IN 95

WL-EMP-IN-95

SC-RR-71 0475

THE RECEIVING PROPERTIES OF AN ASYMMETRIC CYLINDRICAL ANTENNA

L. D. Licking
Electromagnetic Effects Division, 1426
Sandia Laboratories
Albuquerque, New Mexico 87115

Published
September 1971

antennas, cylinders, receivers

ABSTRACT

Asymmetric cylindrical antennas are interesting models for missiles where the metal skin is separated into two parts by a dielectric spacer. This report develops an analysis for the response of internal wires in such antennas when illuminated by a plane-wave electromagnetic field. Parametric curves showing short-circuit current and open-circuit voltage responses are given.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

THE RECEIVING PROPERTIES OF AN ASYMMETRIC CYLINDRICAL ANTENNA

Introduction

The metal skin of some missiles is separated into two parts by a dielectric spacer. Communication across the gap (dielectric spacer) is accomplished by way of cables connected to the missile skin by some terminating load. It is the purpose of this study to determine the currents in and voltages across the terminating loads when the split-missile structure is illuminated by plane-wave electromagnetic fields.

The system will be modeled by a perfectly conducting cylinder with a gap located a distance g from the center of the cylinder (Figure 1). The two parts of the cylinder are connected by a wire inside the cylinder and terminated at each end by arbitrary resistive loads. It is assumed that the structure is illuminated by a plane-wave electromagnetic field, the electric field of which is parallel to the axis of the cylinder.

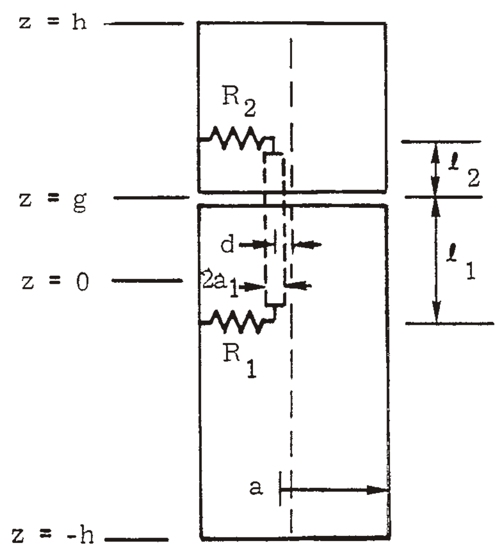


Figure 1. Model of the Split-Missile System

Analysis

The parameters of the system are as follows:

- h is the half-length of the cylinder,
- a is the radius of the cylinder,
- ℓ_1 is the length of the wire below the gap in the cylinder,
- ℓ_2 is the length of wire above the gap in the cylinder,
- a_1 is the radius of the wire,
- d is the distance between the axis of the wire and axis of the cylinder.
- g is the distance between the center of the cylinder and the gap,
- R_1 is the resistance of the load at the bottom of the wire,
- R_2 is the resistance of the top termination, and E_0 is the magnitude of the incident electric field.

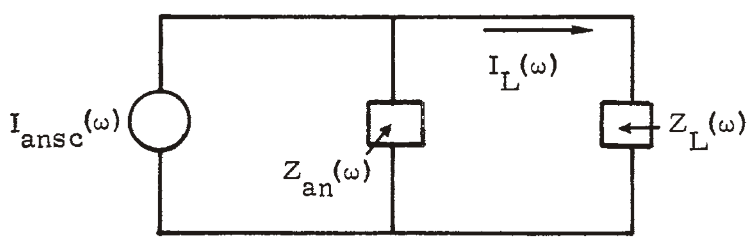


Figure 2. Equivalent Circuit for a Loaded Antenna

Currents in the load of an antenna can be found by using the equivalent circuit shown in Figure 2. In Figure 2, $I_{ansc}(\omega)$ is the current in the shorted terminals of the antenna when it is illuminated by a plane-wave electromagnetic field with a unit magnitude electric field and time variations of the form $e^{j\omega t}$, $Z_{an}(\omega)$ is the antenna impedance, and $Z_L(\omega)$ is the load impedance. For this analysis the load impedance used is the sum of R_1, R_2, and the impedance of the wire between R_1 and R_2. Thus,

$$\begin{aligned} Z_L &= R_1 + R_2 + j\omega(\ell_1 + \ell_2)L \\ &= R + jk_0 \ell Z_0 \ . \end{aligned} \tag{1}$$

Here, $R = R_1 + R_2$, $\ell = \ell_1 + \ell_2$, $k_0 = \omega/c$, L is the inductance per unit length along the wire, and

$$\begin{aligned} Z_0 &= cL \\ &= 60 \cosh^{-1}[(a^2 + a_1^2 - d^2)/2aa_1] \ . \end{aligned} \tag{2}$$

From Figure 2,

$$I_L(\omega) = \frac{E_0 Z_{an}(\omega) I_{ansc}(\omega)}{R + j\omega \ell L + Z_{an}(\omega)} \tag{3}$$

or

$$I(k_0 h) = \frac{Z_a(k_0 h) I_{asc}(k_0 h)}{R + j(k_0 h) Z_0 \ell / h + Z_a(k_0 h)} \quad . \tag{4}$$

In Equation 4, $I(\omega h/c) = I_L(\omega)/(hE_0)$, $Z_a(\omega h/c) = Z_{an}(\omega)$, and $I_{asc}(\omega h/c) = I_{ansc}(\omega)/(hE_0)$. Both Z_a and I_{asc} are functions of g/h, $k_0 h$, and $\Omega = 2\,\ell n(2h/a)$ only. From Equation 3, the open-circuit voltage (one end of the wire open and the other end shorted to the cylinder) is

$$V_{Loc}/h = E_0\, Z_a(k_0 h)\, I_{asc}(k_0 h) \quad . \tag{5}$$

The transient load current resulting from a step incident electromagnetic field whose electric field has a magnitude E_1 can be found by multiplying Equation 3 by $E_1/(E_0 j\omega)$ and taking the inverse Fourier transform. Therefore,

$$\begin{aligned} i_L(t)/(E_1 h) &= \frac{-2}{\pi h E_0} \int_0^\infty \mathrm{Im}[I_L(\omega)/j\omega] \sin \omega t \, d\omega \\ &= \frac{2}{\pi} \int_0^\infty \mathrm{Re}[I(\gamma)] \frac{\sin \gamma (tc/h)}{\gamma} d\gamma \, , \quad t > 0 \quad . \end{aligned} \tag{6}$$

The inverse transform $-\frac{2}{\pi}\int_0^\infty \mathrm{Im}[I_L(\omega)/j\omega] \sin \omega t \, d\omega$ was used instead of $\frac{2}{\pi}\int_0^\infty \mathrm{Re}[I_L(\omega)/j\omega] \cos \omega t \, d\omega$ because the imaginary parts of Z_a and I_{asc} are much larger and can be calculated more accurately than their real parts for small values of $k_0 h$.

Examination of Equations 4 and 6 shows that $i_L(t)/(E_1 h)$ is a function of g/h, R, $Z_0 \ell/h$, tc/h, and Ω.

The equivalent circuit of Figure 3 can be used to find I_L when the gap is spanned by a shorting strap. The result is

$$I_L(\omega)/h = \frac{E_0 j k_0 h (L_s c/h) Z_a I_{asc}}{j k_0 h (L_s c/h) Z_a + [R + j k_0 h (Z_0 \ell/h)] [Z_a + j k_0 h (L_s c/h)]} \quad . \tag{7}$$

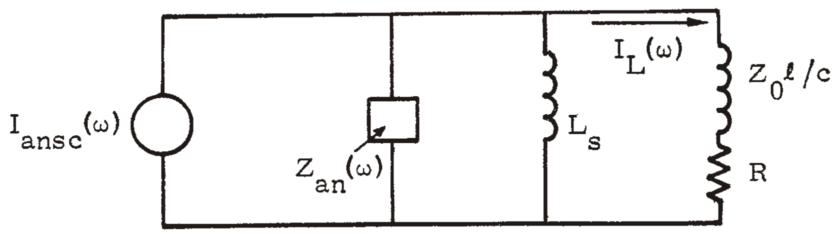


Figure 3. Equivalent Circuit for a Loaded Antenna With Shorting Straps

Here, L_s is inductance of the shorting strap. If $k_0h(L_sc/h) << |Z_a|$ and R = 0, Equation 7 can be written

$$I_L(\omega)/h \simeq \frac{E_0 L_s c/h}{L_s c/h + Z_0\ell/h} I_{asc} . \tag{8}$$

If $|j(k_0h)Z_0\ell/h| << |Z_a|$, comparison of Equations 4 and 8 shows that I_L is reduced by the factor

$$\frac{L_s c/h}{L_s c/h + Z_0\ell/h} . \tag{9}$$

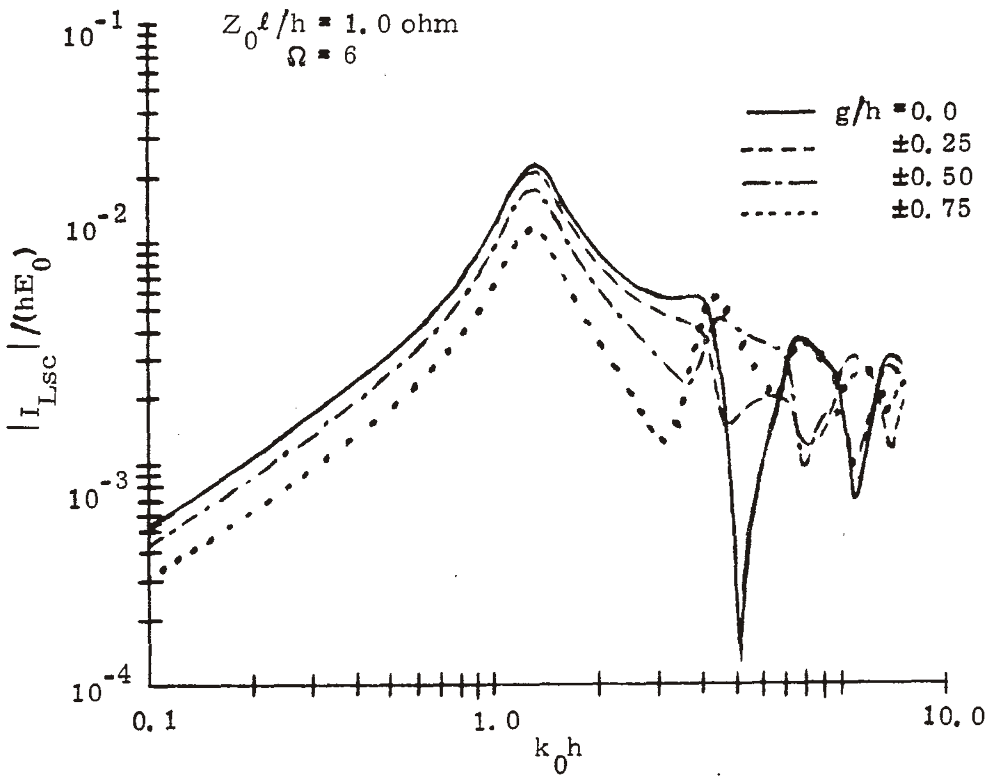
The current would be reduced more if $R \neq 0$.

Results

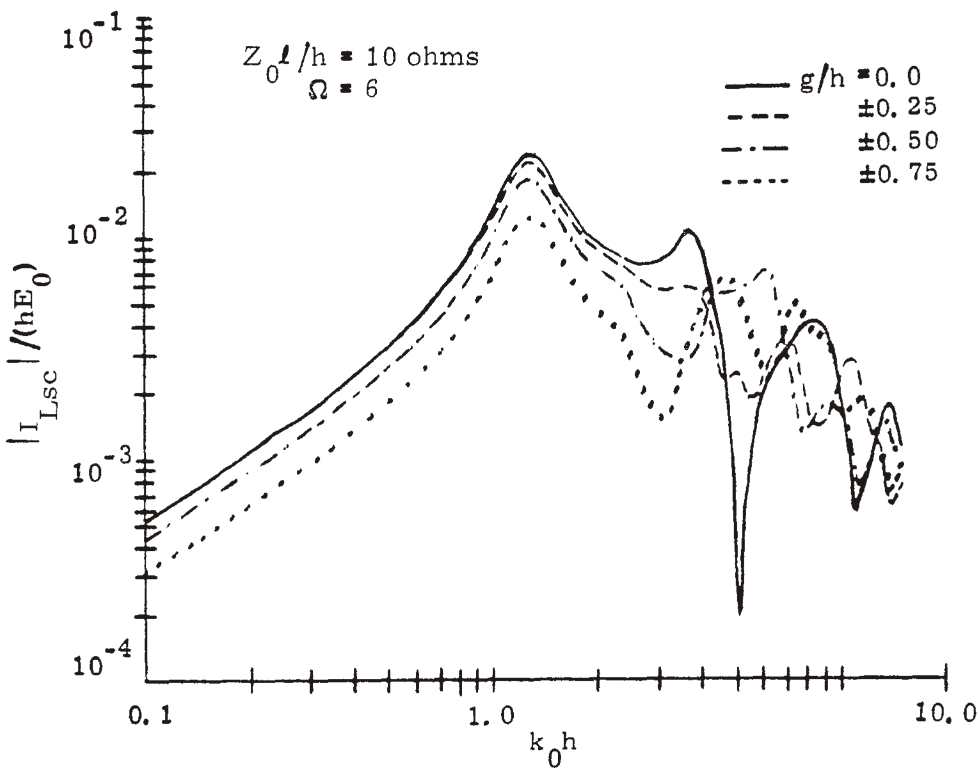
Equations 4, 5, and 6 were evaluated numerically to obtain the results of this section. Z_a was obtained from a program developed in Reference 1 that yields impedances for asymmetric cylindrical antennas.

The current I_{asc} induced at an arbitrary point on a conducting cylinder by a plane wave was obtained from a program developed in Reference 2.

Parametric curves of the short-circuit current at the ends of the wire are shown in Figures 4a, b, c, and d. Each figure has four graphs in which g/h is varied and $Z_0\ell/h$ is held constant. $Z_0\ell/h$ has different values in each figure. Open-circuit voltage is shown in Figure 5, where g/h is varied.

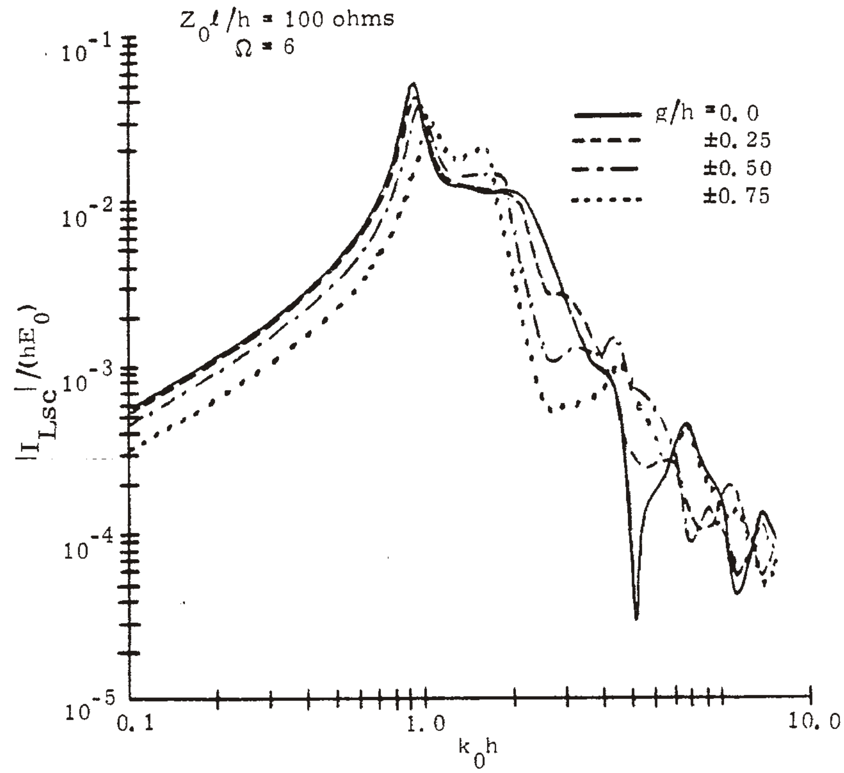


4a

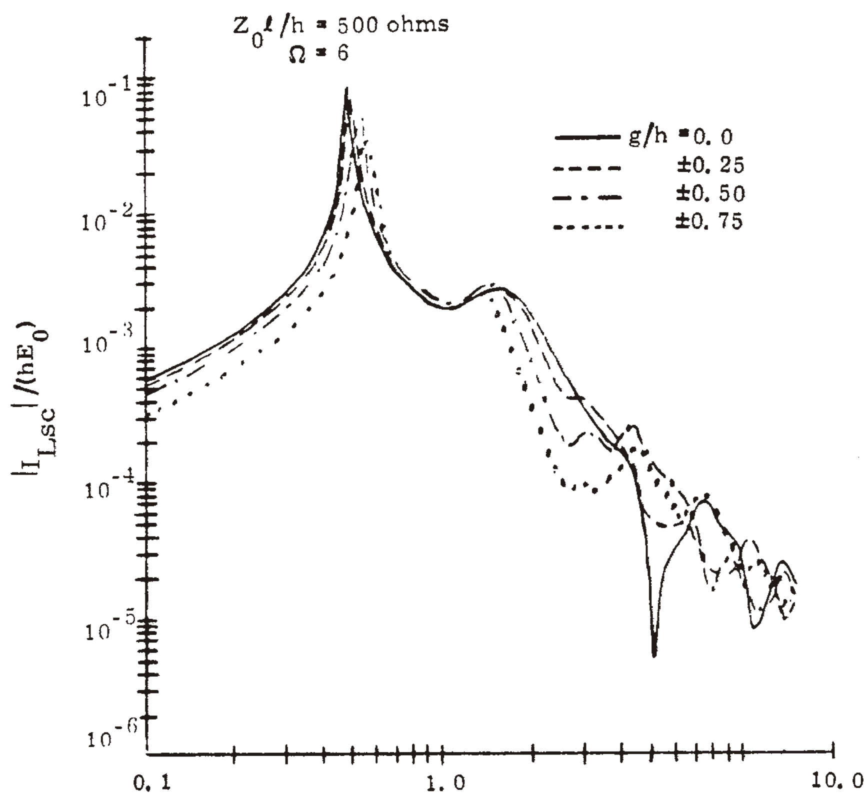


4b

Figure 4. Short-Circuit Current Transfer Function



4c



4d

Figure 4. Short-Circuit Current Transfer Function (cont)

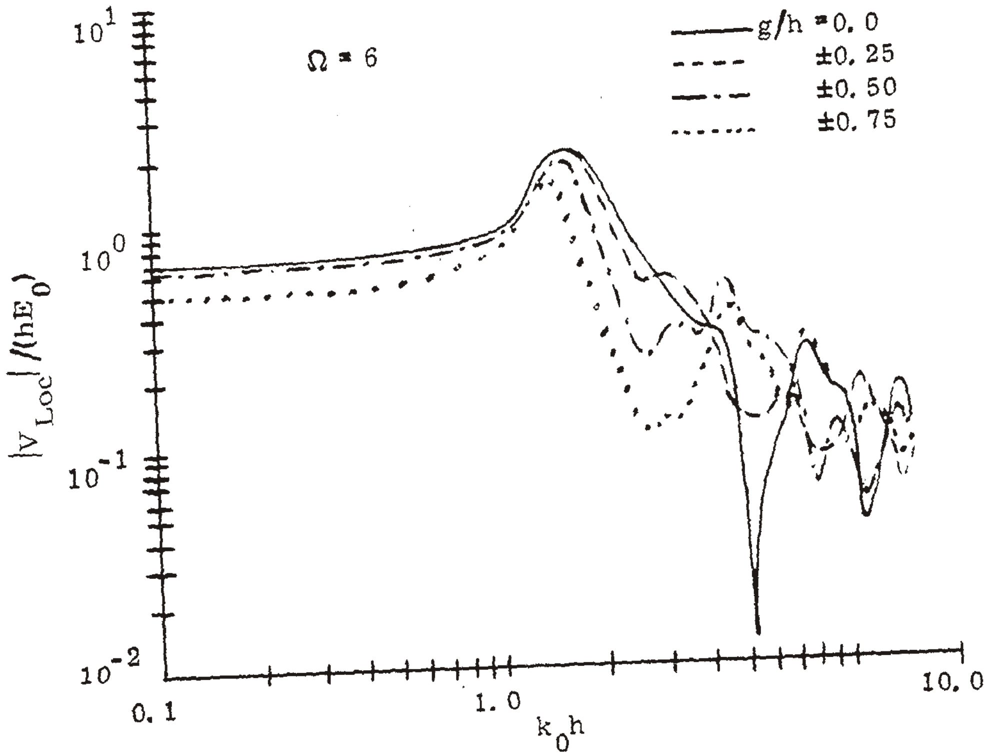
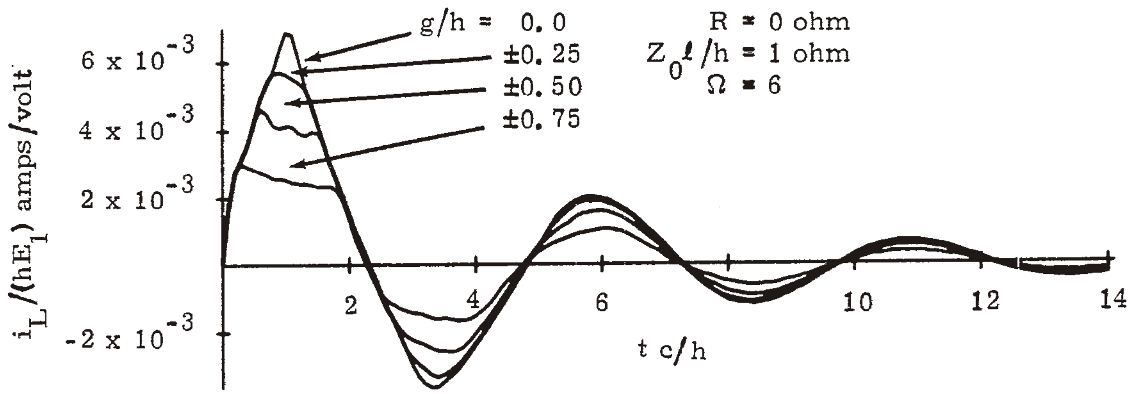
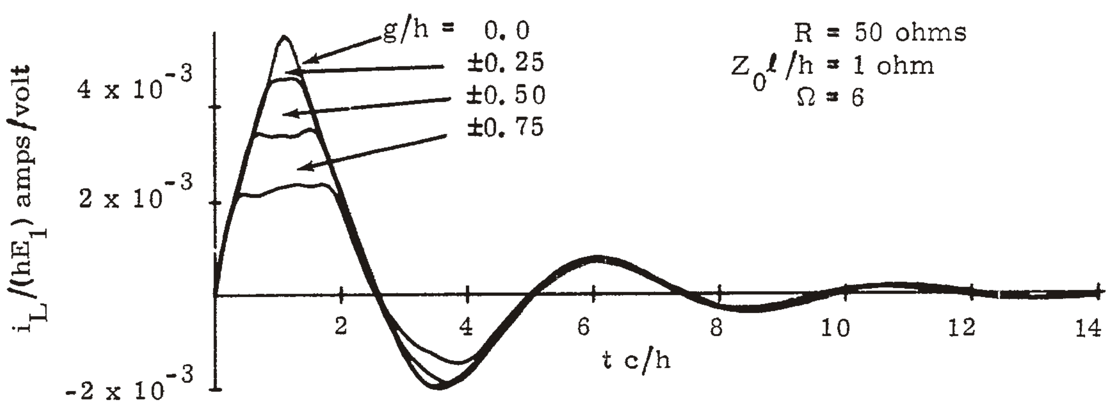


Figure 5. Open-Circuit Voltage Transfer Function

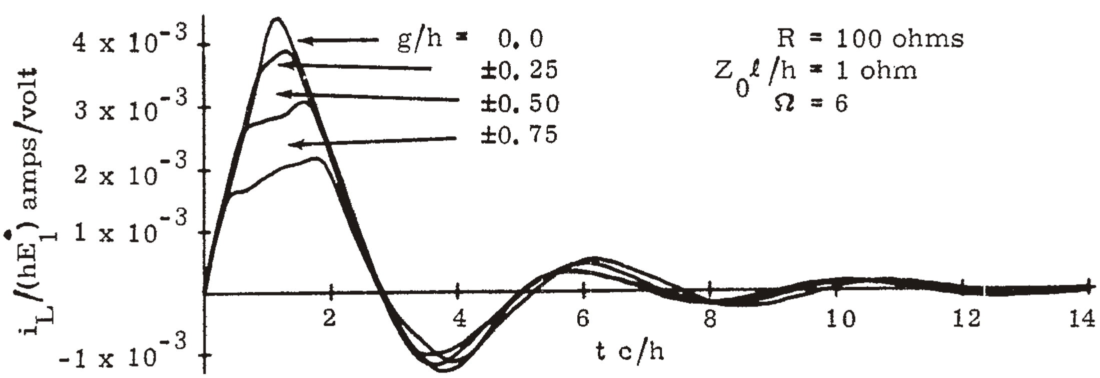
Parametric curves of the transient load current when the structure is illuminated by a step electromagnetic field are shown in Figures 6 through 8. Since the system response is a slowly varying function of Ω, $\Omega = 6$ in all computations.



6a

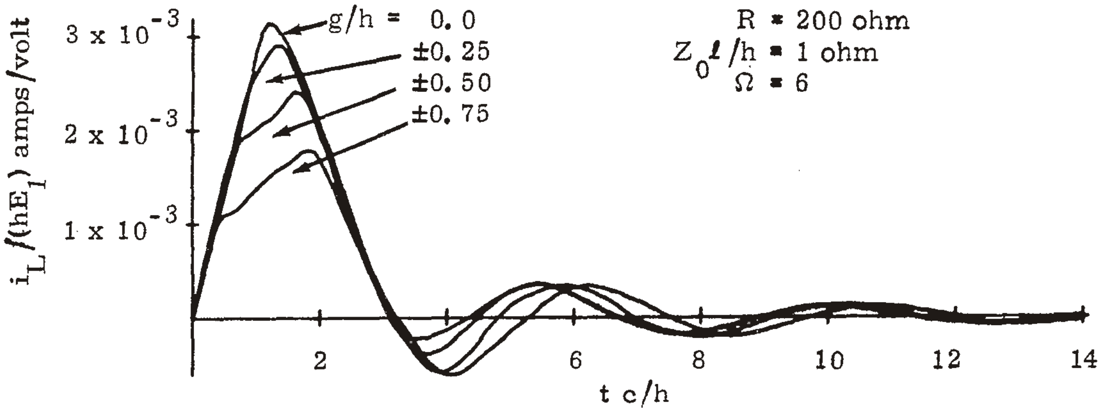


6b

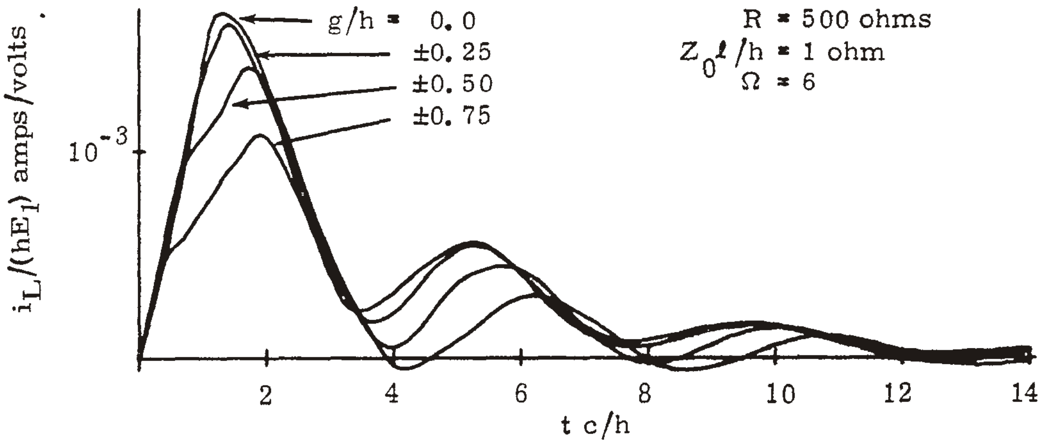


6c

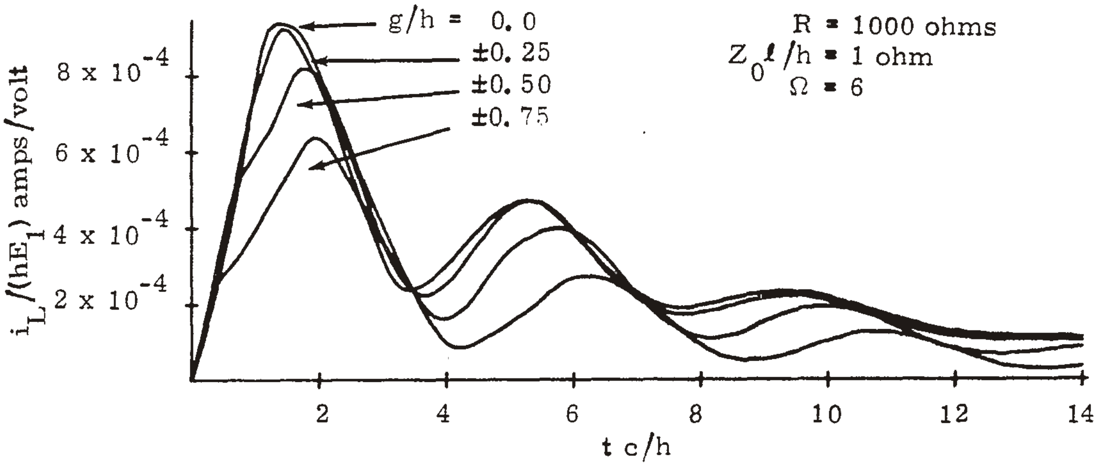
Figure 6. Transient Current in the Load Resistors Due to a Step Incident Field



6d



6e



6f

Figure 6. Transient Current in the Load Resistors Due to a Step Incident Field (cont)

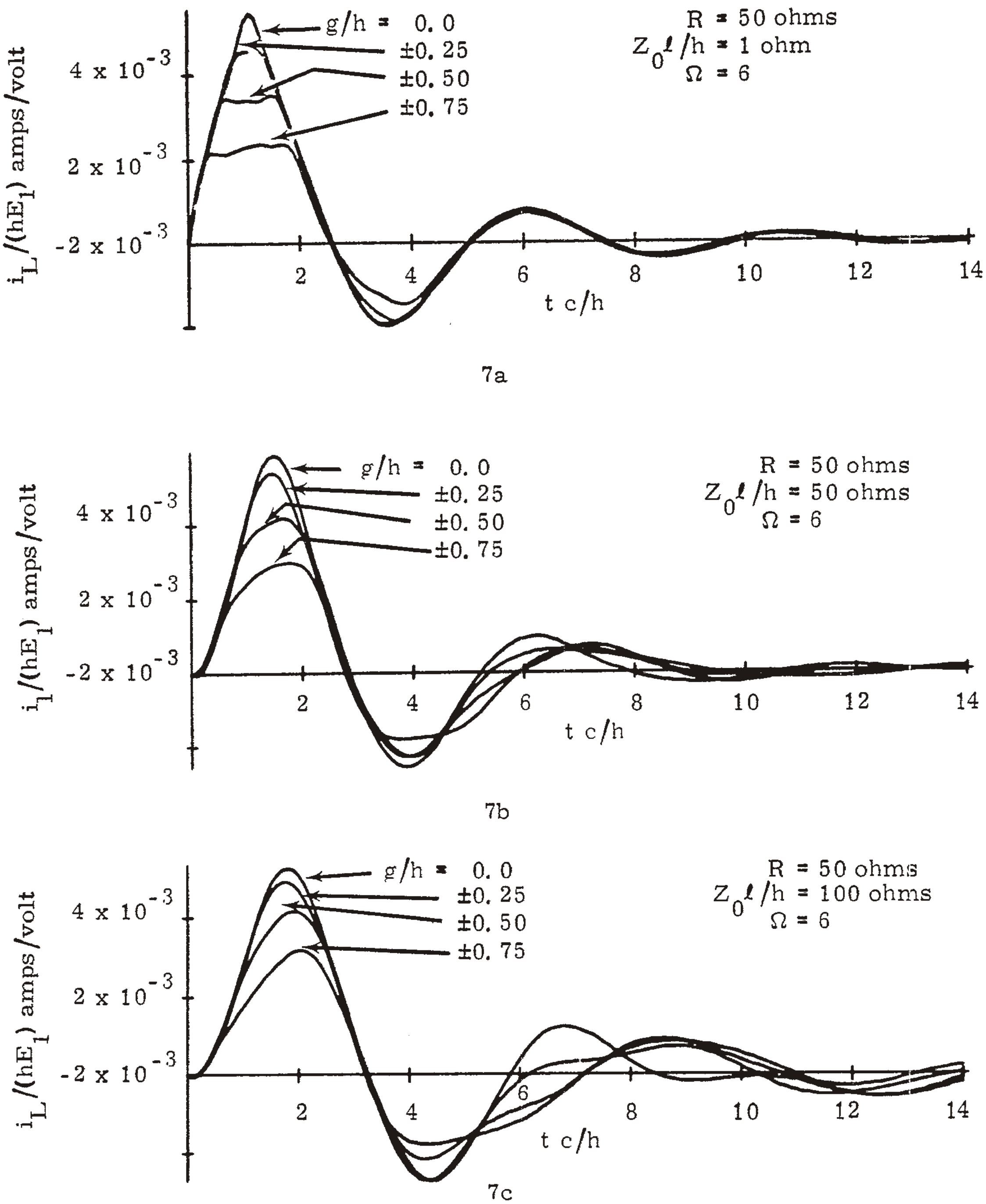
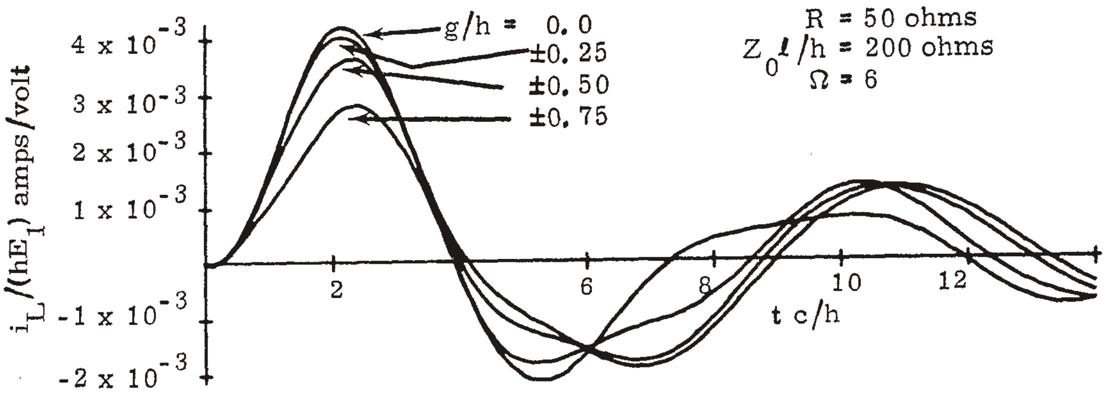
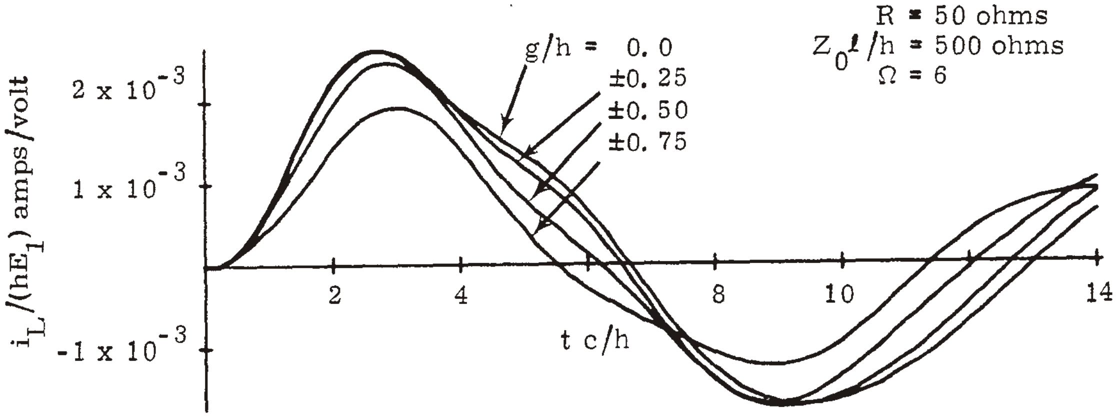


Figure 7. Transient Current in the Load Resistors Due to a Step Incident Field



7d



7e

Figure 7. Transient Current in the Load Resistors Due to a Step Incident Field (cont)

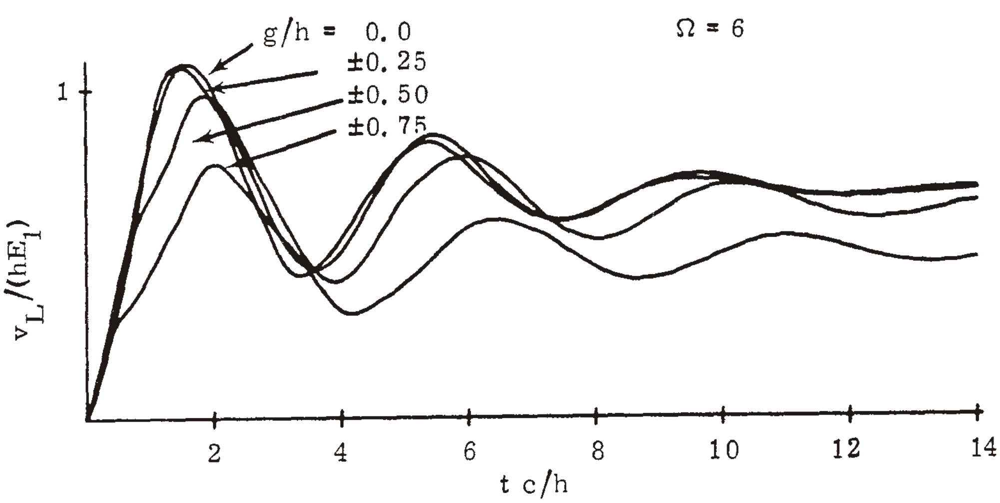


Figure 8. Transient Open-Circuit Voltage Due to a Step Incident Field

An example problem illustrates the use of Figures 4 through 8:

Problem. Find $i_2(t)$ and $v_2(t)$ at t = 30 nanoseconds when a split-missile system is illuminated by a 5-volt/m step incident electric field. Values of the system parameters are h = 5 m, a = 0.5 m, ℓ_1 = 2 m, ℓ_2 = 1 m, a_1 = 0.001 m, d = 0.4 m, g = 0.0 m, R_1 = 10 ohms, and R_2 = 40 ohms.

Solution. Calculate the following:

$$Z_0 = 60 \cosh^{-1} [(0.25 + 10^{-6} - 0.16)/(2 \times 0.5 \times 10^{-3})]$$

$$= 311.6 \text{ ohms},$$

$$R = 10 + 40$$

$$= 50 \text{ ohms},$$

$$\ell = 2 + 1$$

$$= 3 \text{ m},$$

$$Z_0 \ell/h = 311.6 \times 3/5$$

$$\simeq 200 \text{ ohms},$$

$$g/h = 0,$$

$$tc/h = 3 \times 10^{-8} \times 3 \times 10^{8}/5 = 1.8 .$$

From Figure 7d, $i_2/(hE_1) = 4 \times 10^{-3}$ amps/volt. Therefore, at $t = 30$ nanoseconds

$$i_2 = 5 \times 5 \times 4 \times 10^{-3} = 0.1 \text{ amp}$$

and

$$v_2 = 0.1 \times 40 = 4 \text{ volts.}$$

Conclusions

Normalized curves of the CW and transient response to a step incident field of a loaded cylindrical antenna have been presented. The antenna need not be symmetric. In most cases the results of this analysis should be sufficiently accurate to be used to calculate the currents and voltages induced in electronic packages of a split-missile system. However, for long cable runs and large values of k_0h, a more accurate analysis, using transmission line theory, may be necessary.

REFERENCES

1. D. E. Merewether, The Arbitrarily Driven Long Cylindrical Antenna, Sandia Laboratories SC-R-68-1862, November 1968.

2. C. W. Harrison, Jr.; C. D. Taylor; E. E. O'Donnell; and E. A. Aronson, "On Digital Computer Solutions of Fredholm Integral Equations of the First and Second Kind Occurring in Antenna Theory," Radio Science, Vol. 2 (New Series), No. 9, pp 1067-1081, September 1967.