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AXIAL CURRENT INDUCED ON A BODY OF REVOLUTION WITH SURFACE ANOMALIES

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ABSTRACT

An integral equation is derived for the axial current induced on a body of revolution with surface anomalies. In particular, surface protusions, apertures, and connecting wire loops are treated. A general illumination is considered but the diameter of the body of revolution at the surface anomaly is required to be small in terms of the wavelength of the illumination. Further, it is required that the anomaly size be much less than the diameter of the body of revolution. It is shown that in many cases the effect of a surface anomaly is equivalent to an impedance loading.

INTRODUCTION

To determine theoretically the electromagnetic field penetration into the interior of various semi-shielded configurations, often it is only necessary to know the induced surface currents and charges.[1] However, a tractable analysis to obtain these currents and charges requires that most physical configurations be roughly approximated. For example a missile is represented by a right circular cylinder[2] (or more recently by a body of revolution[3]), and an aircraft is represented by perpendicular crossed cylinders.[4] But many physical configurations are quite complex structures. This paper presents an attempt at analysing a class of complex physical structures. An approximate methodology is developed for obtaining the axial current induced on a body of revolution with surface anomalies.

An integral equation may be solved for the axial current induced on a body of revolution with arbitrary excitation. Surface anomalies are treated by using quasi-static type approximations. The electric and magnetic dipole moments of the anomlies are obtained in terms of the surface charge and current that would exist at the positions of the anomalies if they were not present. The radiated fields from these moments are then included with the impressed (or incident) field. Finally, Waterman's[3] extended boundary condition is used to arrive at an integral equation for the axial current.

Due to the aforementioned approximations the following limitations must be imposed: (1) the diameter of the body of revolution must be small in terms of wavelength at the position of the anomaly, and (2) the largest dimension of the anomaly must be small in comparison with the radius of the body of revolution at the position of the anomaly.

Three specific types of anomalies are considered. First surface protrusions are treated using the examples of a hemisphere and a cylindrical stub. Then apertures are considered using the elliptic aperture as an example. Finally, wire loops are treated using as an example the circular wire loop.

PRÉCIS

A general formulation is presented for determining the effects of surface anomalies on the current distribution induced on a conducting body of revolution. In particular the hemispherical anomaly is considered in detail since it possesses comparable electric and magnetic dipole moments. It is found that the effect of the magnetic dipole moment is equivalent to an inductive impedance loading. Whereas the effect of the electric dipole is equivalent to driving the structure with a doublet generator. In many cases only the magnetic dipole mement significantly effects the axial current. However the location of the surface anomialy determines the relative effects of the electric and magnetic dipole moments.

In general the surface anomaly will always have an electric dipole moment but it may or may not have a magnetic dipole moment. Furthermore the electric dipole moment will strongly perturb the local charge distribution but it will have a negligible effect on the local current distribution. The reverse is true of the magnetic dipole moment.

ANALYSIS

The integral equation for the axial current induced on a body of revolution may be obtained by using Waterman's extended boundary conditions in this case, requiring the total electric field to vanish on the axis of revolution within the conducting body (see Figure 1).[3] The result is

$$\int_0^L dz' \, I_t(z')\left(k^2 - \frac{\partial^2}{\partial z \partial z'}\right)K[z - z', a(z')] = -j \, \frac{4\pi k}{\eta} \, E_z^{inc} \, (0,\phi,z) \qquad (1)$$

where

$$K[z - z', a(z')] = \exp[-jk\sqrt{(z-z')^2 + a^2(z')}]/\sqrt{(z-z')^2 + a^2(z')} \qquad (2)$$

$$I_t(z') = a(z') \oint d\phi'[\vec{J}_s(z',\phi') \cdot \hat{t}'] \qquad (3)$$

with $I_t(z')$ as the total current through the cross section at z' with radius a(z'). In the foregoing $k = 2\pi/\lambda$ is the propagation constant, $\eta \simeq 120\pi$ ohms is the intrinsic wave impedance of free space, and E_z^{inc} $(0,\phi,z)$ is the incident (or impressed) electric field evaluated on axis of the body of revolution. To solve the integral equation the method of moments may be applied; although for certain geometries analytic solutions exist, for example when the body of revolution is a sphere.

If the radius of the body of revolution is small compared to the operating wavelength then the unperturbed surface current density is simply

$$\vec{J}_s(z,\phi) \simeq \frac{1}{2\pi a(z)} I_t(z)\, \hat{t} \tag{4}$$

where

$$\hat{t} = \hat{n} \times \hat{\phi}$$

with $\hat{n}$ as the unit vector normal to the surface of the body of revolution. Likewise the unperturbed surface charge density under the same conditions is

$$\rho_s(z) \simeq \frac{j}{\omega} \frac{\sin\theta(z)}{2\pi a(z)} \frac{d}{dz} I_t(z) \tag{5}$$

where

$$\theta(z) = \cos^{-1} (\hat{n} \cdot \hat{z}) \quad .$$

The surface field associated with the foregoing current and charge distributions is considered to be the impressed field on the surface anomalies. The surface electric field is

$$\vec{E}_o = \frac{1}{\varepsilon_o} \rho_s(z)\, \hat{n} \tag{6}$$

and the surface magnetic field is

$$\vec{H}_o = \hat{n} \times \vec{J}_s(z) \tag{7}$$

The presence of surface anomalies may be treated by using the quasi-static approximations. First the impressed field about a given anomaly is considered to be uniform so that the surface currents and charges on the anomaly may be obtained by solving an equivalent problem of the anomaly on an infinite conducting plane with a uniform impressed field. Then the electric and magnetic dipole moments may be readily determined in terms of the impressed field.* Provided the dimensions of the anomaly are small in comparison with the radius of curvature of the body of revolution, the field scattered from the anomaly on the body of revolution may be expressed in terms of the previously obtained electric and magnetic dipole moments. Including this scattered field as part of the incident field on the body of revolution in (1) introduces the effect of the anomaly on the current distribution of the body of revolution.

Upon solving (1) J_s and ρ_s are obtained. In order then to obtain the complete surface current and charge densities the contributions to the surface fields from the dipole moments of the anomlies must be included.

The electromagnetic field about an electric dipole and a magnetic dipole is[1]

$$\vec{E}_A(\vec{r}) = \frac{e^{-jkR}}{4\pi\varepsilon_o} \{[\frac{1}{R^3} + j\,\frac{k}{R^2}][3\,\hat{R}(\hat{R}\cdot\vec{P}_o) - \vec{P}_o] - \frac{k^2}{R}\,[\hat{R}\times(\hat{R}\times\vec{P}_o)]\} + \frac{\eta}{4\pi}\,e^{-jkR}\,[j\,\frac{k}{R^2} - \frac{k^2}{R}]\,(\hat{R}\times\vec{M}_o) \tag{8}$$

*The impressed field at this point is yet an unknown. In order to obtain it a self consistent field solution must be effected.

$$\vec{H}_A(\vec{r}) = \frac{\mu_o e^{-jkR}}{4\pi} \left(\frac{1}{R^3} + j\frac{k}{R^2}\right)[3\,\hat{R}(\hat{R}\cdot\vec{M}_o) - \vec{M}_o]$$

$$- \frac{k^2}{R}\,[\hat{R} \times (\hat{R} \times \vec{M}_o)]\}$$

$$- \frac{\eta}{4\pi}\, e^{-jkR} \left(j\frac{k}{R^2} - \frac{k^2}{R}\right)(\hat{R} \times \vec{P}_o) \qquad (9)$$

where

$$\vec{R} = \vec{r} - \vec{r}_o$$

$$\hat{R} = \vec{R}/R$$

with $\vec{r}$ the radius vector to the field point and $\vec{r}_o$ the radius vector to the dipoles. As pointed out previously the effect of the surface anomaly on the current distribution $J_s(z)$ is obtained by using the following replacement in (1)

$$E_z^{inc}(0,\phi,z) \rightarrow E_z^{inc}(0,\phi,z) + \vec{E}_A(0,\phi,z) \qquad (10)$$

Superposition may be employed when there is more than one anomaly.

According to the aforementioned considerations the dipole moments appearing in (8) and (9) by assuming the charge and current distributions on the anomaly are the same as would occur if the anomaly were in an infinite plane. Using these current and charge distributions the dipole moments are readily obtained. Note that the dipole moment of the anomaly is one-half the dipole moment of the anomaly plus its image in the infinite plane.

EXAMPLES OF ANOMALIES

I. Surface Protrusions

First a hemisphere is considered to be protruding out of the surface of a body of revolution. To obtain the electric and magnetic dipole moments the hemisphere is considered to be protruding from a conducting plane with the impressed uniform field ($\vec{E}_o$, $\vec{H}_o$) as given in (6) and (7). Image theory is used to remove the plate. Provided the radius of sphere is small in terms of wavelength, i.e., $ka_s << 1$, the surface charge distribution (see Figure 2)

$$\rho_{sp} \approx 3\varepsilon_o E_o \cos\theta \tag{11}$$

where θ is the polar angle measured from the direction of $\hat{n}$, the unit normal to the plane. The dipole moment of the hemisphere with the foregoing charge distribution to be used in (10) is[7]

$$\vec{P}_o = 2\pi a_s^3 \varepsilon_o \vec{E}_o \tag{12}$$

where $E_o = E_o n$ and

$$\hat{n} = \hat{r} \sin[\theta(z_o)] + \hat{z} \cos[\theta(z_o)]$$

Here $\hat{r}$ is the radial unit vector of a cylindrical coordinate system, z_o locates the cross section containing the spherical protrustion anomaly, and $\theta(z)$ is the angle between the $\hat{z}$ axis and the normal to the surface of the body of revolution, i.e.

$$\theta(z) = \frac{\pi}{2} - \tan^{-1} \frac{d}{dz} a(z) \tag{13}$$

By following an analogous procedure to the foregoing it is readily shown that the magnetic dipole moment of the hemisphere is[7]

$$\vec{M}_o = - \pi a_s^3 \vec{H}_o$$

Second a short cylindrical stub is considered to be protruding normally from the surface of the body of revolution (see Figure 3). Following the foregoing procedure it is found that the dipole moment for the stub is

$$\vec{P}_o = \frac{\pi h^3 \varepsilon_o E_o}{2(\Omega - 3.39)} \hat{n} \tag{14}$$

where

$$\Omega = 2 \ln(2h/a)$$

with h as the height of the stub and a as the radius. It is readily shown that the magnetic dipole moment of the stub is negligible.

II. Surface Apertures

The problem of small aperture diffraction has been considered by a number of authors.[1,5] Only the results are presented here. For an elliptical aperture both magnetic and electric dipole moments are needed to express the scattered field. They are

$$\vec{M}_o = \vec{\vec{\alpha}} \cdot \vec{H}_o$$

$$\vec{P}_o = \varepsilon_o \vec{\vec{\alpha}} \cdot \vec{E}_o$$

where the elements of the dyadic are

$$\overleftrightarrow{\alpha} = \alpha_{11} \hat{u}_1\hat{u}_1 + \alpha_{22} \hat{u}_2\hat{u}_2 + \alpha_{33} \hat{n} \hat{n}$$

$$\alpha_{11} = \frac{\pi}{3} \frac{\ell_1^3 e^2}{K(e^2) - (1-e^2)K(e^2)}$$

$$\alpha_{22} = \frac{\pi}{3} \frac{\ell_1^3 e^2(1-e^2)}{E(e^2) - (1-e^2)K(e^2)}$$

$$\alpha_{33} = \frac{\pi}{3} \frac{\ell_1^3 (1-e^2)}{E(e^2)}$$

$$e^2 = 1 - (\ell_1/\ell_2)^2$$

Here $\hat{u}_1$ is the unit vector along the major axis of the ellipse, $\hat{u}_2$ is the unit vector along the minor axis, $E(e^2)$ and $K(e^2)$ are elliptic integrals of the first and second kinds,[5] respectively, ℓ_1 is the length of the semi-major axis and ℓ_2 is the length of the semiminor axis (see figure 4).

III. Wire Loops

Wire loops protruding from the surface of a conducting body will have induced electric and magnetic dipole moments. Just as for the hemisphere the currents and charges on the loop are considered to be nearly the same as would occur if the wire were protruding from an infinite plane. For convenience a semicircular loop is considered to extend from the body of revolution so that the plane of the loop is perpendicular to the tangent plane of the body of revolution at the point of the loop attachment. Therefore to obtain the electric and magnetic dipole moments the semicircular loop together with its image forms a complete circular loop (see Figure 5).

The field impressed on the loop is such that the electric field lies in the plane of the loop and the magnetic field forms an angle ψ with the normal to the plane of the loop. According to King and Harrison[8] the induced current in an electrically small loop (unloaded) is

$$I(\phi) \simeq \frac{\pi b H_o \cos\psi}{\ln \frac{8b}{a} - 2} + j \frac{2\pi k b^2 E_o}{(\ln \frac{2b}{a} - \gamma)} \cos\phi$$

where γ is Euler's constant. From the equation of continuity and the foregoing, the charge per unit length induced on the loop is

$$\rho_s(\phi) = \frac{2\pi b \ \varepsilon_o E_o}{\ln \frac{2b}{a} - \gamma} \sin\phi$$

By using the foregoing charge and current distributions the electric and magnetic dipole moments of the semi-circular loop may be readily obtained. They are

$$\left.\begin{aligned} \vec{P}_o &= \frac{\pi}{2} \frac{b^3}{\ln \frac{2b}{a} - \gamma} \varepsilon_o \vec{E}_o \\ \vec{M}_o &= -\frac{\pi}{2} \frac{b^3}{\ln \frac{8b}{a} - 2} \vec{u}\,\vec{u} \cdot \vec{H}_o \end{aligned}\right\} \qquad (15)$$

where $\hat{u}$ is the unit normal to the plane of the loop.

AN ILLUSTRATIVE APPLICATION

In the foregoing various types of surface anomalies are considered. It is of interest to consider a particular geometry and to obtain the integral equation for the current distribution. A convenient geometry to consider is that of a hemispherical protrusion from the lateral surface of a right circular cylinder with sperical end caps (see figure 6). The geometry is simple and the hemispherical anomaly possesses comparable electric and magnetic moments.

The contribution to the axial component of the electric field along the axis of the body is obtained from (9). It is

$$\hat{z} \cdot \vec{E}_A(\rho,\phi,z)\bigg|_{\substack{\rho=0 \\ P_o=0}} = -j\omega \frac{\mu M_o a}{\pi} [jk + \frac{1}{R}] \frac{e^{-jkR_o}}{R_o^2} \tag{16}$$

where

$$R_o = \sqrt{(z-z_o)^2 + a^2} \tag{17}$$

According to the aforementioned considerations

$$\vec{M}_o = -M_o \hat{\phi}$$

where

$$M_o = \frac{a_s^3}{2a} I_t(z_o)$$

If the structure were driven from a magnetic ring source a $z = z_o$ (equivalent to the delta gap source) then the axial component of the source field becomes

$$E_z^{inc}(\rho,\phi,z)\Big|_{\rho=0} = \frac{V_o a^2}{2}\left[jk + \frac{1}{R_o}\right]\frac{e^{-jkR_o}}{R_o^2} \tag{17}$$

Note that (16) and (17) become equivalent when

$$V_o = -j\omega\frac{\mu a_s}{4\pi a^2} I_t(z_o) \tag{18}$$

It may be further noted that (18) may be used to define an equivalent impedance loading for the effect of the magnetic dipole moment of the hemisphere anomaly. It is

$$Z_L = j\omega\frac{\mu a_s^3}{4\pi a^2}$$

Suppose that $ka = 0.5$ and $a_s = a/4$, then

$$Z_L = j\ 0.23 \text{ ohms}$$

Also the hemispherical anomaly has an electric dipole contribution to the axial component of the electric field along the axis of the body. It is

$$\hat{z}\cdot\vec{E}_A(\rho,\phi,z)\Big|_{\substack{P=0\\ M_o=0}} = \frac{a\,P_o}{4\pi\varepsilon_o}\frac{d}{dz}\left\{\left[jk + \frac{1}{R}\right]\frac{e^{-jkR_o}}{R_o}\right\} \tag{19}$$

where $\vec{P}_o = -\frac{1}{j\omega}\frac{a_s^3}{a}\frac{d}{dz}I_t(z_o)$

From (17) and (19) it is observed that the effect of the electric dipole moment is analogous to that obtained by driving the structure from an equivalent doublet generator. That is, it is not possible to represent the effect of the electric dipole moment with an impedance loading.

The obvious question now is what is the relative importance of the electric and magnetic dipole moment contributions. Near the anomaly the electric dipole contribution (19) approaches zero whereas the magnetic dipole contribution approaches the maximum value. At large distances from the anomaly $R_o >> 1$ it is easily shown for the hemispherical anomaly that

$$\frac{\left| \hat{z} \cdot \vec{E}_A(0,\phi,z) \right|_{M_o=0}}{\left| \hat{z} \cdot \vec{E}_A(0,\phi,z) \right|_{P_o=0}} \simeq 2 \left| \frac{E_o}{\eta H_o} \right| \tag{20}$$

Considering the aforementioned geometry

$$\left| \frac{E_o}{\eta H_o} \right| = \left| \frac{\frac{d}{dz} I_t(z_o)}{k I_t(z_o)} \right| \tag{21}$$

Thus at large distances the electric dipole contribution is dominant for anomalies near current nulls and the magnetic dipole contribution is dominant for anomalies near current maxima. Therefore for anomalies near the middle of the structure the magnetic dipole contribution should be the most significant in affecting the current distribution whereas near the structure ends the electric dipole moment should be the most significant.

In any event the integral equation for the current distribution on the aforementioned geometry (figure 6) is

$$\int_0^L dz' \, I_t(z')\left(k^2 - \frac{\partial^2}{\partial z \partial z'}\right) K[z - z', a(z')]$$

$$- \frac{(ka_s)^3}{2} \left[j + \frac{1}{kR_o}\right] \frac{e^{-jkR_o}}{R_o^2} \; I_t(z_o)$$

$$- \frac{(ka_s)^3}{k} \frac{d}{dz}\left(\left[j + \frac{1}{kR_o}\right] \frac{e^{-jkR_o}}{R_o^2} \right) \frac{d}{dz} I_t(z_o)$$

$$= - j \frac{4\pi k}{\eta} E_z^{inc} (0,\phi,z) \tag{22}$$

where

$$\begin{aligned} a(z) &= \sqrt{a^2 - (a-z)^2} & 0 \le z \le a \\ &= a & a \le z \le L - a \\ &= \sqrt{a^2 - (z-L+a)^2} & L - a \le z \le L \end{aligned}$$

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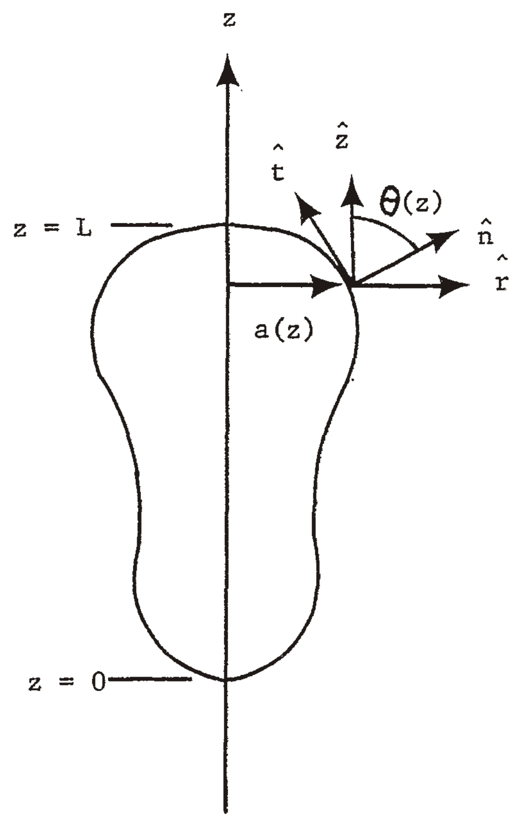

Figure 1: Body of Revolution

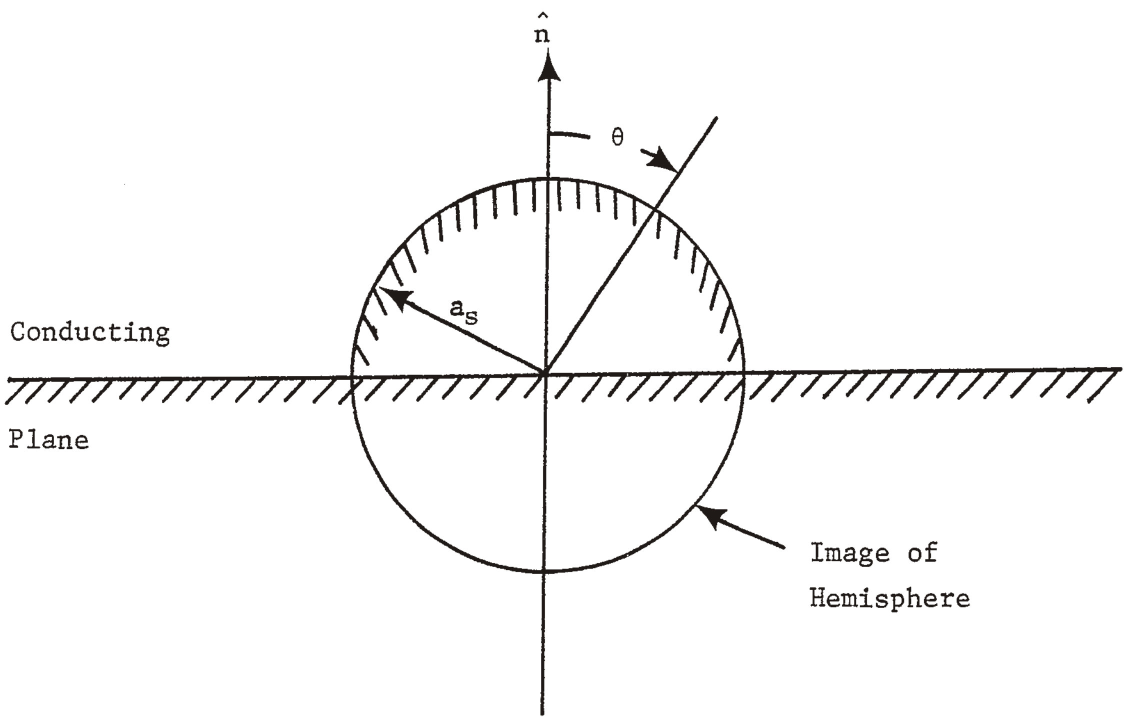

Figure 2: Conducting Hemisphere on a Conducting Plane.

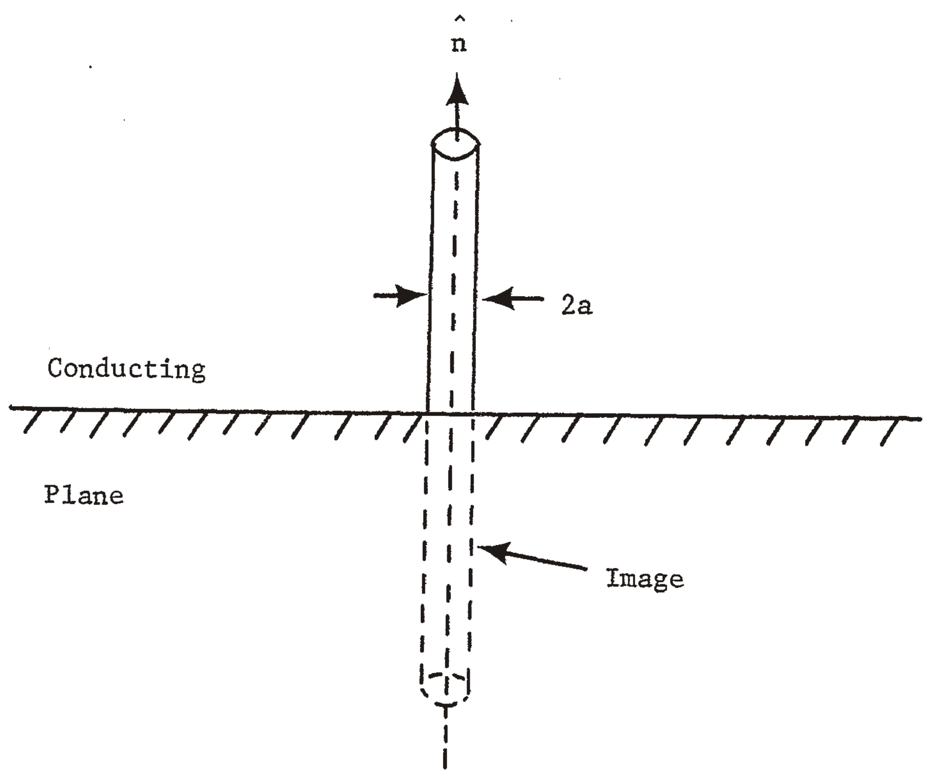

Figure 3: Cylindrical Stub over a Conducting Plane.

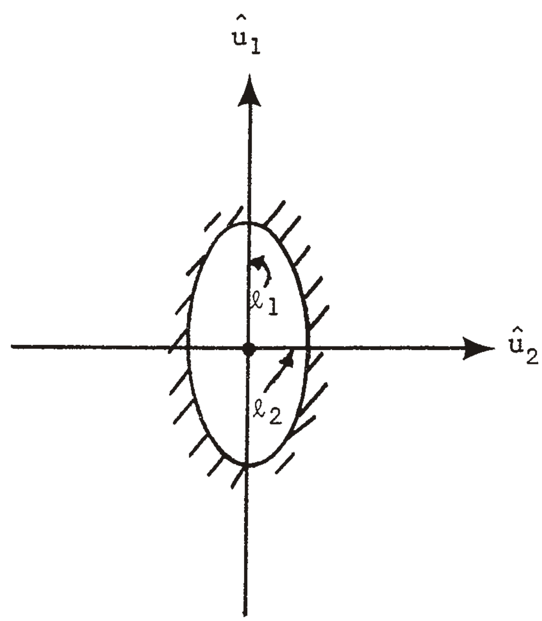

Figure 4: Elliptical Aperture in a Conducting Plane.

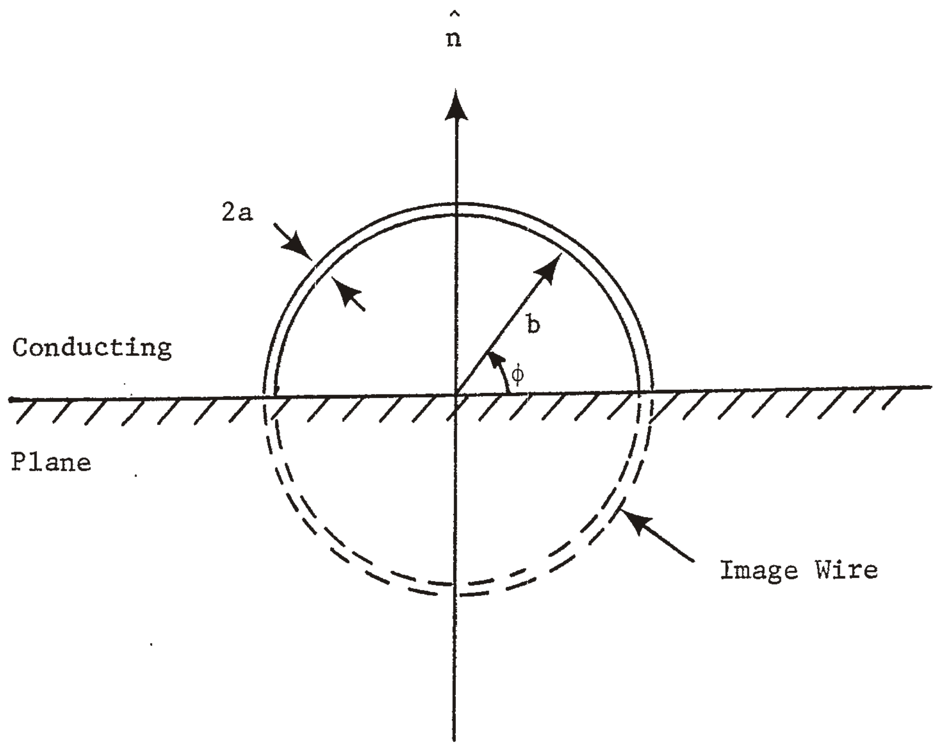

Figure 5: Thin Wire Loop Over a Conducting Plane.

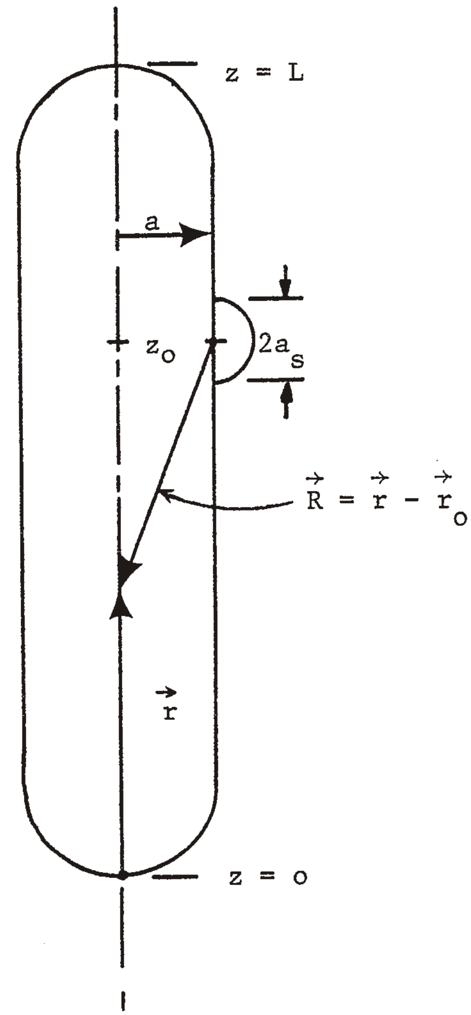

Figure 6: Right Circular Cylinder with Spherical End Caps and a Small Hemispherical Protrusion on its Lateral Surface.