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NUMERICAL SOLUTIONS OF INTEGRAL EQUATIONS FOR CURRENTS INDUCED ON A WIRE MODEL OF A PARKED AIRCRAFT

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ABSTRACT

A set of coupled integral equations of the Hallén type have been derived in an earlier note for a wire model of a parked aircraft. Numerical results for the currents induced on a somewhat simplified model of a parked aircraft are obtained. The three-dimensional structure is assumed to be above a perfectly conducting ground plane and is excited by an incident time-harmonic electromagnetic plane wave.

NUMERICAL SOLUTION OF INTEGRAL EQUATIONS

A wire model of a parked aircraft has been investigated [1] and a set of coupled integral equations of the Hallén type has been derived for the structure subject to illumination by a time-harmonic electromagnetic field. The model, which is placed above a ground plane of infinite extent and grounded, includes representations of the wings, the nose and fuselage sections, the horizontal and vertical stabilizers, and the grounding strap. The axial currents on this model of a parked aircraft can be obtained through the solution of the set of fifteen equations derived in Part I [1]. As an initial step in determining these currents, a numerical solution for the currents on the simplified wire structure of Fig. 1 has been obtained.

As can be readily seen, this wire model differs from that investigated in Part I only in the elimination of the horizontal and vertical stabilizers. The numerical techniques used for solving the more representative structure of Part I present no new problems, but would require a several-fold increase in computer program length and execution time over that for the simplified structure. The solution of the original model has therefore been deferred for treatment by a larger and faster computing machine than is available to the authors.

Formulation

The definitions and assumptions presented in this analysis of Section I hold, of course, for the structure of Figure 1. The analysis for the simpler structure is exactly the same as that for the original model except that no expressions for vector or scalar potential are derived for the horizontal or vertical stabilizers. This leads to three Hallén type coupled integral equations rather than the five equations for the original structure [Eq. 29 (a-f) of [1]]. The equations resulting from physical requirements on the structure are, as a result of fewer wire segments, reduced in number. Since the junction of the fuselage and the stabilizers no longer exists, there are no resulting Kirchoff current equations or continuity of scalar potential equations at the tail.

Continuity of scalar potential is still enforced at the junction of the nose, fuselage, wings, and grounding strap, and the resulting equations are used to constrain the constants which appear in the integral equations. The Kirchoff current law equation at the front junction and the equation requiring scalar potential to be equal to zero at the point where the ground strap joins the ground plane both remain unchanged. The equations necessary for solving for the currents on the structure now become (see notation of [1]);

$$I^{n}(0) + I^{w-}(0) - I^{w+}(0) - I^{f}(0) - I^{g}(0) = 0 \quad (31)$$

$$I^{n}(-n) = 0, \quad (32a)$$

$$I^{w-}(-w) = 0, \quad (32b)$$

$$I^{w+}(w) = 0, \quad (32c)$$

$$I^{f}(f) = 0. \quad (32d)$$

As stated before, the Hallén type equations for the nose-fuselage, wings, and grounding strap are unchanged save for the deletion of the terms involving the stabilizer sections. These equations now assume the form given by Equation (33). The equation containing the scalar potential to zero at (0,0,g) is repeated here as Equation (33d). The list of unknowns now includes five currents and four constants for a total of nine. There are six equations specifying physical requirements (boundary conditions) and three Hallén-type integral equations, also for a total of nine. This is a tractable system of equations and a solution for the axial currents should yield to the method of moments.

$$\int_{x'=-n}^{f} I_x^{fn}(x')G_{fn}(x,0,0;a^{fn})dx' + \int_{\xi=0}^{x}\left\{\int_{y'=-w}^{w} I_y^{w}(y')\frac{\partial}{\partial y}G_w(\xi,0,0;a^{fn})dy' + \int_{z'=0}^{g} I_z^{g}(z')\frac{\partial}{\partial z}G_g(\xi,0,0;a^{fn})dz'\right\}\cos k(z-\xi)d\xi$$

$$= C_x^{fn}\cos kx + B\sin kx - j\frac{k}{\omega}\int_{\xi=0}^{x} E_x^{i}(\xi,0,0;a^{fn})\sin k(x-\xi)d\xi \quad , \quad x\varepsilon(-n,f) \tag{33a}$$

$$\int_{y'=-w}^{w} I_y^{w}(y')G_w(0,y,0;a^{w})dy' + \int_{\xi=0}^{y}\left\{\int_{x'=-n}^{f} I_x^{fn}(x')\frac{\partial}{\partial x}G_{fn}(0,\xi,0;a^{w})dx' + \int_{z'=0}^{g} I_z^{g}(z')\frac{\partial}{\partial z}G_g(0,\xi,0;a^{w})dz'\right\}\cos k(y-\xi)d\xi$$

$$- C_y^{w}\cos ky + B\sin ky - j\frac{k}{\omega}\int_{\xi=0}^{y} E_y^{i}(0,\xi,0;a^{w})\sin k(y-\xi)d\xi \quad , \quad y\varepsilon(-w,w) \tag{33b}$$

$$\int_{z'=0}^{g} I_z^g(z')G_g(0,0,z;a^g)dz' + \int_{\xi=0}^{z} \left\{ \int_{x'=-n}^{f} I_x^{fn}(x') \frac{\partial}{\partial x}G_{fn}(0,0,\xi;a^g)dx' \right.$$

$$\left. + \int_{y'=-w}^{w} I_y^w(y') \frac{\partial}{\partial y}G_w(0,0,\xi;a^g)dy' \right\} \cos k(z-\xi)d\xi$$

$$= C_z^g \cos kz + B \sin kz - j\frac{k}{\omega} \int_{\xi=0}^{z} E_z^i(0,0,\xi;a^g) \sin k(z-\xi)d\xi \ , \ z\varepsilon(0,g) \qquad (33c)$$

$$\int_{\xi=0}^{g} \left\{ \int_{x'=-n}^{f} I_x^{fn}(x') \frac{\partial}{\partial x}G_{fn}(0,0,\xi;a^f)dx' + \int_{y'=-w}^{w} I_y^w(y') \frac{\partial}{\partial y}G_w(0,0,\xi;a^g)dy' \right\} \sin k(g-\xi)d\xi$$

$$= C_z^g \sin kg - B \cos kg + j\frac{k}{\omega} \int_{\xi=0}^{g} E_z^i(0,0,\xi;a^g) \cos k(g-\xi)d\xi \qquad (33d)$$

SOLUTION PROCEDURE

As the initial step in the numerical solution for the currents on the structure in Figure 1, the five unknown currents are expanded in series of piece-wise sinusoidal basis functions. The current expansions are given in Equations (34):

$$I_x^n(x') \doteq \sum_{p=1}^{P} I_p^n f_p(x'), \tag{34a}$$

$$I_x^f(x') \doteq \sum_{p=1}^{Q} I_p^f f_p(x') \tag{34b}$$

$$I_y^{w+}(y') \doteq \sum_{p=1}^{R} I_p^{w+} f_p(y'), \tag{34c}$$

$$I_y^{w-}(y') \doteq \sum_{p=1}^{S} I_p^{w-} f_p(y') \tag{34d}$$

$$I_z^g(z') \doteq \sum_{p=1}^{T} I_p^g f_p(z'), \tag{34e}$$

where f_i is given by

$$f_i(x') = \frac{1}{\Delta x'}\left\{\left[U(x'-x'_{i-1}) - U(x'-x'_i)\right] \sin k(x'-x'_{i-1}) + \left[U(x'-x'_i) - U(x'-x'_{i-1})\right] \sin k(x'_{i+1}-x')\right\} \tag{35}$$

$$\frac{1}{\Delta x'}\sum_{p=1}^{P} I_p^n \int_{x'=-n}^{0} f_p(x')\ G_{fn}(x,0,0;a^{fn})dx' + \frac{1}{\Delta x'}\sum_{p=1}^{Q} I_p^f \int_{x'=0}^{f} f_p(x')\ G_{fn}(x,0,0;a^{fn})dx'$$

$$+ \int_{\xi=0}^{x} \left\{ \frac{1}{\Delta y'}\sum_{p=1}^{R} I_p^{w-} \int_{y'=-w}^{0} f_p(y')\frac{\partial}{\partial y}\ G_w(\xi,0,0;a^{fn})dy' + \frac{1}{\Delta y'}\sum_{p=1}^{S} I_p^{w+} \int_{y'=0}^{w} f_p(y')\frac{\partial}{\partial y}\ G_w(\xi,0,0;a^{fn})dy' \right.$$

$$\left. + \frac{1}{\Delta z'}\sum_{p=1}^{T} I_p^g \int_{z'=0}^{g} f_p(z')\frac{\partial}{\partial z}\ G_g(\xi,0,0;a^{fn})dz' \right\} \cos\ k(x-\xi)d\xi$$

$$= C_x^{fn}\cos\ kx + B\ \sin\ kx\ -j\frac{k}{\omega}\int_{\xi=0} E_x^i(\xi,0,0;a^{fn})\sin\ k(x-\xi)d\xi,\ \ x\varepsilon(-n,f) \qquad (37a)$$

$$\frac{1}{\Delta y'}\sum_{p=1}^{R} I_p^{w-} \int_{y'=-w}^{0} f_p(y')\ G_w(0,y,0;a^w)dy' + \frac{1}{\Delta y'}\sum_{p=1}^{S} I_p^{w+} \int_{y'=0}^{w} f_p(y')\ G_w(0,y,0;a^w)dy'$$

$$+ \int_{\xi=0}^{y} \left\{ \frac{1}{\Delta x'}\sum_{p=1}^{P} I_p^n \int_{x'=-n}^{0} f_p(x')\frac{\partial}{\partial x}\ G_{fn}(0,\xi,0;a^w)dx' + \frac{1}{\Delta x'}\sum_{p=1}^{Q} I_p^f \int_{x'=0}^{f} f_p(x')\frac{\partial}{\partial x}\ G_{fn}(0,\xi,0,a^w)dx' \right.$$

$$\left. + \frac{1}{\Delta z'}\sum_{p=1}^{T} I_p^g \int_{z'=0}^{g} f_p(z')\frac{\partial}{\partial z}\ G_g(0,\xi,0;a^w)dz' \right\} \cos\ k(y-\xi)d\xi$$

$$= C_y^w\cos\ ky + B\ \sin\ ky\ -j\frac{k}{\omega}\int_{\xi=0}^{y} E_y^i(0,\xi,0;a^w)\sin\ k(y-\xi)d\ \ ,\ y\varepsilon(-w,w) \qquad (37b)$$

$$\frac{1}{\Delta z'} \sum_{p=1}^{T} I_p^g \int_{z'=0}^{g} f_p(z') G_g(0,0,z;a^g)dz' + \int_{\xi=0}^{z} \Bigg\{ \frac{1}{\Delta x'} \sum_{p=1}^{P} I_p^n \int_{x'=-n}^{0} f_p(x') \frac{\partial}{\partial x} G_{fn}(0,0,\xi;a^g)dx'$$

$$+ \frac{1}{\Delta x'} \sum_{p=1}^{Q} I_p^f \int_{x'=0}^{f} f_p(x') \frac{\partial}{\partial x} G_{fn}(0,0,\xi;a^g)dx' + \frac{1}{\Delta y'} \sum_{p=1}^{R} I_p^{w-} \int_{y'=-w}^{0} f_p(y') \frac{\partial}{\partial y} G_w(0,0,\xi;a^g)dy'$$

$$+ \frac{1}{\Delta y'} \sum_{p=1}^{S} I_p^{w+} \int_{y'=0}^{w} f_p(y') \frac{\partial}{\partial y} G_w(0,0,\xi;a^g)dy' \Bigg\} \cos k(z-\xi)d\xi$$

$$= C_z^g \cos kz + B \sin kz - j\frac{k}{\omega} \int_{\xi=0}^{z} E_z^i(0,0,\xi;a^g) \sin k(z-\xi)d \ , \ z \varepsilon (0,g) \tag{37c}$$

$$\int_{\xi=0}^{g} \Bigg\{ \frac{1}{\Delta x'} \sum_{p=1}^{P} I_p^n \int_{x'=-n}^{0} f_p(x') \frac{\partial}{\partial x} G_{fn}(0,0,\xi;a^g)dx' + \frac{1}{\Delta x'} \sum_{p=1}^{Q} I_p^f \int_{x'=0}^{f} f_p(x') \frac{\partial}{\partial x} G_{fn}(0,0,\xi;a^g)dx'$$

$$+ \frac{1}{\Delta y'} \sum_{p=1}^{R} I_p^{w-} \int_{y'=-w}^{0} f_p(y') \frac{\partial}{\partial y} G_w(0,0,\xi;a^g)dy' + \frac{1}{\Delta y'} \sum_{p=1}^{S} I_p^{w+} \int_{y'=0}^{w} f_p(y') \frac{\partial}{\partial y} G_w(0,0,\xi;a^g)dy'$$

$$\Bigg\} \sin k(g-\xi)d\xi$$

$$= C_z^g \sin kg - B \cos kg + j\frac{k}{\omega} \int_{\xi=0}^{g} E_z^i(0,0,\xi;a^g) \cos k(g-\xi)d\xi \tag{37d}$$

with $\Delta x' = \sin k\, |x'_{i+1} - x'_i|$

and U(x) is the familiar unit step:

$$U(x) = \begin{cases} 1 & x > 0 \\ 0 & x < 0. \end{cases} \tag{36}$$

An illustration of the piece-wise sinusoidal basis is given in Figure 2. The distribution of piece-wise sinusoids on the structure of Figure 1 is illustrated in Figure 3.

The method of collocation is employed to generate a set of linear equations which can be solved through the use of well-known numerical methods. The boundary condition constraining the currents to be zero at the free wire ends is invoked by setting the magnitude of the piece-wise sinusoidal components at those points equal to zero and not including them in the array of unknown constants (See Figure 2). The array of unknowns includes the coefficients of the piece-wise sinusoidal expansions for the currents and the four unknown constants arising in the Hallén type integral equations.

Subject to the approximate representations of Equations (34), Equations (33) can be written in the form given by Equations (37). The following definitions are useful in describing subsequent matrix equations:

$$S^{n}_{n_p}(x) = \frac{1}{\Delta x'} \int_{x'=-n}^{0} f_p(x') G_{fn}(x,0,0;a^{fn}) dx', \quad x \varepsilon (-n,0), \quad x' \varepsilon (-n,0), \tag{38}$$

$$\gamma^{n}_{w_p}{}^{-}(x) = \frac{1}{\Delta y'} \int_{\xi=0}^{x} \int_{y'=-w}^{0} f_p(y') \frac{\partial}{\partial y} G_{w^-}(\xi,0,0;a^{fn}) \cos k(x-\xi) dy' d\xi, \quad x \varepsilon (-n,0), \quad y' \varepsilon (-w,0) \tag{39}$$

$$v^{n}(x) = j\frac{k}{\omega} \int_{\xi=0}^{x} E^{i}_{x}(\xi,0,0;a^{fn}) \sin k(x-\xi) d\xi, \quad x \varepsilon (-n,0). \tag{40}$$

In Equation (38) the superscript n on S denotes the wire element on which the match point is located, the subscript n refers to the wire element on which the source current is located. The same convention holds for Equations (39) and (40). The E^{i}_{x} is Equation (40) is the electric field impressed on the structure by the induced currents. For a scatterer, E^{i}_{x} is the negative of the incident electric field component polorized in the x direction.

The number of linear equations resulting from the use of collocation will depend upon the number of match points specified on each of the wires. For this structure, it will be assumed that there are D match points on the nose element, E match points on the fuselage element, F match points on each of the wings, and H match points on the ground strap. Employing the following definitions:

$$S^{n}_{n_{mp}} = s^{n}_{n_p}(x^{n}_{m}), \tag{41a}$$

$$\Gamma^{n}_{w-_{mp}} = \gamma^{n}_{w-_{p}}(x^{n}_{m}), \tag{41b}$$

$$V^{n}_{m} = v^{n}(x^{n}_{m}), \tag{41c}$$

where x^{n}_{m}, y^{w}_{m}, and z^{g}_{m} are match points on the x, y, and z-directed wires respectively, one arrives at the set of equations:

$$\sum_{m=1}^{D}\left\{\sum_{p=1}^{P} I^{n}_{p} S^{n}_{n_{mp}} + \sum_{p=1}^{Q} I^{f}_{p} S^{n}_{f_{mp}} + \sum_{p=1}^{R} I^{w-}_{p} \Gamma^{n}_{w-_{mp}} + \sum_{p=1}^{S} I^{w+}_{p} \Gamma^{n}_{w+_{mp}} + \sum_{p=1}^{T} I^{g}_{p} \Gamma^{n}_{g_{mp}} = C^{fn}_{x} \cos kx^{n}_{m} + B \sin kx^{n}_{m} - V^{n}_{m}\right\}, \tag{42a}$$

$$\sum_{m=1}^{E}\left\{\sum_{p=1}^{P} I^{n}_{p} S^{f}_{n_{mp}} + \sum_{p=1}^{Q} I^{f}_{p} S^{f}_{f_{mp}} + \sum_{p=1}^{R} I^{w-}_{p} \Gamma^{f}_{w-_{mp}} + \sum_{p=1}^{S} I^{w+}_{p} \Gamma^{f}_{w+_{mp}} + \sum_{p=1}^{T} I^{g}_{p} \Gamma^{f}_{g_{mp}} = C^{fn}_{x} \cos kx^{f}_{m} + b \sin kx^{f}_{m} - V^{f}_{m}\right\}, \tag{42b}$$

$$\sum_{m=1}^{F}\left\{\sum_{p=1}^{P} I^{n}_{p} \Gamma^{w-}_{n_{mp}} + \sum_{p=1}^{Q} I^{f}_{p} \Gamma^{w-}_{f_{mp}} + \sum_{p=1}^{R} I^{w-}_{p} S^{w-}_{w-_{mp}} + \sum_{p=1}^{S} I^{w+}_{p} S^{w-}_{w+_{mp}} + \sum_{p=1}^{T} I^{g}_{p} \Gamma^{w-}_{g_{mp}} = C^{w}_{y} \cos ky^{w-}_{m} + B \sin ky^{w-}_{m} - V^{w-}_{m}\right\}, \tag{42c}$$

$$\sum_{m=1}^{F}\left\{\sum_{p=1}^{P} I^{n}_{p} \Gamma^{w+}_{n_{mp}} + \sum_{p=1}^{Q} I^{f}_{p} \Gamma^{w+}_{f_{mp}} + \sum_{p=1}^{R} I^{w-}_{p} S^{w+}_{w-_{mp}} + \sum_{p=1}^{S} I^{w+}_{p} S^{w+}_{w+_{mp}} + \sum_{p=1}^{T} I^{g}_{p} \Gamma^{w+}_{g_{mp}} = C^{w}_{y} \cos ky^{w+}_{m} + B \sin ky^{w+}_{m} - V^{w+}_{m}\right\}, \tag{42d}$$

$$\sum_{m=1}^{H} \left\{ \sum_{p=1}^{P} I_p^n \Gamma_{n_{mp}}^g + \sum_{p=1}^{Q} I_p^f \Gamma_{f_{mp}}^g + \sum_{p=1}^{R} I_p^{w-} \Gamma_{w-_{mp}}^g + \sum_{p=1}^{S} I_p^{w+} \Gamma_{w+_{mp}}^g \right.$$

$$\left. + \sum_{p=1}^{T} I_p^g S_{g_{mp}}^g = C_z^g \cos kz_m^g + B \sin kz_m^g - V_m^g \right\} . \qquad (42c)$$

Equation (33d) can now be rewritten as

$$\sum_{p=1}^{P} I_p^n \Gamma_{n_{mp}}^g + \sum_{p=1}^{Q} I_p^f \Gamma_{f_{mp}}^g + \sum_{p=1}^{R} I_p^{w-} \Gamma_{w-_{mp}}^g + \sum_{p=1}^{S} I_p^{w+} \Gamma_{w+_{mp}}^g$$

$$= C_z^g \sin kg - B \cos kg + V_s^g \quad , \qquad (43)$$

where $V_s^g = j\frac{k}{\omega} \int_{\xi=0}^{g} E_z^i(0,0,\xi;a^g) \cos k(g-\xi) d\xi .$ (44)

Equations (42), (43), and (31) can be written in matrix form,

$$[A]\ [I] = [V], \qquad (45)$$

where [A], [I], and [V] are given by Equations (46). This matrix equation (45) may be solved to obtain the column vector of unknown current coefficients and constants [I] (46c) by regular matrix inversion:

$$[I] = [A]^{-1}\ [V].$$

$$
\left[\begin{array}{cccccccccccccc}
S^{n}_{n_{11}}\cdots & S^{n}_{n_{1P}} & S^{n}_{f_{11}}\cdots & S^{n}_{f_{1Q}} & r^{n}_{w_{\bar{1}1}}\cdots & r^{n}_{w_{\bar{1}R}} & r^{n}_{w_{\overset{+}{1}1}}\cdots & r^{n}_{w_{\overset{+}{1}S}} & r^{n}_{g_{11}}\cdots & r^{n}_{g_{1T}} & \cos kx^{n}_{1} & 0 & 0 & \sin kx^{n}_{1} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
S^{n}_{n_{D1}}\cdots & S^{n}_{n_{DP}} & S^{n}_{f_{D1}}\cdots & S^{n}_{f_{DQ}} & r^{n}_{w_{\bar{D}1}}\cdots & r^{n}_{w_{\bar{D}R}} & r^{n}_{w_{\overset{+}{D}1}}\cdots & r^{n}_{w_{\overset{+}{D}S}} & r^{n}_{g_{D1}}\cdots & r^{n}_{g_{DT}} & \cos kx^{n}_{D} & 0 & 0 & \sin kx^{n}_{D} \\
S^{f}_{n_{D1}}\cdots & S^{f}_{n_{1P}} & S^{f}_{f_{11}}\cdots & S^{f}_{f_{1Q}} & r^{f}_{w_{\bar{1}1}}\cdots & r^{f}_{w_{\bar{1}R}} & r^{f}_{w_{\overset{+}{1}1}}\cdots & r^{f}_{w_{\overset{+}{1}S}} & r^{f}_{g_{11}}\cdots & r^{f}_{g_{1T}} & \cos kx^{f}_{1} & 0 & 0 & \sin kx^{f}_{1} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
S^{f}_{n_{E1}}\cdots & S^{f}_{n_{1P}} & S^{f}_{f_{E1}}\cdots & S^{f}_{f_{EQ}} & r^{f}_{w_{\bar{E}1}}\cdots & r^{f}_{w_{\bar{E}1}} & r^{f}_{w_{\overset{+}{E}R}}\cdots & r^{f}_{w_{\overset{+}{E}S}} & r^{f}_{g_{E1}}\cdots & r^{f}_{g_{ET}} & \cos kx^{f}_{E} & 0 & 0 & \sin kx^{f}_{E} \\
r^{w-}_{n_{11}}\cdots & r^{w-}_{n_{1P}} & r^{w-}_{f_{11}}\cdots & r^{w-}_{f_{1Q}} & S^{w-}_{w_{\bar{1}1}}\cdots & S^{w-}_{w_{\bar{1}R}} & S^{w-}_{w_{\overset{+}{1}1}}\cdots & S^{w-}_{w_{\overset{+}{1}S}} & r^{w-}_{g_{11}}\cdots & r^{w-}_{g_{1T}} & 0 & \cos ky^{w-}_{1} & 0 & \sin ky^{w-}_{1} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
r^{w-}_{n_{F1}}\cdots & r^{w-}_{n_{FP}} & r^{w-}_{f_{F1}}\cdots & r^{w-}_{f_{FQ}} & S^{w-}_{w_{\bar{F}1}}\cdots & S^{w-}_{w_{\bar{F}R}} & S^{w-}_{w_{\overset{+}{F}1}}\cdots & S^{w-}_{w_{\overset{+}{F}S}} & r^{w-}_{g_{F1}}\cdots & r^{w-}_{g_{FT}} & 0 & \cos ky^{w-}_{F} & 0 & \sin ky^{w-}_{F} \\
r^{w+}_{n_{11}}\cdots & r^{w+}_{n_{1P}} & r^{w+}_{f_{11}}\cdots & r^{w+}_{f_{1Q}} & S^{w+}_{w_{\bar{1}1}}\cdots & S^{w+}_{w_{\bar{1}R}} & S^{w+}_{w_{\overset{+}{1}1}}\cdots & S^{w+}_{w_{\overset{+}{1}S}} & r^{w+}_{g_{11}}\cdots & r^{w+}_{g_{1T}} & 0 & \cos ky^{w+} & 0 & \sin ky^{w+}_{1} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
r^{w+}_{n_{F1}}\cdots & r^{w+}_{n_{FP}} & r^{w+}_{f_{F1}}\cdots & r^{w+}_{f_{FQ}} & S^{w+}_{w_{\bar{F}1}}\cdots & S^{w+}_{w_{\bar{F}R}} & S^{w+}_{w_{\overset{+}{F}1}}\cdots & S^{w+}_{w_{\overset{+}{F}S}} & r^{w+}_{g_{F1}}\cdots & r^{w+}_{g_{FT}} & 0 & \cos ky^{w+}_{F} & 0 & \sin ky^{w+}_{F} \\
r^{g}_{n_{11}}\cdots & r^{g}_{n_{1P}} & r^{g}_{f_{11}}\cdots & r^{g}_{f_{1Q}} & r^{g}_{w_{\bar{1}1}}\cdots & r^{g}_{w_{\bar{1}R}} & r^{g}_{w_{\overset{+}{1}1}}\cdots & r^{g}_{w_{\overset{+}{1}S}} & S^{g}_{g_{11}}\cdots & S^{g}_{g_{1T}} & 0 & 0 & \cos kz^{g}_{1} & \sin kz^{g}_{1} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
r^{g}_{n_{H1}}\cdots & r^{g}_{n_{HP}} & r^{g}_{f_{H1}}\cdots & r^{g}_{f_{HQ}} & r^{g}_{w_{\bar{H}1}}\cdots & r^{g}_{w_{\bar{H}R}} & r^{g}_{w_{\overset{+}{H}1}}\cdots & r^{g}_{w_{\overset{+}{H}S}} & S^{g}_{g_{H1}}\cdots & S^{g}_{HT} & 0 & 0 & \cos kz^{g}_{H} & \sin kz^{g}_{H} \\
r^{g}_{n_{M1}}\cdots & r^{g}_{n_{MP}} & r^{g}_{f_{M1}}\cdots & r^{g}_{f_{MQ}} & r^{g}_{w_{\bar{M}1}}\cdots & r^{g}_{w_{\bar{M}R}} & r^{g}_{w_{\overset{+}{M}1}}\cdots & r^{g}_{w_{\overset{+}{M}s}} & 0\cdots & 0 & 0 & 0 & \sin kg & -\cos kg \\
0\cdots & 0\ \ 1\ \ -1 & 0\cdots & 0 & 0\cdots & 0\ \ 1\ \ -1 & 0\cdots & 0\ \ -1 & 0\cdots & 0 & 0 & 0 & 0 & 0
\end{array}\right]
$$

One should note that the next to last line of the matrix [A] in Equation (46a) represents Equation (43) and the last line represents Equation (31), the Kirchoff's law equation.

$$[I] = \begin{bmatrix} I_1^n \\ \vdots \\ I_p^n \\ I_1^f \\ \vdots \\ I_Q^f \\ I_1^{w-} \\ \vdots \\ I_R^{w-} \\ I_1^{w+} \\ \vdots \\ I^{w+} \\ I_1^g \\ \vdots \\ I_T^g \\ C_x^{fn} \\ C_y^w \\ C_z^g \\ B \end{bmatrix} \quad (46b), \text{ and } [V] = \begin{bmatrix} V_1^n \\ \vdots \\ V_D^n \\ V_1^f \\ \vdots \\ V_E^f \\ V_1^{w-} \\ \vdots \\ V_F^{w-} \\ V_1^{w+} \\ \vdots \\ V_F^{w+} \\ V_1^g \\ \vdots \\ V_H^g \\ V_S^g \\ 0 \end{bmatrix}$$

DETERMINING THE [V] VECTOR

The structure is treated as a scatterer with illumination provided by a plan wave polarized in a specified direction. The boundary condition that tangential electric field on the surface of a perfect conductor be zero is invoked. For plane wave incidence, this involves substituting the negative of the x polarized field component into the integral of Equation (40) and performing similar operations for the y and z polarized components in order to determine the elements of the [V] vector.

In the case treated numerically, the incident field is considered to be polarized in the x direction and propagating in the y-z plane. The total incident electric field along the x-directed wire is given by

$$\bar{E}_x^i = \bar{E}_x^d + \bar{E}_x^r \tag{47}$$

where

$$\bar{E}_x^d = 1\ e^{-j\beta s}\hat{u}_x \tag{48}$$

is the directly incident field component and

$$\bar{E}_x^r = \bar{E}_x^d\ \Gamma_r\ e^{-j\beta T}\hat{u}_x \tag{49}$$

is the reflected incident field component. The quantity s is measured along the direction of propagation and is equal to zero on a plane passing through the wire junction, Γ_r is the reflection coefficient of the ground plane which equals -1 for a perfect conductor, and βT is the phase delay of the reflected wave. The incident field is illustrated in Figure 4 showing both the directly incident and the reflected field components. The phase delay βT is seen to be equal to

$$\begin{aligned} \beta T &= \beta(Q + R) \\ &= \beta \; R\cos 2\theta + \frac{D}{\cos\theta} \\ &= \frac{\beta D}{\cos\theta} (1 + \cos 2\theta) \end{aligned} \tag{50}$$

where $$\beta = \frac{2\pi}{\lambda} \; .$$

Results of the Investigation

Solutions for a representative structure have been obtained for two different length of ground strap and two directions of propagation of the incident plane wave. The resulting current distributions are presented in Figures 5, 6, and 7.

References

[1] Butler, C. M., "Integral Equations for Currents Induced on a Wire Model of a Parked Aircraft," Interaction Note 139, AFWL EMP Note Series, January, 1972.

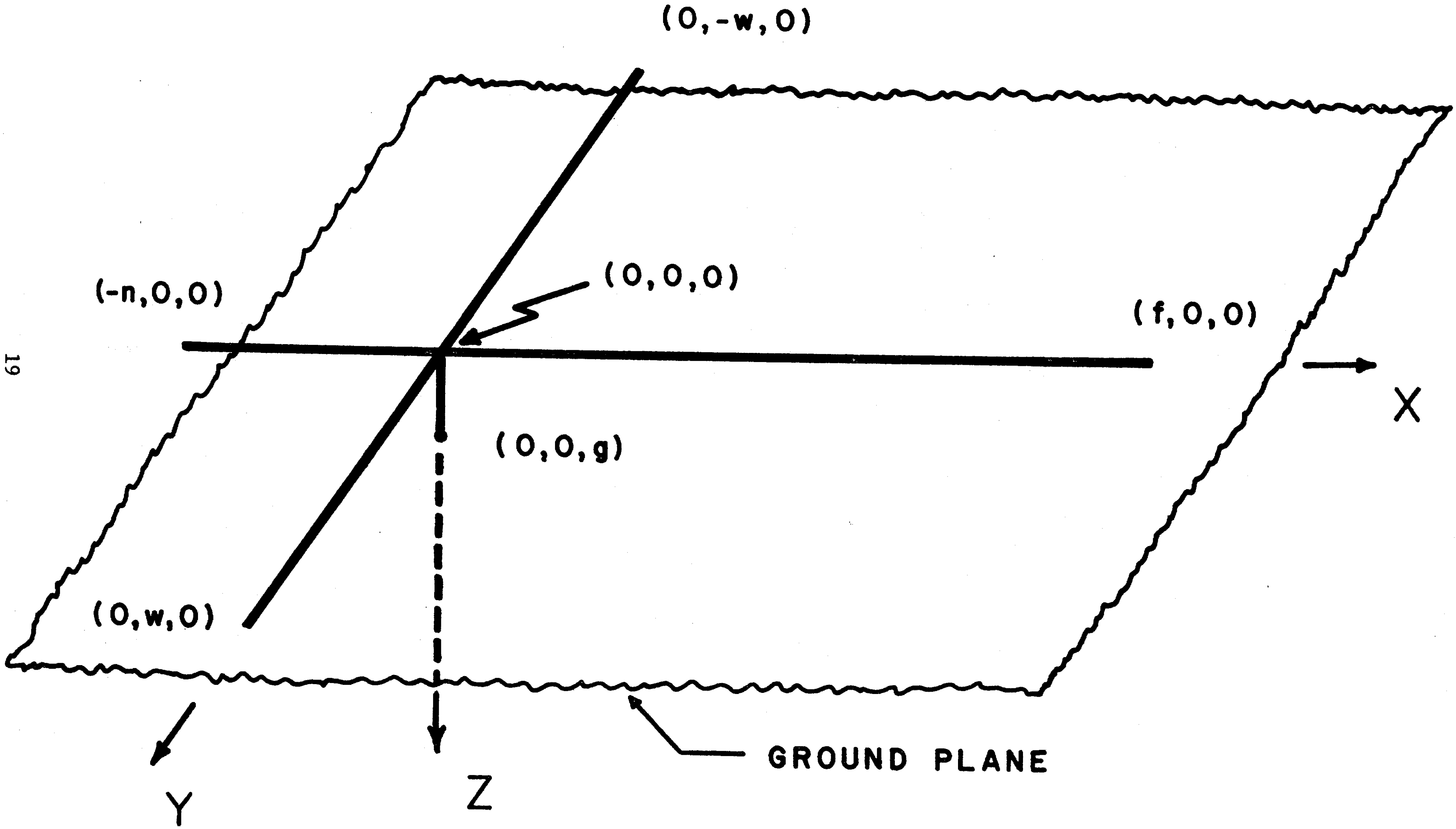


FIGURE 1. Simplified Wire Model of Parked Aircraft

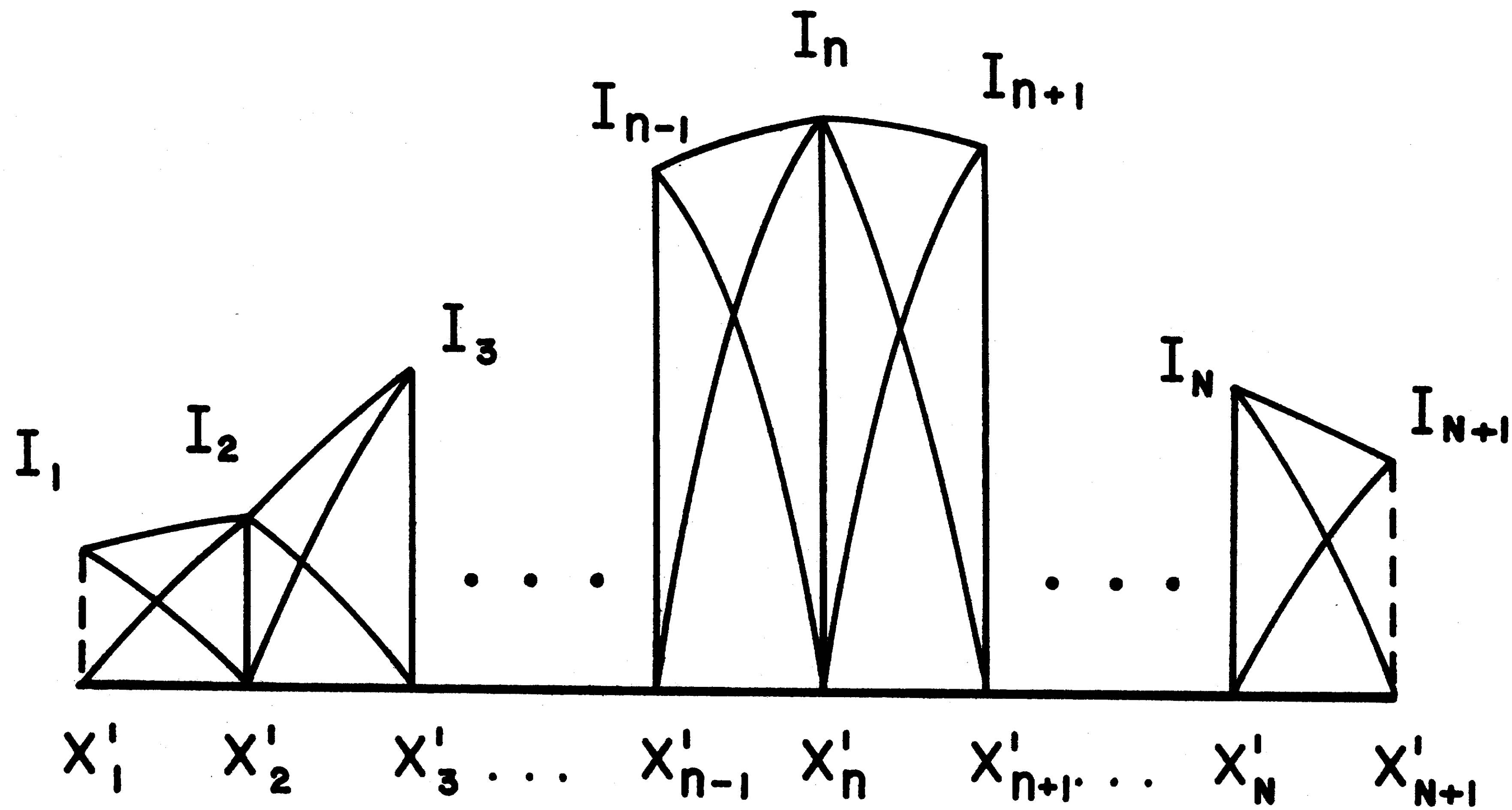


FIGURE 2. Piece-Wise Sinusoidal Basis Set

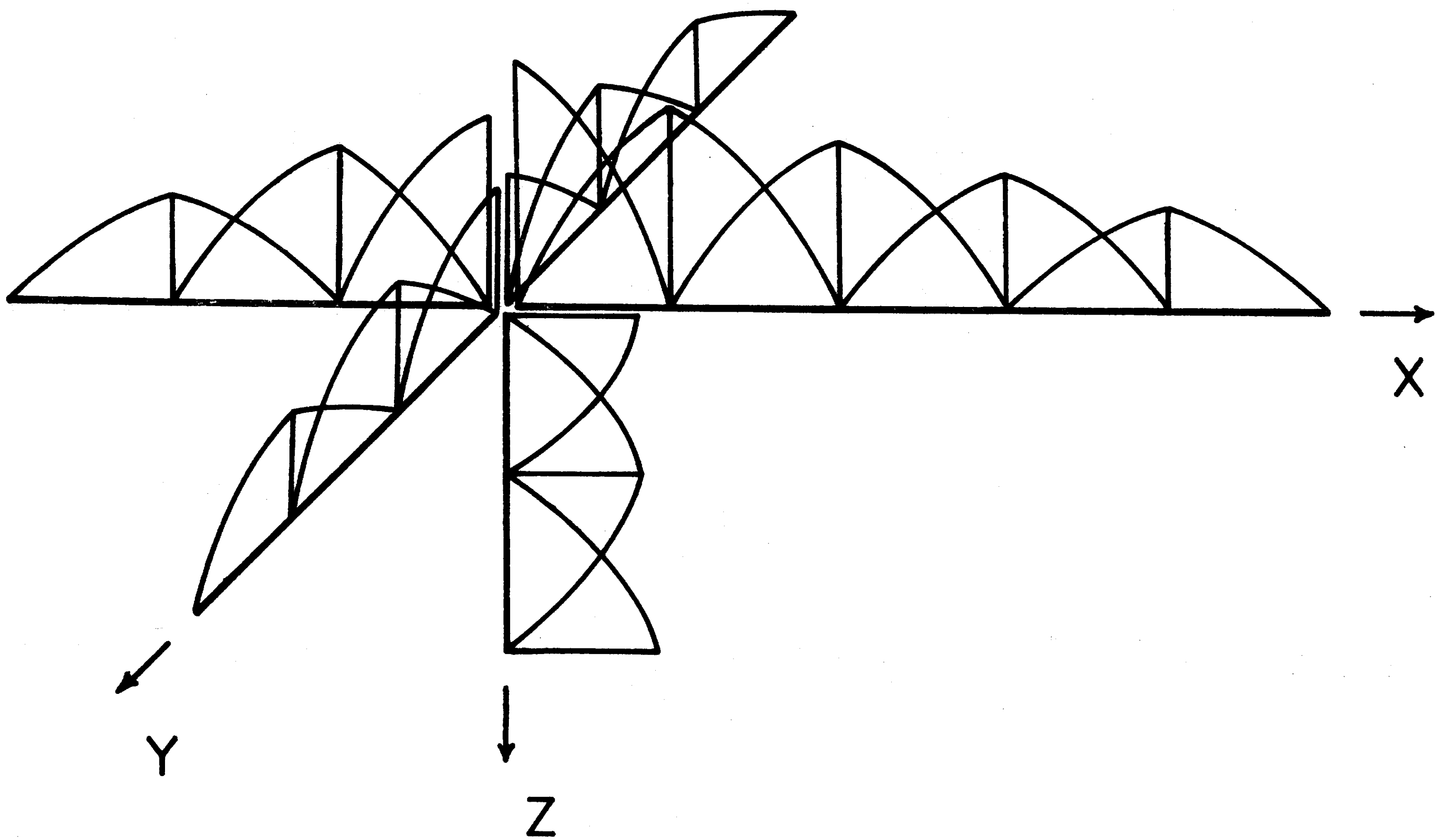


FIGURE 3. Distribution of Piece-Wise Sinusoids on Wire Model of Parked Aircraft

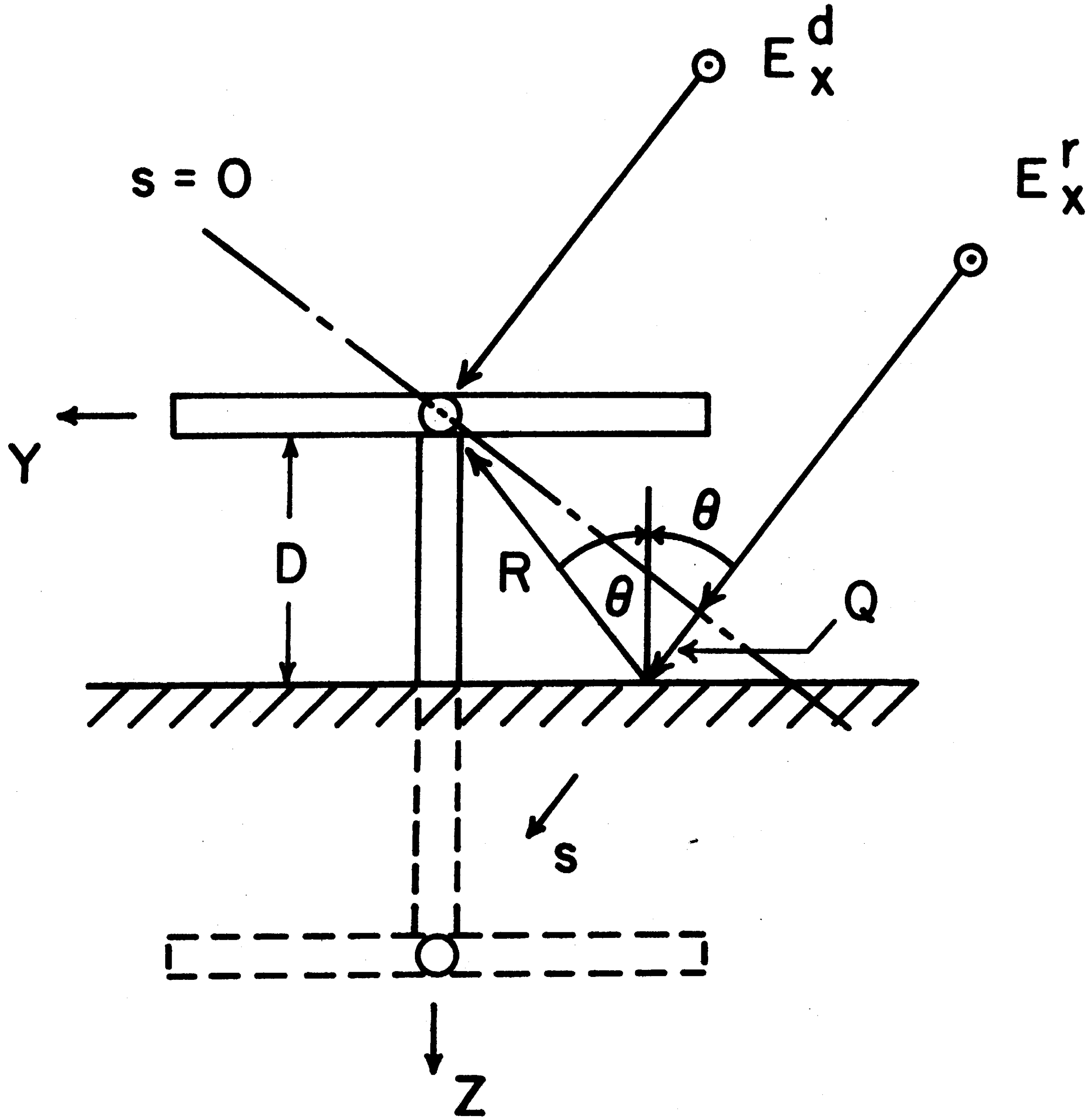


FIGURE 4.

Incident X-Polarized Plane Wave at Surface
of X-Directed Wire

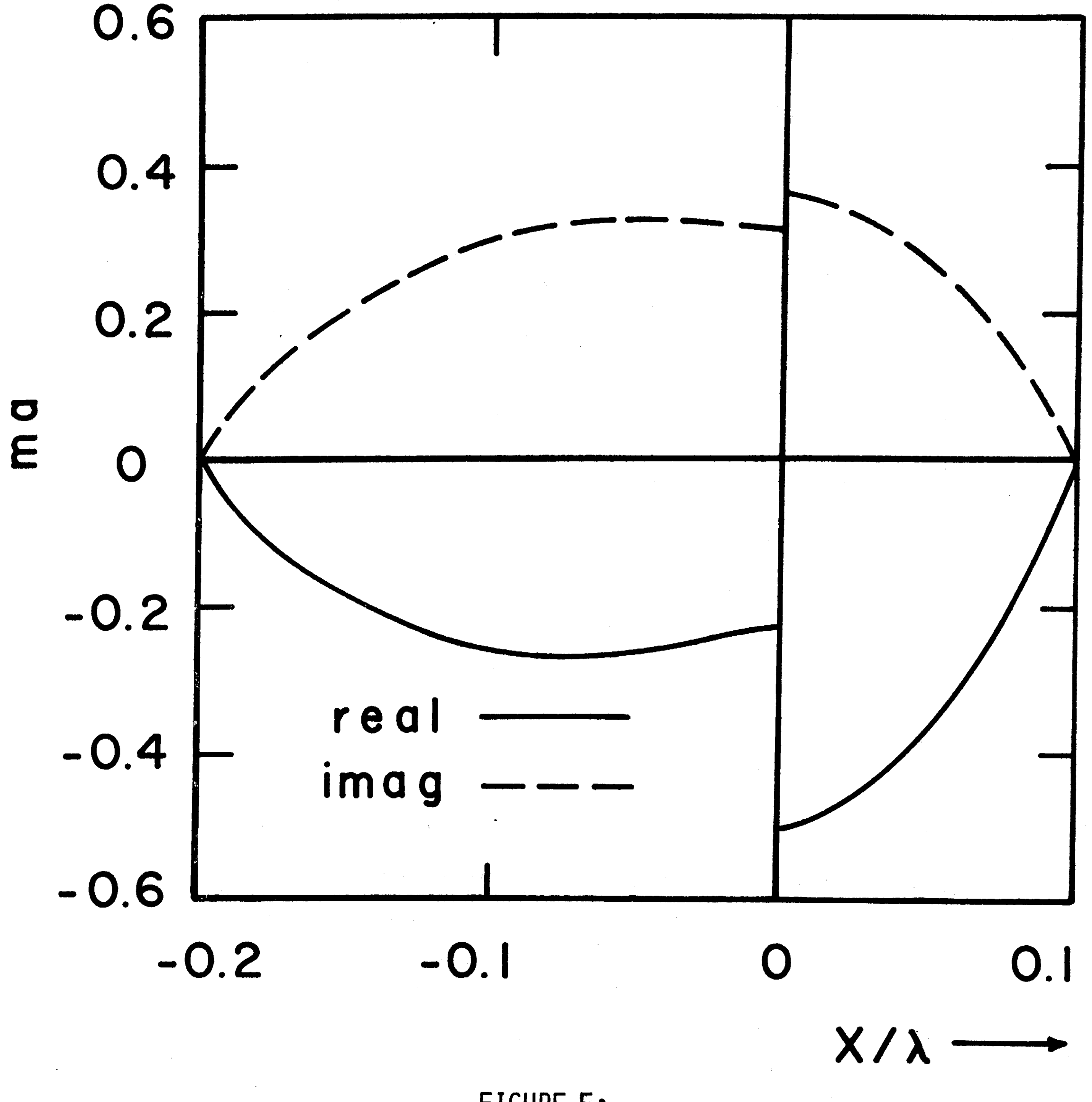
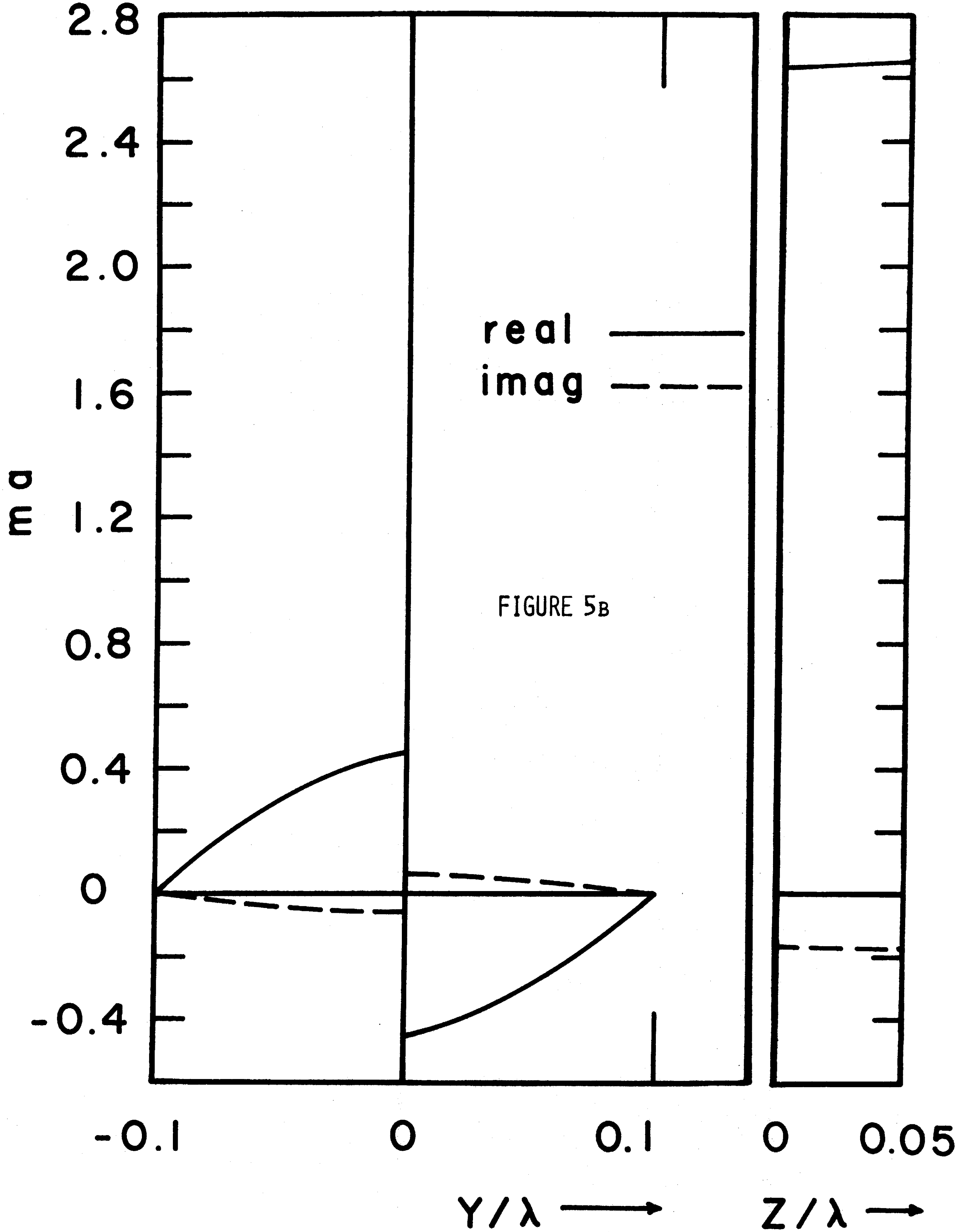


FIGURE 5A

Induced Axial Current Distribution on X-Directed Wire: $n/\lambda = 0.1$, $f/\lambda = 0.2$, $w/\lambda = 0.1$, $g/\lambda = 0.05$, $a^{fn} = a^{w} = a^{g} = 0.002\ \lambda$

Incident field is X-Polarized with normal incidence.



Induced Axial Current Distribution on Y-Directed Wire and Z-Directed Wire

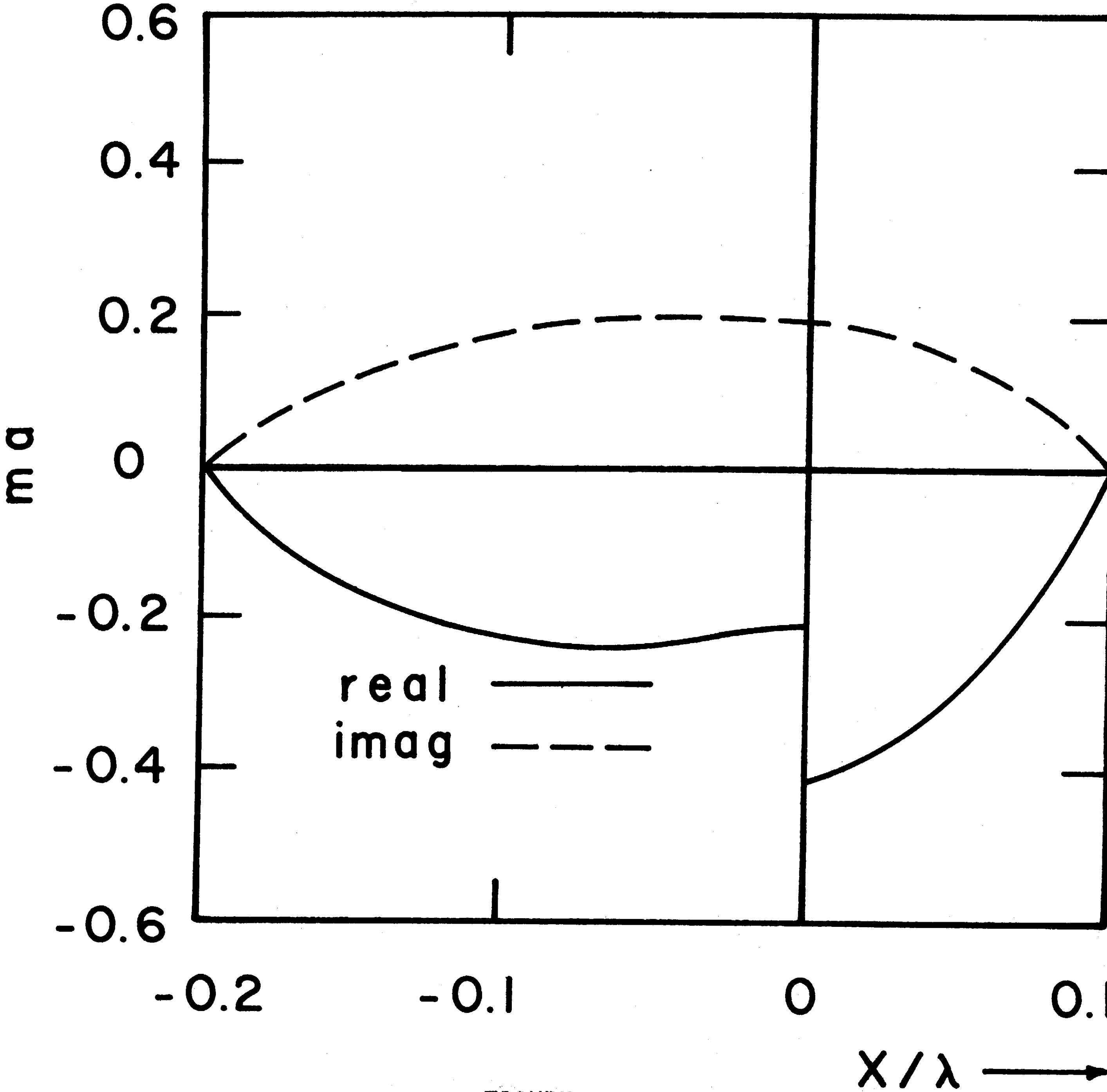
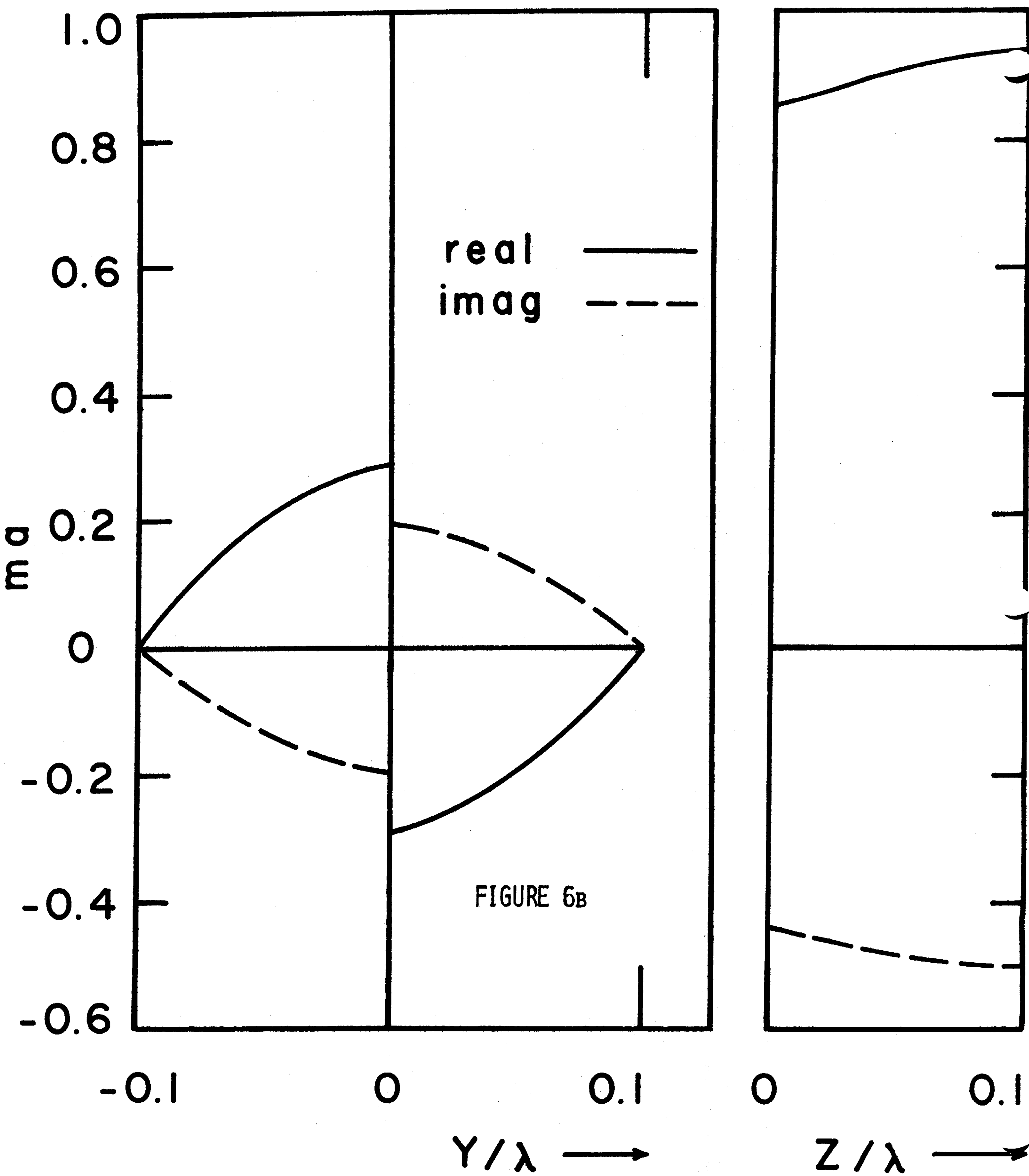


FIGURE 6A

Induced Axial Current Distribution on X-Directed Wire: $n/\lambda = 0.1$, $f/\lambda = 0.2$, $w/\lambda = 0.1$, $g/\lambda = 0.1$, $a^m/\lambda = a^w/\lambda = a^g/\lambda = 0.002$
Incident field is X-polarized with normal incidence.



Induced Axial Current Distribution on Y-Directed Wire and on Z-Directed Wire

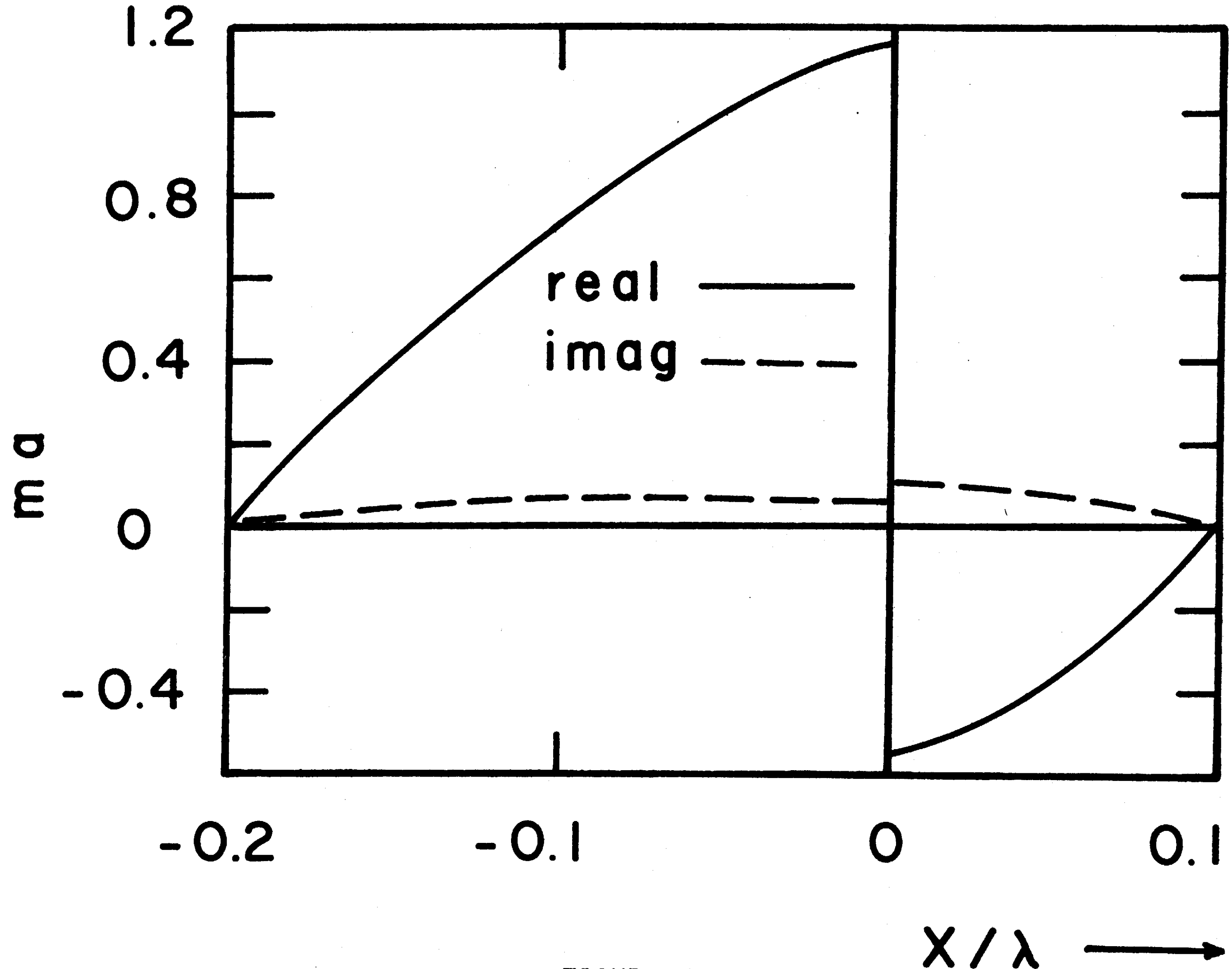
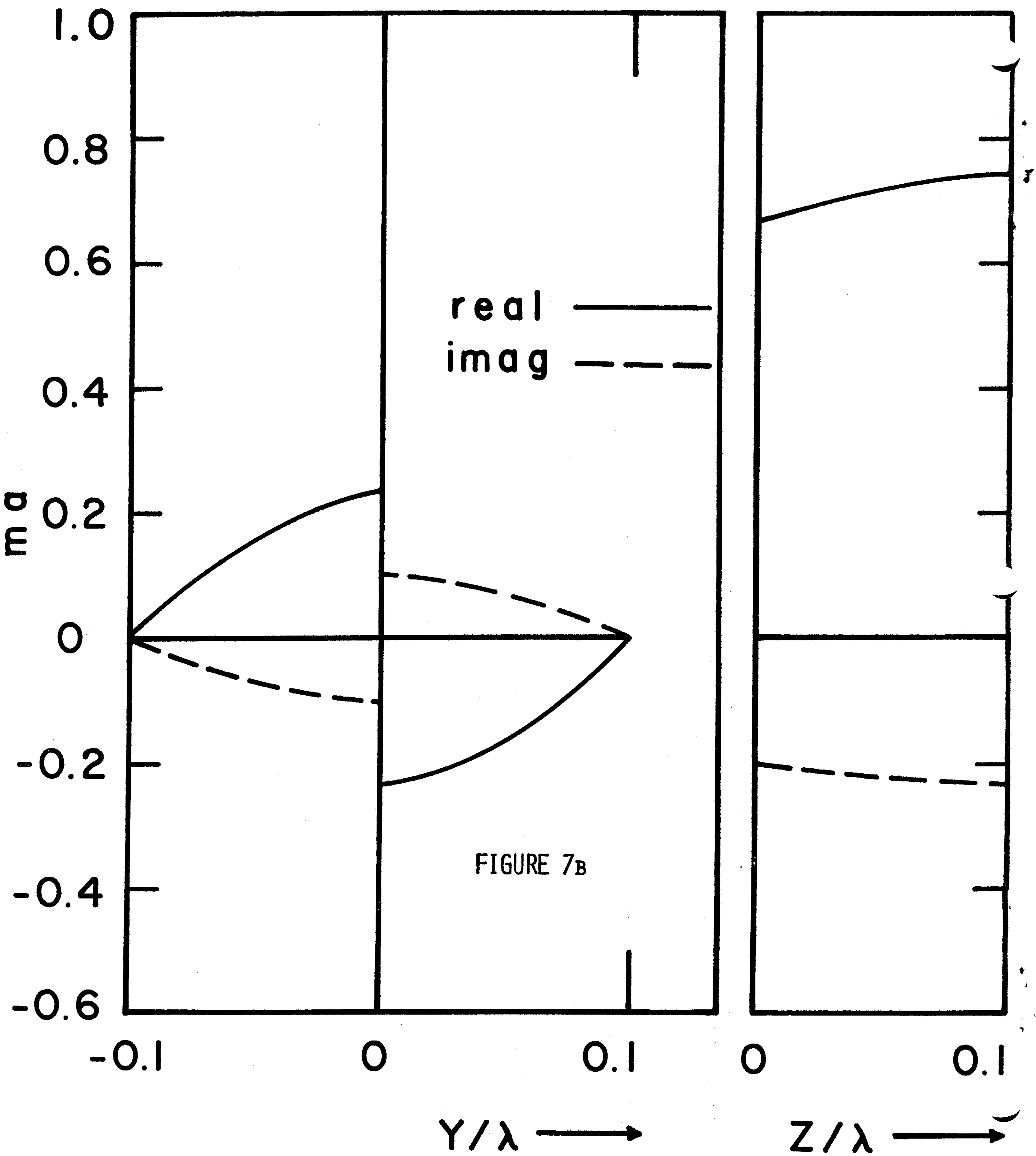


FIGURE 7A

Induced Axial Current Distribution on X-Directed Wire: $n/\lambda = 0.1$, $f/\lambda = 0.2$, $w/\lambda = 0.1$, $g/\lambda = 0.1$, $a^{fn} = a^{w} = a^{g} = 0.002\ \lambda$
Incident field is X-Polarized with 45 degree from normal incidence.

153 (28)



Induced Axial Current Distribution on Y-Directed Wire and Z-Directed Wire