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EXCITATION OF A COAXIAL LINE BY THE PROPAGATION OF AN ELECTROMAGNETIC FIELD THROUGH A TRANSVERSE SLOT IN THE SHEATH

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ABSTRACT

A rocket with removed access plate is simulated by a section of coaxial transmission line with a transverse elliptical slot cut in its sheath. The internal circuit consists of two arbitrary impedances in series with the inner conductor at its ends. The object is to find the currents in these impedances when the cylinder is illuminated from the outside by an electromagnetic field that enters the aperture and excites the internal circuit.

The problem is solved by application of the reciprocal theorem. The current in a dipole antenna is determined when this is in the far field maintained by the slotted coaxial line when driven by a generator in series with one of the load impedances. The field in the aperture is replaced by equivalent electric and magnetic dipoles. The reciprocal theorem gives the current in the load impedance when the distant dipole is driven. A numerical example is given.

INTRODUCTION

In Fig. 1 is shown a simplified rocket in the form of an aluminum tube with closed ends, radius b and negligible wall thickness which extends from $z = 0$ to $z = s$ along the axis of the cylindrical coordinate system (ρ,ϕ,z). The open access door is simulated by a transverse elliptical slot with minor axis which extends from $z = \ell - W/2$ to $z = \ell + W/2$ in the axial direction and major axis from $\phi = \pi - \Psi/2$ to $\phi = \pi + \Psi/2$ laterally. The center of the slot is at $\rho = b$, $\phi = \pi$, and $z = \ell$. The internal circuit consists simply of a copper coaxial conductor terminated at $z = 0$ in the impedance Z_0, at $z = s$ in the impedance Z_s. The tube is illuminated from the outside by the far field (incident plane wave) of a distant antenna, which is conveniently taken to be a dipole at a distance r from the point on the axis opposite the center of the slot. The origin of the polar coordinates (r,Θ,Φ) is at the center of the aperture. The problem is to determine the currents $I_z(0)$ and $I_z(s)$ in the impedances Z_0 and Z_s.

The procedure to be followed is to remove the generator from the center of the distant dipole and connect it in series with the impedance Z_0 (or Z_s) and then to calculate the current I_d at the center of the dipole. According to the reciprocal theorem, $I_z(0) = I_d$ (or $I_z(s) = I_d$).

In order to determine I_d it is necessary to obtain the electromagnetic field maintained at the dipole by the driven, slotted, coaxial cylinder. This involves determination of the current in the coaxial line and the field at the center of the slot when metallically closed as intermediate steps.

THE DISTRIBUTION OF CURRENT ALONG THE CENTER CONDUCTOR OF THE COAXIAL SECTION OF TRANSMISSION LINE

When the coaxial line is terminated at $z = s$ in the impedance Z_s and driven by the emf V_0^e in series with the impedance Z_0 at $z = 0$ (as shown in Fig. 1), the current in the center conductor is

$$I_z(z) = V_0^e \, F(w)/D \quad ; \quad V(z)/Z_c = V_0^e \, G(w)/D \tag{1a}$$

with

$$F(w) = Z_c \cosh \gamma w + Z_s \sinh \gamma w \tag{1b}$$

$$G(w) = Z_c \sinh \gamma w + Z_s \cosh \gamma w \tag{1c}$$

$$D = Z_c(Z_0 + Z_s) \cosh \gamma s + (Z_c^2 + Z_0 Z_s) \sinh \gamma s \tag{1d}$$

In (1a-d), $w = s - z$ is the distance from the load Z_s to the point z where the current is determined. It is assumed that the presence of the electrically small aperture at $z = \ell$ has a negligible effect on the distribution of current. The complex propagation constant γ and characteristic impedance Z_c of the line are defined by

$$\gamma = \alpha + j\beta = \sqrt{(z^i + j\omega\ell^e)(g + j\omega c)} \tag{2}$$

$$Z_c = \sqrt{(z^i + j\omega\ell^e)/(g + j\omega c)} \tag{3}$$

The internal impedance per loop unit length of the line is

$$z^i = \frac{(1 + j)}{2\pi} \sqrt{\frac{\omega\mu}{2}} \left[\frac{1}{a\sqrt{\sigma_a}} + \frac{1}{b\sqrt{\sigma_b}} \right] \tag{4}$$

where σ_a and σ_b are the conductivities, respectively, of the inner and outer conductors - in this case copper ($\sigma_a = 5.8 \times 10^7$ mhos/m) and aluminum ($\sigma_b = 3.72 \times 10^7$ mhos/m). This formula assumes the frequency and the conductivity to be high enough so that the skin depth $d_s = \sqrt{2/\omega\mu\sigma}$ is small compared with the radius a and the wall thickness of the outer conductor. More general formulas that apply when this is not the case are in the literature [1]. The other line constants have the well-known forms:

$$\ell^e = (\mu/2\pi)\ln(b/a) \quad ; \quad g = 2\pi\sigma_d/\ln(b/a) \quad ; \quad c = 2\pi\epsilon_d/\ln(b/a) \qquad (5)$$

where σ_d and $\epsilon_d = \epsilon_0\epsilon_r$ are the conductivity and permittivity of the dielectric in the coaxial line. In this case with air as the dielectric, $\sigma_d \doteq 0$, $\epsilon_r \doteq 1$. As usual, $\mu = \mu_0\mu_r$ where $\mu_0 = 4\pi \times 10^{-7}$ henries/m and $\epsilon_0 = 8.85 \times 10^{-12}$ farads/m. If the dissipation in the line is neglected, $\gamma = j\beta_0 = \omega\sqrt{\ell^e c}$, $Z_c = \sqrt{\ell^e/c} = (\zeta_0/2\pi)\ln(b/a)$ where $\zeta_0 = \sqrt{\mu_0/\epsilon_0} \doteq 120\pi$ ohms.

APPROXIMATION OF THE FIELD IN THE SLOT BY EQUIVALENT DIPOLES

It was shown by Bethe [2] and Bonwkamp [3] for a circular hole and by Collin [4] for an elliptic aperture in a perfectly conducting, infinitely thin screen that the radiation field on one side of the screen when this is illuminated on the other side by a normally incident plane wave may be calculated from suitably defined electric and magnetic dipoles located at the center of the aperture. In the derivation it is assumed that the dimensions of the aperture are electrically very small. If the screen is not plane, its radius of curvature must be large compared with the size of the hole.

In order to apply the theory to the elliptic aperture centered at $\rho = b$, $\phi = \pi$, $z = \ell$, in the coaxial sheath, it is necessary to determine the electric and magnetic fields at this point when the aperture is metallically closed. In this case the fields are rotationally symmetrical and given by

$$B_\phi(b,\pi,\ell) = \frac{\mu_0 I_z(\ell)}{2\pi b} \tag{6a}$$

$$E_\rho(b,\pi,\ell) = \frac{q(\ell)}{2\pi\varepsilon_0 b} = \frac{V(\ell)\zeta_0}{2\pi b Z_c} \tag{6b}$$

where $\zeta_0 = \sqrt{\mu_0/\varepsilon_0} \doteq 120\pi$ ohms and $Z_c = (\zeta_0/2\pi)\ln(b/a)$ is the characteristic impedance of the line. If the semi-major axis of the elliptic aperture is a_e and the semi-minor axis b_e, the central axial length W and transverse width $b\Psi$ of the slot are

$$W = 2b_e \quad , \quad b\Psi = 2a_e \tag{7}$$

as shown in Fig. 1. It is assumed that the following inequalities are satisfied:

$$2a_e << b \quad \text{or} \quad \Psi << 1 \tag{8a}$$

$$\beta_0 b = 2\pi b/\lambda << 1 \tag{8b}$$

According to Collin's generalization [4] of Bethe's theory the electromagnetic field at electrically large distances from the aperture may be calculated from an electric doublet with moment p_ρ and magnetic doublet with moment m_ϕ given by

$$p_\rho = \alpha_e \varepsilon_0 E_\rho(b,\pi,\ell) \quad , \quad \alpha_e = \frac{\pi a_e^3(1-k)}{3E(k)} \tag{9a}$$

$$m_\phi = -\alpha_m B_\phi(b,\pi,\ell)/\mu_0 \quad , \quad \alpha_m = -\frac{\pi a_e^3 k}{3[K(k) - E(k)]} \tag{9b}$$

The more general form for m given by Collin reduces to (9b) when the unperturbed magnetic field is directed parallel to the major axis of the elliptical aperture as in the present case. The parameter k and the eccentricity e of the elliptic aperture are defined by $k \equiv e^2 = (1 - b_e^2/a_e^2) = 1 - (W/b\Psi)^2$; $K(k)$ and $E(k)$ are the complete elliptic integrals of the first and second kinds, respectively. They are defined by

$$K(k) = \int_0^{\pi/2} (1 - k \sin^2 \theta)^{-1/2} \, d\theta \quad , \quad E(k) = \int_0^{\pi/2} (1 - k \sin^2 \theta)^{1/2} \, d\theta$$

When k is small,

$$k < 1: \quad E(k) \doteq \frac{\pi}{2} (1 - k/4) \quad , \quad K(k) \doteq \frac{\pi}{2} (1 + k/4) \quad ; \tag{10a}$$

when k is near one,

$$k \sim 1: \quad E(k) \doteq 1 \quad , \quad K(k) \doteq \ln(4a_e/b_e) \tag{10b}$$

THE RADIATION FIELD OF THE DIPOLES

The electric field in the radiation zone of an electrically short electric or magnetic dipole is readily calculated from the appropriate Hertz potential. The Hertz potentials due to an electric dipole with moment $\underset{\sim}{p} = \hat{x}p_x$ and a magnetic dipole with moment $\underset{\sim}{m} = \hat{y}m_y$ are, respectively, $\underset{\sim}{\Pi}_e = \hat{x}\Pi_{ex}$ and $\underset{\sim}{\Pi}_m = \hat{y}\Pi_{my}$ with

$$\Pi_{ex} = \frac{p_x}{4\pi\epsilon_0} \, \frac{e^{-j\beta_0 r}}{r} \quad , \quad \Pi_{my} = \frac{\mu_0 m_y}{4\pi} \, \frac{e^{-j\beta_0 r}}{r} \tag{11}$$

where r is the distance from the center of the dipole to the point of calculation so that $r = (x'^2 + y^2 + z^2)^{1/2}$ with $x' = x + b \doteq x$, since in the far zone $|x| >> b$ and $\beta_0 b << 1$.

The far-zone electric fields are readily obtained from the Hertz potentials. They are given in the literature [5] for electric and magnetic dipoles oriented along the z-axis. By simple permutation of the Cartesian coordinates* the following formulas are obtained:

*(see next page for footnote).

*The available formulas express the Cartesian components of the electric field in terms of dipoles with moments p_z and m_z. In order to obtain fields due to the moments p_x and m_y the following permutations are made: To obtain $\tilde{E}$ for p_x from $\tilde{E}$ for p_z, change x to y, y to z, z to x in all formulas and subscripts; to obtain $\tilde{E}$ for m_y from $\tilde{E}$ for m_z, change x to z, y to x, z to y in all formulas and subscripts - this involves two permutations.

$$E_x^r = K_1(r)\left[p_x \frac{(r^2 - x^2)}{r^2} + \frac{m_y}{c}\frac{z}{r}\right] \quad (12a)$$

$$E_y^r = -K_1(r)\left[p_x \frac{xy}{r^2}\right] \quad (12b)$$

$$E_z^r = -K_1(r)\left[p_x \frac{zx}{r^2} + \frac{m_y}{c}\frac{x}{r}\right] \quad (12c)$$

where $c = \omega/\beta_0$ is the velocity of light and

$$K_1(r) = \frac{\beta_0^2}{4\pi\varepsilon_0} \frac{e^{-j\beta_0 r}}{r} \quad (13)$$

The far-zone field is most conveniently expressed in spherical coordinates (r,Θ,Φ) with $x = r \sin\Theta \cos\Phi$, $y = r \sin\Theta \sin\Phi$ and $z = r \cos\Theta$. The spherical components of the field are:

$$E_r^r = E_x \cos\Phi \sin\Theta + E_y \sin\Phi \sin\Theta + E_z \cos\Theta \quad (14a)$$

$$E_\Theta^r = E_x \cos\Phi \cos\Theta + E_y \sin\Phi \cos\Theta - E_z \sin\Theta \quad (14b)$$

$$E_\Phi^r = -E_x \sin\Phi + E_y \cos\Phi \quad (14c)$$

When (12a-c) are substituted in (14a-c), the results are:

$$E_r^r = 0 \quad (15a)$$

$$E_\Theta^r = K_1(r)[p_x \cos\Theta + m_y/c]\cos\Phi \quad (15b)$$

$$E_\Phi^r = -K_1(r)[p_x + (m_y/c)\cos\Theta]\sin\Phi \quad (15c)$$

With (6a,b) and (9a,b) the dipole moments in (15a-c) are:

$$p_x = -p_\rho = -\alpha_e\varepsilon_0 E_\rho(b,\pi,\ell) = -\frac{\pi a_e^3(1 - k)}{3E(k)} \frac{V(\ell)}{2\pi b c Z_c} \quad (16a)$$

$$\frac{m_y}{c} = -\frac{m_\phi}{c} = \frac{\alpha_m B_\phi(b,\pi,\ell)}{\mu_0 c} = -\frac{\pi a_e^3 k}{3[K(k) - E(k)]}\ \frac{I_z(\ell)}{2\pi bc} \tag{16b}$$

It follows from (1a-d) that

$$I_z(\ell) = V_0^e\, F(s - \ell)/D \quad , \quad V(\ell)/Z_c = V_0^e\, G(s - \ell)/D \tag{17}$$

Hence, with $\zeta_0 = 1/c\varepsilon_0 = \sqrt{\mu_0/\varepsilon_0} \doteq 120\pi$ ohms,

$$p_x K_1(r) = -V_0^e L(r) \left[\frac{(1 - k)G(s - \ell)}{E(k)}\right] \tag{18a}$$

$$\frac{m_y K_1(r)}{c} = -V_0^e L(r) \left[\frac{kF(s - \ell)}{K(k) - E(k)}\right] \tag{18b}$$

with

$$L(r) = \frac{a_e^3 \zeta_0 \beta_0^2}{24\pi bD}\ \frac{e^{-j\beta_0 r}}{r} \tag{19}$$

The use of (18a,b) in (15a-c) gives:

$$E_r^r = 0 \tag{20a}$$

$$E_\Theta^r = -V_0^e L(r) \left\{\left[\frac{(1 - k)G(s - \ell)}{E(k)}\right] \cos\Theta + \frac{kF(s - \ell)}{K(k) - E(k)}\right\} \cos\Phi \tag{20b}$$

$$E_\Phi^r = V_0^e L(r) \left\{\frac{(1 - k)G(s - \ell)}{E(k)} + \frac{kF(s - \ell)}{K(k) - E(k)} \cos\Theta\right\} \sin\Phi \tag{20c}$$

In the equatorial plane, $\Theta = \pi/2$; in the direction through the center of the aperture, $\Theta = \pi/2$, $\Phi = \pi$. In this latter case,

$$\Theta = \pi/2,\ \Phi = \pi:\quad E_r^r = 0 \quad ; \quad E_\Theta^r = V_0^e L(r)\, \frac{kF(s - \ell)}{K(k) - E(k)} \quad , \quad E_\Phi^r = 0 \tag{21}$$

CURRENT IN THE DIPOLE: APPLICATION OF THE RECIPROCAL THEOREM

Let a dipole antenna be placed in the far-field of the slotted cylinder with its axis parallel to $E_\Theta^r(r,\Theta,\Phi)$. The current at the center of the dipole is [6]

$$I_d(0) = -2h_{e\Theta}(\pi/2)Y_A E_\Theta^r(r,\Theta,\Phi) \tag{22}$$

where Y_A is the admittance of the antenna and $2h_{e\Theta}(\pi/2)$ is the complex effective length of the dipole when it is parallel to the incident component $E_\Theta^r(r,\Theta,\Phi)$ which is given by (20b). Tables of $h_{e\Theta}(\Theta)$ are in the literature [7].

According to the Rayleigh-Carson reciprocal theorem [8], the impedanceless generator V_0^e can be moved to the center of the dipole from its position in series with the impedance Z_0 at $z = 0$ in the coaxial line, and the current $I_z(0)$ will then equal the current $I_d(0)$ given by (22). Thus, with (20b),

$$I_z(0) = 2h_{e\Theta}(\pi/2)Y_A V_0^e L(r)\left\{\frac{(1-k)G(s-\ell)}{E(k)}\cos\Theta + \frac{kF(s-\ell)}{K(k)-E(k)}\right\}\cos\Phi \tag{23}$$

The electric field actually maintained by the dipole at the center of the aperture in the cylinder is

$$E^{inc} = \frac{j\zeta_0 V_0^e}{2\pi}\,\frac{e^{-j\beta_0 r}}{r}\,\frac{\beta_0 h_{e\Theta}(\pi/2)}{Z_A} \tag{24}$$

since, by the reciprocal theorem, the electrical effective half-length is the same as the far-field factor.

If (24) is used to eliminate V_0^e from (23), the following general formula for the current in Z_0 is obtained:

$$I_z(0) = -\frac{jE^{inc}a_e^3\beta_0}{6bD}\left\{\frac{(1-k)G(s-\ell)}{E(k)}\cos\Theta + \frac{kF(s-\ell)}{K(k)-E(k)}\right\}\cos\Phi \tag{25}$$

This is a satisfactory approximation when $0 < \Theta < \pi$ and $\pi/2 < \Phi < 3\pi/2$.

Specifically, Θ may not approach close to either 0 or π and Φ may not be near $\pi/2$ or $3\pi/2$. The best approximation is with normal incidence when $\Theta = \pi/2$ and $\Phi = \pi$. In this case $E^{inc} = -E_z^{inc}$ and

$$I_z(0) = - \frac{jE_z^{inc} a_e^3 \beta_0}{6bD} \frac{kF(s - \ell)}{K(k) - E(k)} \tag{26}$$

These formulas give the desired solution for the current in the impedance Z_0 as a result of the excitation of the coaxial line by an incident plane wave. This has its wave front perpendicular to a radial line outward from the center of the aperture. The radial line is inclined at an angle Θ from the z-axis in the range $0 < \Theta < \pi$ and its projection on the equatorial plane is at an angle Φ with the x-axis in the range $\pi/2 < \Phi < 3\pi/2$. The electric field is polarized in the plane containing the axis of the cylinder and the radial line. [The current resulting from an incident field polarized perpendicular to this plane can be obtained in a similar manner from E_Φ^r given in (20c).] The current $I_z(s)$ in the load impedance Z_s is obtained by a simple interchange of ends.

GENERALIZATION TO A CYLINDER OF FINITE LENGTH

In the determination of the current $I_z(0)$ as given in (25) and (26) the field radiated from the slotted cylinder has been approximated by the field maintained by a pair of fictitious dipoles located at the center of the slot. This approximation is a good one when the aperture is electrically very small and the adjacent metallic surfaces are large compared with the aperture and substantially plane over distances from the aperture in which significant surface currents and charges are found. In general, these are local and restricted to the edges of and a small area surrounding the aperture. When this is true, the actual extent and shape of the metal surface beyond this area is of no importance since it carries no significant currents or charges. A possible exception to this general rule is when the surface is finite and of such size and shape that resonance can occur. In the case at hand, the transverse dimensions of the cylinder are required to be electrically small (although large compared to the aperture) so that resonant transverse currents are not possible. On the other hand, the cylinder may be long enough to have axially resonant currents which amplify the effective externally maintained field in the slot. Note that these possible resonances are in the currents induced on the outside surface of the cylinder. Internal resonances in the coaxial cavity are also possible and potentially much more important in affecting the currents in the loads Z_0 and Z_s. Complete and accurate account of all internal conditions ranging from a matched line to a resonant line are included in the transmission-line equations (1a-d).

Up to this point in the analysis it has been assumed that the external length of the cylinder and the internal length of the coaxial line are the same and equal to s. Actually, the finite length of the external surface has been ignored and, in effect, it has been assumed to be infinitely long. The length s has occurred explicitly only as the internal length of the coaxial line. Since with suitably located internal walls the length of the coaxial line can be made arbitrarily shorter than the external length of the cylinder, it is convenient to introduce the length $2h \geq s$ for the cylinder and retain s for the length of the coaxial line. For simplicity it will be assumed that the coaxial line is centered in the cylinder so that the coordinates $z_0 = 0$ and $z = s/2$ define the same cross section through the middle of both the cylinder and the coaxial cavity it contains.

In order to introduce a correction factor for possible axial resonances along the outside surface, it is convenient to regard the cylinder as a parasitic antenna of length 2h in a normally incident field. The axial coordinate $z_0 = z - s/2$ is referred to the center of the cylinder. The effective externally maintained field in the aperture is approximately proportional to the total axial current induced on the cylinder. Hence, the ratio of the field in the aperture of a cylinder of finite length 2h to that in the aperture of a cylinder of infinite length is approximately proportional to the ratio of the total axial current $I_u(z_0)$ at $z_0 = \ell - s/2$ or $z = \ell$ on the outside surface of the finite cylinder to the corresponding current I_∞ on the surface of an otherwise similar infinitely long cylinder. This ratio is readily determined. Thus, the current at a distance z_0 from the center of an unloaded receiving antenna of length 2h that is parallel to the incident electric field E_z^{inc} is [9]

$$I_u(z_0) = \frac{j4\pi E_z^{inc}}{\beta_0\zeta_0}\left[\frac{\cos\beta_0 z_0 - \cos\beta_0 h}{\Psi_{dU}\cos\beta_0 h - \Psi_U(h)}\right] \tag{27}$$

where the parameters Ψ_{dU} and $\Psi_U(h)$ are defined in the Appendix. The current at any point along an infinitely long antenna oriented parallel to the incident field is readily obtained from a formula for the electrically very long receiving antenna given by Wu and Chen [10]. The result is

$$I_\infty = \frac{-j2\pi E_z^{inc}}{\zeta_0\beta_0[\ln(2/\beta_0 b) - 0.5772 - j\pi/2]} \tag{28}$$

The desired ratio at $z_0 = \ell - s/2$ along a cylinder of length 2h is

$$t = \frac{I_u(\ell - s/2)}{I_\infty} = -\frac{[2\ln(2/\beta_0 b) - 1.1544 - j\pi][\cos\beta_0(\ell - s/2) - \cos\beta_0 h]}{\Psi_{dU}\cos\beta_0 h - \Psi_U(h)} \tag{29}$$

This ratio may be used to multiply (26) in order to correct this for a cylinder of finite length. Note that when the length of the coaxial cavity is the same as the length of the cylinder, $s = 2h$. A more general ratio that includes the angle Θ can be derived, but for present purposes (26) combined with (29) is adequate to determine the magnitude of the maximum current $I_z(0)$ to be expected in Z_0 inside the coaxial line.

SPECIAL CASE AND ILLUSTRATIVE EXAMPLE

When the terminations at both ends are matched so that $Z_0 = Z_s = Z_c = (\zeta_0/2\pi)\ln(b/a)$, the line is perfectly conducting with $\gamma = j\beta_0$, and the aperture is at the center of the coaxial section where $z = w = \ell = s/2$, it follows that

$$F(s - \ell) = G(s - \ell) = Z_c e^{j\beta_0\ell} \quad ; \quad D = 2Z_c^2 e^{j2\beta_0\ell}$$

and

$$F(s - \ell)/D = (1/2Z_c)e^{-j\beta_0\ell}$$

Let $f = 1$ MHz so that $\lambda = 300$ m and $\beta_0 = 2\pi/\lambda = 0.0209$. Let the coaxial shield have the radius $b = 1$ m and the inner conductor the radius $a = 10^{-3}$ m; it follows that $Z_c = 414.48$ ohms. The dimensions of the elliptical aperture are chosen to be $W = 2b_e = 0.25$ m and $b\Psi = 2a_e = 0.5$ m; the square of the eccentricity is $e^2 = 1 - 0.25 = 0.75$. The complete elliptic functions [11] are $K(k) = 2.1565$ and $E(k) = 1.2111$ so that $K(k) - E(k) = 0.9454$.

If these several quantities are substituted in the formula (26) for a normally incident plane wave with the electric vector parallel to the axis of the coaxial line, the following result is obtained:

$$|I_z(0)/E_z^{inc}| = \frac{(0.25)^3 \times 0.0209 \times 0.75}{6 \times 1 \times 2 \times 414.48 \times 0.9454} = 0.052 \ \mu\text{Am/volt} \tag{30}$$

Thus, if $E_z^{inc} = 10^5$ volts/m, $|I_z(0)| = 5.2$ mA. In this example the dimensions of the slot are $W = 0.25$ m (9.84 in) along the axial direction and $b\Psi = 0.5$ m (19.69 in) along the circumference. Note that $\beta_0 W = 0.00525 << 1$ and $\beta_0 b\Psi =$

0.0105 << 1.

In an earlier report [12] this same problem was analyzed by an entirely different method in which the axial electric field in the aperture was first determined and from this the far-field of an infinite cylinder. For the same illustrative example but with a rectangular rather than elliptical aperture with the same values of W and $b\Psi$ this alternative approach gives (when a numerical error in kb is corrected) $|I_z(0)/E_z^{inc}| = 0.057$ μAm/volt in remarkably good agreement with (30).

The correction factor t in (29) for a cylinder of finite length is easily evaluated for the special case considered above when the cylinder is near resonance with $2\beta_0 h = \beta_0 s = \pi$. In this case

$$t = \frac{[2\, \ell n(2/\beta_0 b) - 1.1544 - j\pi]}{\Psi_U(\lambda/4)}$$

With $\Omega = 2\, \ell n(s/b) = 2\, \ell n\, 150 = 10.02$ and $b/\lambda = 0.00333$, $\Psi_U(\lambda/4) = C_b(\lambda/4,\lambda/4) = 0.688 - j1.218$; $2\, \ell n(2/\beta_0 b) = 2\, \ell n(2/0.0209) = 2\, \ell n\, 95.69 = 9.122$,

$$|t| = \left| \frac{7.968 - j3.142}{0.688 - j1.218} \right| = \sqrt{\frac{73.36}{1.957}} = 6.12 \qquad (31)$$

Thus, the magnitude of the current on the surface of the nearly resonant cylinder is 6.12 times that on the infinite cylinder in the same field. It may be expected that the ratio $|I_z(0)/E_z^{inc}|$ for the current in the load Z_0 is also increased by about this factor to

$$t\, |I_z(0)/E_z^{inc}| \doteq 0.321 \text{ μAm/volt} \qquad (32)$$

When the cylinder is near antiresonance with $2\beta_0 h = \beta_0 s = 2\pi$,

$$t = \frac{2[2\,\ell n(2/\beta_0 b) - 1.1544 - j\pi]}{\Psi_{dU} + \Psi_U(\lambda/2)}$$

where

$$\Psi_{dU} = \frac{1}{2}\,[C_b(\lambda/2,0) - C_b(\lambda/2,\lambda/2) + E_b(\lambda/2,0) - E_b(\lambda/2,\lambda/2)]$$

$$\doteq 7.60 - j2.23$$

$$\Psi_U(\lambda/2) = C_b(\lambda/2,\lambda/2) + E_b(\lambda/2,\lambda/2) \doteq -0.882 - j0.672$$

It follows that

$$|t| = 2\left|\frac{7.968 - j3.142}{6.72 - j2.90}\right| = 2.34 \tag{33}$$

Hence, the magnitude of the current on the surface of the nearly antiresonant cylinder is 2.34 times that on an infinite cylinder in the same normally incident field. It may be expected that $|I_z(0)/E_z^{inc}|$ for the current in the load Z_0 is also increased by this factor to

$$t\,|I_z(0)/E_z^{inc}| \doteq 0.123\ \mu\text{Am/volt} \tag{34}$$

If the termination Z_s is set equal to zero instead of Z_c while all other parameters are unchanged, the currents in the coaxial line and its terminations are very different. The formula (26) for $I_z(0)$ still applies multiplied by the ratio t given in (29), but $F(s - \ell)/D = F(s/2)/D$ now becomes

$$\frac{F(s/2)}{D} = \frac{\cos(\beta_0 s/2)}{Z_0 \cos\beta_0 s + jZ_c \sin\beta_0 s}$$

instead of $\exp(-j\beta_0 s/2)/2Z_c$. It follows that when $\beta_0 s = \pi$, $I_z(0) = 0$. This result is readily understood if it is noted that the field maintained by the aperture acts like a generator at a quarter wavelength from a short circuit. Since it sees an infinite impedance, the induced current vanishes [13]. If the aperture is at a different distance from the short-circuited end, $I_z(0)$ will, of course, not be zero.

On the other hand, when $\beta_0 s = 2\pi$, $F(s/2)/D = -1/Z_0$ so that with (34)

$$t\,|I_z(0)/E_z^{inc}| = 0.123 \times 2Z_c/Z_0 = 101.7/Z_0 \ \mu\text{Am/volt}$$

Thus, when $Z_0 = 1$ ohm and $E_z^{inc} = 10^5$ volt/m, $I_z(0) = 10.17$ A. This large current indicates clearly that internal (coaxial line) resonance is much more important in determining the magnitude of the current in the load than the external (antenna) resonance.

If Z_s is open-circuited ($Z_s = \infty$) instead of zero or Z_c, while all other parameters of the coaxial line are unchanged,

$$\frac{F(s/2)}{D} = \frac{j \sin(\beta_0 s/2)}{Z_c \cos \beta_0 s + jZ_0 \sin \beta_0 s}$$

If it is now assumed that the internal length s of the coaxial cavity is one-half the length 2h of the cylinder, it follows that when $2\beta_0 h = \pi$ and $\beta_0 s = \pi/2$, $F(s/2)/D = 1/Z_0\sqrt{2}$ so that with (32) [which applies here since $2\beta_0 h = \pi$],

$$t\,|I_z(0)/E_z^{inc}| = 0.321 \times 2Z_c/Z_0\sqrt{2} = 188.1/Z_0 \ \mu\text{Am/volt}$$

When $Z_0 = 1$ ohm and $E_z^{inc} = 10^5$ volt/m, $I_z(0) = 18.81$ A. This large current is a consequence of simultaneous external and internal resonances.

On the other hand, when $2\beta_0 h \geq \beta_0 s = 2\pi$, $I_z(0) = 0$. In this case the effective generator is at a half wavelength from an open circuit in the coaxial line, so that it sees an infinite impedance and maintains a zero current.

CONCLUSION

A rocket with its access plate removed has been approximated by a large coaxial cable with an arbitrarily located electrically small slot of elliptical shape cut in the sheath. The interior circuit is represented by the inner conductor with a load impedance in series at each end. Approximate formulas have been derived for the currents in the load impedances in terms of the magnitude of the electric field of an incident plane wave. A correction factor has been given to take account of the axial resonance of the currents induced on the outer surface of the cylinder. Internal resonances in the coaxial line are included in the formulation. For simplicity it has been assumed that the center of the internal coaxial line is the same as that of the outer surface of the cylinder. If required, the analysis is easily generalized to include an inner cavity that has its center displaced with respect to that of the outer surface of the cylinder.

APPENDIX

The functions Ψ_{dU} and $\Psi_U(h)$ which appear in (27) are defined as follows:

$$\Psi_{dU} = (1 - \cos \beta_0 h)^{-1} \int_{-h}^{h} (\cos \beta_0 z' - \cos \beta_0 h)[K(0,z') - K(h,z')]\, dz' \qquad \text{(A-1)}$$

$$\Psi_U(h) = \int_{-h}^{h} (\cos \beta z' - \cos \beta h)\, K(h,z')\, dz' \qquad \text{(A-2)}$$

with

$$K(z,z') = \frac{e^{-j\beta_0 R}}{R}$$

$$R = [(z - z')^2 + a^2]^{1/2}$$

They are readily expressed in terms of the tabulated generalized sine and cosine integral functions or evaluated by computer.

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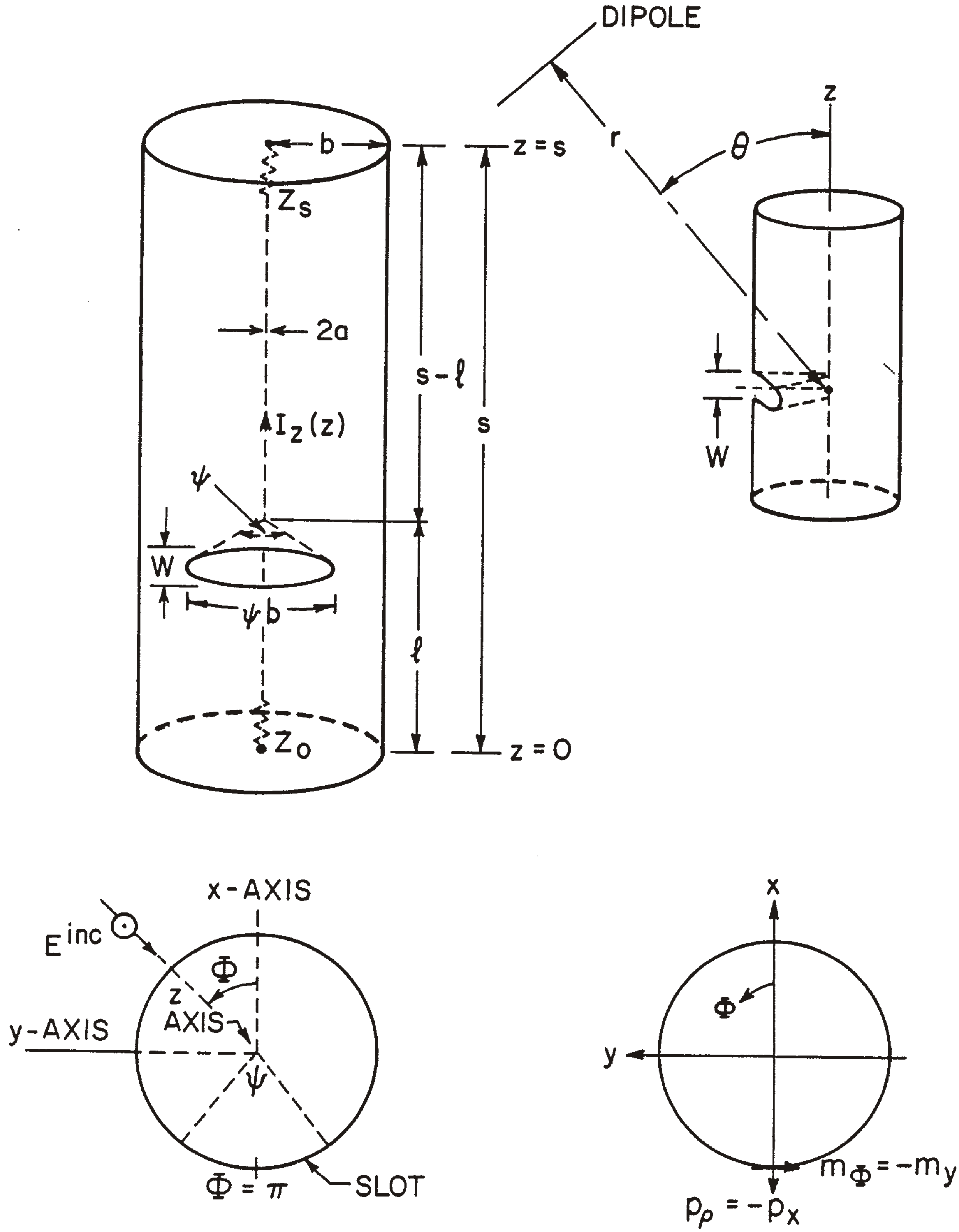

Fig. 1. Coordinates and Parameters for Internally Loaded Cylinder with Aperture.