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THE RESPONSE OF A TERMINATED TWO-WIRE TRANSMISSION LINE EXCITED BY A NONUNIFORM ELECTROMAGNETIC FIELD

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The Response of a Terminated Two-Wire Transmission Line Excited by a Nonuniform Electromagnetic Field

The present communication is a sequel to an earlier investigation relating to the response of transmission lines excited by the nonuniform resultant electric field in proximity to a cylindrical scatterer of finite length.[1] The procedure followed in the paper was, first, to find the resultant electric field ' the vicinity of the cylinder; second, to .place the differential electric field acting along the line with equivalent point generators; and third, to integrate the generator contributions along the length of the line to determine the terminating impedance load currents. In the approach to the problem used in this communication, the transmission-line differential equations, including appropriate source terms, are derived and these equations (both of which are nonhomogeneous) are solved subject to the boundary conditions. Evidently, the two formulations must be equivalent.

Consider a two-wire transmission line, illustrated in Fig. 1, lying in the xz plane terminated in impedances Z_0 and Z_s at $z=0$ and $z=s$, respectively. It is assumed that the source of line excitation is a nonuniform electromagnetic field. Without loss of generality the B field is taken in the y direction and the E field in the xz plane. The time dependence assumed (but suppressed) in the following development is $\exp(j\omega t)$. The notation employed in this communication is that commonly appearing in the literature.

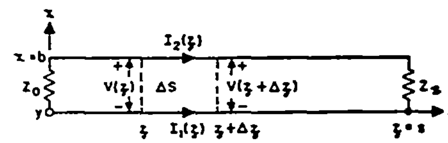

Fig. 1. Terminated two-wire transmission line excited by an arbitrary electromagnetic field.

In order to obtain expressions for the line voltage and current it is convenient to integrate curl $E=-j\omega B$ by using Stokes' theorem to yield

$$\oint E \cdot dl = -j\omega \int_{\Delta S} B \cdot \hat{n} dS. \qquad (1)$$

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Consider the surface ΔS to lie in the xz plane bounded by $x=0$, b and $z=z$, $z+\Delta z$, as shown in Fig. 1. If n is considered to be parallel to the y-axis then (1) yields

$$\int_0^b [E_x(x, z+\Delta z) - E_x(x, z)]dx - \int_z^{z+\Delta z} [E_z(b, z) - E_z(0, z)]dz = -j\omega \int_z^{z+\Delta z} \int_0^b B_y(x, z)dxdz. \quad (2)$$

The foregoing field components represent the total electromagnetic field about the transmission line.

$$-\frac{\partial V(z)}{\partial z} = \lim_{\Delta z \to 0} \frac{1}{\Delta z} \int_0^b \cdot [E_x(x, z+\Delta z) - E_x(x, z)]dx. \quad (3)$$

The total electric field components in the z direction on the conductors are

$$\left.\begin{aligned} E_z(b, z) &= Z_2^i I_2(z) \\ E_z(0, z) &= Z_1^i I_1(z) \end{aligned}\right\} \quad (4)$$

where Z_2^i and Z_1^i are the internal impedance per unit length of the upper and lower wires, respectively, and $I_2(z)$ and $I_1(z)$ are the axial currents in the wires. By using (4) it is easily exhibited that

$$\lim_{\Delta z \to 0} \frac{1}{\Delta z} \int_z^{z+\Delta z} [E_z(b, z) - E_z(0, z)]dz = Z_1^i [I_2(z) - I_1(z)] \quad (5)$$

when it is assumed that the upper and lower conductors are identical. In general, transmission-line and antenna currents are excited on the wires. The first are bidirectional; the latter are codirectional, respectively.

The general expressions for the line currents may be written

$$\left.\begin{aligned} I_2(z) &= I^d(z) + I(z) \\ I_1(z) &= I^d(z) - I(z) \end{aligned}\right\}. \quad (6)$$

Here $I^d(z)$ is the contribution to the current from the dipole mode (or the symmetric component of the current) and $I(z)$ is the contribution to the current from the transmission-line mode (or the antisymmetric component of the current). By using (6) in (5) it is readily shown that

$$Z^i I(z) = \lim_{\Delta z \to 0} \frac{1}{\Delta z} \int_z^{z+\Delta z} \cdot [E_z(b, z) - E_z(0, z)]dz \quad (7)$$

where $Z^i = 2Z_1^i$. Substituting (3) and (7) in (2) yields

$$\frac{\partial V(z)}{\partial z} + Z^i I(z) = j\omega \int_0^b B_y(x, z)dx. \quad (8)$$

Neglecting displacement current the Maxwell-Ampere law gives

$$\int_0^b B_y^s(x, z)dx \simeq -l^e I(z) \quad (9)$$

where $l^e = (\mu/\pi)\ln b/a$, $\mu = 4\pi \times 10^{-7}$ H/m, and a is the conductor radius. The superscript s denotes the scattered magnetic field component. The first transmission-line equation is obtained by substituting (9) into (8). It is

$$\frac{\partial V(z)}{\partial z} + ZI(z) = j\omega \int_0^b B_y^i(x, z)dx \quad (10)$$

where $Z = Z^i + j\omega l^e$ is the distributed series impedance of the transmission line. The superscript i used in (10) and in subsequent equations denotes the incident-field components.

To obtain the other transmission-line equation it is convenient to begin with the expression for the line voltage

$$V(z) = -\int_0^b E_x(x, z)dx. \quad (11)$$

Since

$$E_x(x, z) = \frac{j\omega}{k^2} \frac{\partial B_y}{\partial z},$$

$$V(z) = \frac{\sinh \gamma z}{\sinh \gamma s} V_s - V_0 \frac{\sinh \gamma(z-s)}{\sinh \gamma s} + \frac{Z^i}{Z_c \sinh \gamma s} \cdot \left[\sinh \gamma(z-s) \int_0^z K_v(u) \sinh \gamma u du + \sinh \gamma z \int_z^s K_v(u) \sinh \gamma(u-s) du\right] \quad (16a)$$

$$I(z) = I_s \frac{\sinh \gamma z}{\sinh \gamma s} - I_0 \frac{\sinh \gamma(z-s)}{\sinh \gamma s} - \frac{1}{Z_c \sinh \gamma s} \cdot \left[\sinh \gamma(z-s) \int_0^z K(u) \sinh \gamma u du + \sinh \gamma z \int_z^s K(u) \sinh \gamma(u-s) du\right], \quad (16b)$$

where

$$k = \omega \sqrt{\mu\left(\epsilon - j\frac{\sigma}{\omega}\right)}$$

is the propagation constant of the medium about the transmission line,

$$V(z) = -j\frac{\omega}{k^2} \frac{\partial}{\partial z} \int_0^b B_y(x, z)dx. \quad (12)$$

$$\frac{\partial^2}{\partial z^2} V(z) - YZV(z) = Z^i Y \int_0^b E_z^i(x, z)dx \quad (14a)$$

$$\frac{\partial^2}{\partial z^2} I(z) - YZI(z) = -Y[E_z^i(b, z) - E_z^i(0, z)]. \quad (14b)$$

The terminal conditions are

$$\left.\begin{aligned} V(0) &= -I(0)Z_0 \\ V(s) &= I(s)Z_s \end{aligned}\right\}. \quad (15)$$

The solutions of (14a) and (14b) may be obtained by using conventional techniques. These are

where

$$K_v(z) = \int_0^b E_z^i(x, z)dx$$

$$K(z) = [E_z^i(b, z) - E_z^i(0, z)],$$

with $\gamma^2 = YZ$ and $Z_c = \sqrt{Z/Y}$ as the characteristic impedance of the transmission li[ne]. The terminal impedance currents I_0 and I_s may be obtained from (10), (13), and (15). They are

$$I_0 = -\frac{Z_s}{D}\int_0^s K(z) \sinh \gamma(z-s)dz - \frac{Z_c}{D\gamma}\int_0^s \frac{d}{dz} K(z) \sinh \gamma(z-s)dz + \frac{Z_c}{D\gamma} K(0) \sinh \gamma s - \frac{Z_c}{D}\int_0^b E_x^i(x, s)\, dx + \frac{Z_c \cosh \gamma s + Z_s \sinh \gamma s}{D} \int_0^b E_x^i(x, 0)dx \quad (17)$$

$$I_s = \frac{Z_0 \cosh \gamma s + Z_c \sinh \gamma s}{D} \int_0^s K(z) \sinh \gamma(z-s)dz - \frac{(Z_c \cosh \gamma s + Z_0 \sinh \gamma s)}{D\gamma} \int_0^s \frac{d}{dz} K(z) \sinh \gamma(z-s)dz + \frac{K(0)}{D\gamma}(Z_c \cosh \gamma s + Z_0 \sinh \gamma s) \sinh \gamma s + \frac{Z_c}{D}\int_0^b E_x^i(x, 0)dx - \frac{(Z_c \cosh \gamma s + Z_0 \sinh \gamma s)}{D} \int_0^b E_x^i(x, s)dx, \quad (18)$$

The substitution of (9) into (12) leads to the second transmission-line equation. It is

$$\frac{\partial I(z)}{\partial z} + YV(z) = -Y\int_0^b E_z^i(x, z)dx \quad (13)$$

where $Y = j(k^2/\omega l^e)$ is the distributed shunt admittance of the line conductors.

The coupled first-order equations for the current and voltage may be combined to obtain the following uncoupled second-order differential equations

where

$$D = (Z_c Z_0 + Z_s Z_c) \cosh \gamma s + (Z_c^2 + Z_s Z_0) \sinh \gamma s.$$

Note that the currents in the loads are pr[o]duced by only the E_z^i field along the line a[nd] by only E_x^i at the terminations. If the contributions from E_x^i at the terminations are neglected and the $K(z)$ appropriate to a cylindrical scatterer of finite length is employed, the currents in the load impedances are found to be identical to those obtained by Harrison[1] using a different approach in which the effect of the electric field transverse to the lines was omitted. In general, this is not negligible and the previously obtained currents are not the total currents.

[1] C. W. Harrison, Jr., "Response of transmission lines excited by the nonuniform resultant field in proximity to a cylindrical scatterer of finite length," Sandia Corporation, Sandia Base, Albuquerque, N. Mex., Reprint SCR-65-978, August 1965.

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