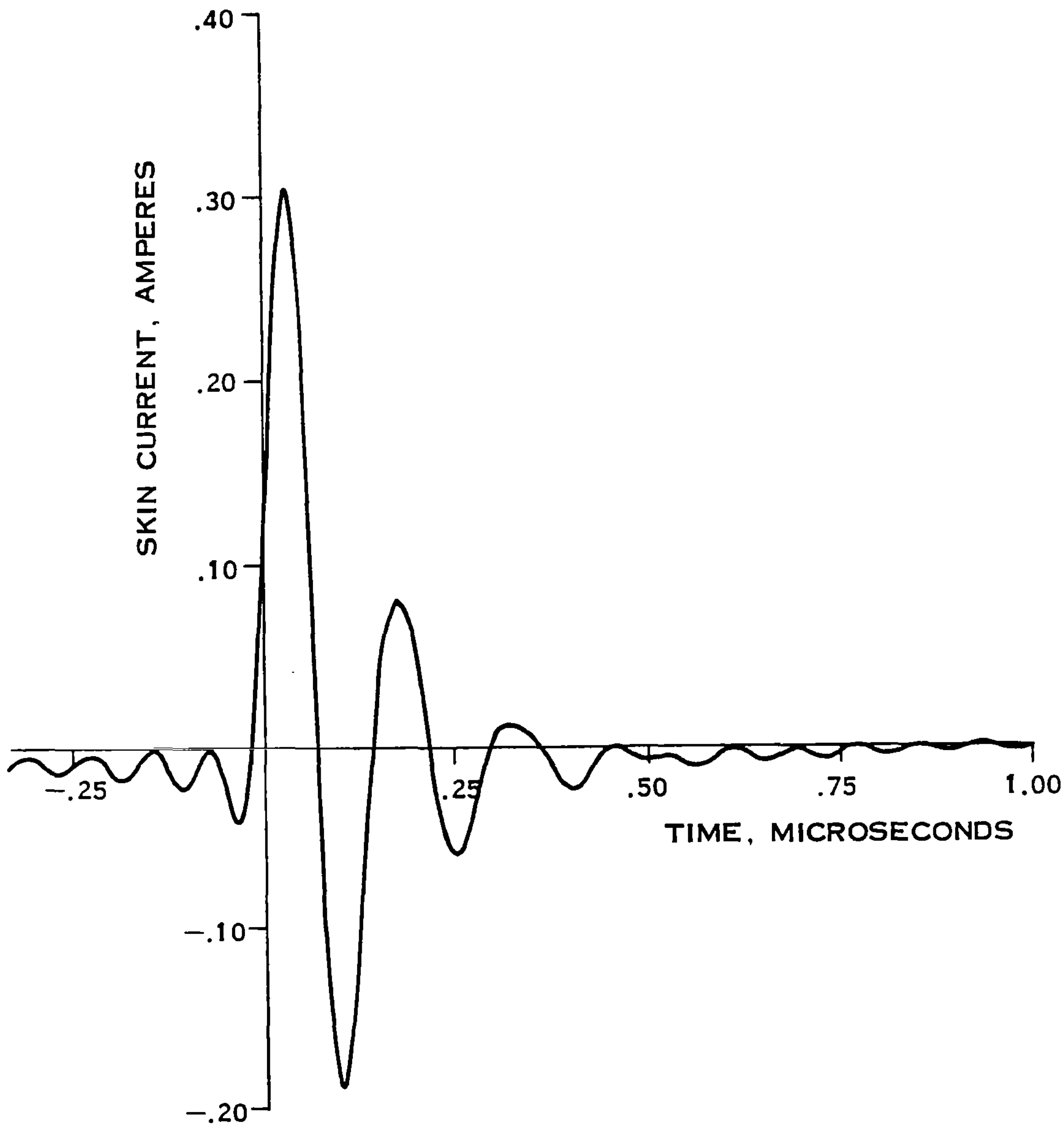


FIGURE 8

SKIN CURRENT VS TIME AT CENTER OF MISSILE WITH EXHAUST PLUME. UNIT STEP DRIVING FUNCTION.

$\sigma_o = .1$

$a = .5393$

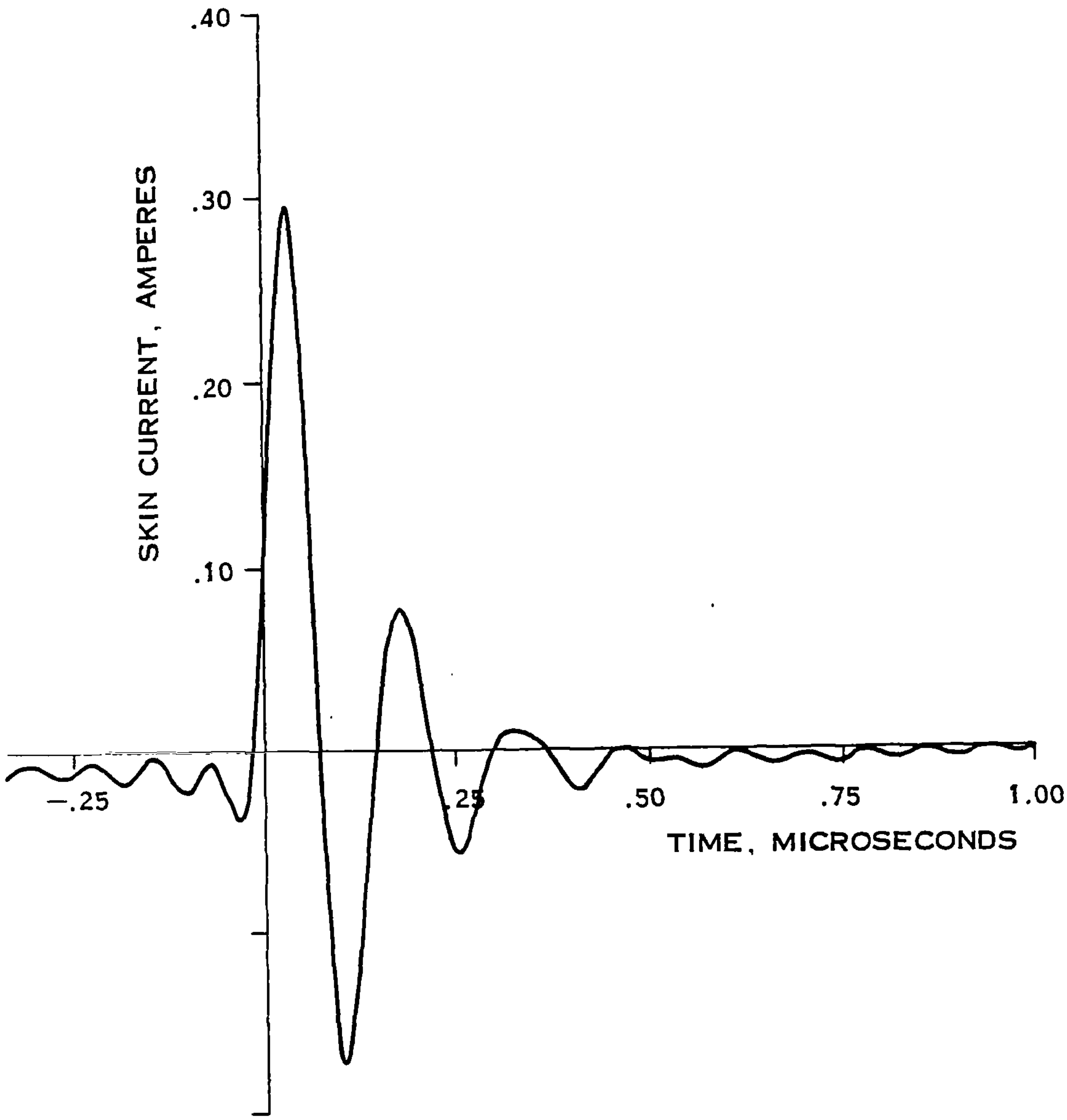


FIGURE 9

SKIN CURRENT VS TIME AT POINT 1/4 MISSILE LENGTH FROM NOZZLE. UNIT STEP DRIVING FUNCTION.

$\alpha = .5393$

$\sigma_o = 0.1$

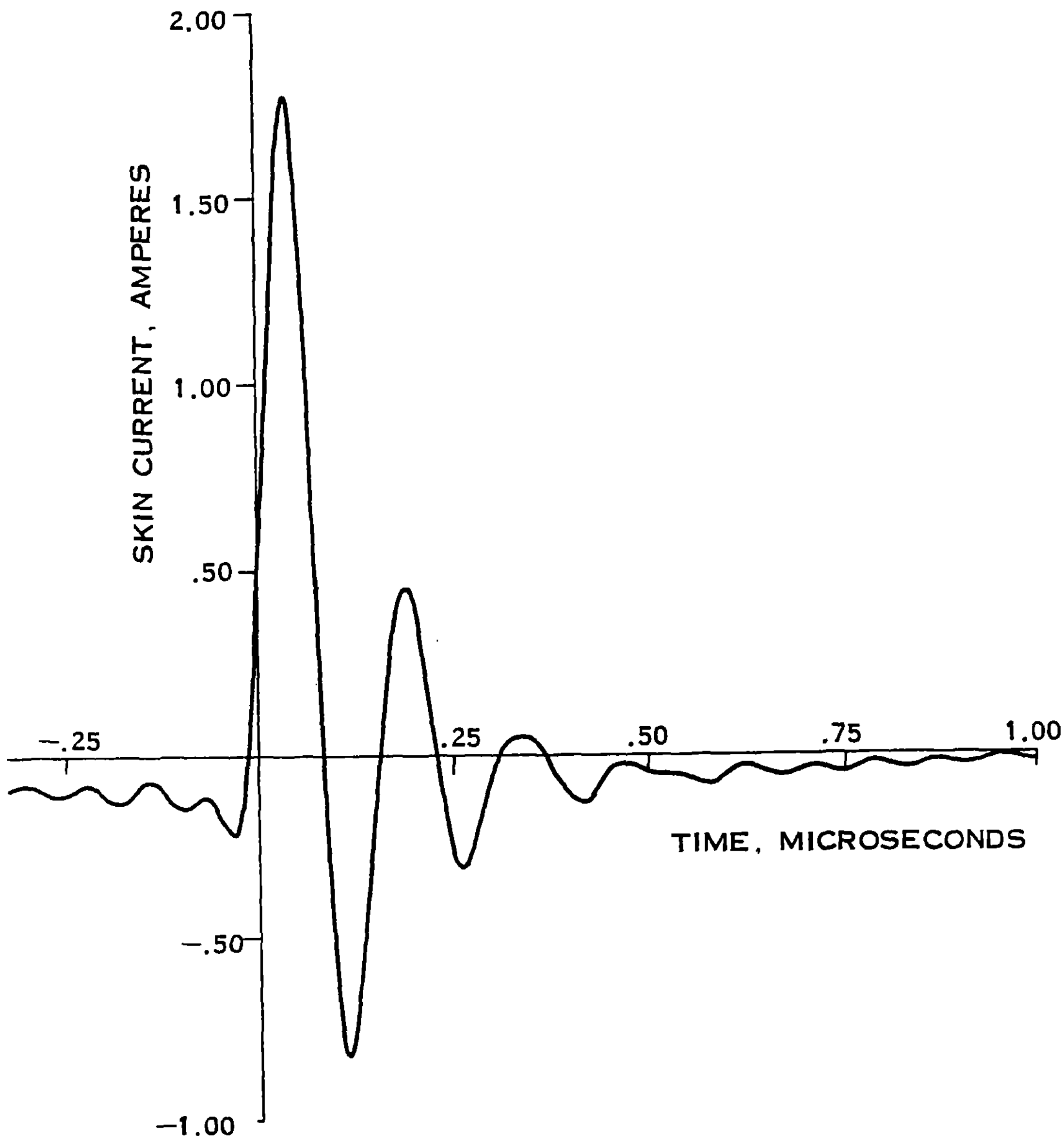


FIGURE 10

SKIN CURRENT VS TIME AT NOZZLE OF MISSILE WITH EXHAUST PLUME. UNIT STEP DRIVING FUNCTION.

$\alpha = .5393$

$\sigma_o = .1$

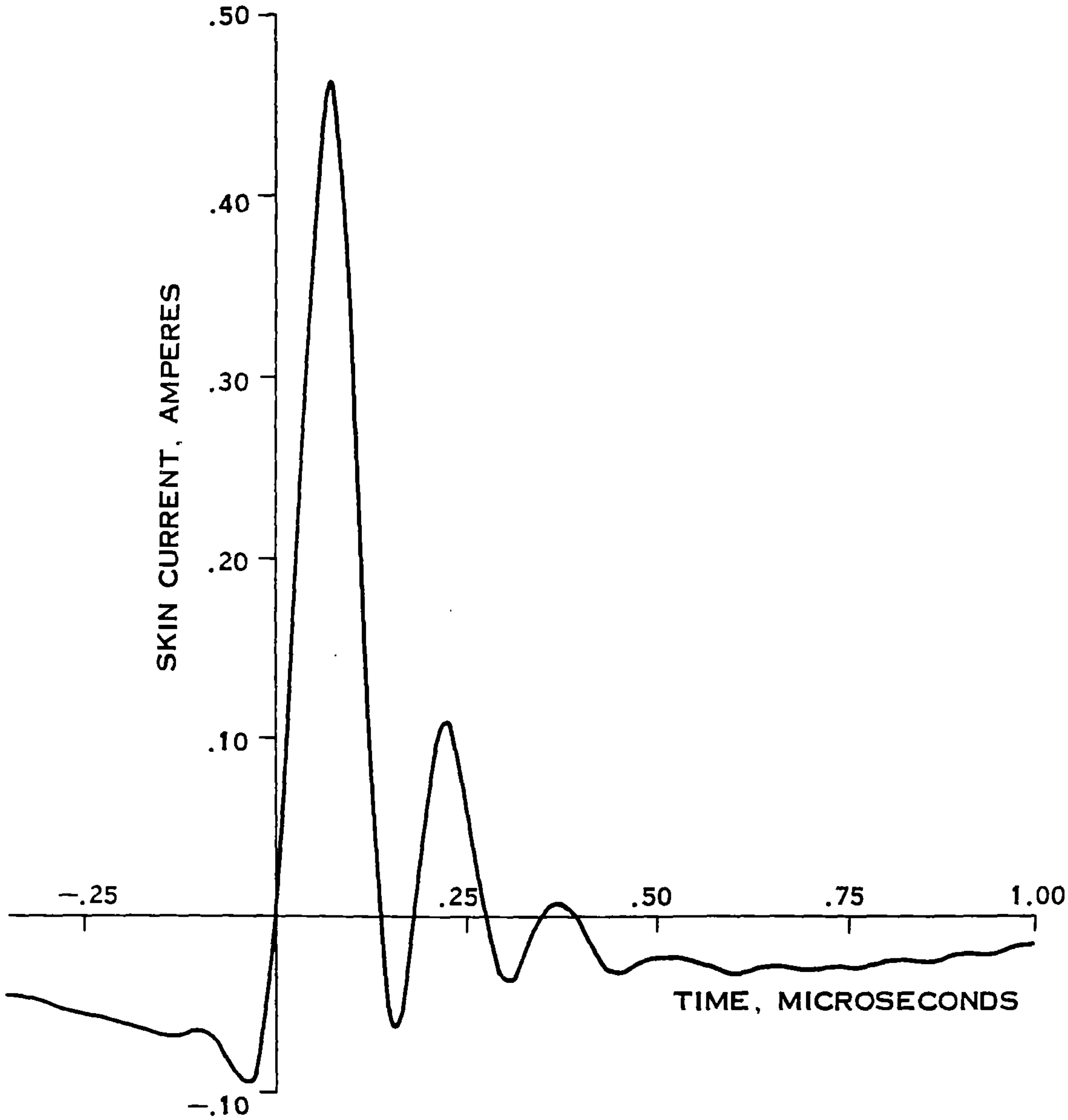


FIGURE 11

SKIN CURRENT VS TIME AT POINT ON EXHAUST PLUME.
UNIT STEP DRIVING FUNCTION.

$\sigma_o = .1$

$\alpha = .5393$

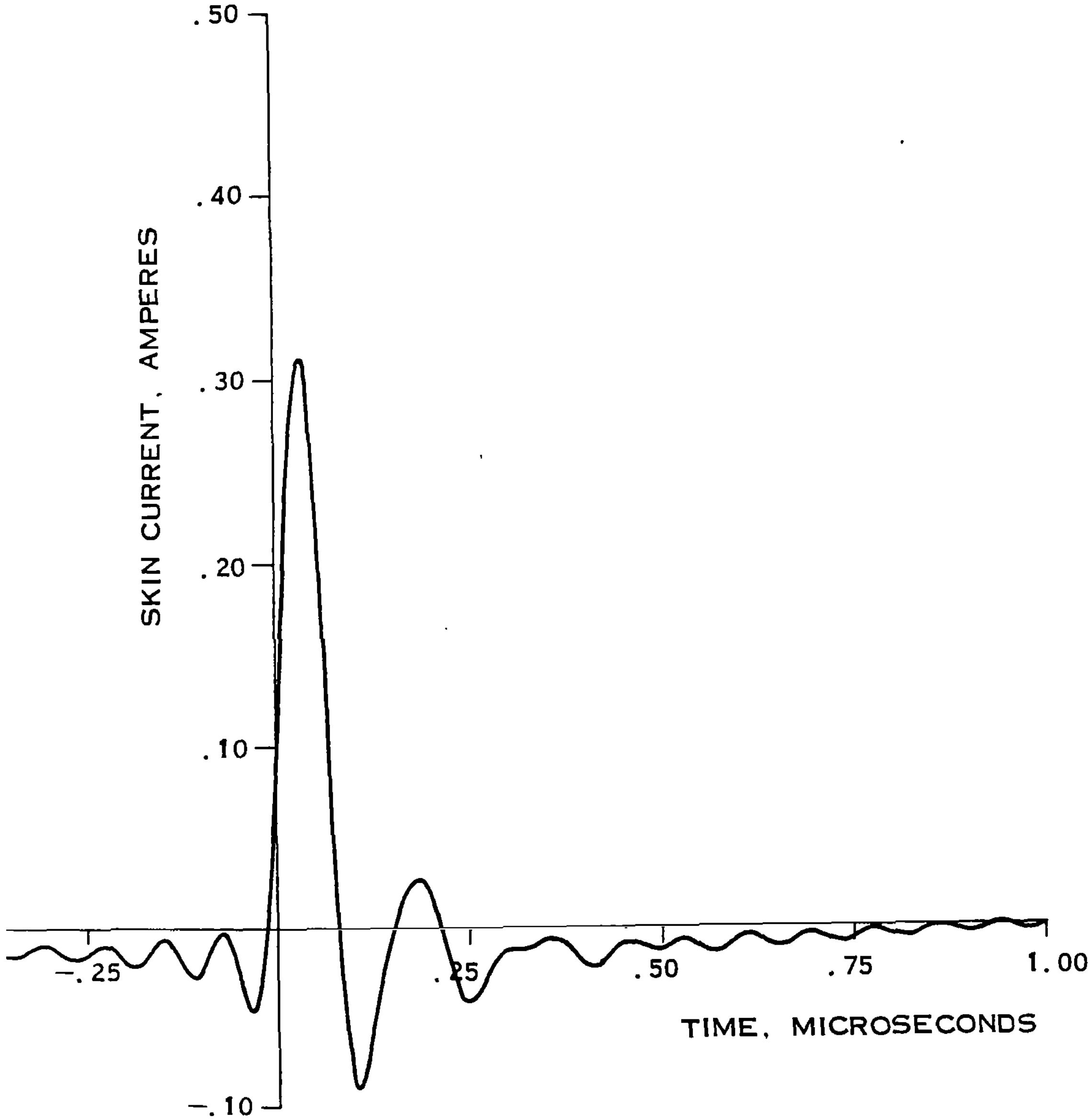


FIGURE 12

SKIN CURRENT VS TIME AT CENTER AT MISSILE
WITH EXHAUST PLUME, UNIT STEP DRIVING FUNCTION

$a = 0$

$\sigma_o = .1$

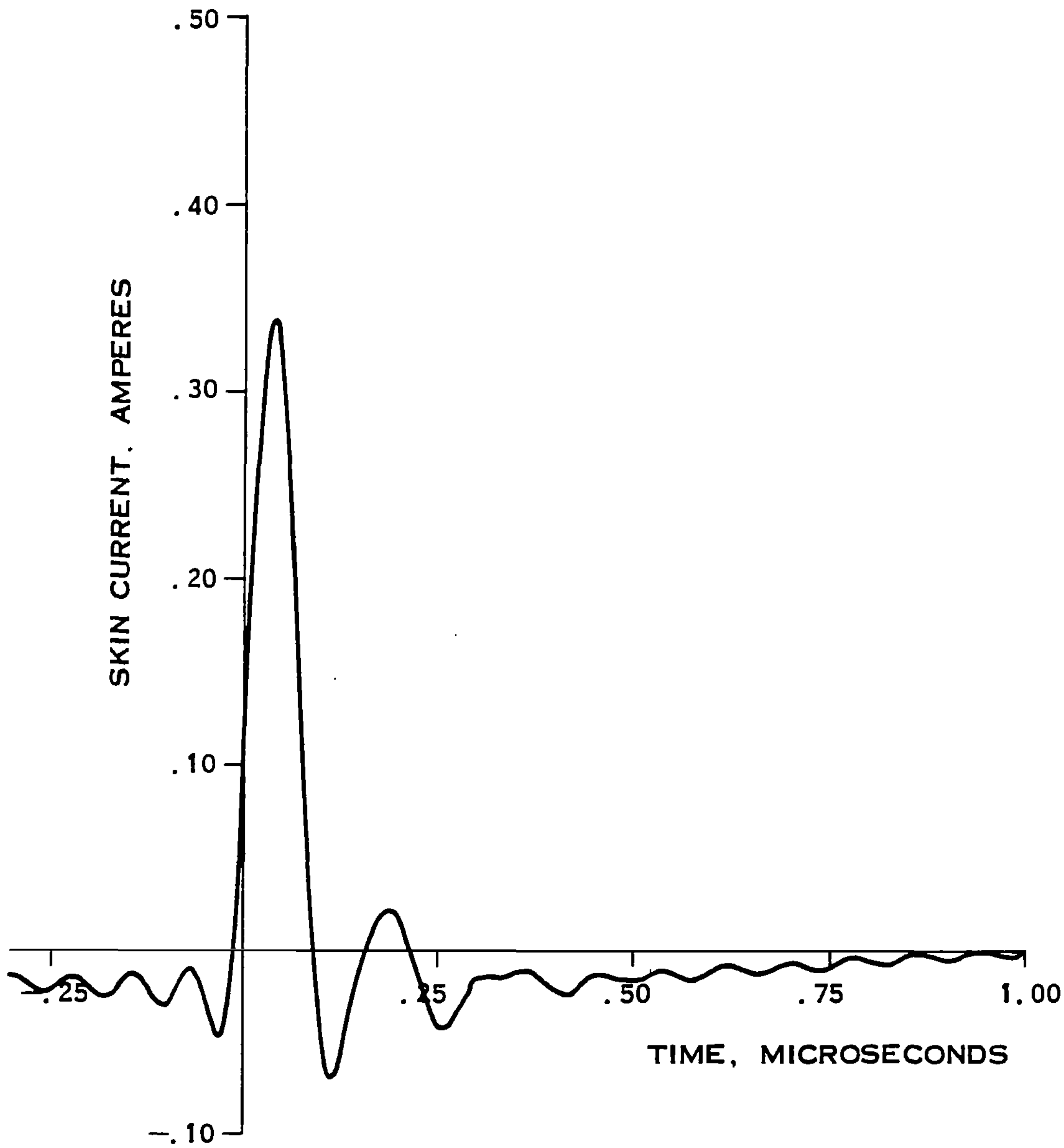


FIGURE 13

SKIN CURRENT VS TIME AT POINT 1/4 MISSILE LENGTH FROM NOZZLE. UNIT STEP DRIVING FUNCTION

$\alpha = 0$

$\sigma_e = .1$

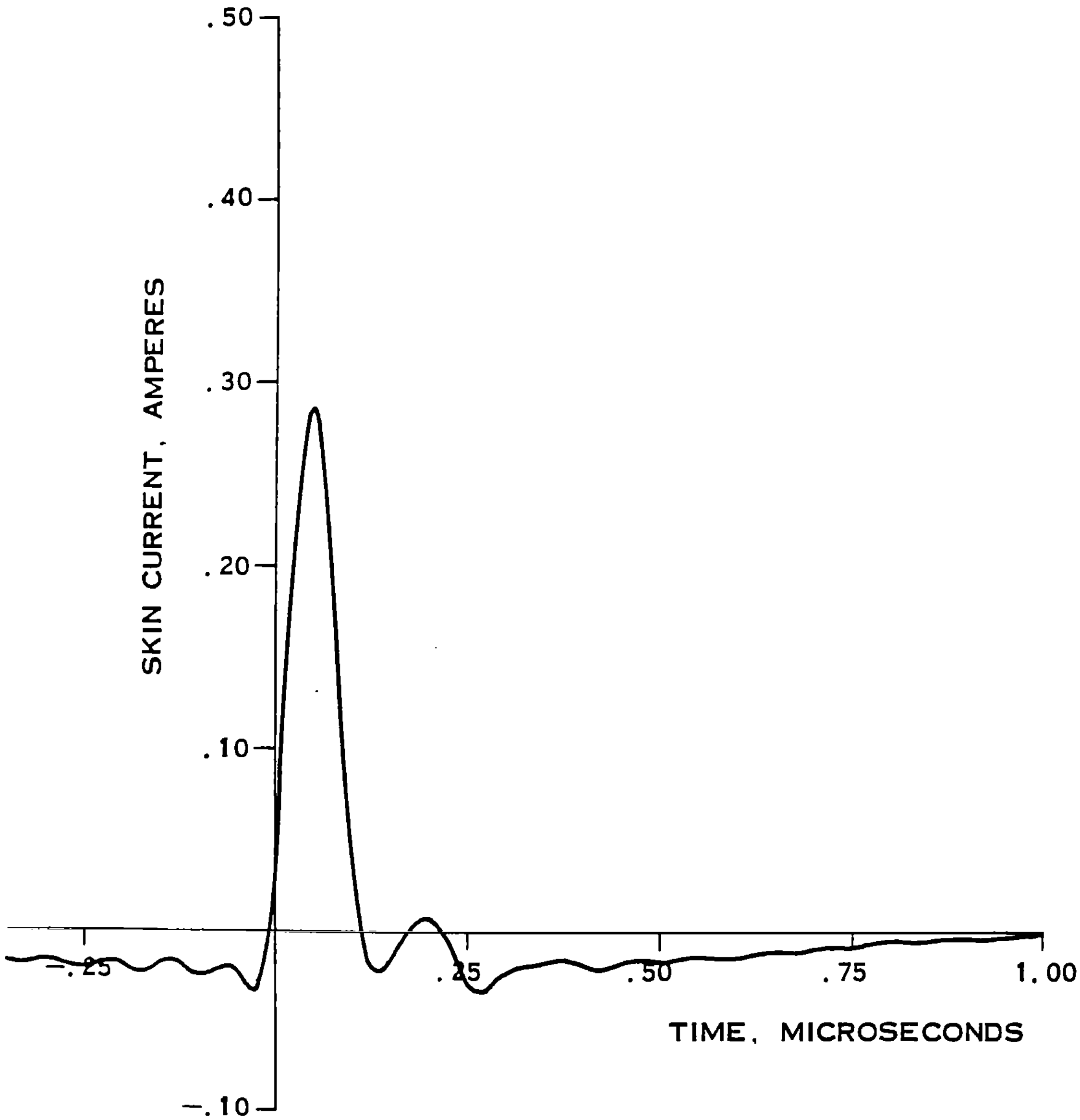


FIGURE 14

SKIN CURRENT VS TIME AT NOZZLE OF MISSILE WITH EXHAUST PLUME. UNIT STEP DRIVING FUNCTION

$\alpha = 0$

$\sigma_0 = .1$

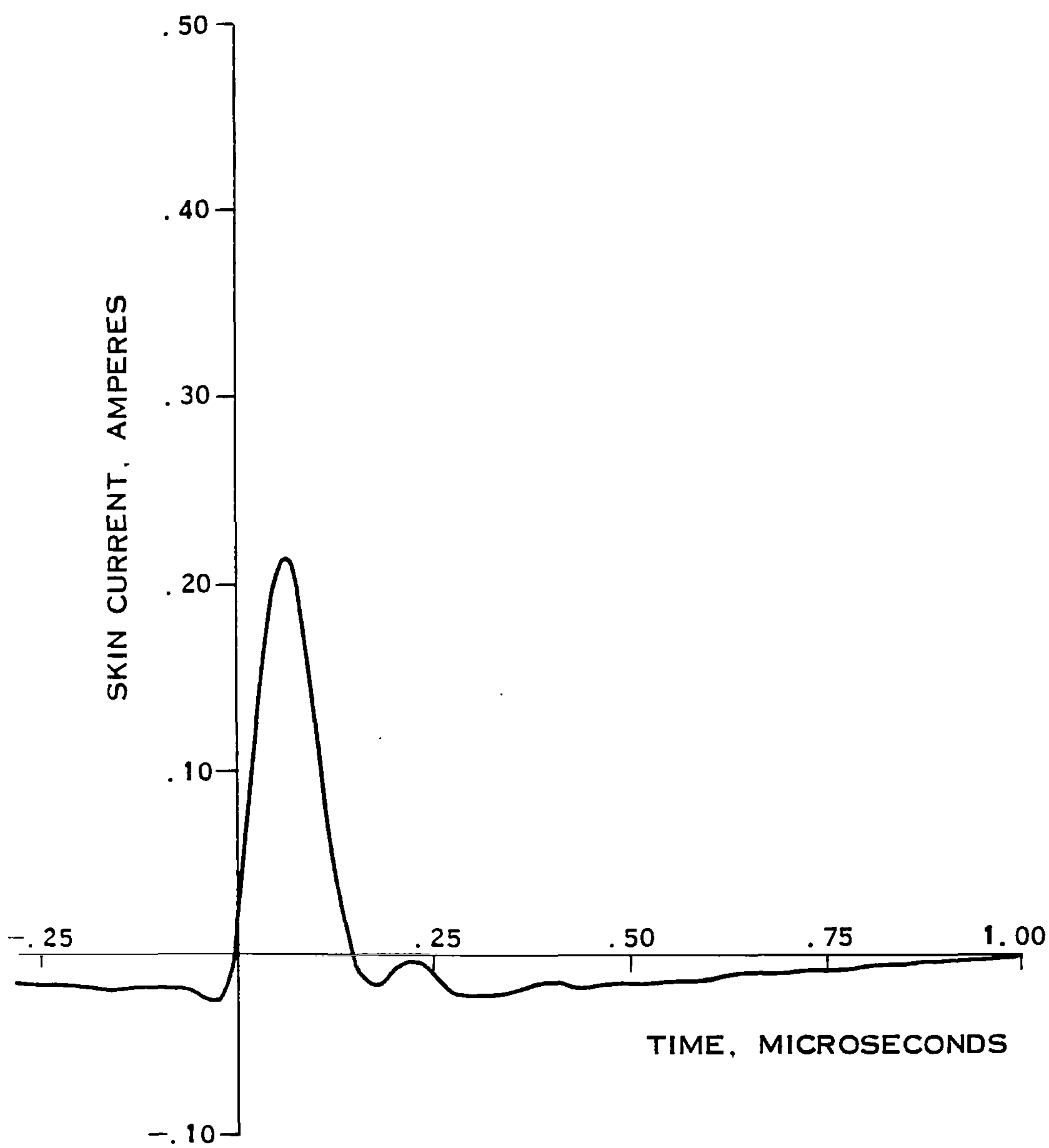


FIGURE 15

SKIN CURRENT VS TIME AT POINT ON EXHAUST PLUME
UNIT STEP DRIVING FUNCTION

$\alpha = 0$

$\sigma_o = .1$

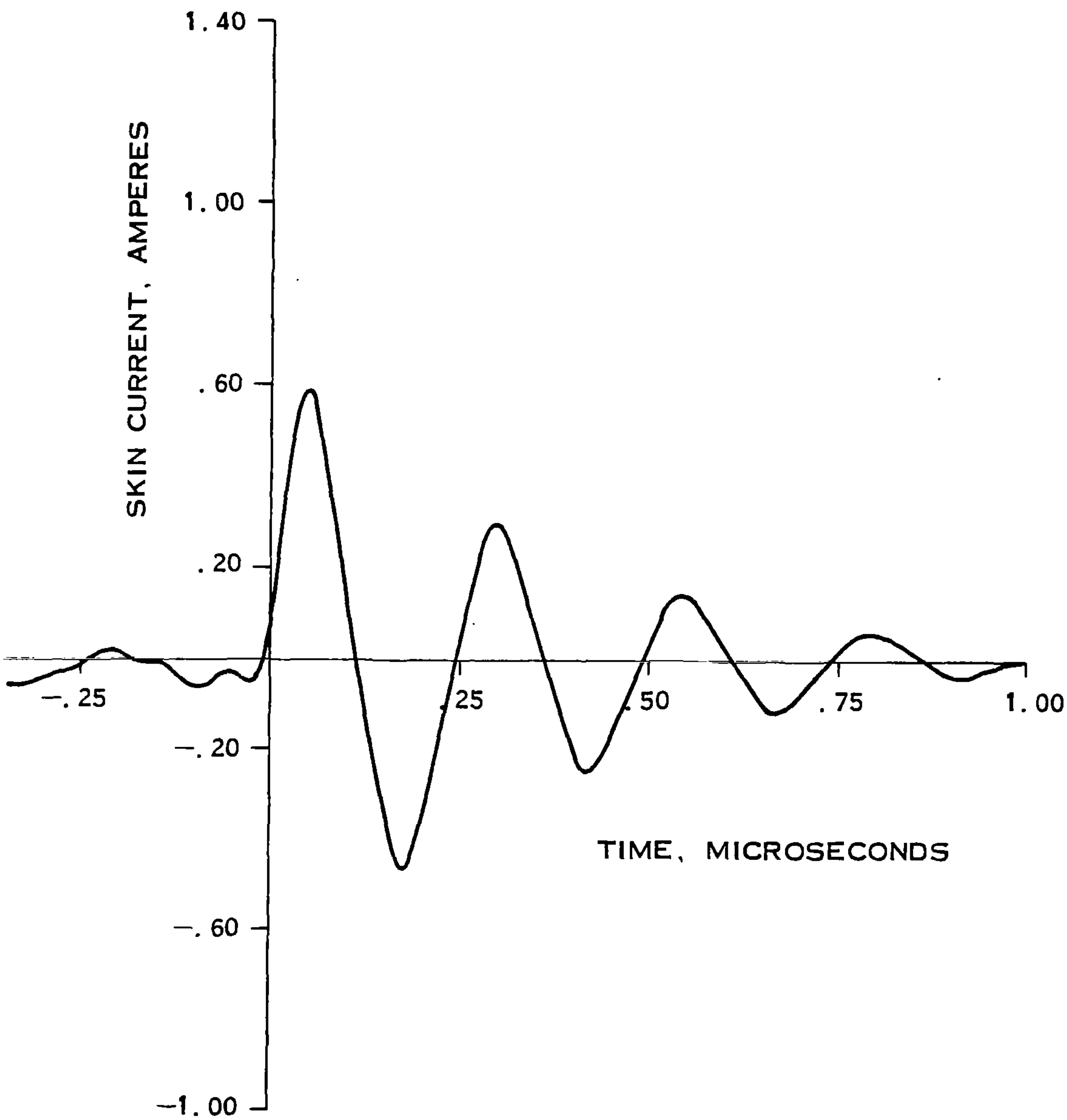


FIGURE 16

SKIN CURRENT VS TIME AT CENTER OF BASE LOADED MONOPOLE

UNIT STEP DRIVING FUNCTION

HB = HP = 17.08

OM = 10

4. DISCUSSION

The data presented above are results for a specific plume and missile model and a specific driving function. Since the transient response of an antenna is dependent upon the driving function, these results should not be considered to be generally applicable. With this disclaimer in mind, the following conclusions are offered:

1. The principle effect of a weakly conducting ($\sigma \lesssim .1$) plume is to increase the current between the nozzle and the missile center. The amplitude of the maximum skin current and the resonant frequency are not changed greatly.

2. The technique of simulating the plume with the image in a conducting ground plane may lead to an overtest in the peak skin current at any point on the missile. However, the resonant frequency will be different from the case of a missile with an imperfectly conducting plume. Since a theoretical transfer function from missile skin current to cable and wire currents is not within the state of the art, the value of such a simulation is questionable.

Since the merit in grounding the missile to the ground plane is open to question, it is reasonable to ask whether some other experiment in a simulation facility makes sense. The most logical experiment would consist of connecting the missile to the ground plane through some impedance Z. Quite possibly with a judicious selection of Z, the skin current of the missile-plume combination could be adequately simulated. The addition of a lumped load Z between the missile and the ground plane can be accounted for in the present computer code.

In the loaded dipole model of the missile-plume combination, it is assumed that the missile and plume diameters are identical. In reality, particularly at high altitudes, the plume can expand significantly. Since the DC resistance of the plume is inversely proportional to the square of the radius, the expansion of the plume may reduce the loading on the dipole. Even if the plume conductivity were quite small, the plume could then have a significant influence on the missile skin current. The effects of the expanding exhaust plume are currently being investigated.

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1. R. W. P. King and T. T. Wu, "The Cylindrical Antenna with Arbitrary Driving Point (U)", Cruft Laboratory Report No. 455, Oct. 1964. UNCLASSIFIED.

2. C. D. Taylor, "Electromagnetic Scattering by Thin Inhomogeneous Circular Cylinders (U)", Radio Science Vol. 2 (New Series) No. 7, page 729-738, July 1967. UNCLASSIFIED.