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Analysis of the Shielding Characteristics of Saturable Ferromagnetic Cable Shields

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Abstract—A numerical technique is developed that may be used to analyze the shielding characteristics of a cable with a saturable ferromagnetic outer sheath in an intense low-frequency environment. The solution relates the center conductor current to the total current exciting a coaxial cable with a ferromagnetic sheath. Several examples are given to illustrate how saturation effects the shielding properties of a ferromagnetic sheath.

Introduction

THE USE of tubular shields encasing cables has long been an accepted means of extending the Faraday shielding concept to physically distributed electronic systems [1], [2]. Recently, in an effort to obtain better shielding at lower frequencies without greatly increasing the shield weight or decreasing its flexibility, some cable shields have been fabricated from ferromagnetic materials. At low levels of excitation, the predicted high level of attenuation is attained. However, when the level of excitation is increased, the saturation of the ferromagnetic material reduces the effectiveness of the shield.

In this study, a numerical solution was evolved for assessing the shielding provided by a tubular shield of a saturable ferromagnetic material to intense levels of external interference. The technique used to solve the appropriate nonlinear equations is similar to that used in a previous paper dealing with the transmission of pulses through an infinite sheet of saturable ferromagnetic material [3]. The application of this technique to the cable shielding problem is new, and the theory developed here is useful in analyzing the effectiveness of a ferromagnetic cable shield.

Formulation

The theoretical model considered is a thin coaxial cable supported between two perfectly conducting infinite plates (Fig. 1). The center conductor of the cable is also assumed to be perfectly conducting. The outer sheath of the cable is made of a saturable ferromagnetic material in which the directions of the b and h vectors coincide while the amplitudes are related by a single-valued function (Fig. 2). The material exhibits saturation, but hysteresis has been neglected.

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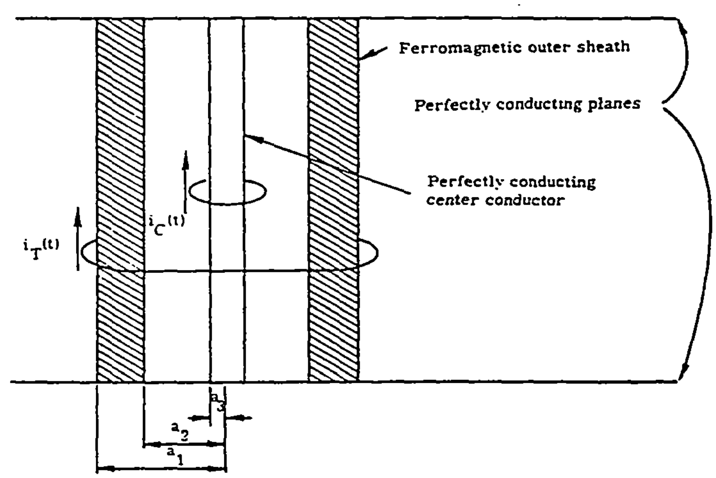

Fig. 1. Model coaxial cable.

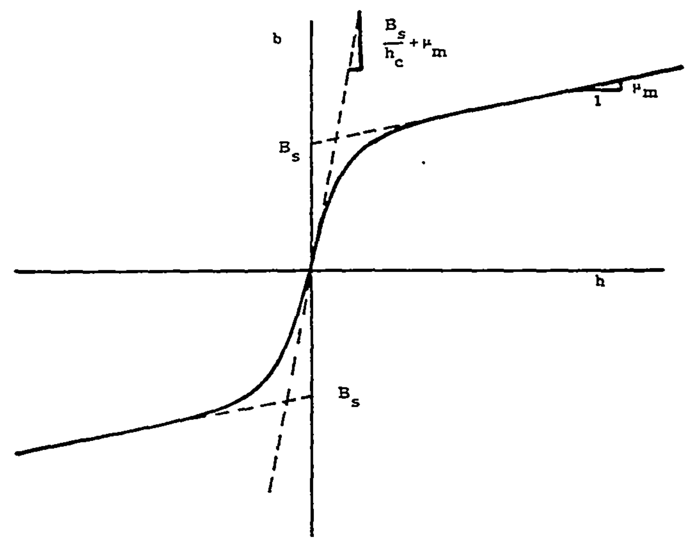

Fig. 2. Magnetic characteristics of ferromagnetic material.

The interfering source is impressed between the two plates, giving rise to a total cable current $i_T(t)$ uniform along the cable. For simplicity, it is assumed that the driving current is a causal function, i.e., $i_T(t) = 0$, if $t \leq 0$.

The problem is to find the relation between the center conductor current $i_C(t)$ and the impressed current $i_T(t)$. The center conductor of a coaxial line short circuited at each end is not equivalent to a multiconductor cable with many separate load impedances; however, this simplified model is adequate for the consideration of the effects of material saturation on the shielding effectiveness of the ferromagnetic sheath.

In both the outer sheath and the separating dielectric (Fig. 1), the field components must satisfy Maxwell's equations,

$$\nabla \times e = -\frac{\partial b}{\partial t} \quad (1)$$

and

$$\nabla \times h = \sigma e + \varepsilon_0 \frac{\partial e}{\partial t}. \quad (2)$$

For the physical configuration considered, the total cable current is uniform along the length of the cable; therefore, the only nonzero components of the fields are e_z, h_ϕ, and b_ϕ (in cylindrical coordinates). The curl equations reduce to

$$\frac{\partial e_z}{\partial r} = \frac{\partial b_\phi}{\partial t} \quad (3)$$

and

$$\frac{1}{r}\left[\frac{\partial(rh_\phi)}{\partial r}\right] = \sigma e_z + \varepsilon_0 \frac{\partial e_z}{\partial t}. \quad (4)$$

In the separating dielectric, the conductivity σ is zero; for a thin cable, accurate approximations of the fields in this region are

$$h_\phi = \frac{i_C(t)}{2\pi r} \quad (5)$$

$$e_z = \frac{\mu_0}{2\pi}\frac{\partial i_C(t)}{\partial t} \ln \frac{r}{a_3} \quad (6)$$

for $a_2 \geq r \geq a_3$. Here μ_0 is the permeability of free space, $\mu_0 = 4\pi \times 10^{-7}$ H/m.

In the ferromagnetic sheath, if the displacement current is neglected, (3) and (4) may be combined into one second-order equation,

$$\frac{\partial^2 h_\phi}{\partial r^2} + \frac{1}{r}\frac{\partial h_\phi}{\partial r} - \frac{h_\phi}{r^2} = \sigma\mu(h_\phi)\frac{\partial h_\phi}{\partial t}. \quad (7)$$

Here

$$\mu(h_\phi) \equiv \frac{\partial b_\phi}{\partial h_\phi}.$$

The boundary conditions that e_z and h_ϕ must be continuous at the inner surface of the sheath yield, from (4), (5), and (6), the single mixed boundary condition

$$\left.\frac{h_\phi}{a_2} + \frac{\partial h_\phi}{\partial r} = \sigma\mu_0 a_2 \frac{\partial h_\phi}{\partial t} \ln \frac{a_2}{a_3}\right|_{r=a_2}. \quad (8)$$

The total cable current is the driving function and is related to the magnetic field at $r = a_1$ by

$$h_\phi|_{r=a_1} = \frac{i_T(t)}{2\pi a_1}. \quad (9)$$

Equations (7), (8), and (9) complete the formulation, and the problem reduces to finding an appropriate solution to the nonlinear diffusion equation (7) subject to the boundary conditions (8) and (9).

Numerical Solution

To effect a numerical solution of (7) with boundary conditions (8) and (9), a rectangular mesh of points is introduced into the r–t plane:

$$r_j = a_2 + (j-1)\Delta r, \quad j = 1, \cdots, J \quad (10)$$

$$t_k = (k-1)\Delta t, \quad k = 1, \cdots. \quad (11)$$

Here J is the number of nodes introduced between $r = a_1$ and $r = a_2$; $\Delta r = (a_1 - a_2)/(J-1)$; and Δt is the selected time increment. The derivatives in (7) and (8) are replaced by the difference approximations

$$\frac{h_{j+1}^{k+1} - 2h_j^{k+1} + h_{j-1}^{k+1}}{\Delta r^2} + \frac{1}{r_j}\frac{h_{j+1}^{k+1} - h_{j-1}^{k+1}}{2\Delta r} - \frac{h_j^{k+1}}{r_j^2} = \sigma\mu(h_j^{k+1/2})\frac{h_j^{k+1} - h_j^k}{\Delta t} \quad (12)$$

where $j = 2, 3, \cdots, (J-1)$ and

$$\frac{h_1^{k+1}}{a_2} - \frac{3h_1^{k+1} - 4h_2^{k+1} + h_3^{k+1}}{2\Delta r} = \sigma\mu_0 a_2 \ln\left(\frac{a_2}{a_3}\right)\frac{h_1^{k+1} - h_1^k}{\Delta t}. \quad (13)$$

From (9) the final equation is obtained,

$$h_J^{k+1} = \frac{1}{2\pi a_1} i_T(t_{k+1}). \quad (14)$$

Here $h_j^k \equiv h(t_k, r_j)$. An implicit differencing scheme (backward time differencing) was employed because the difference equations so obtained are stable for all values of Δt and Δr [4]. The value of μ used in the calculation was the value obtained at

$$h_j^{k+1/2} \equiv (h_j^k + h_j^{k+1})/2.$$

Equations (12), (13), and (14) constitute a system of J nonlinear equations in the unknowns h_j^{k+1}, $j = 1, 2, \cdots, J$. Since $i_T(t)$ is assumed to be causal, the magnetic field is initially zero throughout the material $h_j^1 = 0$. The solution is effected by proceeding sequentially through the t_k, solving the system of J nonlinear equations for h_j^{k+1} from the previously determined values of h_j^k.

When the equations are written in matrix form, every row of the coefficient matrix has only three elements, centered on a dominant diagonal term. The Gauss–Seidel iteration procedure has been found to be an effective technique of solving the system of equations since the iteration needed to account for the nonlinear permeability may be incorporated into the normal iteration of the solution.

Sample Calculation

To illustrate how saturation affects the shielding action of a ferromagnetic cable shield, the transient response of a thin annealed-steel sheath was considered.

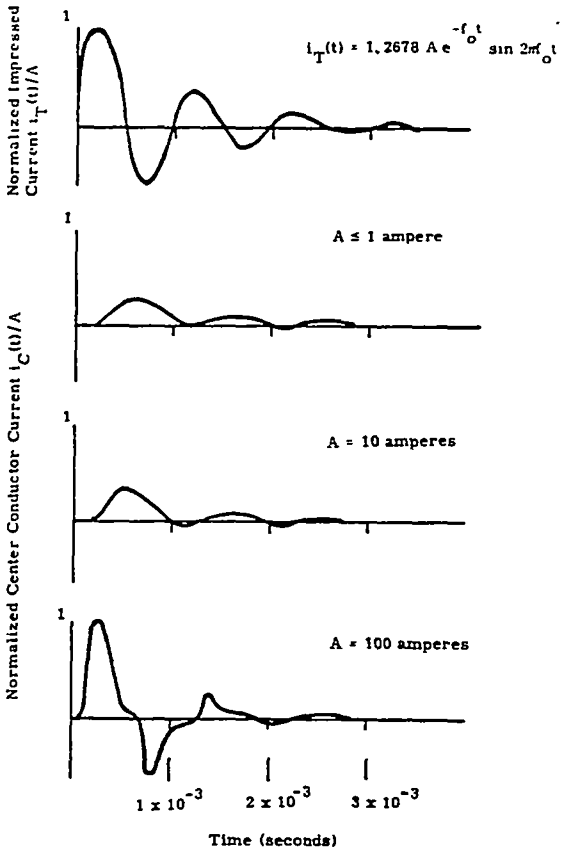

Fig. 3. Normalized center conductor current for a damped sine wave impressed current $f_0 = 1$ kHz.

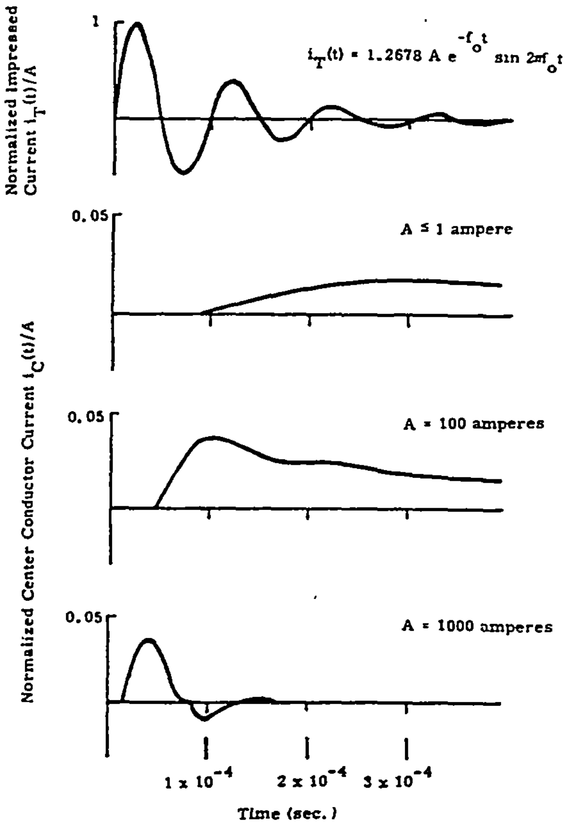

Fig. 4. Normalized center conductor current for a damped sine wave impressed current $f_0 = 10$ kHz.

TABLE I

PARAMETRIC VALUES USED IN NUMERICAL CALCULATIONS

$\mu_m = 1.67 \times 10^{-4}$ H/m
$B_S = 1.53$ Wb/m² (15.3 kG)
$h_c = 120$ A/m (1.51 O)
$\sigma = 10^7$ mho/m
$a_1 = 0.25$ inch (6.35×10^{-3} meters)
$a_2 = 0.245$ inch (6.223×10^{-3} meters)
$a_3 = 0.1064$ inch (2.7045×10^{-3} meters)
$J = 21$
$\Delta t = \dfrac{1}{800 f_0}$ second

The driving current was taken to be a damped sinusoid; sources generating this type of waveshape are often used in experiments requiring intense electrical transients. Results computed for damped sinusoids of two different predominate frequencies are given: $f_0 = 1$ kHz and $f_0 = 10$ kHz (Figs. 3 and 4).

The magnetic parameters in Table I were chosen to approximate the magnetization curve for annealed steel given by Stratton [5].

For each value of f_0, the lowest level of impressed current considered was 1-ampere peak. At this level of excitation, the material response is essentially linear: the center conductor current $i_C(t)$ normalized by the amplitude of the excitation A is identical for all similar inputs of lower amplitude. The results given agree with the results of a linear-medium study, using Fourier transform techniques and a Bessel function expansion of the fields in the various regions, following Harrison [6].

The amplitude A was increased until the material saturated throughout the sheath,

$$(h_j^k \geq 2h_c, \quad j = 2,3,\cdots,(J-1))$$

during some portion of the pulse. During this time interval, the behavior of the material is again almost linear and the results given may be applied to higher level impressed currents. The primary effect of increasing the amplitude is to widen the time interval during which the sheath is totally saturated.

DISCUSSION OF RESULTS

Although no figure is included, the case $f_0 = 100$ Hz was also considered. The frequency content of that transient is so low that the sheath offers almost no attenuation, whether the material is saturated or not; all the impressed current flows on the inside conductor.

For $f_0 = 1$ kHz, the ferromagnetic sheath does offer some attenuation at low levels (Fig. 3). As the level of excitation is increased, the effectiveness of the shield decreases. For $A = 100$ amperes the material is saturated during a portion of each of several half-cycles. During the interval that the total shield is saturated, the sheath behaves as a linear medium with $\mu = \mu_m$. For $f_0 = 1$ kHz the saturated shield offers no significant attenuation.

For $f_0 = 10$ kHz, the ferromagnetic sheath provides increased attenuation, at low levels (Fig. 4). In this case the impressed current wave has significant high-frequency content; consequently, considerable smoothing of the center conductor current is obtained.

When the level of the excitation is increased until the material saturates all the way through, the center conductor current again resembles the input pulse.

It should be noted that the level required to saturate the sheath in the second case is ten times that required in the first case; the proportionality of the saturating current to frequency is an important consideration in the design of ferromagnetic shields.

Conclusion

The effect of saturation on the shielding characteristics of a ferromagnetic cable shield can be assessed using the numerical solution presented.

When the sheath is not saturated all the way through, Norton's equivalence theorem of circuit theory can be used to obtain the voltage across a resistor terminating one end of the coaxial line if needed.

The numerical examples given illustrate the behavior to be expected. As the level of the impressed current is increased, the shielding provided decreases gradually until the level is high enough that the sheath saturates all the way through during some portion of the impressed current wave. During this time interval, the response is again linear, the permeability of the material being the limiting value $\mu = \mu_m$ (Fig. 2).

It should be mentioned that the saturation flux density of shielding grade ferromagnetic materials is lower than the value of saturation flux density used in the examples; therefore, the nonlinear effects illustrated can be expected to occur at proportionally lower levels.

This model is primarily intended for use in studying the saturation effects associated with low-frequency impressed currents. The application of this model to high-frequency phenomena could result in erroneous conclusions. One reason is that hysteresis and the time delays associated with the flux switching of individual domains has been neglected. A second reason is that computational difficulties arise if the shielding effectiveness is very high. From the author's experience with a CDC-3600 computer (~12 significant figures) shielding attenuation exceeding 140 dB ($Ic(t)/A < 10^{-7}$) could not be evaluated using single-precision arithmetic.

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